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DEVISING A METHOD FOR DETERMINING THE MOISTURE CONDUCTIVITY COEFFICIENT OF SUBGRADE SOILS TAKING INTO ACCOUNT EUROPEAN APPROACHES AND STANDARDS

Andrii Bubela
Doctor of Technical Sciences, Professor, Dean*
Liudmyla Bondarenko
PhD, Associate Professor*
Yevheniia Kvatadze
Corresponding author
PhD Student*
E-mail: kvatadzeev@gmail.com
Vitalii Stozhka
PhD, Associate Professor*
Andrii Ivko
PhD, Associate Professor
Department of Project Management
Kyiv National University of Construction and Architecture
Povitrianykh Syl ave., 31, Kyiv, Ukraine, 03037
*Department of Transport Construction and Property Management
National Transport University
Omelyanovicha-Pavlenka str., 1, Kyiv, Ukraine, 01010

The object of this study is the theoretical and methodological approaches to determining the coefficient of moisture conductivity of subgrade soils. The work focuses on considering the European approaches and standards when devising a method for determining the coefficient of moisture conductivity of soils.

In the course of the research, a method was developed for determining the coefficient of moisture conductivity of soils K_1, which characterizes the diffusion movement of water, through the filtration coefficient K_0, calculated in accordance with European requirements based on laboratory test data.

The proposed method is based on a mathematical model built on the basis of the differential equation of changes in soil moisture. The model is special in that, unlike existing ones, the movement of water was modeled from the bottom up, which reflects the process of moisture accumulation in the lower layers of the subgrade from groundwater or topwater.

A good agreement of the mathematical model with the data by other authors was obtained (the relative error did not exceed 12.98 %).

A direct relationship between the moisture conductivity coefficient of soils K_1 and their initial moisture content W_0 and an inverse relationship between K_1 and the total moisture capacity of soils W_{FH} were established in the paper. Dependences were derived in the range of changes in initial soil moisture W_0 from 0.08 to 0.15 and W_{FH} from 0.15 to 0.5. It was found that the values of the moisture conductivity coefficient of soils K_1 increase from $4.64 \cdot 10^{-6}$ to $3.81 \cdot 10^{-5}$ m^2/h with an increase in their initial moisture content and with a decrease in total moisture capacity.

From the point of view of engineering practice of road construction, the proposed method makes it possible to predict seasonal changes in soil moisture in the subgrade, to determine the strength of the road structure. This makes it possible to make sound design decisions on the installation of drainage systems on roads in order to extend their service life

Keywords: soil moisture, coefficient of soil moisture conductivity, subgrade, road structure

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1. Introduction

One of the most effective ways to reduce the cost of road construction is the intensive use of local soils for the arrangement of subgrade. The importance of this task is due to the lack of a detailed theoretical analysis of water-thermal processes occurring in the body of the earth bed, as well as due to issues related to the strength and stability of the corresponding road structures.

The question of the maximum permissible moisture content of local soils when they are arranged in an embankment is of great importance since the possibility of mechanization of construction works, the compactness of the embankment, and the duration of preliminary drainage will depend on this moisture content.

Thus, the use of local soils for the arrangement of road subgrades is impossible without a detailed study of their properties, in particular moisture conductivity, for the further development of measures to regulate the humidity of these soils.

The relevance of related research is based on determining the physical and mechanical characteristics of soils in accordance with European requirements and standards, which differ significantly from the outdated methods used in Ukraine. This requires the establishment of a relationship between the indicators, in particular the coefficient of moisture conductivity, determined according to the current DSTU, and the European ISO.

2. Literature review and problem statement

Moisture is one of the most important factors affecting soil strength. Paper [1] examines the influence of moisture and density on the shear strength parameters of silty embankment soil in dry, wet (partially saturated) and saturated conditions for slope stability analysis. Wide distribution of silty soil in Ethiopia poses challenges in many construction fields such as earthworks, stability of slopes and foundations, in understanding their engineering characteristics, especially behavioral changes under dry and saturated conditions. As a result, the change in density and humidity significantly affects the properties of silty soil. The soil sorting proposed in work [1] is a technique for modifying its physical and mechanical properties. But in order to improve the stability of the slope of the embankment, reduce its height, and prevent landslides, the need to predict the moisture regime of the road structure based on such an indicator as the moisture conductivity coefficient of the soil remains an open question. The definition of this indicator for the purpose of predicting soil moisture is not considered by the authors.

Work [2] reports the results of research on soil moisture and soil strength to control the quality of construction of earthen structures, as well as the relationship between its structural properties. Research was carried out by means of experimental tests at the construction site. An acceptable level of soil compaction is achieved using a dynamic cone penetrometer in combination with density and moisture measurements. This improves the quality control of works but does not fully make it possible to determine the design parameter taking into account the moisture conductivity of the soil during the construction of earthen structures. This would reduce the quality of the next forecast regarding changes in their strength characteristics during the annual cycle due to the influence of soil moisture.

The soil moisture level of the subgrade during the compaction process plays a decisive role in ensuring the durability of the entire road structure [3]. Modern methods involve manual sampling of small soil samples from several isolated locations. This makes it difficult to monitor the spatial fluctuations of the moisture content along the entire construction section of the road during its compaction. Paper [3] demonstrated the effectiveness of using a passive L-band microwave radiometer to measure soil moisture in the context of optimal compaction of road construction materials. But it is not enough to find a simple and effective approach to measuring soil moisture since its change is determined by such an indicator as the coefficient of moisture conductivity of the soil, the methods for determining which require clarification.

