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IDENTIFYING THE PARAMETERS AND OPERATION MODES OF THE CAVITATION APPARATUS TAKING INTO ACCOUNT THE INFLUENCE OF THE PROCESSING MATERIAL

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The object of this study is dynamic pressure that takes into account the main parameters of technological process. Under real conditions of cavitation apparatus operation, a discrepancy between actual and calculated parameters of 30–50 % was found, which significantly reduces the efficiency and quality of material processing. The problem was solved through a joint study of the movement of the acoustic apparatus and the technological environment as a single structured system taking into account the influence of materials. This is a peculiarity of the approach and distinctive features of the results in comparison with existing ones, in which the research was conducted separately for the processing material and the cavitation apparatus. The model proposed in the work reflects the elastic, inertial, and dissipative parameters in the equations of motion, provided that their changes are taken into account, both in the acoustic apparatus and in the technological material. This approach has made it possible to reveal the physical essence of the interaction and analytically describe their joint movement. The resulting analytical dependences made it possible to calculate and propose numerical values of vibration amplitudes in the range of 4.0...20.0 μm, sound pressure values in the range of 5.0...30.0·10^5 Pa for media with a viscosity of (10–200)·10^{-3} Pa·s. With the set value of the amplitude, it is possible to assign the necessary parameters for the implementation of the cavitation process. The developed calculation algorithm ensures the reliability of the accepted models and parameters of the cavitation apparatus. The proposed approach of joint study of the movement of the acoustic apparatus and the environment is expedient to use for the practical implementation of ultrasonic treatment. In particular, such processes as dispersion, emulsification, mixing, extraction, and others, in order to increase their efficiency

Keywords: cavitation apparatus, discrete-continuous model, contact zone, pressure, displacement amplitude, wave coefficient, synergy coefficient

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1. Introduction

The effectiveness of cavitation processing of materials largely depends on the dynamic pressure, the key parameters of which are the amplitude and frequency of oscillations. Their required values completely depend on the accepted calculation model, which should reflect the real process of materials processing. However, until now there is no generally accepted calculation model of the process of processing stages. Such a problem holds back the widespread adoption of ultrasound equipment and technology. The technological environment, as a rule, is represented by some attached mass to the radiation surface of the cavitation apparatus. And the change in the properties and parameters of the technological environment during its processing is a proven fact. Therefore, if such an approach can be used, then only within the framework of those regimes and parameters under which the research was conducted. Therefore, there is a need to carry out scientific research on the parameters of the cavitation apparatus based on a model that takes into account the properties of the technological environment during its processing. In practice, it is possible to implement a high-quality process of ultrasonic cavitation treatment of technological environments by using the algorithm and determined numerical values of the parameters. Thus, matching the characteristics of the "apparatus – environment" system and determining the parameters and modes of the cavitation apparatus with the minimization of energy costs is a relevant area of our studies.

2. Literature review and problem statement

Determining the parameters and modes of operation of the cavitation apparatus with the minimization of energy costs for the implementation of a high-quality process of ultrasonic cavitation treatment of technological environments is one of the key conditions of modern requirements for the design and technological parameters of this class of technological equipment. In works [1, 2], the characteristics and parameters of the stages of nucleation, development, and bursting of bubbles in the created cavitation region of the medium are considered. The impact on the cavitation process of the threshold amplitude of the sound pressure, which depends on the sound frequency, the type of liquid, the amount of dissolved gas and impurities, has been established. But the questions related to the calculation of the parameters of the cavitation apparatus remained unresolved. Work [3] reports a parametric review of ultrasonic cavitation and related sonochemistry, in particular taking into account the pressure amplitude, frequency, and reactor design. The consideration of parameters was carried out in the plane of influence on cavitation bubbles and, as a result, on sonochemical activity. However, the calculation of the parameters of drivers of the process is not given. The efficient use of energy given in [4] is proposed to be implemented by means of discrete pulsed energy supply. The dominant role of pulsation of turbulent flow speed fluctuations is indicated, which depends on the density of the technological medium and the interfacial tension coefficient. It is obvious that such a statement is possible and valid in the studied hydromechanical processes of emulsification, but the influence of such a mode on the parameters of the working body has not been considered. Work [5] reports analysis of the possible mechanism for the erosive effect of cavitation bubbles on the working surfaces of the equipment. Modeling of the cavitation process was carried out using the method of pressure fluctuations, which makes it possible to calculate the pressure in each zone of the research area. The volume and size of the vapor-gas phase formed in the hydrodynamic type and the chemical effects of cavitation were theoretically determined and experimentally confirmed. However, the interaction between the cause of cavitation and the solution of the investigated process, which needs to be resolved, is not indicated. In [6], an assessment of resonance frequencies and output sound pressure of a transducer design with a concave acoustic resonator with a spherical cavity was carried out. The ultrasonic transducer is designed for processing delicate surfaces at low frequencies. The importance of the work is the approach of using resonance, but the effectiveness of the interaction is not given. Low frequencies were studied, which requires additional research under conditions of cavitation action.

