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ASSESSMENT OF THE DEGREE OF RIPENESS OF ORIENTAL PERSIMMON FRUITS (DİOSPYROS kaki L.) BY TENSOR STRESS

Melahet Ismayilova
PhD Student
Department of Mathematical Analysis
Ganja State University
H. Aliyev ave., 429, Ganja, Azerbaijan, AZ 2000
Mushfiq Khalilov
Corresponding author
Doctor of Philosophy in Technics, Associate Professor*
E-mail: fmim@list.ru
Maryam Mammadaliyeva
Doctor of Philosophy in Biology, Lecturer
Department of Engineering and Applied Sciences
Azerbaijan State University of Economics (UNEC)
Istiqlaliyyat str., 6, Baku, Azerbaijan, AZ 1001
İlhama Kazimova
Doctor of Philosophy in Technics, Senior Lecturer**
Sevinj Maharramova
Doctor of Philosophy in Biology, Associate Professor**
Afet Gasimova
Doctor of Philosophy in Technics, Associate Professor*
*Department of Food Engineering and Expertise
University of Technology of Azerbaijan
Sh. I. Khatai ave., 103, Ganja, Azerbaijan, AZ 2011
**Department of Engineering and Applied Sciences
Azerbaijan State University of Economics (UNEC)
Istiqlaliyyat str., 6, Baku, Azerbaijan, AZ 1001

The object of the study is persimmon fruits, which occupy an important place among subtropical crops and have a wide development prospect. Since these fruits are difficult to process, they are mainly used fresh. The usefulness of these fruits is associated with their chemical composition. This composition includes biologically active substances, microelements, various mono- and polysaccharides, saturated and unsaturated fatty acids, etc. The complexity of the technological processing of persimmon fruits is associated with its astringent taste, which is determined by the amount of polyphenolic compounds. In general, the strength of raw materials is manifested not only in the degree of ripeness, but also in its technological processing processes, which are the object of the study. From this point of view, the hardness of persimmon fruits acts as a subject of study. Data on the property of fruits and vegetables associated with the stress-hard state is a solution to the problems that arise when expanding the range of finished products. For example, it has been established that at the stage of commercial ripeness, the hardness of persimmon fruits is no more than 12.3 kg/cm^2. And this indicator changes downwards over time, i.e. to 1.5÷2.0 kg/cm^2. Consequently, the possibility of using fruits for the production of various food products is expanding.

The study of raw materials according to the laws of solid state physics is explained by its polymer structure. Therefore, the ripening of raw materials depends on the monomerization of this structure. In such decomposition, a condition is created for the combination of various mono-substances, for example, in persimmon fruits, monophenols combine with monosaccharides, which results in a decrease or disappearance of the tart taste of the raw material. Therefore, determining the degree of raw material ripeness by changes in stress will allow to predict its destruction in advance

Keywords: penetrometer, astringent taste, hardness, diversity, stress tensors

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1. Introduction

As is known, fruits and vegetables, depending on the degree of ripeness, differ from each other in mechanical structure and chemical composition. In technical ripeness, raw materials are used for technological needs, since in this state the raw materials have high technological processing qualities. And in consumer ripeness, fruits and vegetables have the maximum amount of nutrients and are used mainly fresh. Physiologically ripe raw materials have a soft structure, spoiled appearance, unpleasant taste and smell. With early use of persimmon fruits, the required results cannot be achieved because the fruits have not yet completed their development, contain a lot of polymer carbohydrates, due to which they have a hard frame. Without the necessary nutrients, such prematurely harvested fruits do not form characteristic taste qualities and appearance. As a result of accelerated respiration, the nutritional value of unripe fruits is sharply reduced. At the last stages of ripeness, persimmon fruits increase in weight (1–2 %), and become so soft that they can be eaten with a spoon.

As is known, in Azerbaijan, persimmon fruits ripen from October to January. It is noteworthy that these fruits continue to ripen better during storage, and as a result of the transition of monomeric phenols in polymer forms, their astringent taste disappears, they become very sweet and soft, due to the decomposition of polysaccharides.

The layered structure of persimmon fruits, which is associated with their mechanical strength, allows them to approach the ripening process as a continuous medium. And this approach makes it possible to analyze raw materials from a mathematical point of view. Because, having a large number of polymer compounds in their composition, persimmon fruits exhibit more or less resistance to external influences, depending on the degree of ripeness. Their mechanical strength and durability are determined by these stress resistances. The solution to the problem with this approach can be applied to other fruits and vegetables.

In general, the stresses that arise when loading layered materials do not depend on the number of layers, if the total thickness of the material does not change. Delamination at the boundary of layers and splitting along the polymers of the raw material are the main mechanisms of destruction [1, 2].

Prediction of the deformation properties of materials plays an important role in determining the ripeness of raw materials. Therefore, the issue of developing existing methods for calculating stresses in layered materials is relevant.