The high level of groundwater and the humidity of road construction materials significantly reduce the service life of the road surface. In [4], two types of non-intrusive devices were tested for their potential to detect moisture in road structures: a passive microwave radiometer and a ground-penetrating radar. The measurements showed that the scanner detects moisture fluctuations in the upper layers of the road structure (0.4 m), and the GPR clearly detects the level of groundwater at a greater depth. This confirms the effectiveness of both tools in practice but does not completely solve the problem of determining the change in soil moisture to prevent unwanted moisture accumulation in the subgrade body. The change in soil moisture is determined by the coefficient of moisture conductivity, which is not determined.

Paper [5] also considered the main problems of highway construction in the seasonally frozen regions of northeastern China. Using the method of combined field and laboratory tests, the influence of the plant system on the temperature and humidity of the soil massif, especially the root soil system during the freezing process, was analyzed. Studies have shown that soil with plants contained less moisture and froze more slowly. This confirms the effectiveness of plants in improving moist and thermal conditions on slopes, which can reduce the destruction of embankment slopes and trenches during freezing and thawing. However, the reported methods do not fully make it possible to analyze the process of moisture migration during freezing and thawing of the subgrade soil in the absence of determination of the moisture conductivity coefficient.

Maintenance and repair of roads caused by damage is costly to both the economy and the environment [6]. Subgrade soils are subject to large stress fluctuations, deformations due to the influence of rolling stock and natural conditions. In addition, the constant change in soil moisture causes a decrease in the strength and stability of the ground surface, which leads to damage to the road structure. Engineered water repellent technology to control soil moisture can extend road life, reduce recurring maintenance costs, and reduce CO_2 emissions. But the proposed approach is not justified by the term of possible maintenance of the same conditions of soil moisture and the coefficient of its moisture conductivity during the annual cycle.

In work [7], an analysis of existing approaches and methods for determining the coefficient of moisture conductivity of soils for the calculation of seasonal moisture accumulation in the subsoil was carried out. It is shown that the calculated values and measurement units of soil moisture conductivity coefficients, obtained within the requirements of the European standard ISO 17892-11:2019 (E) [8], do not correspond to those provided by existing calculation methods and models [9]. However, the problem of the analytical relationship between the considered parameters of the subsurface soil remained unsolved.

This predetermines the relevant task to harmonize different approaches to determining the coefficient of moisture conductivity of soils and adapt existing methods for forecasting moisture accumulation in road structures to European construction standards.

3. The aim and objectives of the study

The purpose of our work is to devise methods for determining the coefficient of moisture conductivity of subgrade soils in order to adapt it to European construction standards. This will make it possible to move away from outdated methods and solve a number of important issues related to increasing the stability of road structures and extending the term of their operation under modern climatic and ecological conditions.

To achieve the goal, the following tasks were solved:

– to perform experimental studies of soil moisture conductivity processes on a measuring bench, in accordance with European approaches and standards;

– on the basis of mathematical modeling, to determine the analytical relationship between the coefficient of moisture conductivity of soils K_1 (m^2/s) and the coefficient of moisture conductivity of soils K_0 (m/s), obtained from the data of laboratory tests of soils;

– to conduct a numerical experiment to study the influence of initial soil moisture and its total moisture content on the values of moisture conductivity coefficients;

– to compare the values of moisture conductivity coefficients of sandy soil K_1 (m^2/s), obtained on the basis of the proposed mathematical model, and on the basis of the results of research by other authors.

4. The study materials and methods

The object of research in this paper is the theoretical and methodological approaches to determining the coefficient of moisture conductivity of subsurface soils.

The subject of the study is a method for determining the coefficient of moisture conductivity of soils, taking into account European approaches and standards.

The research is based on the hypothesis that there is a functional relationship between the coefficient of moisture conductivity of soils K_1 (m^2/s) and the coefficient of moisture conductivity of soils K_0 (m/s), which can be described by a certain mathematical model. The coefficient K_1 (m^2/s) characterizes the diffusion transfer of water movement, and the coefficient K_0 (m/s) is determined according to laboratory test data within the requirements of the European standard ISO 17892-11:2019 (E) [8].

At the initial stage of the work, experimental studies of soil moisture conductivity were conducted to determine the coefficient of moisture conductivity K_0 (m/s). Research was conducted at the educational-scientific laboratory of the Department of Transport Construction and Property Management, the National Transport University (Kyiv, Ukraine), on a measuring bench in accordance with the requirements of the European standard ISO 17892-11:2019 (E) [8]. The setup of laboratory equipment for testing soil samples is shown in Fig. 1.

The device was prepared for operation in the following sequence (Fig. 1):

1) with the help of screws (7), mold (1) was separated into two parts: upper (10) and lower (11);

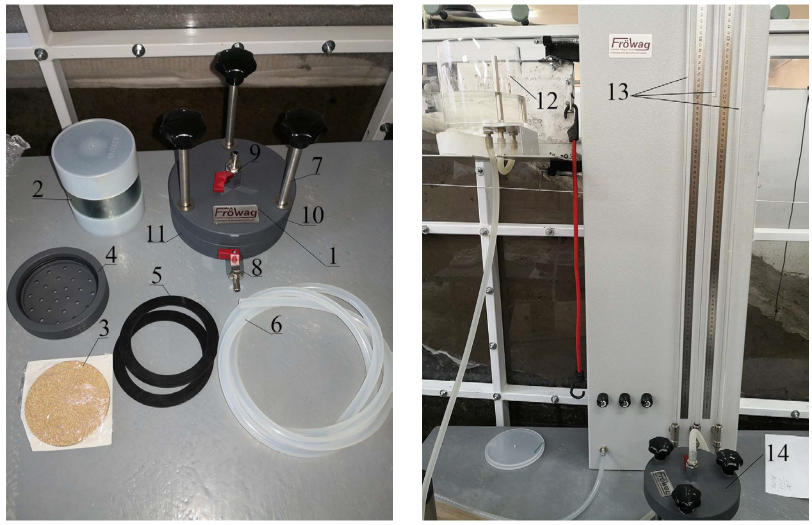
2) drain valve (9) and supply valve (8) were set to the "closed" position;

