The object of research is coal enterprises where powerful electrical equipment is used (underground installations up to 3000 kW, tunneling complexes of 500–1500 kW, technological complexes of 6–10/0.4 kV, etc.). The deficit of generated capacities, caused by growing energy consumption, can be reduced by regulating power consumption modes.

The relevance of this issue is determined by the need to conduct a research of the electricity consumption system, to determine the qualitative and quantitative characteristics of electricity consumption using mathematical models.

The lack of mathematical models makes it difficult to analyze energy intensity and consumption modes of each technological operation in the overall balance of electricity consumption of coal enterprises.

The article considers the structure and classification of the main technological groups of energy consumers, the development of mathematical models for each type of load modes, as well as a generalized model of electricity consumption of coal enterprises, with the use of mathematical apparatus of probability theory and mathematical statistics.

As a result of the work, mathematical models of the electricity consumption process for the main technological groups and for models of daily electricity consumption of coal enterprises as a whole were developed, and it was also established that technological objects of electricity consumption are divided into three different values, in terms of power consumption modes: constant, uniform and pulse. For each class of consumers their statistical characteristics were obtained.

The work results can be applied for managing power consumption modes of coal enterprises

Keywords: energy consumption, probability theory, mathematical models, power consumption modes, daily consumption, mathematical statistics

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DEVELOPMENT OF MATHEMATICAL MODELS OF POWER CONSUMPTION AT COAL PLANTS

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1. Introduction

For efficient and reliable operation of the power supply system for underground mining, it is necessary to timely identify and eliminate the mode of excess consumption of active power and penalties from the power system. It is known that up to 80 % of violations occur due to non-compliance with the specified power consumption modes.

To date, mainly the modes and standards of power supply for general industrial enterprises have been studied, and for coal enterprises, the modes of power consumption management have been studied to an insufficient degree, there are no modes of power consumption for energy-intensive installations, the existing methods of analysis and calculation of justified values of non-standardized parameters of power consumption for coal enterprises are not effective enough. It can also be noted that these methods control only the modes of power consumption at the main inputs of feeding substations, without affecting the features of the modes of power consumption of individual technological groups.

The listed shortcomings make it relevant to develop mathematical models and control algorithms for energy-intensive installations based on measuring the active power of electrical energy for individual technological consumption, which takes into account both the mode of electrical consumption and its volume for various technological groups.

In this regard, research on development of mathematical models of expected power consumption modes for coal enter-

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prises in general and for individual technological groups of consumers is relevant.

2. Literary review and problem statement

The article [1] presents the results of a studying the problem of optimizing power systems by smoothing the load profile. There is proposed an algorithm for optimizing the modes of power systems, taking into account simple and functional limitations in the form of inequalities in the conditions of the probabilistic nature of the initial node information, based on the use of matrices for calculating the objective function and constraint functions.

One of the disadvantages of this study is the use of a simplified mathematical model that includes only two consumers and two thermal power plants. Such a model does not adequately reflect the complexity of real power systems, where the number of consumers can reach several dozen.

In the article [2], the authors present the results of an experimental study of the dynamics of changes in specific energy consumption depending on the volume of extracted rock per shift and the rate of excavation. Energy technology profiles of energy characteristics have been obtained to assess the efficiency of tunneling operations.

It should be noted that the profiles created are applicable only for estimating the energy consumption of mining equipment. However, in order to comprehensively assess the energy consumption of a coal mine, it is necessary to expand the study and include an analysis of the energy consumption of processes such as mining, transportation, ventilation and lifting.

The article [3] presents the results studying the optimization of production processes in conditions of limited peak capacity of the workshop using a mathematical model and heuristic solutions.

One of the disadvantages of this study is that the results obtained are applicable only to one workshop with a single power supply source and a limited set of technological processes and does not consider more complex systems with multiple power sources.

The article [4] presents the results of a study in which, using a multi-criteria optimization algorithm, it was possible to reduce the energy consumption of CNC machines by optimizing milling parameters. One of the disadvantages of this study is the study of one technological consumer (a CNC machine). The model does not take into account the complexity of real enterprises with many consumers that require an integrated approach to optimizing energy consumption.

The article [5] presents the results of a study of an optimization model for regulating a ventilation system using mixed integer nonlinear programming. It is shown that mathematical models take into account changes in the geometric parameters of the mine, meteorological conditions and other factors. One of the disadvantages of this work is the study of the operating modes of a low-power fan (70–75 kW). This model does not take into account the specifics of industrial enterprises, where the total capacity of ventilation systems can reach 4–6 MW.

The article [6] presents the results of a study in which, using mathematical models, the dependence of the maximum water flow on the diameter of the water meter and other factors was studied. Time series analysis methods were used to analyze hourly fluctuations and determine peak loads. In general, the approaches used in this work can be adapted to our task, although we are dealing with other objects and types of energy.

The article [7] presents the results of a study of the operating modes of complex mechanized stopping faces. The analysis of the interrelationships between various factors affecting power consumption, using statistical methods, allowed to create a mathematical model that optimizes the operating modes of complex mechanized stopping faces. The disadvantage of the work is that the developed methods are applicable only to mining combines, whose share in the total energy consumption of the mine is only 2–5 %, which limits their impact on optimizing the energy consumption of the mine as a whole.

The authors [8] use simple and multiple linear regression analysis in their work, as well as quadratic regression analysis to hourly and daily data from the research center. The disadvantage of this work is the narrow specialization of the proposed models, which are applicable only to residential buildings. Coal enterprises represent a much more complex system for analysis, which requires the development of specific models.

The article [9] presents the results of a study of various strategies for reducing energy consumption using the example of a coal mine in Mpumalanga (South Africa). The lack of a detailed analysis of power consumption modes and insufficient control at the level of individual processes are significant disadvantages of this work.

The article [10] presents the results of a study of simulation models for optimizing the operation of a complex mechanized coal mine face. Prototypes of combine harvester, conveyor, crusher and loader models have been obtained. The disadvantage of the work is that the analysis was carried out only for the complex mechanized face of a coal mine, without a detailed consideration of the impact of various types of equipment on energy consumption.

Thus, the considered works mainly studied the modes and norms of power supply for general industrial enterprises, shops, and for coal enterprises the modes of power consumption control are studied insufficiently, there are no modes of power consumption for all energy-intensive installations, and the existing methods of analysis and calculation of justified values of non-standardized parameters of power consumption for coal enterprises are not effective enough.

The foregoing justifies the need to develop mathematical models for the research of power consumption modes of mining enterprises using the probability theory and mathematical statistics.