Mechanical processing of persimmon fruits requires large energy costs. Therefore, the deformation characteristics of the raw material must be taken into account in process calculations.

As is known, the mechanical properties of raw materials are identified on the basis of experimental data and theories. The result is a comprehensive characteristic of the elastic-plastic materials is obtained.

High-molecular organic compounds, including pectin substances, cellulose, hemicellulose, polyterpenoids, etc. have a complex structure. Therefore, it can be assumed that such compounds can be formed in several polymorphic modifications in the case of a change in the aggregate states. The reason for this is intermolecular interaction, and the hydrogen bonds formed in this case affect both the conformational state of the above-mentioned polymer compounds and the mutual formation of their molecular chains [3]. Considering the above, research into the stress state of persimmon fruits as they ripen is relevant in the creation of new food products based on this raw material.

2. Literature review and problem statement

Persimmon fruits are mainly distributed in tropical and subtropical regions of the globe, including in the Republic of Azerbaijan, various varieties of these fruits are grown. This is a perennial plant with a productivity cycle of 50–60 years. Persimmon fruits belong to the genus Diospyros and have 200 species [3].

The use of innovative techniques in various sectors of the food industry is very relevant. And in the presented work, the results of the approach to persimmon fruits as a smooth variety are presented. Such consideration can be considered primary. In this case, the chemical structure of the raw material is taken into account, consisting in the primary stages of ripeness of polymeric compounds such as polysaccharides, polyphenols, carotenoids, etc. There are literary data on mechanical changes occurring in polymeric compounds, but such data are absent in relation to fruits.

It should be noted that for polymeric compounds, including polysaccharides, anisotropy is a characteristic feature, but this feature is manifested only in single crystals. The sequential arrangement of individual molecules of these compounds is also the cause of its anisotropy. Therefore, the primary mechanical action is resisted by the surface crystalline structures. In such crystals, the molecules are linked to each other by Van der Waals forces, and hydrogen bonds prevail inside. It should be noted that any targeted effect on polymeric substances is not the same for their properties. Analyzing the source [3], it is worth noting that the study explains the cause of the anisotropy of polymer structures, but the issues related to the change in the hardness of the raw material as it matures remain unresolved. The reason for this may be the researchers' approach to the raw material only from a chemical point of view. It has been shown that the deformation of the studied raw material contributes to the degradation of its components [4, 5]. But the issues related to clarifying the cause of such deformations remain unresolved. The tensor approach to the process may be an option for overcoming the corresponding difficulties, since with the help of tensors it is possible to correctly track the change in the deformed state. It is in the work [6] that the deformation properties of the product, depending on the type of plant materials, are noted, but the reasons for such differences are not specified. Since tensor analysis deals with objects and properties that do not depend on the choice of coordinate system, the tensor approach to studying the deformations of persimmon fruits allows for a better analysis of the elastic properties of this raw material. It is known that the hardness of fruit and vegetable raw materials is associated with its polymer chemical components. As is known, unripe fruits and vegetables contain a large number of polymer compounds that give the products mechanical strength.

Research conducted on the mechanics of deformable solids is being further developed with the use of invariant tensor notations. Despite the fact that the works [6, 7] show dynamic failures of brittle materials, their results on the formation of cracks in the objects under study can be partially applied to solid materials. Consideration of the object under study as a manifold (topological manifold) is due to the fact that this mathematical concept generalizes the concepts of lines and surfaces that do not contain singular points to any number of dimensions. An example of a one-dimensional manifold was a cylinder, for each point of which there is a neighborhood that is a one-to-one and continuous image of an interval [8, 9]. A manifold is a space that is locally similar to Euclidean space. So-called smooth manifolds are usually considered. In such manifolds, one can talk about tangent vectors and tangent spaces. However, all these concepts related only to solids, and there was no information regarding plant materials. Multilayer polymer structures have interlayer boundaries that are destroyed under the action of tangential stresses during bending, shear or stretching. Intercalates also contribute to the splitting off of fragments of the layered matrix.

Stratification is a triple consisting of a topological space, a base and a projection of the stratification. The tangent stratification of a smooth manifold is a smooth vector stratification. In [10], it is shown that a vector stratification is a geometric construction corresponding to a family of vector spaces parameterized by another space. Therefore, given that the material under study has a layered structure, this structure can be approached as a stratification space.

All this allows to assert that the tensor approach to the stress state of raw materials is appropriate for further research.

3. The aim and objectives of the study

The aim of the study is to generalize the deformation properties of persimmon fruits of the Hyakume, Hachia, Zenji-Maru, and Emon varieties by the degree of ripeness. This will allow to prevent the astringent taste of the raw material in time.

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To achieve the aim, the following objectives were set:

– to determine the degree of ripeness of the fruit by changing the stress state of the raw material;

– to study the raw material as a solid material or diversity, observing the stratification that occurs as it ripens;

– to identify the causes of tensor stresses in persimmon fruits.