3) mold (1) was filled with filters (3): for the lower part, filter (3) was arranged so that the inner surface of the mold was flat; for the upper one, filter (3) was arranged in ring (4) so that the soil did not leave the cylinder during the experiment;

4) sealing rubber rings (5) were arranged between parts (10) and (11) of mold (1) to ensure tight contact between the touching surfaces;

5) the arrangement of cylinder (2) in mold (1) took place in the following sequence: the lower part of mold (11) with filter (3) and sealing ring (5), cylinder (2), ring (4) with filter (3), sealing ring (5), upper part of mold (10);

6) then mold (1) was tightened with bolts (7) to achieve tightness.



a *b*

Fig. 1. Setup for soil testing in a cylindrical permeameter with rigid walls: *a* — permeameter in disassembled form; *b* — measuring bench for determining moisture conductivity of soils; 1 — mold; 2 — cylinder; 3 — filters — porous disks; 4 — transitional ring; 5 — sealing rings; 6 — inlet tube; 7 — connecting bolts; 8 — water inlet valve; 9 — water outlet valve; 10 — the upper part of the mold; 11 — the lower part of the mold; 12 — pressure tank with water; 13 — piezometric tubes with a diameter of 4, 6, 8 mm; 14 — permeameter assembly

During the experiment, the movement of water took place from the bottom up, which reflected the process of moisture accumulation in the lower layers of the subsoil.

The bench made it possible to create water pressure with a variable hydraulic gradient due to the use of three hydraulic tubes of different diameters. Also, the water tank could be used to create pressure with a constant hydraulic gradient.

During the constant pressure test, the water level at the permeameter inlet and outlet was kept constant. In the falling pressure test, the inlet water level dropped through the inlet burette as the water passed through the sample. This caused a change in the hydraulic gradient during the test.

The minimum amount of water flow, or the change in its level during each measurement was determined taking into account the accuracy of the measuring devices. At the same time, the measurement period was assumed to be long enough, and the hydraulic gradient was high enough to ensure the reliability and representativeness of the results.

Determination of the coefficient of moisture conductivity of the soil was carried out in the following sequence:

1. For testing, a soil sample was taken and weighed (Fig. 2, *a*).

The soil sample was taken using a soil-carrying cylinder (hereinafter the cylinder) with a compaction factor of 0.95.

2. The type of soil and its moisture content were determined (Fig. 3).

3. The cylinder was fitted to the mold to determine the moisture conductivity coefficient of the soil (Fig. 1).

4. The assembled mold was connected to piezometric tubes and a feeding tank with water (Fig. 4).

5. The feeding tank was filled with water and, after stabilizing the water level in it, its initial level was determined using a water measuring scale.

6. Water supply was ensured due to the fact that supply valve (8) was in the "open" position.

7. On the device, where the dosing valves and piezometric tubes are located, the necessary valve was brought to the "open" position.

After that, the air outlet was provided due to the fact that drain valve (9) was brought to the "open" position and the timer started.

a *b*

Fig. 2. Weighing the soil sample before and after water saturation: *a* – before water saturation; *b* – after water saturation

Fig. 3. Determination of soil type and moisture content

Fig. 4. Water supply connection diagram

When any amount of water was absorbed, its level in the feed tank decreased, which was immediately compensated by the arrival of the same amount of water. Thus, the water level in the feed tank was maintained at a constant level. When the wetting front approached the upper end of the sample, the experiment was stopped, after which the final water level H_f in the feed tank was marked on the water measuring scale. The amount of water that was absorbed into the soil sample of water q was determined by the difference between the H_f and H_s indicators. The sample moistening time t was determined according to the timer readings and was automatically stopped when the humidity sensor was activated.

To repeat the test with the next soil sample, the filter was replaced and, if necessary, water was added to the feed tank.

When the water conductivity measurement was finished, the water inlet and outlet valves were closed and the permeameter was disconnected from the hydraulic pressure sources.

For each of n experimental tests of the same type of soil, the amount of water absorbed by the sample and the time it was absorbed, as well as the water level indicators of the piezometer, were determined.

The results of laboratory studies on soil samples were used for further mathematical treatment and determination of the coefficient of moisture conductivity of the studied type of soil according to the procedure from [9]. The methodology is based on the general requirements of the European standard ISO 17892-11:2019 (E) for determining the coefficient of moisture conductivity of soils [8]. The time of moistening of the soil sample was taken as that required for the distribution of moisture in it from initial to full moisture content. Wetting of the sample took place from the water level maintained from below.

The method for determining the coefficient of moisture conductivity of the soil implied the fulfillment of the following boundary and initial conditions:

1. The initial moisture content and density of the soil sample are uniformly distributed over the volume.

2. When the sample is moistened through the lower surface, no changes in humidity are allowed on its upper surface when the wetting front approaches it.

3. The sample must be moistened under a pressure-free mode.

5. Results of experimental studies on soil moisture conductivity processes and determination of soil moisture conductivity coefficient

5. 1. Determination of soil moisture conductivity coefficient based on laboratory measurement data

Data from laboratory tests of soil samples, obtained on the measuring bench described above, underwent further mathematical treatment to determine their coefficients of moisture conductivity (m/s) according to the procedure from [9].