3. The aim and objectives of the study

The aim of the study is to develop a comprehensive system of mathematical models for the power consumption of coal enterprises, allowing for detailed analysis, real-time forecasting, and long-term planning of energy consumption modes for coal enterprises, power supply centers, technological processes, and individual consumers:

– develop mathematical models for the process of active power consumption for the main technological groups of a coal enterprise, taking into account their specific characteristics and operating modes;

– develop a generalized mathematical model of the electrical load of a coal enterprise, combining the models of individual technological groups and considering the interaction between them;

– develop a mathematical model of the daily electricity consumption of coal enterprises;

– construct daily load curves and histograms of the electrical load distribution for various technological groups and the enterprise as a whole, and analyze their features.

4. Materials and methods of research

The object of research is energy-intensive electricity consumers of coal enterprises, for which it is proposed to determine the energy intensity and consumption modes of each technological operation in the overall electricity consumption balance of coal enterprises, as well as to develop mathematical models of the electricity consumption process. To simplify the analysis, numerous underground electric consumers are united into technological groups.

Among power consumers under consideration, two main groups can be distinguished: power consumers who consume electricity continuously and power consumers who consume electricity at discrete moments. The first group includes: main ventilation units, vacuum pumping stations, boiler rooms, surface transformers and underground load [11]. The second group of energy receivers includes skip and cage hoists. The main indicators of power consumption at the Kostenko coal mine in Karaganda coal basin are shown in Table 1.

Table 1

Main indicators of power consumption in a coal mine

Energy receivers	*S*, kVAr	PF	*P*, kW	PF	*Q*, kVAr	PF	cosφ
Ventilation	9366	0.169	2300	0.311	627	0.146	0.910
Cage hoist	6388	0.115	840	0.143	567	0.132	0.828
Skip hoist	8390	0.151	1400	0.189	949	0.221	0.847
Vacuum room	1120	0.020	175	0.034	156	0.036	0.891
Surface transformers	7610	0.137	583	0.039	426	0.099	0.760
Boiler room	2800	0.050	112	0.015	92	0.021	0.821
Underground load	19845	0.357	1986	0.268	1485	0.345	0.748
Mine	55519	–	7396	–	4302	–	0.825

Note: S – installed capacity, kVA; PF – participation factor; P – active power, kW; Q – reactive power, kVAr; cosφ – power coefficient.

Some of the energy consumers under consideration include dozens of independent energy receivers. For example, the underground load is determined by hundreds of machines: mining and tunneling combines, manual and column electric drills, winches, conveyor lines, electrical haulage, etc. Some power receivers consist of one or two powerful energy consumers, for example, a boiler room.

4. 2. Ventilation plant

Ventilation of cleaning and development faces is one of the most energy-intensive processes. Power losses during mines ventilation reach significant amounts. Electrical consumption for mines ventilation of with high methane abundance of coal beds in the overall mine energy balance is 30–40 %. The main and local ventilation units consist of two independent units, one of which is a stand-by unit. Powerful synchronous motors are usually used to drive main ventilation fans, which are used to increase the overall power factor of the mine (cosφ) [12].

4. 3. Surface transformers

The group of energy receivers supplied with 380/660 V voltage from transformers installed on the mine surface includes various groups of mechanisms, for example, technological complex, providing transportation and loading of coal into railroad cars, administrative and household facilities, auxiliary needs of stationary installations, etc. If to consider each group of energy receivers separately, they have very different operating modes depending on a number of reasons of technological, climatic and technical nature. In the future, it is possible to consider them as a single group of energy receivers, generalizing them according to the principle of territoriality of energy supply. Analyzing the graph of power consumption, it is possible to say that the load on 6 kV feeders supplying this group of power consumers is a continuous value changing in time [12].

4. 4. Underground load

The underground load is a large group of individual energy receivers (up to several hundred in the largest mines), including transport mechanisms (conveyor units, electric locomotives, winches), mining and tunneling equipment, etc. Magnitude of underground load and its share in the overall energy balance of the mine depend on many factors, the main ones are the number of mined beds, their depth, mining technology, etc. Some energy receivers (drainage, conveyor transport) have a relatively continuous schedule of energy consumption, the rest (electric drives of mining and tunneling equipment, winches, electric locomotive transport) belong to the group of indexing mechanisms. The total electrical load for the entire group is a continuous random value [12].

4. 5. Boiler room

The boiler installation is a continuous energy consumer with a pronounced seasonal cycle, which is caused by heating of mine buildings and air heating entering into the mine during fall and winter. In addition to seasonal fluctuations in the level of power consumption, boiler installations have daily fluctuations related to shift work (showers, laundries) [12].

4. 6. Vacuum pump stations

Vacuum pump stations (VPS), designed for degassing coal beds are installed on the mine surface. The VPS operating mode is constant. The amount of electrical load on the feeders feeding vacuum pumping stations depends on the number of simultaneously operating pumps (usually 1–3 pcs.). Changes usually occur when the production unit transfers to a new longwall.

Thus, the amount of power consumption for the VPS is a continuous and constant value, with unchanged topological structure of the mine field. VPS power consumption planning is carried out in accordance with the mining plan for each bed [12].

4. 7. Lifting installations

Lifting installations, by nature of their electrical load over time, can be classified as cyclic intermittent mechanisms. In particular, cage hoisting installations must provide men hoisting (for example, in a 4-shift operating mode – up to 8 hours a day), hoisting auxiliary materials into the mine, rock and mechanisms hoisting need to be repaired. For each cage hoisting installation, a work schedule shall be drawn up in accordance with the daily operating schedule of the entire mine.

Operating mode of the skip hoisting installation is directly dependent on the amount of coal mined. The work schedule of skip hoisting also provides for the time required for carrying out preventive and repair operations (usually during a repair shift provided for by the general mine daily cycle of work) [11, 12].

5. Research results of power consumption modes for technological groups in underground mining and at coal enterprises in general

5. 1. Development of mathematical models for the process of active power consumption for the main technological groups of a coal enterprise

All technological consumers of electricity from the point of view of statistical characteristics can be divided into 2 groups:

– continuous consumption mode, which in turn can be divided into 2 types: with uniform (UD) and normal (ND) distribution;

– pulse distribution mode (ID).

UD group are vacuum and ventilation installations. The amount of power consumption is a randomly varying value within small limits, and distribution may be considered uniform [12]. Then, for the UD group the distribution function will obey the following law:

$$f_{UD}(x)=\begin{cases}0, \text{for } x<P_{\min},\\ 1/(P_{\max}-P_{\min}), \text{for } P_{\min}<x<P_{\max},\\ 0, \text{for } x>P_{\max},\end{cases} \quad (1)$$

where $P_{\max}$ – maximum power value; $P_{\min}$ – minimal power value.

The ND group is boiler rooms, surface transformers, underground load. The results of statistical data processing [13, 14] show that the power consumption mode of this group has a normal distribution law of load amplitude on the feeders supplying power receivers of this group:

$$f_{ND}(x)=\frac{1}{\sigma_{ND}\sqrt{2\pi}}e^{\frac{-(x-m_x)^2}{2\sigma_{ND}^2}}, \quad (2)$$

where δ_{ND} – mean-square deviation of power consumers of ND group; m_x – expected value.