4. Materials and methods of the study

The object of the study is persimmon fruits. It is known that, depending on the structural and mechanical properties, plant materials resist external influences differently. The degree of ripeness of fruits and vegetables is determined by these characteristics. To study the rigidity of persimmon fruits, penetrometers are mainly used as a criterion for the degree of ripeness. The standard of penetrometers is the study of the rheological properties of substances by measuring the depth of penetration of a standardized form of working fluid into the environment. Penetration is a method used to measure consistency, i.e. the resistance of a sample to deformation under the action of applied external forces. Penetrometers measure the density, compaction, consistency or permeability of a wide range of solid, semi-solid, food and non-food products [5, 11, 12]. The operating principle of penetrometers is based on the fact that the raw material under study, when pressed into it by a conical rod or needle, resists the penetration of the latter. As a result, the depth of penetration of a cone or needle into materials with different mechanical properties over the same period of time is not the same. The depth of penetration of a cone or needle into raw materials is characterized by the degree of penetration.

The study of the hardness of persimmon fruits as indicators of the degree of ripeness was carried out on 15 samples for each variety. Fruits at the technical stage of ripeness had an average hardness of 10.9±12.3 kg/cm^2. As they ripen, this hardness decreases, since the resistance of the raw material decreases. Fruits with less hardness are well suited to technological processing, do not have a tart taste, and do not form a lump during heat treatment.

The rationale for choosing the penetration method was to study the change in the hardness of the raw material occurring in the fruits during their ripening. And the tensor approach to the causes of such changes simplifies the understanding of these same changes.

Based on the generalized Hooke's law, the raw material was considered as a material with uniform stress in all directions and each part of which is in a state of static equilibrium. If to select a geometric figure of the cube type from this material, it is possible to conduct an analysis on this cube (Fig. 1). For this, let's make the assumption that the sides of the cube, constructed in the Cartesian coordinate system, are parallel to the coordinate axes [13].

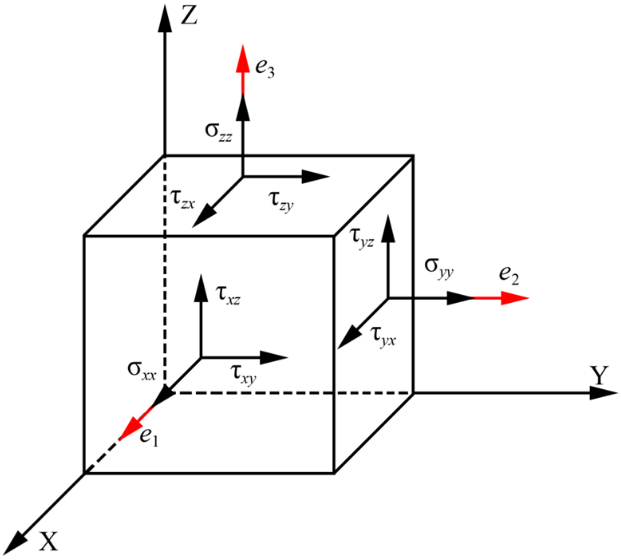


Fig. 1. Stress tensor of a volumetric body

The sign of the stress σ_{ij} indicates the force acting in the direction of the i-axis on a unit area perpendicular to the j-axis – this is a component of the stress tensor T_σ. The trace of the tensor is understood as the principal diagonal components of the stress σ_{ii}, more precisely σ_{xx}, σ_{yy} and σ_{zz}. The components of the tangential force or the components of the displacement are denoted as τ_{ij}. The normals are shown as e_1, e_2 and e_3. If the components are shown as a matrix (1), then they actually represent the stress at any given point in the material. For volumetric bodies, tensors are depicted as a 3×3 matrix:

$$T_\sigma = \begin{vmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{vmatrix} = \sigma_{ij}, \quad i,j = x,y,z. \tag{1}$$

The above stress tensor T_σ is a second-rank tensor consisting of three normal and six tangential stresses. Since the stress $\sigma_{ij}=\sigma_{ji}$, only six of the nine components of the tensor are free (independent), i.e. the stress tensor is symmetric [14]. In addition, if to consider a pair of shear stresses, the following equations (2) are true:

$$\tau_{xy} = \tau_{yx}; \quad \tau_{yz} = \tau_{zy}; \quad \tau_{xz} = \tau_{zx}. \tag{2}$$

When the material is completely compressed, shear stresses do not arise and for the component σ_{ij} there is an inequality $i \neq j$. When $i=j$, the stress is equal to σ_{ii} or σ_{jj}.