Work [7] shows the use of high-intensity ultrasonic vibrations for the implementation of the degassing process of liquid media. The influence of ultrasound intensity on the efficiency of technological action was studied. It is noted that the intensity of ultrasound is proportional to the sound pressure. However, the influence of the environment on the choice of parameters was not considered. Taking into account the hydrodynamic and heat-mass exchange aspects of cavitation as a single approach to the intensification of technological processes is suggested in [8]. A model is proposed that describes the behavior of bubbles and their ensembles in boiling and cavitation processes. In this work, the dynamics of bubbles in a compressible liquid is analyzed, but there is no information about the effect on the source of excitation. Worthy of attention is work [9], which proposes a complex method of radial ultrasonic electrochemical micromachining, where an ultrasonic field is added to a cylindrical surface. At the same time, considering the influence of standard and system parameters on the intensity of cavitation processes, their possible influence on the generator of ultrasonic waves is not indicated. In [10], the main calculation and design elements of the acoustic system, which is used in the structure of the air filter, are given. The work is focused on the design of the main part of the ultrasonic system represented by the ultrasonic transducer, but at the same time, the physics and conditions for assessing the influence of the environment on the transducer are not disclosed. In [11], a comprehensive review of the implementation of experimental methods in the production of hydrogen using ultrasound was carried out. The process of bubble collapse is described in detail, which is not only a high-speed burst, but also a high-energy action. It is indicated that there is a significant increase in temperature and pressure. At the same time, the opinion of the authors about the joint presentation of the "ultrasound installation – hydrogen production process" system is not given. In [12] it is noted that in the process of ultrasonic dispersion, the effect of ultrasonic cavitation can seriously affect the efficiency of dispersion of magnetorheological polishing liquid. A dynamic model of the cavitation bubble was built and calculated by considering the continuity equation and the viscosity equation. However, under what conditions this model correlates with the cause of oscillations is not stated. A numerical study of the effect of liquid compressibility on the acoustic vaporization of droplets is presented in [13]. The influence of the pressure amplitude of the acoustic pulse on the stages of nucleation, development, and bursting of bubbles was studied. It is noted that the compressibility of the liquid significantly inhibits the growth of the bubble during its collapse. Only the interaction between the bubbles was considered, but the questions related to the interaction with the working body remained unresolved. In [14] it is noted that predicting the maximum cavitation damage of hydraulic machines in cryogenic engineering, such as turbopumps in liquid rockets, is possible by determining the achievable bubble collapse intensity. All subsequent studies exclusively consider the consideration of the processes of bubble dynamics.

Therefore, the main focus of the cited works is research into the technological aspects of the cavitation process of formation and development of bubbles in the processing of various media, including liquid-dispersed ones. At the same time, there are no studies on the interaction of the cavitation apparatus and the technological environment during the ultrasonic cavitation treatment of liquid-disperse media. All this gives reason to assert that it is expedient to conduct a study of the acoustic apparatus and the technological environment as a single structured system. At the same time, the model of such a system should take into account elastic, inertial, and dissipative properties both in the acoustic apparatus and in the technological environment. An option to overcome the difficulties is to consider [15, 16]. However, in these works, considerable attention is paid to the technological aspects of the process. The considered processes are aimed at the search for rational parameters, definition of models, classification of environments based on taking into account the viscous properties of environments. Therefore, within the framework of these studies, determining the parameters of the cavitation apparatus is a logical stage of these studies.

3. The aim and objectives of the study

The purpose of our work is to determine the parameters and modes of the cavitation apparatus under the conditions of minimizing the energy during material processing. This will make it possible to determine parameters for the cavitation device and implement a high-quality process of ultrasonic cavitation treatment of materials.

To achieve the goal, the following tasks were defined:

– to research and determine the rational parameters and modes of operation of the cavitation apparatus;

– to propose an algorithm for calculating the parameters of the cavitation device, taking into account the influence of the processing material.

4. The study materials and methods

The object of our study is dynamic pressure, which takes into account the main parameters of the processing technological process. These are amplitude, speed of oscillations, and intensity. Determination of the change in pressure over time in the contact zone is due to the fact that the cavitation region originates and is formed under the influence of an external source of oscillations, which forms the pressure. Therefore, the following research hypothesis is accepted in the work. To implement construction of such a model, the scientific idea of considering the interaction of the "cavitation device – technological environment" system is proposed, by determining the balance of the force pressure of the device and stresses and considering the environment model as a system with distributed parameters. The environment model, as a system with distributed parameters, makes it possible to reveal the physical essence of interaction, to develop proposals for improving the processing technology of technological environments. The nature of the pressure change and its quantitative values in the contact zone provide an opportunity to evaluate the effects of interaction and energy transfer from the acoustic device to the processing medium. At the same time, the following provisions have been adopted to ensure the efficiency of the cavitation apparatus:

– the maximum possible extraction of energy from the source of oscillations;

– minimal dissipation of energy in the design elements of the technological apparatus;

– the greatest use of acoustic energy introduced into the processed environment to ensure the flow of this technological process;

– the maximum stability of parameters of the acoustic apparatus in their values and modes of operation of the acoustic apparatus set in advance by the technology.

From the point of view of the greatest transmission of acoustic energy into the processed environment, it can be achieved by selecting such parameters that will ensure that the resistance of the technological environment to the movement of the acoustic apparatus is reduced. This condition is achieved by examining the pressure in the contact zone, and the cavitation area is generated and formed under the influence of an external source of vibrations of the general structure of the ultrasonic technological system (Fig. 1).

A typical ultrasonic oscillating system (Fig. 2) consists of electronic-acoustic transducer 1, housing 2, support 3, concentrator 4, and emitter 5, which transmits energy to the processing medium.