The most important characteristic of power consumers of the ND group is the correlation function. This characteristic is necessary to determine the probability of load surges occurring. The correlation functions of power consumers of the ND group are described with sufficient accuracy by an exponential dependence such as [15]:

$$r(\tau)=e^{-\alpha^2\tau^2}\cos\beta\tau, \quad (3)$$

where α, β – correlation coefficients of load values; τ – correlation interval.

The ID group are power receivers operating in pulse mode (skip and cage hoisting installations), which use up to 20 % of all electricity consumed by a coal mine. Therefore, even a slight reduction in irrational energy consumption per ton-kilometer of lifted load will save tens and hundreds of thousands of kilowatt-hours per year at each installation.

Electrical consumption by skip and cage hoisting is an intermittent process. Periods of continuous operation of hoists (t_{ct} – cycle time) alternate with periods of downtime (t_{bc} – time between cycles) (Fig. 1), i. e. hoisting operates in "blocks" of cycles. The number of cycles in a "block" (t_w – block operating time) and the time between "blocks" (t_b – downtime) are determined largely at random (Fig. 2).

It is very difficult to obtain complete information about the process of electrical consumption in real conditions. According to the results of measurements (hourly load) the condition of hoisting unit (operates – does not operates) and power consumption are known. The task is to determine complete information about the power consumption of hoists based on the results of hourly measurements.

The process of hourly load measurements can also be represented as a pulse deterministic process according to Fig. 3.

The pulses height of a single consumption flow of skip and cage hoists is deterministic $\left(h_i \approx P_i^{SH,\,CH}\right)$, i. e. the pulses height is equal to the probability of operation of skip and cage units.

The duration of time between adjacent pulses of this process is 60 minutes, Fig. 3. The control measurement of consumed electricity actually occurs in a very short period of time (can be read instantly). However, for further operation with the generalized mathematical model, it is necessary to determine the maximum time t_3, during which, with a high probability, no significant change in the observed process occurs.

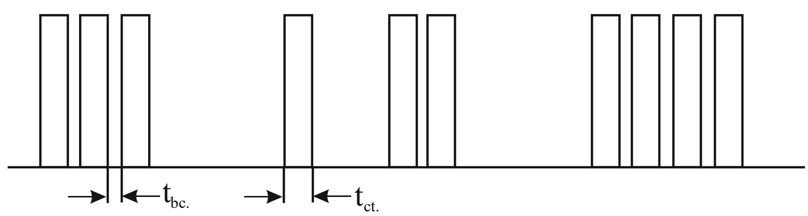

Fig. 1. Periods of continuous operation of hoists with downtime periods

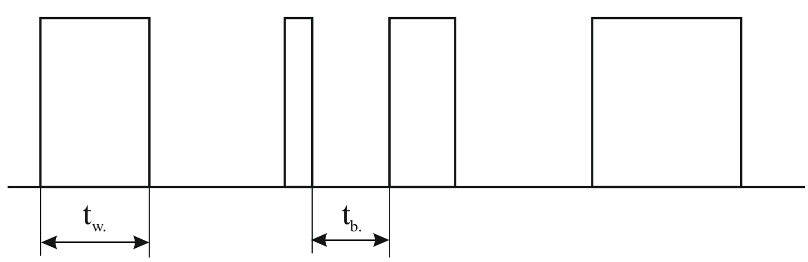

Fig. 2. Cycles operation in "blocks"

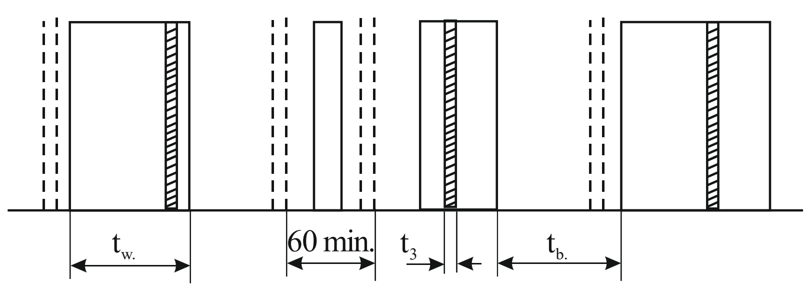

Fig. 3. Electrical load pulses of the hoist system

For processes of the UD and ND groups, this time is determined by the type of correlation function and, within the framework of the adopted model (the process under study is assumed to be constant between hourly measurements), is equal to at least 60 minutes.

For the ID group processes, a different qualitative and quantitative picture is observed. Here time $t_3$is determined by the probability of pulse-pause and pause-pulse transition. Let's determine time t_3 for pulse processes.

Let the beginning of measurement be the point t (Fig. 4), which randomly falls on a pulse process with the following characteristics:

$$f_1(t)=\frac{1}{\bar{t}_{w_1}}e^{-\frac{t}{\bar{t}_{w_1}}}, \tag{4}$$

$$g_1(t)=\frac{1}{\bar{t}_{b_1}}e^{-\frac{t}{\bar{t}_{b_1}}}, \tag{5}$$

where $f_1(t)$ and $g_1(t)$ – distribution laws of impulses and flow pauses, respectively; $\bar{t}_{w_1}$, $\bar{t}_{b_1}$ – average flow pulse and pause time.

For example, this point fell at the base of pulse. Then distribution of random values t_1 and t_2 is determined by a two-dimensional probability density [15, 16]:

$$F(t_1,t_2)=\frac{1}{\bar{t}_w}\alpha(t_1+t_2), \tag{6}$$

where $\alpha(t)$ – pulse duration lighting.

Distribution of random values $\alpha_1(t_1)$ and $\alpha_2(t_2)$ identical and found by calculating the integral [15]:

$$\alpha_1(t_1)=\alpha_2(t_2)=\int_0^{\infty}F(t_1,t_2)\mathrm{d}t_1=\frac{1}{\bar{t}_n}\int_{t_1}^{\infty}\alpha(t)\mathrm{d}t. \tag{7}$$

In the case of exponential distribution:

$$\alpha_1(t_1)=f_1(t). \tag{8}$$

Then the probability that the value of t_1 will be at least t_3, is equal to:

$$P=\frac{1}{\bar{t}_w}\int_{t_3}^{\infty}\mathrm{d}t_1*\int_{t_1}^{\infty}f_1(t)\mathrm{d}t=e^{-\frac{t_3}{\bar{t}_w}}. \tag{9}$$

With the value $P=0.9$ in (9) let's obtain $t_3\approx 0.1*\bar{t}_w$ (or for a pause $t_3\approx 0.1*\bar{t}_b$). According to data [15], the values $\bar{t}_w$ and $\bar{t}_b$ are on average 10 min, whence $t_3\approx 1$ min. Thus, let's obtain a maximum sample interval equal to one minute.