Experiments show that the degree of destruction of each material strongly depends on the change in its shape due to deformation. Therefore, when speaking about the deformation of a material, it is necessary to consider the components that depend on the change in its volume and shape. Consequently, the total stress tensor can be represented as the sum of the spherical tensor S_σ and the stress deviator D_σ (3):

$$T_\sigma = S_\sigma + D_\sigma, \tag{3}$$

$$S_\sigma = \begin{vmatrix} \sigma_\alpha & 0 & 0 \\ 0 & \sigma_\alpha & 0 \\ 0 & 0 & \sigma_\alpha \end{vmatrix},$$

$$D_\sigma = \begin{vmatrix} \sigma_{xx}-\sigma_\alpha & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy}-\sigma_\alpha & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz}-\sigma_\alpha \end{vmatrix}.$$

The spherical stress tensor corresponds to the average stress (4)):

$$\sigma_\alpha = \frac{1}{3}\left(\sigma_{xx}+\sigma_{yy}+\sigma_{zz}\right). \tag{4}$$

If the sum of the three principal stresses is zero, then only the volume of the cube will change, and the shape will remain constant [14, 15].

The magnitude of the normal (5) and tangential stresses (6) acting on the main tangent surface is found as:

$$\sigma_{xy} = \frac{\sigma_x+\sigma_y}{2};\quad \sigma_{yz} = \frac{\sigma_y+\sigma_z}{2};\quad \sigma_{zx} = \frac{\sigma_z+\sigma_x}{2}. \tag{5}$$

$$\tau_{xy} = \frac{\sigma_x-\sigma_y}{2};\quad \tau_{yz} = \frac{\sigma_y-\sigma_z}{2};\quad \tau_{zx} = \frac{\sigma_z-\sigma_x}{2}. \tag{6}$$

Thus, to confirm the research results, the use of the tensor approach to the analysis of ripening raw materials is justified, forming a corresponding hypothesis about the possibility of generalizing the deformation properties by the degree of ripeness.

5. Results of the study of the stress state of persimmon fruits

5. 1. Study of the degree of ripeness of persimmon fruits by changing its stress state

The degree of ripeness of the raw material is determined mainly by its hardness. For such a determination, different devices are used, for example, penetrometers were used in the presented study. Depending on the degree of ripeness, the hardness indicator varies. And this indicator is also inherent in different varieties of fruits. In order to have a general idea, four varieties of persimmon fruits were subjected to research. At the stage of commercial ripeness, the hardness of persimmon fruits on average by varieties was no more than 12.3 kg/cm^2. It is difficult and even impossible to obtain juice from such raw materials. And this indicator according to the degree of ripening of the raw materials began to change gradually and eventually amounted to 1.5÷2.0 kg cm^2. Penetrometry of the fruits was carried out every fifth day and recorded. From Fig. 2 it is clear that after a while the strength of the raw materials decreases, and in the end this process accelerates

Persimmon fruits at the technical stage of ripeness have a brownish-yellow color. They vary in shape by variety. There are some that look like apples, pears, quince, tomatoes, etc. But all of them, basically, have a fleshy structure. The mass of spherical or oval fruits reaches, even up to 500 g has a mass of up to 500 g. As they ripen, the fruits acquire a color from yellow to red-orange.

The dry matter of persimmon fruits at the technical stage of ripeness in the Hachiya varieties is 20.7 %, in Hyakume – 21.9 %, in Zenji-Maru – 22.5 %, in Emon – 22.9 %. And at the physiological stage of ripeness, the dry matter of persimmon fruits in the Hachiya varieties is 25 %, in Hyakume – 25 %, in Zenji-Maru – 26 %, in Emon – 25.1 %. These substances are mainly represented by carbohydrates. Persimmon is characterized by a large number of polysaccharides, such as pectin substances (in Hachiya 0.75 %, Hyakume 0.83 %, Zenji-Maru 0.76 %, Emon 0.92 %) and cellulose (in Hachiya 0.84 %, Hyakume 1.30 %, Zenji-Maru 1.12 %, Emon 1.0 %). Therefore, polymeric compounds of carbohydrate origin mainly showed resistance to external influences, because they can be found in fruits, also in a bound form with other chemical compounds. Such a connection, for example, with polyphenols also helps to reduce the astringency of persimmon fruits [3, 16]. When a penetrometer is introduced into the raw material, certain stresses and deformations are created in it. This feature is associated with the resistance of the material to external destructive effects. The cause of deformation is a change in the ratio of forces between the molecules of the chemical components that make up the material, under the action of a force, the unit of measurement of which is stress, that is, the ratio of the applied force to the surface area. Such a displacement of atoms leads to elastic deformation. In this case, the magnitude of the displacement of an atom from equilibrium does not exceed the distance between it and neighboring atoms. As a result, a system of forces arises that balances the forces of external action. Due to the inability of these forces to return the atoms to their previous equilibrium state, plastic deformation occurs, that is, the atoms take on a new state of equilibrium. During plastic deformation, the bonds between atoms are not broken, but the shape changes. And so, as a result of such deformation, the crystalline structure changes [16, 17]. Considering that in the conducted research persimmon fruits are considered as a solid body or manifold, its deformation is described by nonlinear relations. These relations include nonlinear descriptions of finite deformations of bodies.