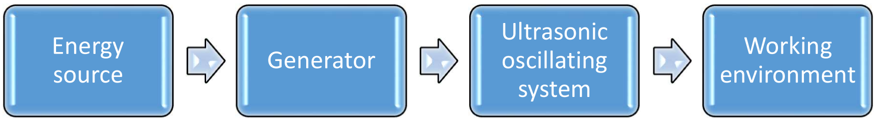


Fig. 1. Structural diagram of the ultrasonic technological system

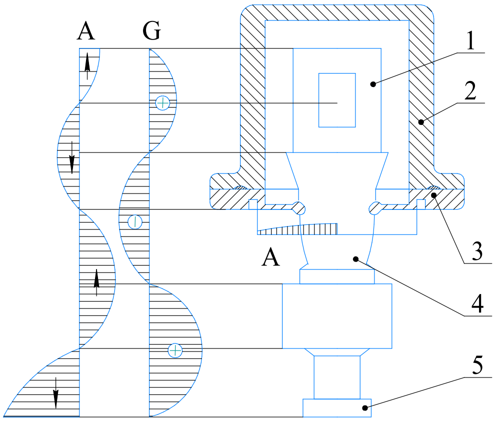
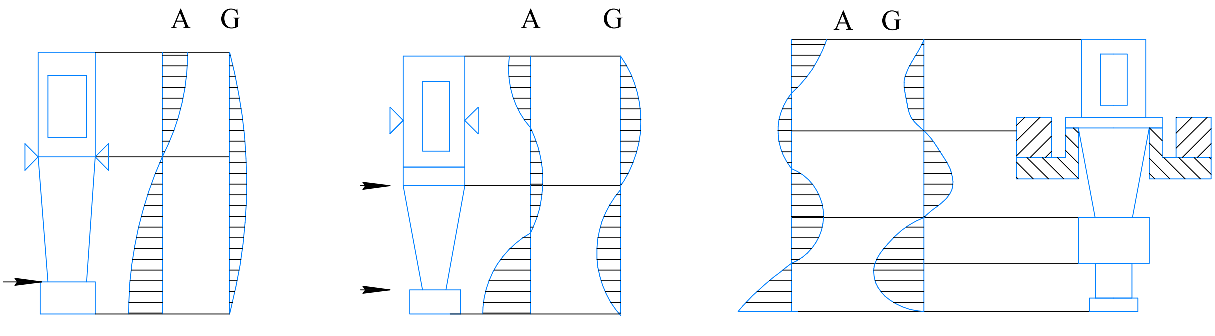


Fig. 2. A typical ultrasonic oscillating system

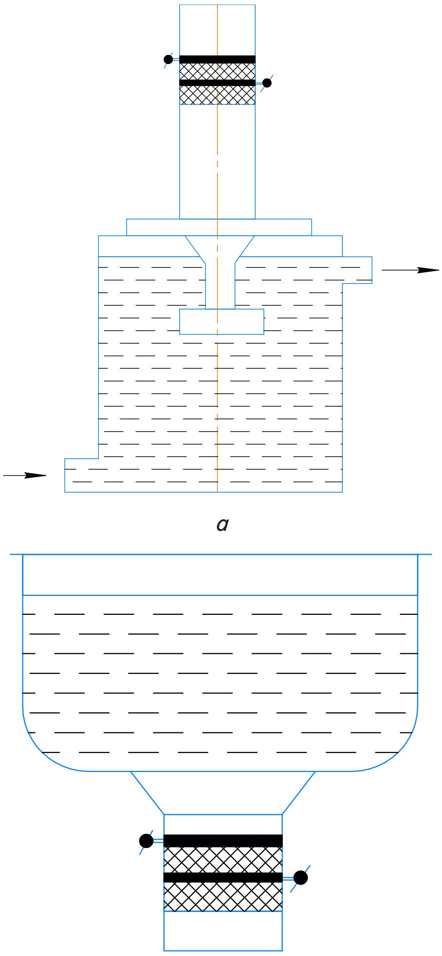
On the left are the possible distributions of the amplitude of oscillations A and internal stresses G from the transducer to the working body, provided that dissipative forces are not taken into account. In this case, in accordance with the provisions of the theory of oscillations, the distribution of amplitudes of oscillations A and internal stresses G is represented by standing waves. The maximum values of amplitudes of oscillations A correspond to the minimum values of internal stresses G and vice versa. Of course, such a distribution is not exact, but it can serve for the analysis and selection of a research scheme for parameters of the working process of ultrasonic oscillating systems.

The schemes (Fig. 3) testify to the possible implementation of certain modes, but this can be achieved under the obvious condition of researching the interactions of the acoustic apparatus with the processing environment, which was not carried out before. Therefore, the distribution of amplitudes of oscillations and stresses is one of the tasks of research and determination of rational parameters and modes of operation of the cavitation apparatus. Based on the possible variants of the location of the emitter of the cavitation device (Fig. 4), the scheme (Fig. 4, *a*) is adopted in the work, which is one of the most used in the technology of acoustic processing of materials.



a *b* *c*

Fig. 3. Schemes of ultrasonic oscillating systems according to the scheme of distribution of amplitudes of oscillations and stresses along the axial line: *a* – ¼-wave converter and ¼-wave concentrator; *b* – 1-wave system; *c* – 1½-wave system



a

b

Fig. 4. Examples of circuitry for ultrasonic cavitation devices: *a* – cavitation chamber with a drive-radiator located on the upper surface; *b* – cavitation tank with a radiator drive on the bottom surface

The following simplifications and assumptions are adopted in our studies:

– the model of the cavitation apparatus is represented by discrete parameters;

– the processing material is a continuous medium, the parameters of which are distributed;

– the centers of gravity of all layers of material lie on the same straight line passing through the center of gravity of the system;

– the values of the modulus of elasticity and the energy dissipation coefficient are considered averaged in the volume of the treated medium;

– the process is considered as a condition of maximum energy concentration;

– oscillations of the contact zone are common. The study of the movement of the acoustic apparatus was performed on the basis of a dynamic analysis (Transient Analysis) at a given oscillation frequency.

5. Results of research on the rational parameters and modes of operation of the ultrasonic oscillating system

5. 1. Research and determination of rational parameters and modes of operation of the cavitation apparatus

Analytical solution and research of the parameters of the "acoustic device – environment" system (Fig. 5, *a*) was carried out when cavitation device 2 was placed above processed technological environment 1. The calculation scheme of such a system and acting forces and movements are shown in Fig. 5, *b*.

The condition of joint motion at the first stage of energy transfer to the environment by the cavitation is as follows:

$$\sigma < \frac{Q_T + P_{cp}}{S}, \tag{1}$$

where σ is the dynamic stress in the contact zone; Q_T is the total weight of the medium and apparatus; P_{cp} is the coupling forces in the contact zone of the system; S is the area of the radiation surface of the acoustic device.

The movement of the medium is described by the wave equation:

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c(1+i\gamma)} \cdot \frac{\partial^2 u}{\partial t^2}, \tag{2}$$

where u – displacement of the cross sections of the medium; x – coordinate, determination of the location of the intersection in question; t – current time; c – speed of wave propagation; γ – resistance coefficient;

When $x=0$ (contact zone), the amplitudes of vibrations of the medium A and the radiation surface of the device U are the same: $U=A$.