Within the framework of this methodology, the values of the soil moisture conductivity coefficient K_0 were taken on the basis of a statistical generalization of the results of n tests of samples of the same type of soil. The K_0 coefficient was determined at the significance level α with the confidence probability P_C, subject to a one-sided (upward) limitation of the confidence interval, m/s:

$$K_0 = K_{0avrg} + \frac{t_\alpha \cdot \sigma_n}{\sqrt{n}}, \quad (1)$$

where K_{0avrg} is the average value of the soil moisture conductivity coefficient for n tests (after excluding possible measurement errors):

$$K_{0avrg} = \frac{1}{n}\sum_{i=1}^{n} K_{0i}, \quad (2)$$

where t_α is the t-test for the level of significance α (accepted: α=0.05, which corresponds to the confidence probability P_C=0.95) for the one-sided critical region and the number of degrees of freedom n–1;

σ_n is the mean squared deviation of the results of n tests of soil samples:

$$\sigma_n = \sqrt{\frac{\sum_{i=1}^{n}\left(K_{0i} - K_{0avrg}\right)^2}{n-1}}, \quad (3)$$

where K_{0i} is the value of the moisture conductivity coefficient for the i-th test.

The minimum number of tests of the same type of soil was accepted depending on the level of confidence probability (reliability) P_C of the desired value of the coefficient of moisture conductivity according to the formula:

$$n_{min} = \frac{t_\alpha^2 \sigma_n^2}{\varepsilon^2}, \quad (4)$$

where ε is the marginal error of determination of soil moisture conductivity coefficient (estimation accuracy).

The minimum number of tests of the same type of soil was accepted according to the data in Table 1 [9].

Table 1

Minimum required number of soil tests

Marginal error of estimation in fractions σ_n (β)	Confidence probability P_C				
	0.7	0.8	0.9	0.95	0.99
1	3	4	5	7	11
0.5	6	9	13	18	31
0.4	8	12	19	27	46
0.3	13	20	32	46	78
0.2	29	43	70	99	171

Confidence probability was taken at the level of P_C=0.95...0.99.

The cross-sectional area of the experimental soil sample in the flow direction was determined from the following formula, m²:

$$A = \frac{\pi d^2}{4}, \quad (5)$$

where d is the diameter of the experimental sample, m.

Next, the coefficient of moisture conductivity of the soil (m/s) was determined depending on the conditions of the experiment.

For the conditions of the experimental test "At constant head and constant flow", the coefficient of moisture conductivity of the soil sample was determined from the following formula, m/s:

$$K_0 = \frac{Q}{A \cdot i}, \quad (6)$$

where Q is the flow rate of water passing through the sample per unit of time, g/s;

A is the cross-sectional area of the experimental sample, m²;

i is the hydraulic gradient of the sample.

The consumption of water passing through the sample per unit of time was determined from the following formula, g/s:

$$Q = \frac{\Delta m}{\Delta t \cdot \rho} = \frac{m_1 - m_0}{\Delta t \cdot \rho}, \quad (7)$$

where m_0, m_1 are, respectively, the initial mass and the mass of the water-saturated sample, g;

Δt is the water saturation time of the sample, s;

ρ is water density, g/m³.

The hydraulic gradient of the sample included in formula (6) was determined from the following formula:

$$i = \frac{\Delta h}{l}, \quad (8)$$

where Δh is the difference in water pressure between measurement points 1 and 2, m.

For the conditions of the experimental test "Growing downstream under decreasing head", the coefficient of moisture conductivity of the soil sample was determined from the following formula, m/s:

$$K_0 = \frac{a_{in} \cdot a_{out}}{\left(a_{in} + a_{out}\right)} \cdot \frac{l}{A \cdot \Delta t} \cdot \ln\left(\frac{\Delta h_{t_1}}{\Delta h_{t_2}}\right), \quad (9)$$

where α_{in}, α_{out} are the cross-sectional areas of the inlet and outlet pipes, respectively, m²;

Δh_{t1}, Δh_{t2}– water pressure above the leakage level, respectively, at time points t_1, t_2, m.

Since the diameter of the intake (exhaust) pipes was known, their cross-sectional area was determined from the following formula, m:

$$a_{in(out)} = \frac{\pi d_{in}^2}{4}. \quad (10)$$

For the conditions of the experimental test "Constant downstream under falling head", the coefficient of moisture conductivity of the soil sample was determined from the following formula, m/s:

$$K_0 = \frac{a_{in} \cdot l}{A \cdot \Delta t} \cdot \ln\left(\frac{\Delta h_{t1}}{\Delta h_{t2}}\right), \quad (11)$$

where l is the length of the experimental soil sample, m.

In order to exclude gross measurement errors (misses), the smallest (biggest) values of tests of the same type of soil suspected of being wrong were evaluated according to the Dixon criterion (maximum difference method) according to the formulas:

$$r_{min} = \frac{K_{02} - K_{01}}{K_{0n} - K_{01}};$$

$$r_{max} = \frac{K_{0n} - K_{0(n-1)}}{K_{0n} - K_{01}}, \quad (12)$$

where K_{01}, K_{02}, K_{03}, ..., $K_{0(n-2)}$, $K_{0(n-1)}$, K_{0n} are the values of moisture conductivity coefficients of the same type of soil, arranged in ascending order.

The calculated values of the Dixon coefficients $r_{\min}$ ($r_{\max}$) were compared with their critical values r_α.

If the inequality $r_{\min(\max)}<r_\alpha$ was fulfilled, then it was considered that there were no gross errors (mistakes) of the tests.

If $r_{\min(\max)}>r_\alpha$, then the smallest (largest) test values examined for gross errors (misses) were excluded from the statistical series for further research.

If the calculated values of $r_{\min(\max)}$ coincided with the table value of r_α, then the questionable test result was not excluded, and the experiment was repeated.

The coefficient of moisture conductivity of the soil, determined for the test conducted with the temperature of the penetrating water T_{test}, was converted into the corresponding coefficient of moisture conductivity for an arbitrary temperature T using the correction coefficient α from the following formula, m/s:

$$K_{0T}=K_{0test}\cdot\alpha, \tag{13}$$

$$\alpha=\frac{\eta_{test}}{\eta_T}, \tag{14}$$

where η_T is the viscosity of water for a given temperature T, determined from the following formula, mPa·s:

$$\eta_T=0.02414\cdot10^{247.8/(T+133)}. \tag{15}$$

Calculation formulas (1) to (15) made it possible to determine the soil moisture conductivity coefficient in accordance with the requirements of the European standard ISO 17892-11:2019 (E).