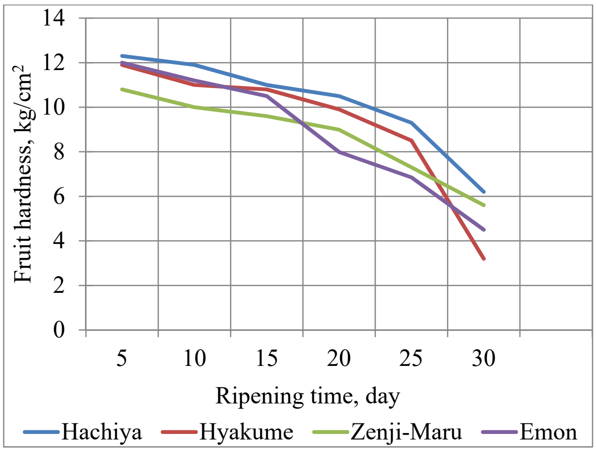


Fig. 2. Change in mechanical strength of fruits by ripeness

The reason for the layered internal structure of the raw material is the polymer substances that make up its composition. These substances are located in different directions and form the structure of the material. Such compounds are in connection with the components corresponding to them and other various groups. It is for this reason that each polymer creates its own layer. Therefore, the study of layers changing depending on the ripeness of the raw material can be approached as a topological diversity. This approach makes it possible to determine the optimal parameters of the raw material processing mode.

The approach to the destruction of the material from the energy point of view allows to better explain the stratification occurring inside the material caused by stresses [2]. The general thermodynamic criterion for such destruction is given by the following formula. In this case, differentiation is made either by time or by the area and length of the crack, which are the parameters of destruction (8):

$$dA \geq dU + dR + dT, \tag{8}$$

where dA – the work done on the material or the energy source; dU – the sum of the increment of elastic energy; dR – the work spent on destruction, R=γS, γ – the specific destruction coefficient; S – the area of the destroyed surface; dT – the energy dissipation.

When taking into account the stresses on the material, it is necessary to take into account [2] that the accumulated energy is spent on destruction, i.e. $U=R$. Then:

$$\sigma = \sqrt{\frac{2E\gamma}{L}}, \tag{9}$$

where E – Young's modulus or elasticity; L – the length of the material under stress.

The critical stress for interlayer shear strength can be represented as follows:

$$\tau = \frac{h}{L}\sqrt{\frac{3E\gamma}{2h}}, \tag{10}$$

where h – the material thickness. As this thickness increases, the critical stress becomes lower.

Since further processing of persimmon is associated with thermal processes, its physicochemical changes are also affected by temperature. In other words, it is the temperature that causes deformation of the material and subsequent destruction of its structure. In this case, the Gibbs formula for a unit volume takes the form (11):

$$dU = TdS + \sigma_{ij} + d\varepsilon_{ij}, \tag{11}$$

where ε_{ij} – the thermal expansion tensor, $\varepsilon_{ij}=\alpha_{ij}\Delta T$, α_{ij} – the linear thermal expansion tensor); dS – the entropy.

Deformation in fibrous and dense materials consists of irreversible and elastic stresses. Irreversible elasticity is due to the fact that the fibers become more stable. At low pressures, bending stress predominates in the fibers. At higher pressures, the areas of interfiber exchange and the number of particles increase [17].

5. 2. Consideration of raw materials as a variety, observing their stratification during the ripeness

To interpret the strength, it is necessary to take into account many different mechanisms of destruction of the surface and pulp of the raw material. The considered raw material is taken as a unidirectional fibrous material. When compressed along the fibers, there is a loss of stability, stratification, bending or destruction of the fibers with the formation of a fold. For such an impact of external forces, there are critical conditions and requirements for the properties of the matrix (binder), the fiber-matrix interface, etc. [9, 18].

Expression of the occurrence of tangential or vector stratification by tensor stresses when exposed to external forces on the raw material is considered more practical. Then, studying the stratification as the raw material mixtures from a vector point of view will allow to explain how deformation occurs due to external influences. This is the peculiarity and distinctive feature of the obtained results. Determining the degree of ripeness of the raw material allows to prevent astringent taste in time in further processing of the raw material. And this makes it possible to expand the use of these fruits as the main component in the creation of new assortments of canned products.

It should also be noted that persimmon and other fruits and vegetables at a certain level of ripeness undergo changes in the degree of stratification. To conduct such studies, it is necessary to approach the material as a variety.

In general, stratification is microdefects that occur during the destruction of the material, caused by bending, compression and stretching of the material under the action of external forces. In this case, interlayer cracks appear, the reasons for the formation of which are associated with the fact that free elastic energy during deformation is spent on destruction. The layers of persimmon, as well as other fruits and vegetables, are multicomponent. Since such raw materials include various polymers (pectin substances, cellulose, pentosans, etc.), conditions are created for the formation of various interlayer bonds. From this point of view, such bonds have a significant impact on the process of technological processing of raw materials (Fig. 3).