The Fourier method was used to solve equation (2), according to which the solution takes the form:

$$u(x,t) = U_x e^{i\omega t}, \tag{3}$$

where

$$U_x = Ce^{x(\alpha+i\beta)} + Be^{-x(\alpha+i\beta)}. \quad (4)$$

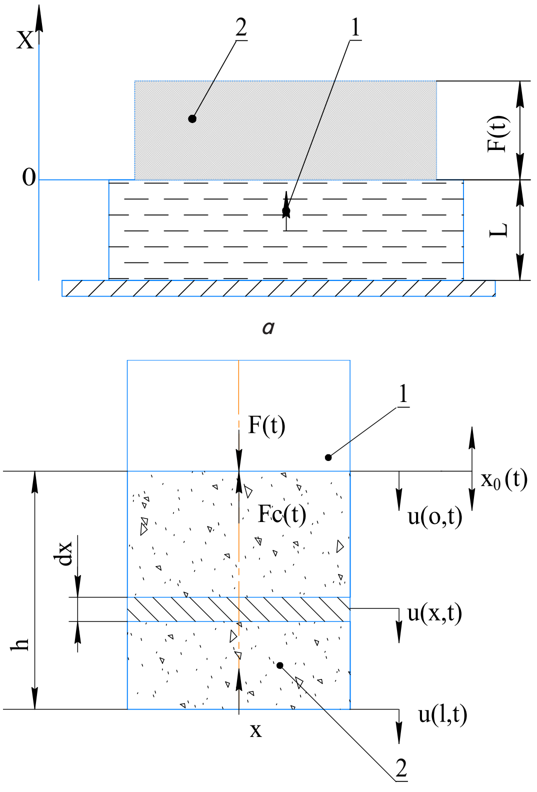


Fig. 5. System “acoustic device - environment”: *a* — arrangement diagram of the cavitation device; *b* — calculation scheme; *h* — height of the technological medium to be processed; X — coordinate that determines the direction of action; *F*(*t*) — contact force of the acoustic device, which is equal to the resistance force of the medium $F_c(t)$ in the contact zone; $x_0(t)$ — amplitude of contact zone oscillations

To determine the coefficients α and β, the value of the second-order derivatives was found:

$$\frac{\partial^2 u}{\partial x^2} = (\alpha + i\beta)\left(Ce^{x(\alpha+i\beta)} + Be^{-x(\alpha+i\beta)}\right); \quad (5)$$

$$\frac{\partial^2 u}{\partial t^2} = -\omega^2\left[Ce^{x(\alpha+i\beta)} + Be^{-x(\alpha+i\beta)}\right]. \quad (6)$$

By substituting (5) and (6) in (2) and after simple transformations, the following is obtained:

$$(\alpha+\beta)^2 = -\frac{\omega^2}{c^2\left(1+i\gamma^2\right)}. \quad (7)$$

By multiplying the numerator and denominator of the right-hand side of (7) by (1+*i*γ), the dependence for determining the coefficients in a complex form is obtained:

$$\alpha^2 - \beta^2 + 2\alpha\beta i = \frac{\omega^2}{C^2\left(1+\varphi^2\right)} + i\frac{\omega^2\varphi}{C^2\left(1+\varphi^2\right)}. \quad (8)$$

By dividing the conditional and real parts of dependence (8), they are obtained explicitly:

$$\alpha^2 + \beta^2 = \frac{\omega^2}{c^2\left(1+\gamma^2\right)}; \; 2\alpha\beta = \frac{\omega^2\varphi}{c^2\left(1+\gamma^2\right)}. \quad (9)$$

Hence:

$$\alpha = \mu_i \cdot \omega/c; \;\; \beta = \upsilon\omega/c, \quad (10)$$

where:

$$\mu_1 = \sqrt{\frac{\sqrt{1+\gamma^2}}{2\left(1+\gamma^2\right)}}; \;\; \upsilon = \sqrt{\frac{\sqrt{1+\gamma^2}+1}{2\left(1+\gamma^2\right)}}. \quad (11)$$

As follows from (11), the coefficients μ_1 and υ take into account energy dissipation in the processing environment. For the solution of equation (2), the adopted boundary conditions were used:

$$x = h; \;\; U_{mk} = 0. \quad (12)$$

Using the wave equation (2), the expression for the coefficients *C* and *B* is found:

$$C = -Be^{-2h(\alpha+i\beta)}. \quad (13)$$

Since in the contact zone the displacements of the medium and the cavitation apparatus are equal to each other, the following expression is obtained:

$$U = A = [C+B]e^{i\omega}. \quad (14)$$

Now it is necessary to obtain the second equation to determine the coefficients *C* and *B*. For this purpose, the condition of equilibrium of the forces of the cavitation apparatus and the medium in the zone of their contact is considered:

$$M\frac{\partial^2 U}{\partial t^2}\Big| = Fe^{\omega t} + ES[1+i\gamma]\frac{\partial U}{\partial x}\Big| + k(Z-U). \quad (15)$$

In (15), *E* is the modulus of elasticity of the medium, and *k* is the coefficient of elasticity of the radiation surface of the cavitator.