The main problem of further practical application of the calculated values of soil moisture conductivity coefficients for predicting moisture accumulation in road construction is the non-compliance of their values and dimensions with those provided by existing calculation models. This creates significant difficulties in the development of infrastructure projects and necessitated the construction of a mathematical model to eliminate this discrepancy, which predetermined the next stage of our work.

5. 2. Determining the soil moisture conductivity coefficient based on mathematical modeling of moisture accumulation processes

The subsoil is moistened by two sources:

– passing rainwater;

– the rise of water in the subsoil layer from the groundwater level.

In work [10], an equation was derived for determining the change in humidity in the soil under the condition that the action of capillary forces and gravity forces coincide (infiltration of water into the subsoil from above). For the practice of road construction, studies on moisture accumulation in the soils of the lower part of the subsoil from the level of groundwater or water table, that is, when water moves from the bottom to the top, are of no less interest. It is this physical process for determining the coefficient of moisture conductivity K_0 that was simulated at the NTU laboratory and described above.

In this case, the original differential equation of soil moisture change was written in the form:

$$\frac{\partial W}{\partial t}=-K_1\frac{\partial^2 w}{\partial x^2}+K_0\frac{\partial W}{\partial x}. \tag{16}$$

The K_1 coefficient characterizes the diffusion transfer in relation to the movement of water in the capillary-porous body, the soil. The filtration coefficients determined for completely water-saturated soils by general conventional methods will not be identical to the filtration coefficients K_0, which characterize the movement of water in the complex process of continuous changes in soil moisture percolating from the bottom up.

Boundary conditions were adopted for the solution to equation (16), based on the conditions of conducting experimental tests of soil samples:

$$W(x,0)=W_0, \tag{17}$$

$$W(0,t)=W_{FH}, \tag{18}$$

$$W(H,\infty)=W_{FH}. \tag{19}$$

Equation (16) under boundary conditions (17) to (19) was solved by the operational method.

Using the Laplace transformation with respect to the variable t, the following relationship is obtained:

$$L\left[\frac{\partial W}{\partial t}\right]=K_0L\left[\frac{\partial W}{\partial x}\right]-K_1L\left[\frac{\partial^2 W}{\partial x^2}\right]. \tag{20}$$

Since the transformation is applied with respect to the variable t, the transformation L from derivatives by coordinates is equal to the derivative of the mapping, i.e.:

$$L\left[\frac{\partial W}{\partial t}\right]=K_0\frac{\partial}{\partial x}\{L[W(x,t)]\}-$$
$$-K_1\frac{\partial^2}{\partial x^2}\{L[W(x,t)]\}. \tag{21}$$

To display the transformed function, the notation $L[W(x,\tau)]=T(x,s)$ is adopted.

Then, in accordance with the order of setting the mapping for the derivative of the sought function, the ratio is obtained:

$$L\left[\frac{\partial W}{\partial t}\right]=sT(x,s)-W(x,t=0). \tag{22}$$

The value W in relation (22) characterizes the increase in soil moisture compared to the initial moisture $W(x,0)=0$.

Substituting (22) into (21), we get:

$$-K_1\frac{d^2T(x,s)}{dx^2}+K_0\frac{dT(x,s)}{dx}-sT(x,s)=0. \tag{23}$$

For further research, equation (23) was represented in the form:

$$-K_1T''(x,s)+K_0T'(x,s)-sT(x,s)=0. \tag{24}$$

The differential equation (24) for displaying the desired function is a linear equation with constant coefficients.

The characteristic equation for (24) is defined as:

$$K_1\alpha^2-K_0\alpha+s=0. \tag{25}$$

The roots of quadratic equation (25) are:

$$\alpha_{1,2}=+\frac{K_0}{2K_1}\pm\sqrt{\left(\frac{K_0}{2K_1}\right)^2-\frac{s}{K_1}}. \quad (26)$$

Since the sought-after function must increase with depth (as wetting occurs from the bottom up), only one root will satisfy the condition of the problem to be solved:

$$\alpha_1=+\frac{K_0}{2K_1}+\sqrt{\left(\frac{K_0}{2K_1}\right)^2-\frac{s}{K_1}}. \quad (27)$$

In this case, the expression for the function mapping was written as follows:

$$T=C\cdot e^{+\left(\frac{\sqrt{K_0}}{2K_1}+\sqrt{\left(\frac{K_0}{2K_1}\right)^2-\frac{s}{K_1}}\right)x}. \quad (28)$$

Coefficient C is determined on the basis of boundary condition (18), written in relation to the increase in soil moisture content:

$$\Delta W(0,t)=\Delta W_{FH}= \\ =(W_{FH}-W_S)-(W_0-W_S)=W_{FH}-W_0. \quad (29)$$

It is obvious from equation (27) that $C=T(0, s)$. However, the mapping from the constant A is equal to A/S. Accordingly, $T(0,s)=\frac{\Delta W_{FH}}{s}=C$. Taking into account the adopted notation $\frac{K_0}{2K_1}=Q$, ratio (28) took the form:

$$T=\frac{\Delta W_{FH}}{s}\cdot e^{+x\left(Q+\sqrt{Q^2-\frac{s}{K_1}}\right)}. \quad (30)$$

For further research, dependence (30) is reduced to the form:

$$T=\Delta W_{FH}e^{+Qx}\cdot\frac{e^{+\frac{x}{\sqrt{K_1}}\sqrt{Q^2K_1-s}}}{s}. \quad (31)$$

The original of the function in the mapping described by dependence (31) is defined as the relation:

$$\Delta W_{x,t}=\frac{\Delta W_{FH}e^{+Qx}}{2}\left[\begin{array}{l} e^{Qx}erfc\left(\frac{x}{2\sqrt{K_1t}}+Q\sqrt{k_1t}\right)+ \\ +e^{-Qx}erfc\left(\frac{x}{2\sqrt{K_1t}}-Q\sqrt{K_1t}\right)\end{array}\right]. \quad (32)$$

Under dependence (32):

$$erfc\left(\frac{x}{2\sqrt{K_1t}}\pm Q\sqrt{K_1t}\right)=1-erf\left(\frac{x}{2\sqrt{K_1t}}\pm Q\sqrt{K_1t}\right), \quad (33)$$

where $erf\left(\frac{x}{2\sqrt{K_1t}}\pm Q\sqrt{K_1t}\right)$ is the Gaussian integral for which there are special tables.