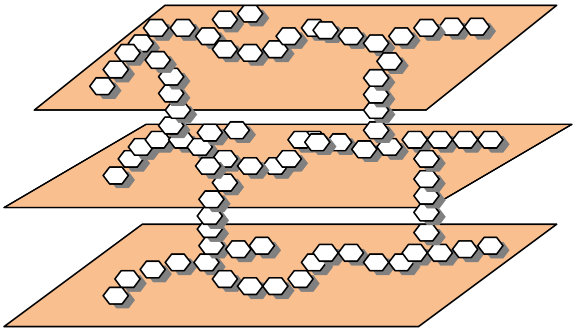


Fig. 3. Structure of polymer compounds in layered form

Since the raw material consists of different types of polymers, this indicates the existence of a phase boundary between these compounds. At such boundaries, the density of the material is lower. In order to fill the spaces at the junction of different structures, the atoms present there have a low directionality.

As mentioned earlier, external influences can cause stratification of the material. The studied raw material, as an arbitrary object, can also be considered as a smooth manifold. The goal is to study what changes can occur in such an object and to explain the deformation stresses mathematically. To do this, first of all, it is necessary to study this manifold in relation to the tangent space.

If to assume that the studied material is an M^n-dimensional manifold, then in this manifold there is a point p, which is its element $\left(p \in M^n\right)$ [10, 19]. If to construct a tangent at this point, then this will be the tangent space at this point (T_pM) (Fig. 3).

A set (TM) consisting of a pair (p, θ) such that $p \in M$ and $\theta \in T_pM$, where θ is a vector in the tangent plane, or more precisely, a tangent vector, was considered. The set TM is called the tangent bundle or vector bundle of the manifold M, and its dimension is equal to twice the dimension of this manifold M. A vector bundle is a special case of a tangent bundle [9, 19]. The manifold itself can be a base of the bundle. In Fig. 4, the normal vector (n), drawn perpendicular to the tangent surface at the point p, forms a normal bundle. This is the orthogonal complement of the tangent surface.

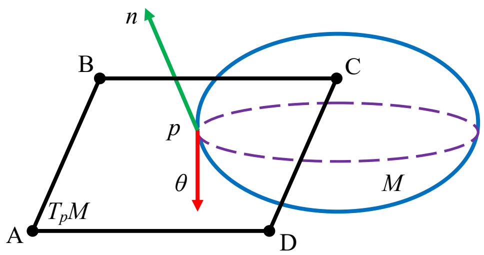


Fig. 4. Tangent plane at a point on the surface: $p \in M$ – an arbitrary point in the manifold; *M* – the manifold; *TpM* – the tangent space at the point *p;* $\theta \in T_pM$ – the tangent vector; *n* – the normal vector

The existence of the manifold M^n means that there also exists an atlas consisting of charts (U_α, φ_α). Here U_α is an open set in the space; φ_α – a homeomorphism into an open set, i. e. φ_α**:** $U_\alpha \to R^n$**.** The set of charts (U_α, φ_α), $\alpha \in A$ is called the atlas of the manifold M^n. This set of charts covers the manifold *M*. If to look at the tangent space from the point *p*, it is possible to observe the formation of a canonical basis in this space [10].

Each chart is given by local coordinates $\left(x_\alpha^1,...,x_\alpha^n\right)$. Then each tangent vector is given by a set of numbers $\left(\theta_\alpha^1,...,\theta_\alpha^n\right)$. These vectors are $\frac{\partial}{\partial_\alpha^1},...,\frac{\partial}{\partial_\alpha^n}$.

If there exists a chart with local coordinates, then conditions are created for the formation of local coordinates for the tangent bundle TM. In this case, the corresponding chart can be represented by the Cartesian product ($U_\alpha \times R^n$**).** In the product, the coordinates for U_α will be $x_\alpha^{/}$, and for R^n will be $\theta_\alpha^{/}$. In this case, the tangent bundle will be isomorphic to the projection $R^{2n} \to R^n$.

Over an *n*-dimensional manifold *M* (dim *M*=*n*), an *m*-dimensional vector bundle has several properties:

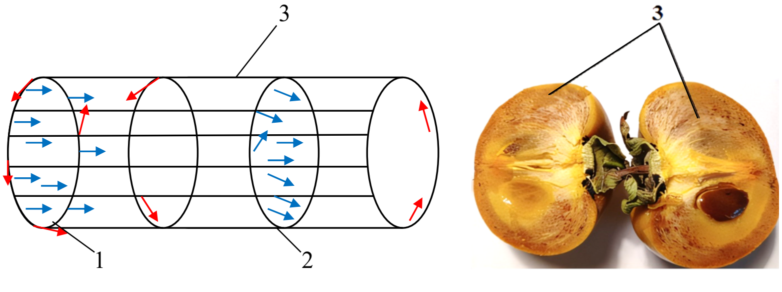
a) the manifold is the total space of the E^{m+n} – dimensional bundle;

b) the smooth mapping π: $E^{m+n} \to M^n$ is a projection of the bundle, i.e., it is a continuous surjective mapping of the bundle, also $\pi(p,\theta)=p$.