To determine the movement *Z*, the equation is used:

$$M\frac{\partial^2 z}{\partial t^2} + k(z - A) = 0. \quad (16)$$

Assuming the condition that the amplitude of oscillations changes according to the law $A = A_n e^{i\varpi t}$, the original equation of motion for displacement is obtained:

$$z + \omega_{ok}^2 z = F, \quad (17)$$

where ω_{ok}^2 is the natural frequency of oscillations of the cavitation apparatus:

$$\omega_{ok}^2 = \frac{k}{M}; \;\; F = \frac{(C+B)k}{M}.$$

The solution to equation (16) is taken in the form:

$$z = Ae^{i}, \quad (18)$$

where

$$A = \frac{F}{\omega_{ok}^2 - \omega^2} = \frac{(C+B)\omega_{ok}^2}{\omega_{ok}^2 - \omega^2}. \qquad (19)$$

Now the equilibrium of forces condition (15) will take the form:

$$M\frac{\partial^2 U}{\partial t^2_{x=0}} - ES(1+i\gamma)\frac{\partial U}{\partial x} - $$
$$-k\left[A_1 - [C+B]\right] = Fe^{i\omega t}, \qquad (20)$$

where:

$$\frac{\partial U}{\partial x}\Big| = (\alpha+i\beta)[C-B]e^{i\omega t}; \quad \frac{\partial^2 U}{\partial t^2}\Big| = -\omega^2[C+B]e^{i\omega t}. \qquad (21)$$

By substituting (18) in (20) after finding the derivatives, the second equation for determining the coefficients *C* and *B* is obtained:

$$-M\omega^2(C+B) - ES(1+i\gamma)\times$$
$$\times(\alpha+i\beta)[C-B] + R(C+B) = F_n, \qquad (22)$$

where:

$$R = \frac{-k\omega^2}{\omega_{ok}^2 - \omega^2}. \qquad (23)$$

Having jointly solved (13) and (22), the expressions for the coefficients *C* and *B* are obtained in the following form:

$$C = -\frac{F_a\dfrac{e^{-h(\alpha+i\beta)}}{e^{h(\alpha+i\beta)}}}{M\omega^2\left(\dfrac{e^{-h(\alpha+i\beta)}}{e^{h(\alpha+i\beta)}}\right) + ES\left(\dfrac{e^{-h(\alpha+i\beta)}}{e^{h(\alpha+i\beta)}} + 1\right)(1+i\gamma)(\alpha+i\beta) - R\left(\dfrac{e^{-h(\alpha+i\beta)}}{e^{h(\alpha+i\beta)}} - 1\right)}; \qquad (24)$$

$$B = -\frac{F_a}{M\omega^2\left(\dfrac{e^{-h(\alpha+i\beta)}}{e^{h(\alpha+i\beta)}}\right) + ES\left(\dfrac{e^{-h(\alpha+i\beta)}}{e^{h(\alpha+i\beta)}} + 1\right)(1+i\gamma)(\alpha+i\beta) - R\left(\dfrac{e^{-h(\alpha+i\beta)}}{e^{h(\alpha+i\beta)}} - 1\right)}. \qquad (25)$$

Considering:

$$E = \rho c^2; \quad S = \frac{m}{\rho h}; \quad (1+i\gamma) = \frac{\omega^2}{c^2(\alpha+i\beta)^2}, \qquad (26)$$

as well as the dependence of coefficients (24) and (25) by substituting them in the general solution (4), an expression is obtained to determine the amplitude of displacement in any layer of the medium:

$$U_x = \frac{F_a\left[\dfrac{e^{x(\alpha+i\beta)}\cdot e^{-2h(\alpha+i\beta)} - e^{-x(\alpha+i\beta)}}{e^{-2h(\alpha+i\beta)} - 1}\right]e^{i\omega t}}{-M\omega^2 + \dfrac{m\omega^2}{h}\cdot\dfrac{\left(e^{-2h(\alpha+i\beta)} + 1\right)}{\left(e^{-2h(\alpha+i\beta)} - 1\right)} + R}. \qquad (27)$$

Taking into account the known relations of hyperbolic functions, the expression in the numerator (27), which is in square brackets, is obtained:

$$\frac{e^{x(\alpha+i\beta)}\cdot e^{-2h(\alpha+i\beta)} - e^{-x(\alpha+i\beta)}}{e^{-2h(\alpha+i\beta)} - 1} = -\frac{sh(x-h)(\alpha+i\beta)}{sh(\alpha+i\beta)h}. \qquad (28)$$

Then the modulus of expression (28) is written as:

$$-\frac{sh(x-h)(\alpha+i\beta)}{sh(\alpha+i\beta)h} =$$
$$= -\sqrt{\frac{ch\alpha(x-h) - \cos 2\beta(x-h)}{ch2\alpha h - \cos 2\beta h}}. \qquad (29)$$

The part of the cofactor of the second term in the denominator (27) is an expression of the hyperbolic cotangent:

$$\frac{\left(e^{-2h(\alpha+i\beta)} + 1\right)}{\left(e^{-2h(\alpha+i\beta)} - 1\right)} = cth(\alpha+i\beta)h. \qquad (30)$$

By substituting expression (30) in the cofactor of the denominator (27) and replacing the cotangent with its complex expression we get:

$$\frac{m\omega^2}{h}\cdot\frac{cth(\alpha+i\beta)h}{(\alpha+i\beta)} =$$
$$= \frac{m\omega^2}{h\left[\dfrac{\alpha\cdot sh2h\alpha + i\beta sh2h\alpha + i\alpha\sin 2h\beta - \beta\sin 2h\beta}{ch2h\alpha + \cos 2h\beta}\right]}. \qquad (31)$$

By designating in (31) the coefficients:

$$a = \frac{h(\alpha sh2\alpha h - \beta\sin 2\beta h)}{ch2\alpha h + \cos 2\beta h};$$

$$b = \frac{h(\alpha\sin 2\beta h + \beta sh2\alpha h)}{ch2\alpha h + \cos 2\beta h}. \qquad (32)$$

Then expression (31) is represented as follows:

$$\frac{m\omega^2 cth(\alpha+i\beta)h}{h(\alpha+i\beta)} = \frac{m\omega^2}{h(a+ib)}. \qquad (33)$$

For convenience, the expression in (29) is denoted by *d*. Then, substituting its value from (33) into (27), dividing the imaginary and real parts, the formula for determining the amplitude of oscillations is obtained:

$$U_x = \frac{F_o}{\left|R - M\omega^2\right|}\sqrt{\frac{(a^2+b^2)d}{\left[a + \dfrac{m\omega^2}{\left|R - M\omega^2\right|}\right]^2 + b^2}}. \qquad (34)$$

It follows from formula (34) that the amplitude of oscillations does not remain constant when the cavitation stage changes. It takes into account the discrete parameters of the cavitation apparatus and the physical and mechanical properties of the material that change during the cavitation process, which are taken into account by the coefficients *a* and *b* (32).