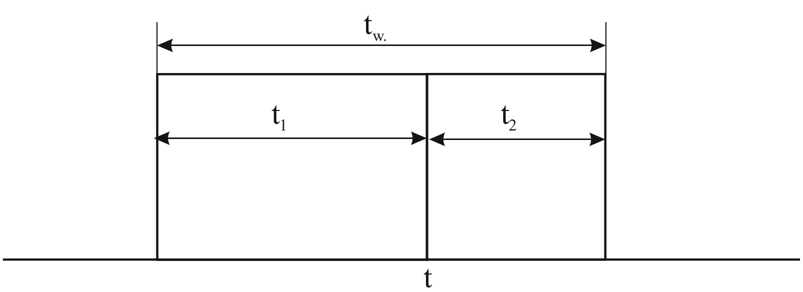

Fig. 4. Changing the electrical load of hoisting units: *a* – electrical load pulses of the hoist system; *b* – hoist operating cycle

The problem of finding characteristics of power consumption of hoisting installations is formalized as follows: according to information about coincidence of two pulse flows, $x_1(t)$ with characteristics (4) and (5), and the second $x_2(t)$ – deterministic pulse flow with characteristics $\bar{t}_{w_2}=t_3=1$ min, $\bar{t}_{b_2}=60-t_3=59$ min, $\bar{t}_{w_1}$, $\bar{t}_{b_1}$ need to be determined of the power flow.

Let's determine characteristics of the processes of coincidence of two independent flows $x_1(t)$ –power consumption flow and $x_2(t)$ – flow of measurements. Duration of pulses and pauses of the measurement flow is fixed and satisfies distributions according to the formulas (10)–(12):

$$f_2(t)=\delta\left(t-\bar{t}_{w_2}\right), \tag{10}$$

$$g_2(t)=\delta\left(t-\bar{t}_{b_2}\right), \tag{11}$$

where δ – delta function; $\bar{t}_{w_2}$, $\bar{t}_{b_2}$ – respectively, duration of pulse and pause of deterministic flow.

In this case, mathematical expectations of pulses and pauses duration of each flow are respectively equal to $\bar{t}_{w_1}$, $\bar{t}_{w_2}$, $\bar{t}_{b_1}$, $\bar{t}_{b_2}$. Pursuant thereto, the average pulse repetition rate of a stationary pulse flow is equal to:

$$\bar{\mu}_i=\frac{1}{\bar{t}_{w_1}+\bar{t}_{b_1}}. \tag{12}$$

Let's consider coincidence of two pulses a success, if their duration is overlapped at least partially. The pulse formed as a result of time overlap of both pulses will be called a coincidence pulse. Since this paragraph examines time characteristics of pulse processes, this allows to consider the shape of each pulse of the flows $x_1(t)$ and $x_2(t)$ to be rectangular, and the amplitude to be equal to one. If the flows are stationary, then at an arbitrary point of time t^* at $t\to\infty$ the equality $x\,(t^*=1)$ is satisfied with the probability [16]:

$$P_i=\bar{\mu}_i*\bar{t}_{w_i}. \tag{13}$$

Using the theory of impulse flows [15], let's obtain formulas for coincidence process of the load flow and measurement flows:

$$\mu_{22}(\delta)=\bar{\mu}_1*\bar{\mu}_2\left(\bar{t}*P_2-1\right), \tag{14}$$

$$P_{22}(\delta)=\bar{\mu}_1*\bar{\mu}_2*\bar{t}*P_1, \tag{15}$$

$$\alpha_{22}(\tau)=\frac{(\bar{t}*P_1+\bar{t}*P_2-1-\tau}{\left(\bar{t}*P_2-1\right)\bar{t}*P_1}e^{\frac{\tau}{\bar{t}*P_1}}, \tag{16}$$

where μ_{22} – pulse repetition rate; P_{22} – probability of coincidence of two pulses; α_{22} – probability density (distribution of pulse duration of the coincidence flow); $\bar{\mu}_1$, $\bar{\mu}_2$ – average pulse repetition rate of two flows; $\bar{t}$ – average measurement time; P_1, P_2 – occurrence probability of the first and, accordingly, the second pulse; t – correlation interval.

When determining the first moments (duration and probability) $\bar{t}*P_1$ and $\bar{t}*P_2$ some certain difficulties arise (only the complex $\bar{\mu}=1/\left(\bar{t}*P_1+\bar{t}*P_2\right)$ is determined), Therefore, in subsequent work with the generalized model, it seems possible to use the probable characteristics P_w of hoist operation and P_b – downtime probability.

Upon receiving information about $\bar{t}*P_1$ and $\bar{t}*P_2$ or the average number of skips in one pulse, it is possible to determine all the characteristics of power consumption processes by hoisting installations.

5. 2. Development of a generalized mathematical model of the electrical load of a coal enterprise

General picture of electrical consumption at mining plants consists of energy consumption by individual technological groups (UD, ND, and ID groups). In each load group, extensive (pulses time characteristics for the ID group) or intensive (amplitude values for the UD, ND groups) characteristics are random values with different distribution laws. Each group of power consumers satisfies the stationarity and ergodicity hypotheses.

There is arise the task to determine the probabilistic characteristics of the energy consumption process at mining plants obtained by summarizing the above-described components of its random processes [15].

Let's consider the physical picture of total load formation in a coal mine, shown in Fig. 5.

The loads of the UD and ND groups are continuous quantities with a random amplitude (Fig. 5). They form the main "constant" level of energy consumption of a coal mine. It can be called "permanent" conditionally, since the amplitude of this level is a random value. Next, the load from intermittent skips (i. e., probability of operation of skip (P_{SH1} and P_{SH2}) and cage (probability of operation of cage hoist P_{CH}) hoists is summed up to a constant level of energy consumption [17]. Therefore, above the constant level, a type of load surges is formed, Fig. 6.

The main task is to determine the probabilistic characteristics of both the main load level and the process of coincidence of the operating moments of skip cage hoists.

Determining the nature of change in the load of power consumers of the ID group is a task for summing up several independent pulse processes of electrical consumption. To solve it, there is necessary to perform a sum operation:

$$Y(t)=\sum_{i=1}^{n}x_1(t), \qquad (17)$$

where $Y(t)$ – total load of power consumers of the ID group; $x_i(t)$– summarized loads of various processes.

Simultaneous coincidence of k pulses from n sequences, calculated under the condition that their coincidence will last at least t, is determined by the correlation [15]:

$$P_{(n,k)}(\tau)=\frac{1}{k!}*\frac{d^k}{d\alpha^k}*\prod_{i=1}^{n}\left[P_{b_i}(\tau)+\alpha P_{w_i}(\tau)\right]_{\alpha=0}, \qquad (18)$$

where $P_{w_i}(\tau)$ – probability of installation operation; $P_{b_i}(\tau)$ – probability of installation downtime (pause); τ – pulse coincidence interval duration.