For the practice of road construction, not only the dynamic change of humidity over time is of interest but also the limit profile of humidity until the end of the period of moisture accumulation. The equation of this curve can be found from expression (32) at $t\to\infty$.

So:

$$\Delta W_{x,t\to\infty}=\frac{\Delta W_{FH}\cdot e^{Qx}}{2}\left[\begin{array}{l} e^{Qx}ercf(0-\infty)+ \\ +e^{-Qx}ercf(0+\infty)\end{array}\right]. \quad (34)$$

Since $ercf(x)$=1-$erf(x)$, then:

$$\begin{cases} erfc(-\infty)=1-erf(-\infty)=1+1=2, \\ erfc(+\infty)=1-erf(+\infty)=1-1=0. \end{cases} \quad (35)$$

Then:

$$\Delta W_{x,t\to\infty}=\frac{\Delta W_{FH}\cdot e^{Qx}}{2}\left[e^{Qx}\cdot 2+e^{-Qx}\cdot 0\right]=\Delta W_{FH}e^{2Qx}. \quad (36)$$

Considering $Q=K_0/2K_1$, from (36) the following relation is obtained:

$$\Delta W_{x,t\to\infty}=\Delta W_{FH}e^{\frac{K_0}{K_1}x}. \quad (37)$$

Based on the conditions of experimental tests, taking into account boundary condition (19), the following was obtained:

$$W_{FH}=(W_{FH}-W_0)e^{\frac{K_0}{K_1}H}. \quad (38)$$

From formula (38), we derived the ratio for determining the coefficient of moisture conductivity (diffusion) of soils, m^2/s:

$$K_1=\frac{K_0\cdot H}{\ln\left(\frac{W_{FH}}{W_{FH}-W_0}\right)}. \quad (39)$$

Formula (39) makes it possible to obtain the value of the soil moisture conductivity coefficient K_1 (m^2/s) based on the values of the soil moisture conductivity coefficient K_0 (m/s). The K_0 coefficient (m/s) is calculated based on laboratory soil test data, using European approaches and standards.

5. 3. Results of a numerical experiment on the study of factors affecting the coefficient of moisture conductivity of the soil

On the basis of our results from experimental studies and the constructed mathematical model (39), a hypothesis was put forward regarding the existence of factors affecting the value of the soil moisture conductivity coefficient, in particular, the initial soil moisture W_0 and its total moisture capacity W_{FH}.

To confirm or refute this hypothesis, a numerical experiment was conducted based on mathematical model (39). The essence of the experiment was to study the influence of the initial humidity W_0 and the total moisture content of soils W_{FH} on the value of their coefficient of moisture conductivity K_1 (m^2/h).

During the numerical experiment, the value of the initial soil moisture W_0 varied in the range from 0.08 to 0.15, and the value of the total moisture content of soil W_{FH} – in the range from 0.15 to 0.5. The choice of such ranges of changes of the investigated factors is due to the fact that they are the most characteristic for the investigated subsurface soils.

A fragment of the results of our numerical experiment is shown in Fig. 5.

Analysis of the dependence of coefficient of moisture conductivity of soils on their total moisture capacity W_{FH} (Fig. 5) makes it possible to draw a conclusion about the inverse relationship between the total moisture capacity of soils W_{FH} and their coefficient of moisture conductivity K_1 (m^2/h). That is, with an increase in the values of the total moisture capacity of soils W_{FH}, the values of their moisture conductivity coefficient K_1 will decrease. This means that heavy clay and loamy soils will have lower values of the coefficient of moisture conductivity K_1 than similar values of light sandy and sandy soils. This feature should be taken into account during the design and construction of the ground surface of the road structure.

At the same time, there is a direct relationship between the initial moisture content of soils W_{FH} and their coefficient of moisture conductivity K_1 (m^2/h). That is, as the value of initial soil moisture W_{FH} increases, the value of its moisture conductivity coefficient K_1 also increases. This feature should be taken into account during the construction and operation of the road structure in different periods of the year.

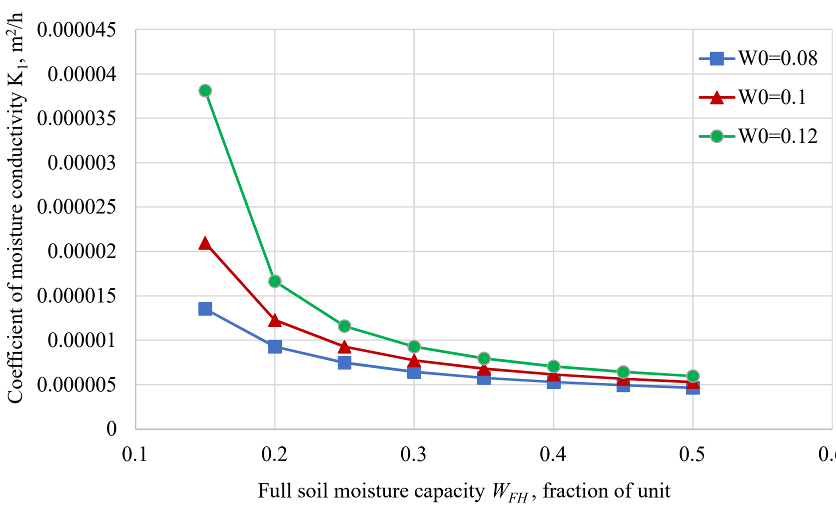


Fig. 5. Dependence of the coefficient of moisture conductivity of soils on their initial moisture W_0 and the total moisture capacity W_{FH}

5. 4. Verification of the mathematical model using an example of determining the coefficient of moisture conductivity of sandy soil

In order to test our mathematical model for determining the coefficient of moisture conductivity of soils (39), a series of test comparisons with data obtained by other authors and given in the literature [10] was carried out.