Next, a possible condition for realizing the resonance of the "cavitator-medium" system is determined.

Provided that *d*=1, an expression for the amplitude of the cavitation apparatus oscillations in the zone of contact with the environment without taking into account dissipation is obtained:

$$A=\frac{A_0}{\frac{m}{M}\frac{1}{\nu tq\nu}-\frac{1}{\eta}}, \tag{35}$$

where:

$$\nu=\left(\omega/_{c}\right)h. \tag{36}$$

As follows from (35), resonance is possible under condition (37):

$$\frac{m}{M}=\frac{\nu tq\nu}{\eta}. \tag{37}$$

For some values of h and η=1, the environment can be considered as a discrete system. Then, based on the resonance condition (37), it is possible to determine approximately the natural frequency of the system by first expanding *tqv* into a series:

$$tq\nu=\nu+\frac{1}{3}\nu^3+\frac{1}{5}\nu^5+\ldots\ldots \tag{38}$$

Taking into account (38), ratio (36) is written in the form:

$$\nu=\sqrt{\frac{(\rho Sh)/M}{1+(\rho Sh)/3M}}. \tag{39}$$

Then the expression for determining the natural frequency of the system "cavitator - medium" will take the form:

$$\omega_{c.m.}=\sqrt{\frac{ES}{h(M+\rho Sh)/3}}. \tag{40}$$

The dependence for determining the amplitude of oscillations is as follows:

$$A=A_0\frac{\omega^2}{\omega_{ci}^2-\omega^2}. \tag{41}$$

The determination of dynamic stress in the medium according to the law of wave motion is carried out (2):

$$\sigma_{dym}=E\varepsilon(1+i\gamma).$$

Using (3) the derivative is found:

$$\frac{\partial u}{\partial x}=(\alpha+i\beta)\left(Ce^{x(\alpha+i\beta)}+Be^{x(\alpha+i\beta)}\right). \tag{42}$$

By substituting their expressions instead of C and B and carrying out the necessary transformations, an expression is obtained that makes it possible to calculate the dynamic pressure in any layer of the medium:

$$\sigma_{dyn}=\rho c^2A\cdot\sqrt{\left(\alpha^2+\beta^2\right)d\cdot d_1}, \tag{43}$$

where d_1 is the wave factor:

$$d_1=\frac{ch2\alpha(x-h)+\cos2\beta(x-h)}{ch2\alpha h-\cos2\beta h}. \tag{44}$$

In the contact zone, where x=0, d_1=1, expression (43) determines the resistance of the medium and the equality of the dynamic pressure to the stress on the contact:

$$\sigma_{dyn}=p_k=\rho c^2A\cdot\sqrt{\left(\alpha^2+\beta^2\right)d}. \tag{45}$$

The uniqueness of formula (45) is in the fact that it takes into account both discrete and continual parameters.

The total value of the pressure, taking into account the static action, can be represented by the expression:

$$\sum p_p=p_{st}+p_{dyn}. \tag{46}$$

where:

$$p_{st}=\frac{M}{S}g+\rho gh_i, \tag{47}$$

where M is the mass of the cavitator emitter, which exerts pressure; h_i is the height measured from the contact zone to the point where the pressure is determined.

5. 2. Development of an algorithm for calculating the parameters of the cavitation apparatus taking into account the influence of the processing material

The resulting expressions (27) and (34) make it possible to calculate the amplitude of oscillations, both in the medium and in the contact zone, or to assign the necessary parameters of the cavitation process based on the given amplitude of oscillations. With the help of formulas (27) and (34), it is possible to determine the zones of amplification or attenuation of the amplitude of oscillations for different frequencies of oscillations, as well as to calculate pressure (43) in any layer of the technological environment.

Calculations were performed using the MATLAB R2023b integrated system. A computer experiment was carried out with a preliminary calculation according to the algorithm (Fig. 6). The essence of the algorithm is the ability to vary not only the initial parameters and the layout of the cavitator relative to the processing environment. It also makes it possible to determine the influence of variable parameters on the maximum value of a particular criterion. Depending on the statement of the problem, a criterion is selected, which is fixed in block 8. Further, in block 2 "Formation of the technological scheme", preliminary calculations are carried out regarding the mode of energy transfer to the environment in accordance with the selected criterion. Blocks 3 and 5, which define the physical and mathematical models, are an important stage of the algorithm. In blocks 4, 6, 7, output data is formed to determine the numerical values of the influence parameters and the limits of their rational use. Completion of the calculation are parameters that serve as initial information for final decision-making on the determination of rational design and technological parameters of the acoustic cavitator. At the same time, it was assumed that the synergy coefficient of the system serves as a condition for the efficiency of the material processing process and is defined as the ratio of the bursting energy of the bubbles to the energy of the primary acoustic wave:

$$k_s=E_s/E_{p.w.}\geq\max, \tag{48}$$

where k_s is the synergy coefficient of the system (the coefficient of cavitation use of acoustic energy); E_s is the energy for the bursting of bubbles (the energy spent on the formation of cavitation in a unit volume of the medium); $E_{p.w.}$ is the density of the added energy of the primary wave.

A modal analysis (Modes Analysis) was carried out, which allowed us to determine the main forms of oscillations and values of frequencies and their corresponding parameters using the integrated system MATLAB R2023b. The methodology for performing the experiment provided for changing the parameters listed in Table 1.

The limits of the numerical values of the initial data were determined taking into account the composition of materials covering a wide range of viscosities of different real environments, the stage nature of the process, and the range of changes in the frequency of oscillations of the acoustic apparatus. This approach is determined by existing data, which made it possible to solve the tasks of this work. The input parameters also include the bubble radius, the value of which was taken depending on the stages of cavitation: initial (single-phase environment), nucleation, development, bursting (two-phase environment). The calculated parameters were as follows: oscillation period, T, s; angular frequency of oscillations, ω, rad/s; amplitude of oscillations, X, m; speed, V, m/s; acceleration a, m/s^2; pressure, P, Pa.