$P_{w_i}(\tau)$ and $P_{b_i}(\tau)$ are determined accordingly by the formulas (19) and (20):

$$P_{b_i}(\delta)=\bar{\mu}\int_0^{\delta}dx\int_x^{\infty}g_i(t)dt, \qquad (19)$$

$$P_{w_i}(\delta)=\bar{\mu}\int_0^{\delta}dx\int_x^{\infty}f_i(t)dt, \qquad (20)$$

where $\bar{\mu}$ – average pulse repetition rate of stationary flow; $g_i(t)$ – flow pause distribution law; $f_i(t)$ – pulse distribution law.

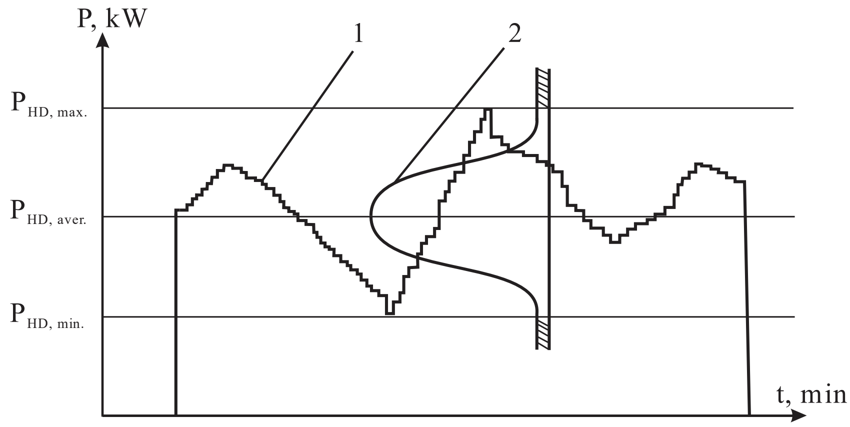

Fig. 5. Total load of coal mine of power consumers of the ND group: 1 – actual electricity consumption; 2 – probabilistic model

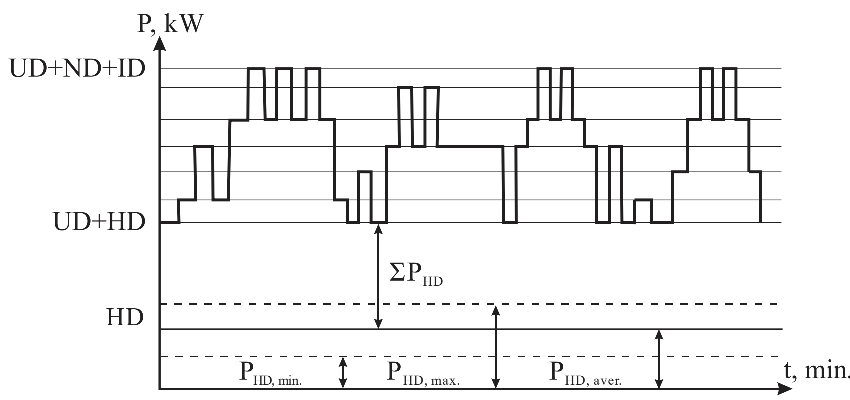

Fig. 6. Total load of coal mine

In addition to the probabilistic characteristics of the coincidence flow, important indicators are the average frequency $\mu_{n,k}(\tau)$ of k coincidence from n sequences, calculated under the assumption that the duration of their coincidence is not less than tand distribution of pulse duration of the coincidence flow $f_{n,k}$(t):

$$\bar{\mu}_{n,k}(\tau)=\frac{1}{k!}\frac{d^{k+1}}{d\alpha^k d_\tau}\prod_{i=1}^{n}\left[P_{b_i}(\tau)+\alpha P_{w_i}(\tau)\right]_{\alpha=0}, \qquad (21)$$

$$\bar{f}_{n,k}(t)=\frac{1}{\bar{\mu}_{n,k}(0)}\frac{d^2}{dt^2}P_{n,k}(t). \qquad (22)$$

And then let's obtain the ID for the energy consumers of the group:

$$P_{n,k}(\tau)=\sum_{\substack{e,m=1\\e+m=n}}^{n}\prod_{i=e}^{e+k}\bar{\mu}_i\bar{t}_{w_i}\left(1-e^{-\frac{\tau}{\bar{t}_{w_i}}}\right)\prod_{j=m}^{n-k}\bar{\mu}_j\bar{t}_{b_j}\left(1-e^{-\frac{\tau}{\bar{t}_{b_i}}}\right), \quad (23)$$

$$\mu_{n,k}(\tau)=-\frac{d}{d_\tau}P_{n,k}(\tau), \quad (24)$$

$$f_{n,k}(\tau)=\frac{\frac{d^2}{dt^2}P_{n,k}(t)}{\sum_{\substack{e,m=1\\e+m=n}}^{n}\prod_{i=e}^{e+k}\bar{\mu}_i\bar{t}_{w_i}\prod_{j=m}^{n-k}\bar{\mu}_j\bar{t}_{b_j}}. \quad (25)$$

Thus, formulas (23)–(25) represent the final probabilistic characteristics of the electrical load of energy consumers of the ID group.

5. 3. Development of a mathematical model for the daily electricity consumption of coal enterprises

To define the daily electrical consumption, let's use the load mode models of the above mentioned mine power consumers. If to represent an hourly load of an individual power consumer as a random function of time $x(t)$ then hourly electrical consumption shall be determined as follows:

$$Y(t)=U*\eta*x(t), \quad (26)$$

where U – voltage level, V; η – standard coefficient (dimensionless value, including the cosφ values, performance factor and constant coefficients).

Daily electrical consumption $W_{day.}$ can be represented as the sum of hourly consumption during a day:

$$W_{day.}=\sum_{t=1}^{24}Y(t). \quad (27)$$

Determination of daily electrical consumption of the ND group consumers appears as summing up the sums of normal values. Daily electrical consumption by energy receivers from the ND group is a random value, which is fully characterized by a mathematical expectation and a correlation function. According to the addition theorem [16], the mathematical expectation of daily electrical consumption by energy receivers from the ND group is as follows:

$$m_{DAY.ND}=24*U*m_{ND_i}, \quad (28)$$

where m_{ND_i} – load value of the i-th power consumer of the ND group. The correlation function [16] of daily electrical consumption can be found as:

$$K_{DAY.ND}(\tau)=24K_{ND_i}(\tau)+ \\ +2\sum_{k=0}^{23}(24-k)K_{ND_i}(\tau+k), \quad (29)$$

where $K_{ND_i}(\tau)$ – autocorrelation function of hourly load values for the ND group.