Below are the results of a comparison of the values of the coefficient of moisture conductivity of sandy soil, obtained on the basis of the proposed mathematical model (39), as well as on the basis of the integration method and the calculation formula given in work [10]:

$$K_1 = \frac{\pi}{4} \cdot \frac{q^2}{\Delta W_{FH}^2 \tau}, \quad (40)$$

where q is the amount of moisture absorbed through a unit of the sample surface in a certain time τ;

$\Delta W_{FH} = \left(W_{FH} - W_0\right)$, W_{FH} – volumetric soil moisture near the surface, through which moistening is carried out;

W_0 is the initial volumetric moisture content of the soil sample.

The following raw data were used for the comparative analysis:

1. The type of soil is light sandy loam.
2. Type of test conditions: "Constant downstream under falling head".
3. Water temperature T_{test}=18 °C.
4. Sample diameter d=0.096 m.
5. The distance between measurement points 1 and 2 in the direction of the flow is l=0.12 m.
6. The diameter of the intake pipe d_{in}=0.006 m.
7. Initial soil moisture W_0=0.1.

For this type of soil (sandy loam), 10 laboratory samples were formed and their experimental study on moisture conductivity was carried out according to the methodology described above.

The minimum number of tests at which the marginal error of the estimate ε will not exceed the mean square deviation σ_n for a single sample at the level of confidence probability P_C=0.95 will be $n_{\min}$=7 (β=1) (Table 1). Therefore, it can be considered that the number of tests n=10 is quite sufficient to obtain the value of the moisture conductivity coefficient with a confidence probability P_C=0.95. The results of experimental measurements and calculated values of soil moisture conductivity coefficients (in ascending order), derived from formula (11), are given in Table 2.

The total soil moisture capacity W_{FH} was determined based on the fact that the compaction coefficient K_y was adopted. Maximum (standard) density of dry soil is $\rho_d^{\max}$ =1.76 g/cm^3. Thus, the density of dry soil was taken as ρ_d=1.67 g/cm^3.

The total soil moisture content W_{FH} was determined from the ratio:

$$W_{FH} = \frac{1}{\rho_{sk}} - \frac{1}{\rho_s}, \quad (41)$$

ρ_s – density of soil particles: for sandy loams – 2.68 g/cm^3; for loams – 2.7 g/cm^3; for clays – 2.72 g/cm^3.

Thus, the full moisture capacity of the studied soil derived from formula (41) is accepted as W_{FH}=0.22.

The initial soil moisture was W_0=0.1.

From formula (39), the value of the moisture conductivity coefficient of the soil (sandy) was determined and compared with the values obtained from formula (40) and on the basis of the integral method [10]. The results of calculations and comparisons are given in Table 3.

Table 3 demonstrates that the values of soil moisture conductivity coefficient (m^2/h) obtained by the proposed mathematical model in the form of analytical formula (39) are in good agreement with similar results obtained by the integral method and from formula (40) [10]. At the same time, better agreement of the proposed model (39) is observed with the integral method (relative error – 4.5 %) than with formula (40) (relative error – 12.98 %).

Table 2

Results of experimental measurements and soil moisture conductivity coefficients calculated from formula (11)

Sample No.	Amount of water absorbed by the sample, g	The time it took for the water to be absorbed by the sample, s	Indicators of the water level of the pesometer at a point in time, m		Calculated coefficient of moisture conductivity, m/s 10^{-8}
			t_1	t_2	
1	96.9	420	0.71	0.6976	1.96788
2	97.3	422	0.71	0.6975	1.96796
3	98.5	427	0.71	0.6974	1.96817
4	99.4	431	0.71	0.6973	1.96834
5	100.3	435	0.71	0.6971	1.9685
6	101.5	442	0.71	0.6970	1.9598
7	103.1	447	0.71	0.6968	1.969
8	103.8	450	0.71	0.6967	1.96913
9	104.7	454	0.71	0.6966	1.96929
10	108.9	472	0.71	0.6961	1.97004
Mean	101.5	440	0.71	0.697	1.96781

Table 3

Calculated values and relative deviations of the coefficient of moisture conductivity of sandy soil, obtained from formula (39), by the integral method, and from formula (40)

Indicator	Formula (39)	Integral method	Formula (40)
Value of soil moisture conductivity coefficient (sandy loam), m^2/h	$1.07909 \cdot 10^{-5}$	$1.03 \cdot 10^{-5}$	$9.39 \cdot 10^{-6}$
Relative deviation, %			
Formula (39)	–	4.5	12.98
Integral method	4.5	–	8.83
Formula (40)	12.98	8.83	–

6. Discussion of results related to the possibilities of using them in the engineering practice for designing and building highways