Fig. 7 shows the program window and charts of changes in the amplitude of oscillation, speed, acceleration, sound pressure, and intensity.

Such a wide range of parameters made it possible to establish patterns of changes in key parameters, which are synthesized in the form of changes in the parameters shown in the figure. Analysis of the resulting charts (Fig. 7) showed the following.

The confirmed wave distribution of the amplitude of oscillations and pressure at the stage of development of the material processing process, as the main stage of formation of the maximum volume of acoustic processing of the material according to criterion (48). That is, the need to consider the technological environment as a system with distributed parameters is confirmed. The law of change of speed and pressure was revealed, and the influence of speed on the nature and magnitude of pressure was determined. It was determined that for the formation of the maximum area (cluster) of bursting bubbles, it is necessary to have the following parameters: amplitude of oscillations within 4.0…20.0 μm, sound pressure value within 5.0…30.0105 Pa. Calculating according to formulas (37) and (40) using the resonance mode, it is possible to obtain the amplification factor of the amplitude of oscillations by 5…7 times, which makes it possible to reduce the energy by 30 % for the execution of the technological process and ensure the stable mode of operation of the acoustic installation.

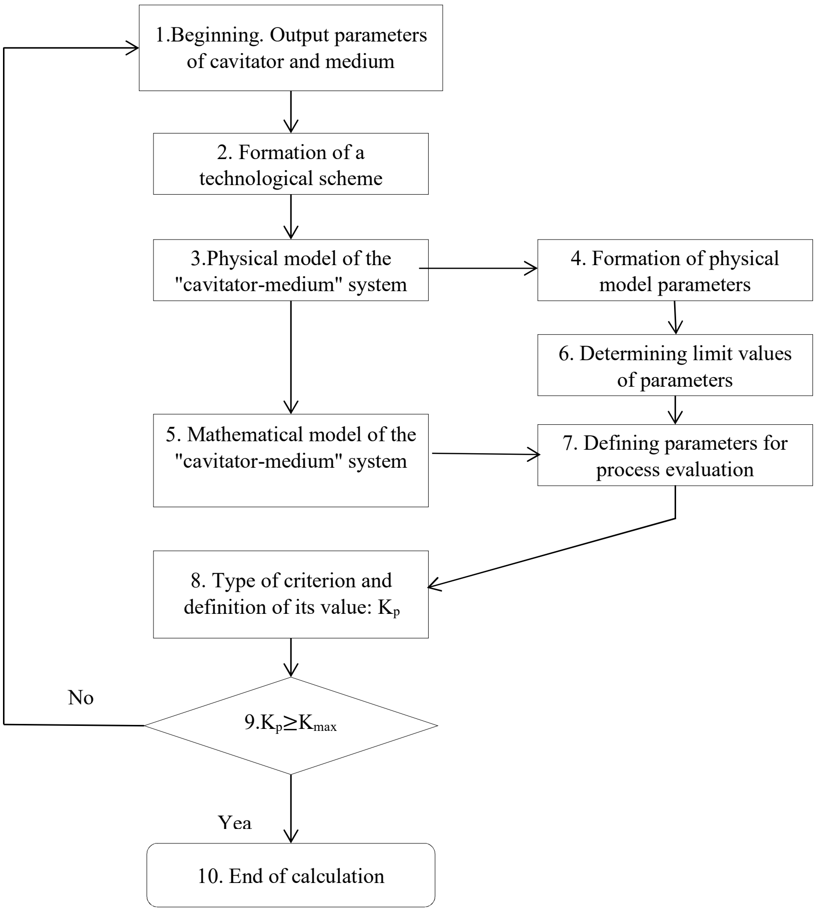
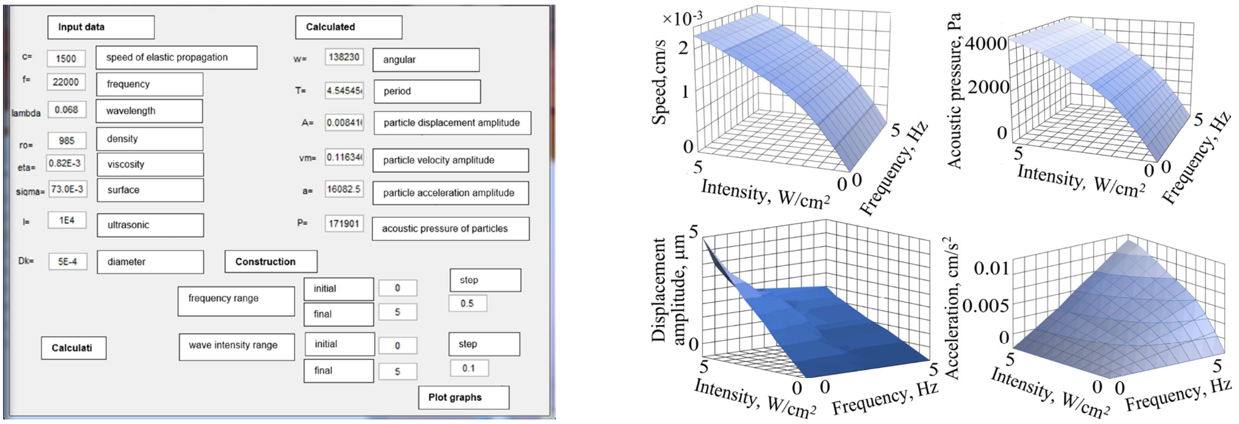


Fig. 6. Calculation algorithm for assessing the reliability of accepted physical and mathematical models



Input data

c= 1500 speed of elastic propagation
f= 22000 frequency
lambda 0.068 wavelength
ro= 985 density
eta= 0.82E-3 viscosity
sigma= 73.0E-3 surface
I= 1E4 ultrasonic
Dk= 5E-4 diameter