For the dispersion of daily flow let's obtain:

$$D_{DAY.ND}= \\ =24D_{ND_i}+2\sum_{k=0}^{23}(24-k)K_{ND_i}(k), \quad (30)$$

where D_{ND_i} – dispersion for groups with normal distribution; $K_{ND_i}(k)$ – autocorrelation function of daily load values for the ND group. Daily consumption of power consumers from the P group is determined by summing up the sums of uniformly distributed values. This operation results in a value with a normal distribution law.

Mathematical expectation and dispersion [16] of daily energy consumption process of power receivers form the P group are determined by the formulas:

$$m_{DAY.UD}=24*U*m_{UD_i}, \quad (31)$$

where m_{UD_i} – load value of the i-th power consumer from the UD group:

$$D_{DAY.UD}=24D_{UD_i}+2\sum_{k=0}^{23}(24-k)K_{UD_i}(k), \quad (32)$$

where D_{UD_i} – dispersion for the UD group; $K_{UD_i}(k)$ – autocorrelation function of daily load values for the UD group.

Due to independence of summarized processes, the characteristics of the distribution law of general mine electrical consumption are determined from the formulas:

$$W_\Sigma=\sum_{i=1}^{n}W_i, \quad (33)$$

$$D_{\Sigma_i}=\sum_{i=1}^{n}D_i, \quad (34)$$

where W_i and $D_{\Sigma i}$ – respectively, the mathematical expectation and dispersion of the i-th group of power consumers.

Since the number of electrical consumption pulses during a day is also a random value, it is possible to come to a model for summing a random number of pulses having a random amplitude. In this case, the daily total electrical consumption by skip and cage hoists is a random value with a normal distribution law. In the absence of a priori information about the duration of current load pulses, the characteristics of the distribution laws of the total amount of electrical consumption by energy receivers from the ID group are determined based on statistical information processing data.

5. 4. The results of the experiment. Construction of daily load curves and histograms of the electrical load distribution for various technological groups and the entire enterprise as a whole

Table 2 shows daily values of electric load by groups of main consumers of the coal mine of the Kostenko coal mine, and Fig. 7 shows their daily schedules.

Table 2

Daily values of electric load of the main consumers of the coal mine of the Kostenko coal processing plant

Energy consumers	Electrical load, kW											
MVF	1020	1000	1160	1230	1160	1200	1000	1140	1140	1140	1300	1160
ST	360	480	520	550	480	670	750	720	500	580	520	550
BR	40	80	30	78	100	90	90	120	130	128	180	120
UL	1000	1600	420	1100	1700	2000	3500	3000	3500	3000	2700	1600
TL	2420	3160	2130	2958	3440	3960	5340	4980	5270	4848	4700	3430
t, h	0	2	4	6	8	10	12	14	16	18	20	22

Note: BR – boiler room; ST – surface transformers; UL – underground load; MVF – main ventilation fans; TL – total load.

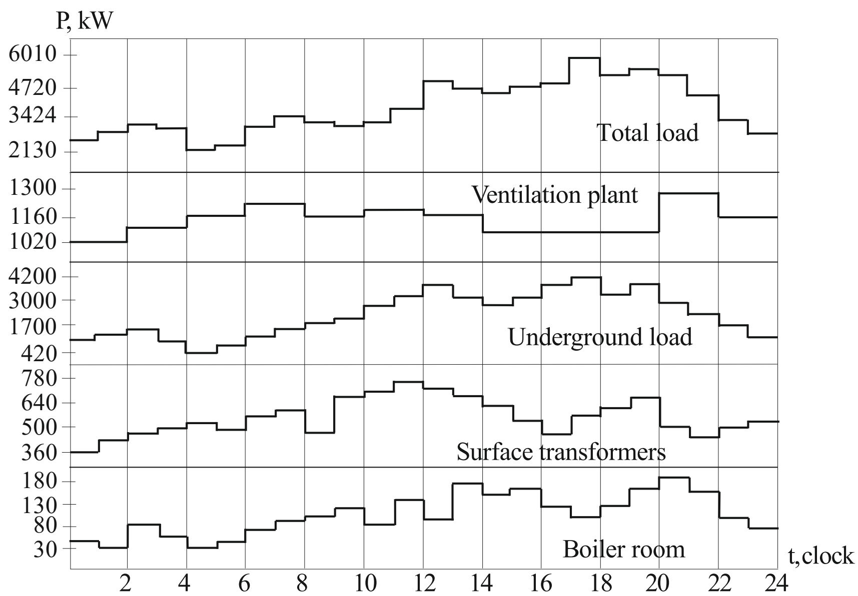

Fig. 7. Daily power consumption schedules of the main consumers of the Kostenko coal mine

The obtained histograms of the distribution of a random value of active electrical power for the main technological processes of the coal mine named after Kostenko is shown in Fig. 8–12.

The obtained results of the experiment and calculation of the values of m_x, D_x, δ, K_v and Pearson's criterion of agreement (λ^2) of the empirical distribution with the normal law of distribution of electric load values are presented in Table 3.