The mapping (π) itself is generally called a bundle. In general, when to speak of a bundle, let's mean a parametrized base M and fibers that are "glued" with the topology $\pi \circ h$ of the tangent space. Then the relation *h*: *M*→*TM* is such that the composition onto the manifold *M* is called a section of the fiber or a vector field. The tangent bundle of a smooth manifold *M* is a vector bundle over this *M* whose fiber is the tangent space T_pM at the point $p \in M$ (Fig. 4, 5).

It is possible to note a certain property of the mapping covering the manifolds *M* as follows: the relations depending over each chart have a structure like the Cartesian product of the vector space of this chart, i. e. π^{-1} **(**U_α**)**$\approx U_\alpha \times R^m$ (the diffeomorphism exists). As is known, the manifold *M* is the base of the bundle, and an arbitrary point p is its element, then the bundle over the point p is expressed as π^{-1} **(**p**)**$=R^m$.

Covering the studied raw material with a cylinder, it is possible to obtain a one-dimensional vector bundle on the circle S^1, i.e. a cylinder in the three-dimensional space R^3 (Fig. 5).



a *b*

Fig. 5. Vector bundle of a manifold in the form: *a* – of a cylinder; *b* – in its natural form; 1 – base, $M=S^1$; 2 – section or vector field; 3 – layers

The cylinder shown in Fig. 3 is a Cartesian product of a circle and a straight line, i.e. if to consider the expressions $E=S^1 \times R$ on a Cartesian product of the type $E=M^n \times R^m$, let's obtain a trivial vector bundle on the manifold M^n.

5. 3. External influences as causes of tensor stresses in persimmon fruits

When studying stress-related changes, one cannot ignore the intrinsic atomic-molecular structure, which plays a major role in the formation of raw materials as a whole material. For example, it is known that pectin polymers are connected to each other by salt bridges (Ca^{2+}, Mg^{2+}). Since such polymers consist of long molecules, carbon atoms play the role of a link between them. These polymers also interact with each other due to Van der Waals forces.

Considering the fact that plant raw materials can be considered as fibrous material, such processes as rupture, stretching, twisting and bending of fibers occur during pressing into raw materials. And for each process there is a certain limit, at different values of the critical state of which, intermolecular bonds are broken or the mechanism is disrupted. In general, external forces acting on a solid are divided into two groups: surface (external) and mass (internal). Surface forces are the result of the interaction of two bodies and are characterized by the force per unit surface area.

Mass forces extend to the entire mass of the body and are expressed as a force per unit volume. Internal forces in a solid material ensure the existence of its strength. These forces are also called volumetric forces [7].

Since the raw material in the degree of technical ripeness is considered an anisotropic material, the force applied to it also determines the mechanical properties of this material.

As can be seen from Table 1, even a small varietal difference affects the stress. According to the research results, it can be noted that a change in the ripeness of the raw material also reduces its resistance to external influences. Such changes make it possible to select raw materials for a certain range of products. For example, raw materials with high resistance can be useful for the production of juices, and raw materials with low deformation – for the preparation of concentrated products (jam, preserves, jam, etc.).

Considering that in the study conducted, persimmon fruits are considered as a solid body or variety, its deformation is described by nonlinear relationships. These relationships include nonlinear descriptions of the final deformations of bodies.

Table 1

Tensor stresses for different varieties of persimmon fruits subject to deformation

Tensor stresses, kg/cm^2	Raw material varieties			
	Hachiya	Hyakume	Zenji-Maru	Emon
Mean normal stress or spherical tensor stress	Technical ripeness			
	11.7	11.6	11.1	11.03
	Technical ripeness			
	1.9	1.8	1.6	1.5
Principal normal stresses acting on the principal tangent surface	Technical ripeness			
	12.1	11.9	11.15	11.3
	11.35	11.4	10.95	10.75
	11.55	11.5	11.1	11.05
	Ripe raw material			
	2.15	2.05	1.8	1.7
	1.75	1.6	1.4	1.3
	1.9	1.75	1.6	1.5
Principal shear stresses acting on the principal tangent surface	Technical ripeness			
	0.2	0.1	0.15	0.3
	0.55	0.4	0.05	0.25
	–0.75	–0.5	–0.2	–0.55
	Ripe raw material			
	0.15	0.15	0.2	0.2
	0.25	0.3	0.2	0.2
	–0.4	–0.45	–0.4	–0.4