A method for determining the coefficient of moisture conductivity of soils reported in our work in the form of mathematical model (39) makes it possible to move away from outdated methods and adapt existing methods for determining moisture accumulation in soils during the annual cycle to European approaches and standards. A special feature of our model, unlike existing ones [10], is that it is based on classical equations of hydrodynamics but constructed under the condition of modeling the movement of water from the bottom up, which reflects the process of moisture accumulation in the lower layers of the subsoil from groundwater or surface water. On the other hand, mathematical model (39) makes it possible to express the soil moisture conductivity coefficient K_1, which characterizes the diffusion movement of water, through the filtration coefficient K_0, calculated in accordance with the requirements of the European standard ISO 17892-11:2019 (E) [8]. The coefficient K_0 was determined based on the data on laboratory tests of soils, which were carried out on a measuring bench to determine the moisture conductivity of soils (Fig. 1–4) in accordance with the requirements of the European standard ISO 17892-11:2019 (E) [8]. The adequacy of mathematical model (39) was confirmed by the results of comparisons with the data by other authors [10] (Table 3). At the same time, a good agreement of mathematical model (39) with the integral method (the relative error did not exceed 4.5 %) and with formula (40) (the relative error did not exceed 12.98 %) was obtained. This gives every reason to recommend the proposed mathematical model (39) for determining the soil moisture conductivity coefficient K_1 (m^2/s) in the engineering practice of road construction.

Based on our numerical experiment, an inverse relationship between the total moisture capacity of soils and their coefficient of moisture conductivity, as well as a direct relationship between the initial moisture of soils and their coefficient of moisture conductivity, was established. Corresponding graphical dependences (Fig. 5) have been built, which could be used during the design of the ground surface of road structures, as well as to determine the main parameters of drainage layers.

However, one of the limitations of the application of the mathematical model (39) built is the equality of the initial soil moisture W_0 to its total moisture capacity W_{FH}. That is, the value of initial soil moisture should be less than its full moisture content $W_0 < W_{FH}$. Also, when conducting laboratory tests of soil samples, according to ISO 17892-11:2019 (E), the maximum size of its particles should not exceed 1/6 of the minimum internal size of the permeameter. If larger particles are present, a larger permeameter should be used or larger particles should be removed [8].

Regarding the shortcomings of the proposed study, the results of determining the coefficient of moisture conductivity K_0 were obtained for the water temperature that corresponds to the conditions of the warm season. In the future, we shall continue research into moisture migration conditions with increased viscosity.

Further area of research is to devise methods for forecasting moisture accumulation in subsurface soils during the annual cycle, taking into account the mathematical model (39). Based on the determined humidity, the stress-deformed state of road structures with drainage layers, calculated using the soil moisture conductivity coefficient (39) in accordance with the requirements of European standards, will be investigated. Difficulties that can be encountered when solving these problems are related to the need to take into account a number of factors, such as the amount of rain runoff, water evaporation, peculiarities of the granulometric composition of local soils, the influence of heavy transport, etc.

From the point of view of engineering practice, the results reported in our work will make it possible to improve the working conditions of the road surface and ensure its more rational use in terms of strength over time due to the regulation of the water-thermal regime of the ground surface. In the end, this will make it possible to increase the strength and extend the service life of the road structure, as well as reduce the final cost of its construction.

7. Conclusions

1. Experimental studies of soil moisture conductivity processes were carried out on a measuring bench, in accordance with the requirements of the European standard ISO 17892-11:2019 (E). Based on the statistical generalization of laboratory test data of soil samples, the value of their coefficients of moisture conductivity K_0 in the range from $2.49 \cdot 10^{-9}$ to $1.75 \cdot 10^{-7}$ m/s was derived. The resulting dimensions of the soil moisture conductivity coefficient K_0 (m/s) do not correspond to those provided by existing calculation models for predicting moisture accumulation in the road

structure K_1 (m^2/s). This necessitated the construction of a mathematical model to eliminate this discrepancy.

2. A mathematical model has been proposed for determining the coefficient of moisture conductivity of soils K_1 (m^2/s) based on the coefficient of moisture conductivity of soils K_0 (m/s), obtained from the data on laboratory tests of soils using the requirements of the European standard ISO 17892-11:2019 (E). The mathematical model was built on the basis of the differential equation of changes in soil moisture during the movement of water from the bottom up, which describes the process of moisture accumulation in the soils of the lower part of the subsoil from the level of supraglacial (soil) waters or head water. The adopted boundary conditions correspond to the physical content of the experiment.

3. It has been established that there is an inverse relationship between the coefficient of soil moisture conductivity K_1 (m^2/h) and its total moisture capacity W_{FH}. At the same time, there is a direct relationship between the coefficient of moisture conductivity of soils K_1 (m^2/h) and their initial moisture content W_0. Dependences were obtained in the range of changes in initial soil moisture W_0 from 0.08 to 0.15 and total soil moisture content W_{FH} from 0.15 to 0.5. It was established that the values of the coefficient of moisture conductivity K_1 of soils increase from $4.64 \cdot 10^{-6}$ to $3.81 \cdot 10^{-5}$ m^2/h with an increase in their initial humidity and a decrease in total moisture capacity. That is, heavy clay and loamy soils have lower values of the coefficient of moisture conductivity than similar values of light sandy and sandy soils. This feature should be taken into account during the design and construction of the subgrade of the road structure, as well as during its operation in different periods of the year.

4. The results of the comparison showed a good agreement of the proposed mathematical model with the integral method (the relative error did not exceed 4.5 %) and with formula (40) (the relative error did not exceed 12.98 %). This makes it possible to recommend the proposed mathematical model for determining the coefficient of moisture conductivity of soils K_1 (m^2/s) in the engineering practice of road construction. The method proposed in our work makes it possible, in the first approximation, to give a forecast of the soil moisture of the ground layer, arranged from local soils; estimate the possible value of its subsidence and give, based on the moisture forecast, an estimate of the possible strength of road structures.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study, as well as the results reported in this paper.

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Data availability

All data are available, either in numerical or graphical form, in the main text of the manuscript.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

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