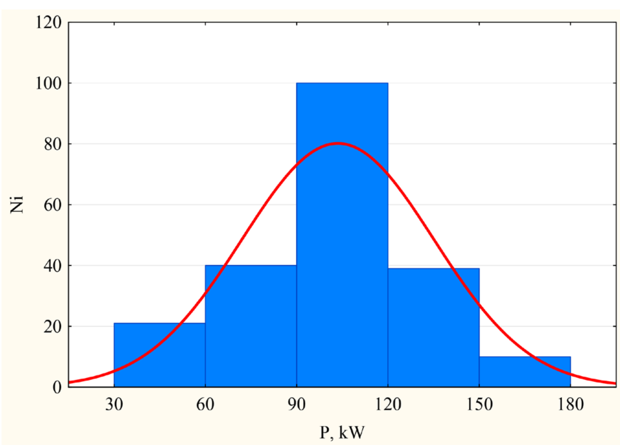

Fig. 8. Histogram of electric boiler load distribution

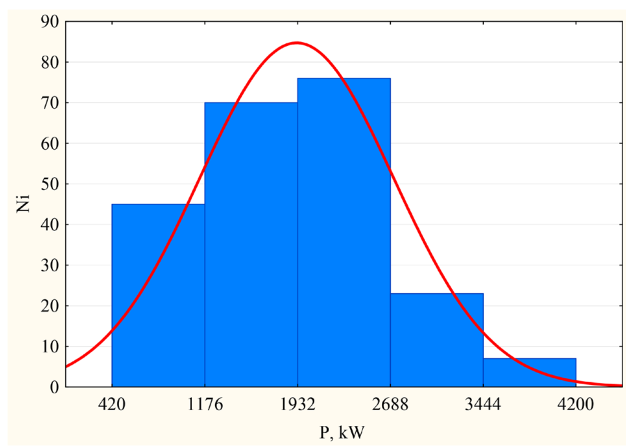

Fig. 10. Histogram of the distribution of electrical underground load

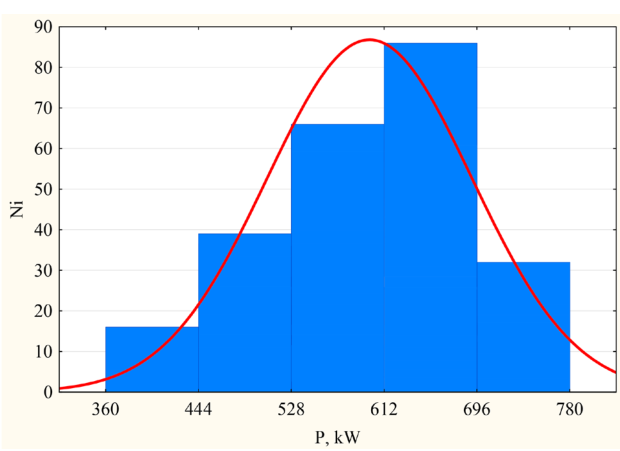

Fig. 9. Histograms of electrical load distribution surface transformers

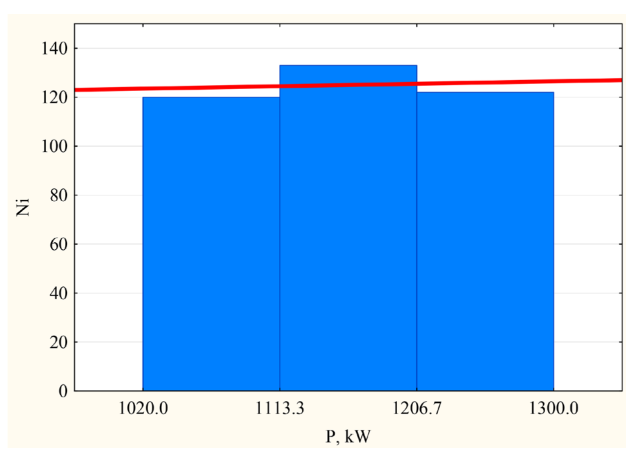

Fig. 11. Histogram of the electrical load distribution of the main ventilation unit

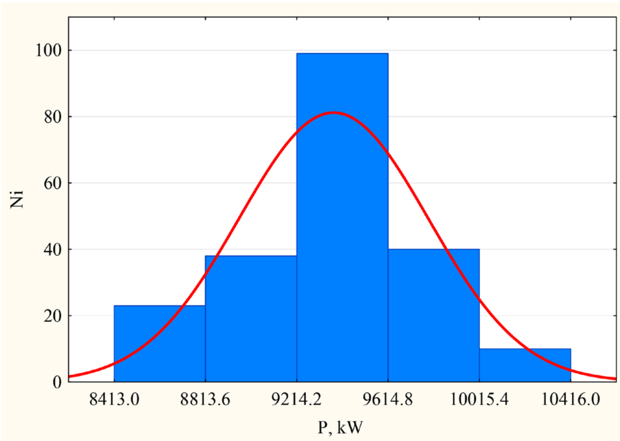

Fig. 12. Histogram of the distribution of the total electrical load of the mine

Table 3

Statistical characteristics of the active load of the mine's energy consumers

Energy consumers	Statistical characteristics			
	m_x, kW	δ_x, kW	K_v	λ^2
Main ventilation fans	1,100	48.1	0.045	18.1
Surface transformers	583	73.8	0.126	17.7
Boiler room	112	20.4	0.182	58.7
Underground load	1,986	654	0.331	16.0

The general form of histograms and statistical characteristics of the active load of mine energy consumers allows to classify boiler plants, surface transformers and underground load as normal distribution law, and main fans as uniform distribution law. Processing of values of skip load and cage hoists was carried out separately due to the pulsed nature of the energy consumption mode by hoisting units.

6. Discussion of the results of developing the mathematical models of electricity consumption of coal enterprises

The work revealed that dozens or even hundreds of individual consumers participate in the process of power consumption of coal enterprises. The main energy-intensive consumers have been identified and classified into groups of technological processes, the power consumption indicators of which are given in Table 1.

Fig. 8–10 show that all technological groups of electric consumers in terms of statistical characteristics can be divided into 2 groups: continuous consumption mode (with normal and uniform distribution) and pulse consumption mode (skip and cage lifting installations). The results of data processing (Table 3) show that for a group with a uniform distribution, the statistical characteristics of power consumption modes are determined by formula (1), for groups with a normal distribution (underground transformers, boiler plants) – by formula (2), for groups with pulse consumption – by formulas (4), (5). Fig. 3, 4 show that the consumption of electrical energy by cage and skip lifting installations is an intermittent process: periods of operation alternate with periods of downtime, which are formed randomly. Formulas (14)–(16) define the process of energy consumption by lifting installations. The total electrical load of a coal mine consists of electricity consumption for individual technological groups, shown in Fig. 6. The loads of groups with uniform and normal distributions are continuous and form the main level of power consumption, after which the loads of lifting installations are summed up to it. The final probable characteristics for the pulse type group are expressed in formulas (23)–(25). Mathematical models of the modes of daily electricity consumption have been developed: for groups with a normal distribution law, they are determined by formulas (28)–(30), for groups with a normal law – by formulas (31), (32). Formulas (33), (34) determine the total energy consumption of the mine as a whole.

In the article [18], the authors analyze the modes of electricity consumption by mining excavators, the data obtained from which can be used to optimize their operation. The research presented in the article [5, 19] is aimed at solving problems of improving the energy efficiency of mine ventilation systems. The authors of the work [20] consider the features of the power consumption of conveyor systems. Unlike these works, which study certain types of mining equipment, our research covers the entire technological process of underground coal mining. The statistical modeling method were used to carry out a comprehensive analysis.

A feature of this study is the possibility of applying the developed mathematical models of power consumption modes for any coal enterprises.

The limitation of this study is that the study was conducted only at the level of technological groups, without detailing to individual mechanisms.

The disadvantage of this study is that it studied only consumers who were connected to outgoing feeders with a voltage of 6 kV of the supply centers and equipped with metering devices for active and reactive electricity.

For the perspective of the development of this study, it is proposed to install a technical control and accounting system for each outgoing feeder, which will allow a detailed analysis of energy consumption for each electric drive of technological installations. The proposed system will allow creating consumption models for each technological process and optimize energy consumption.

For the perspective of the development of this study, it is proposed to install a technical control and accounting system for each outgoing feeder, which will allow a detailed analysis of energy consumption inside the mine. The proposed system will allow creating consumption models for each technological process and optimize energy consumption.