6. Discussion of the results on generalizing the deformation properties of persimmon fruits

The obtained research results showed (Fig. 2) that over time, the strength of the raw material decreases and this process accelerates. This is explained by changes occurring with the chemical components of plant materials, in particular persimmon fruits. As is known, the ripening of fruits depends on their polymer compounds, which, as the raw material ripens, disintegrate into less simple substances. For example, polysaccharides are converted into monosaccharides, alcohols, acids, etc.; polyphenols – into monomeric forms; proteins – into amino acids; lipids – into fatty acids and glycerol; carotenoids – into retinol, flavonoids, etc. Since persimmon fruits are rich in polyphenolic substances, due to which their tart taste appears, this disintegration is strongly felt in ripe fruits. Such a change in taste is explained not only by the conversion of polyphenols into monomeric forms, but also by the formation of compounds between sugars and monomeric polyphenols. Taste changes differ little by variety (Table 1). Generally speaking, all varieties have a moderate change in hardness. It turns out that different varieties of persimmon fruits differ little in chemical composition. This means that the results of the stress state studies for one fruit variety can be applied to all varieties.

A feature of the presented work is that in the literature [4, 8, 10] one can find data only on the study of stresses of non-food materials. But despite the fact that in the literature [6] there are results of deformation properties of various food products, the reasons for such changes are not specified. The chemical (as a polymer substance) and mathematical (as a variety) approach to plant raw materials, in particular to persimmon fruits, will allow in the future to deeply study various food products.

According to the results of Table 1, it is clear that as the raw material approaches a ripe state, its resistance to external influences decreases. The solution to this problematic issue allows choosing raw materials for a certain range of products, because persimmon fruits are very sensitive to heat. By determining the ripeness of processed fruits, it is possible to prevent non-standard taste qualities.

By studying raw materials with a tensor approach, it is possible to determine the critical limit of the material strength failure. Research shows that the degree of destruction of each material strongly depends on the change in its shape as a result of deformation. Therefore, when speaking about the deformation of a material, it is necessary to consider the components that depend on the change in its volume and shape. An unresolved issue – clarifying the causes of deformations requires a tensor approach to the process. And this approach provides the opportunity to correctly track changes in the deformed state.

According to literary data [18], it is known that in solids with large plastic deformations, defects accumulate (dislocations in crystalline bodies, crazes in polymers, etc.), which turn into cracks, which ultimately lead to the destruction of the material. This also happens during the ripening of fruits. These are the limitations inherent in this study. It is difficult to prevent the formation of cracks in solid materials in advance, so it is necessary to determine the critical forces that are difficult to measure. Another feature of the presented study was that for a deep study of the stresses arising in the raw material, it was divided into layers, as a manifold. Vector bundle, as a topological construction, allows to study the geometric properties of the material. The vector bundle itself is a locally trivial bundle, the layer of which is a vector space. The vector field here appears as a section of the object, for which each point corresponds to a vector with the origin at this point. The tangent bundle is the vector bundle with which the raw material is studied.

In the future, it would be necessary to study the stresses arising in the material from an energy point of view. Because all the destruction that occurs from external influences has an energy component. The results of thermodynamic destruction studies will allow to prevent this drawback in advance.

The study of the stress state of plant material from a chemical and mathematical point of view has some difficulties. These difficulties are associated with the fact that with a practical approach to such changes, there are difficulties with the exact definition of these changes. Because the polymeric substances of persimmon fruits, after a period of time, disintegrate, creating various obstacles to determining the voltage, since not all polymers simultaneously transform into monomeric forms. As a result, difficulties arise with accurate data. The development of this study is that the most suitable approach to prevent such difficulties is the tensor approach to the process. Being a mathematical apparatus, this approach can be applied to both dynamic (in this work) and static processes.

7. Conclusions

1. By determining the ripeness of plant materials by changes in stress, it is possible to predict their destruction in advance. It has also been established that as persimmon fruits ripen, their resistance to external influences decreases. When producing products based on these fruits, these results should be relied upon.

2. Due to the polymer structure of persimmon fruits, they were considered as a variety. This material is a layered substance that can be approached as a vector layered space. Such a mathematical approach made it possible to geometrically

explain the structural changes occurring in the raw material during its ripening.

3. Since stratification in fruits is caused by a structural change occurring in the raw material, the use of various tensor stresses such as spherical and deviator stresses makes it possible to determine the strength of the material depending on the degree of ripeness. Studying raw materials with a tensor explanation makes it easier to understand the changes occurring in the internal structure of the raw material. If in the technical ripeness of persimmon fruits by varieties the average normal and main normal stresses on average amounted to 11.34 kg/cm^2 and 11.61 kg/cm^2, respectively, then in ripe fruits these indicators changed, i. e. they amounted to 1.7 kg/cm^2 and 1.9 kg/cm^2, respectively.

Conflict of interest

The authors declare no conflict of interest regarding this study, including financial, personal, authorship or other nature that could affect the research and its results presented in this article.

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Data availability

The manuscript has associated data in the data warehouse.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies in creating the presented work.

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