7. Conclusions

1. Mathematical models for the process of active power consumption for the main technological groups have been developed. For efficient management of power consumption modes in coal enterprises, energy-intensive consumers were classified by types of mining operations into the following technological groups: ventilation installations, surface transformers, underground load, boiler plants, and hoisting installations.

2. A generalized mathematical model of the electrical load of coal enterprises has been developed, based on the summation of different types of technological consumer groups. In this case, pulse power consumption (hoisting installations) generates load surges.

3. A mathematical model of the daily electricity consumption of coal enterprises has been developed, based on the application of probability theory and pulse flows.

4. The analysis of the histograms of electrical load distribution showed that the power consumption modes of various technological groups in coal enterprises can be described by three types of distributions: uniform, normal, and pulse. For hoisting installations, the distributions are characterized by pronounced peaks (pulse), indicating the presence of periodic or random short-term loads. The data analysis showed that the power consumption of surface equipment transformers at the Kostenko mine varies from 360 to 780 kW. The underground load is characterized by a wider range, from 420 to 4200 kW. Boiler plants consume from 30 to 180 kW, and the main ventilation fans consume from 780 to 1200 kW.

Conflict of interest

The authors declare that they have no conflict of interest in relation to this research, whether financial, personal, authorship or otherwise, that could affect the research and its results presented in this paper.

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Data availability

Data will be made available on reasonable request.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

References

1. Reymov, K. M., Turmanova, G. M., Makhmuthonov, S. K., Uzakov, B. A. (2020). Mathematical models and algorithms of optimal load management of electrical consumers. E3S Web of Conferences, 216, 01166. https://doi.org/10.1051/e3sconf/202021601166
2. Petrov, V., Sadridinov, A., Pichuev, A. (2020). Analysis and Modeling of Power Consumption Modes of Tunnelling Complexes in Coal Mines. E3S Web of Conferences, 174, 01006. https://doi.org/10.1051/e3sconf/202017401006
3. Homayouni, S. M., Fontes, D. B. M. M. (2023). Optimizing job shop scheduling with speed-adjustable machines and peak power constraints: A mathematical model and heuristic solutions. International Transactions in Operational Research, 32 (1), 194–220. https://doi.org/10.1111/itor.13414
4. Wang, L., Wei, P., Li, W., Du, L. (2024). Modelling and optimization method for energy saving of computer numerical control machine tools under operating condition. Energy, 306, 132556. https://doi.org/10.1016/j.energy.2024.132556
5. Wen, L., Zhong, D., Bi, L., Wang, L., Liu, Y. (2024). Optimization Method of Mine Ventilation Network Regulation Based on Mixed-Integer Nonlinear Programming. Mathematics, 12 (17), 2632. https://doi.org/10.3390/math12172632
6. Świętochowski, K., Andraka, D., Kalenik, M., Gwoździej-Mazur, J. (2024). The Hourly Peak Coefficient of Single-Family and Multi-Family Buildings in Poland: Support for the Selection of Water Meters and the Construction of a Water Distribution System Model. Water, 16 (8), 1077. https://doi.org/10.3390/w16081077
7. Reshetnyak, S., Golubov, E. (2024). Methods of power consumption in conditions of high-productive areas of coal mines. BIO Web of Conferences, 84, 05008. https://doi.org/10.1051/bioconf/20248405008
8. Fumo, N., Rafe Biswas, M. A. (2015). Regression analysis for prediction of residential energy consumption. Renewable and Sustainable Energy Reviews, 47, 332–343. https://doi.org/10.1016/j.rser.2015.03.035
9. Majola, C. M. D., Langerman, K. E. (2023). Energy efficiency in the South African mining sector: A case study at a coal mine in Mpumalanga. Journal of the Southern African Institute of Mining and Metallurgy, 123 (9), 451–462. https://doi.org/10.17159/2411-9717/1788/2023
10. Kubrin, S., Reshetnyak, S., Zakorshmenny, I., Karpenko, S. (2022). Simulation modeling of equipment operating modes of complex mechanized coal mine face. Sustainable Development of Mountain Territories, 14 (2), 286–294. https://doi.org/10.21177/1998-4502-2022-14-2-286-294
11. Nurmaganbetova, G., Issenov, S., Kaverin, V., Issenov, Z. (2023). Development of a virtual hardware temperature observer for frequency-controlled asynchronous electric motors. Eastern-European Journal of Enterprise Technologies, 3 (1 (123)), 68–75. https://doi.org/10.15587/1729-4061.2023.280357
12. Avdeev, L. A. (2018). Energy-saving technologies in coal mines. Karaganda: KarSTU Publishing House, 159.
13. Avdeyev, L. A., Kokin, S. E., Telbayeva, S. Z. (2024). Research of electric load schedules in the conditions of the Karaganda coal basin. Bulletin of Toraighyrov University. Energetics Series, 1, 6–21. https://doi.org/10.48081/rzhd2721
14. Avdeyev, L., Kaverin, V., Sychev, Y., Telbayeva, S. (2024). Research of the Power Consumption System Coal Mine. Trudy Universiteta, 1. https://doi.org/10.52209/1609-1825_2024_1_445
15. Enatskaya, N. Y., Hakimullin, E. R. (2023). Probability theory and mathematical statistics for engineering and technical areas. Moscow: YURAYT Publishing House, 399.

16. Enatskaya, N. Y. (2024). Mathematical statistics and random processes. Moscow: YURAYT Publishing House, 201.
17. Druzhinin, V., Sivyakova, G., Kalinin, A., Tytiuk, V., Nikolenko, A., Kuznetsov, V., Kuzmenko, M. (2022). Preventing the development of emergency modes of interlocked electric drives of a rolling mill under the impact loads. Diagnostyka, 24 (1), 1–13. https://doi.org/10.29354/diag/157089
18. Manusov, V. Z., Orlov, D. V., Sultonov, S., Ahyoev, J., Bumtsend, U. (2022). Analysis of electricity consumption of electrical machines of a coal industry enterprise using the wavelet transform. IOP Conference Series: Earth and Environmental Science, 1070 (1), 012002. https://doi.org/10.1088/1755-1315/1070/1/012002
19. Issenov, S. S., Issenov, Zh. S., Nurlan, N. N., Mendybaev, S. A. (2017). Development of algorithm flow graph, mealy automaton graph and mathematical models of microprogram control mealy automaton for microprocessor control device. 2017 International Siberian Conference on Control and Communications (SIBCON), 12, 1–6. https://doi.org/10.1109/sibcon.2017.7998502
20. Ji, J., Miao, C., Li, X. (2020). Research on the energy-saving control strategy of a belt conveyor with variable belt speed based on the material flow rate. PLOS ONE, 15 (1), e0227992. https://doi.org/10.1371/journal.pone.0227992