Calculated

w= 138230 angular
T= 4.54545 period
A= 0.00841 particle displacement amplitude
vm= 0.11634 particle velocity amplitude
a= 16082.5 particle acceleration amplitude
P= 171901 acoustic pressure of particles

Construction

frequency range initial 0 final 5 step 0.5
wave intensity range initial 0 final 5 step 0.1

a

b

Fig. 7. Results of our study: *a* – window of the calculation program; *b* – charts of changes in acoustic parameters

Table 1

Output parameters of the program

Medium density ρ, kg/m^3	Wave speed c, m/s	Oscillation frequency f, kHz	Intensity I, W/cm^2	Viscosity of the medium β, 10^{-3} Pa*s	Surface tension σ, 10^{-3} N/m
1,000–6,000	1,500–2,000	20–40	1.5–50.0	10–200	70–25

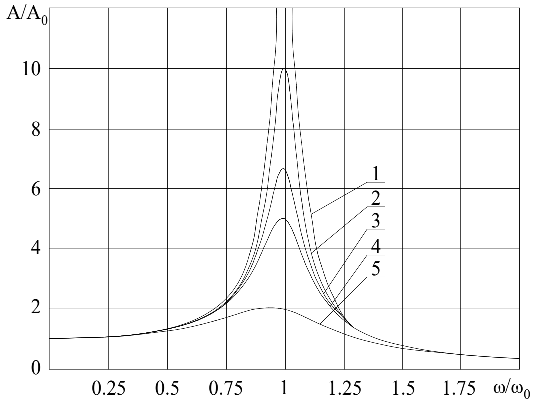


Fig. 8. Amplitude-frequency characteristics of the movement of the "cavitator - environment" system depending on the ratio of forced and natural frequencies when the damping coefficient changes as follows:
1 — 0; 2 — 0.10; 3 — 0.15; 4 — 0.20; 5 — 0.5

Based on the obtained parameter values, the amplitude-frequency characteristic of the movement of the "cavitator-medium" system is constructed (Fig. 8). As follows from the given plot, taking into account the dissipative forces under the operating mode of the acoustic device is a necessary condition. An important result of the given plot is also the definition and implementation of the resonance condition.

6. Discussion of results based on determining the parameters of the cavitation apparatus, taking into account the influence of the processing material.

Our results relating to the determined rational parameters based on taking into account the joint movement of the acoustic apparatus and the technological environment as a single system are reliable. The proof is that the formulas for the amplitude of vibrations of the acoustic apparatus (34) and dynamic pressure (45) take into account the real physical and rheological properties obtained by the analysis of existing technological processes. Using the methods of direct and inverse solution and calculations on the basis of a computer experiment, the verification of the obtained results is ensured. This conclusion is also confirmed by the presented charts and the parameter check using the criteria in the proposed algorithm. Owing to the adopted model, in which the processing material is represented by a continuous medium, in contrast to the cited works [6, 7], in which the material is taken into account according to the empirical coefficient of the so-called "attached mass", it was possible to solve the problem of calculating the parameters of the cavitation apparatus taking into account the influence of the processing material.

A feature of the adopted methodology and research results is the consideration of real discrete and distributed parameters in the proposed formulas. After all, for example, ensuring the operation of the acoustic apparatus under the resonant mode and its stability is due to the simultaneous consideration of the parameters of the acoustic apparatus and the material. The use of generally accepted provisions from the classical theory of mechanical oscillations and the theory of continuous media made it possible to obtain important analytical dependences for this specific study. These provisions are implemented in formulas (34), (37), (43), and (45). These results have a potentially expected effect in terms of determining the zones of amplification of the amplitude of oscillations for different frequencies of oscillations, as well as calculating the pressure. An important result of the research is also for determining the natural frequency of oscillations of the "cavitator - medium" system. This made it possible to obtain the amplification factor of the amplitude of the cavitation installation by 5...7 times by means of calculations according to formula (40) and, as a result, to reduce the energy by 30 % and ensure the stable mode of operation of the acoustic installation.

There are limitations to this study. The model involves considering the joint oscillation of the "cavitator–medium" system in the contact zone. That is, the uninterrupted mode of movement of the medium and the working body of the cavitation apparatus is considered. Such a limitation has a positive part. Thus, at the first initial stage of formation of the cavitation region, it is possible to guarantee the appropriate numerical value of the energy intensity, which is the main criterion for effective treatment of the environment. The components of this energy are the amplitude and frequency of oscillations, the research results are obtained in the work and their functional dependences are shown in the corresponding charts. The obtained values are synthesized in accordance with the specific values of viscosity, which is a key parameter of the modern representation of the classification features of environments. And all this calculation technology is provided by the algorithm developed in the work.

The disadvantage of our work is the lack of formulas for energy assessment of the process, which will be implemented in further research. The proposed model of the "cavitator - environment" system, the obtained results and formulas could be used in the research and calculation of installations in the processes of grinding, sorting materials, and compaction. Although these processes are different from the ones carried out, they have vibration generators and processed material. A similar scientific and practical area of research into the parameters of crushing, sorting, and compacting plants is very important. Such processes are widely used in road construction.

7. Conclusions

1. The rational parameters and modes of operation of the cavitation apparatus, which are the amplitude of oscillations and pressure, have been studied and determined. These parameters take into account input and output properties and allow us to calculate the amplitude of oscillations, both in the medium and in the contact zone for the implementation of stable and resonant modes. The input parameters include the density and viscosity of the medium, the speed of wave propagation, which was previously not taken into account in the empirical dependences. It was established that the amplitude of vibrations of the acoustic device

should be within 4.0...20.0 μm, the sound pressure value within $5.0...30.0 \cdot 10^5$ Pa, depending on the composition and viscosity of the materials. For the resonance mode, the amplification factor of the amplitude of oscillations is obtained by 5...7 times, which allows one to reduce the energy by 30 % and ensure a stable mode of operation of the acoustic installation. Previously, due to the lack of accurate consideration of the above-defined properties of the device and the environment, the resonant mode could not be implemented at all.

2. An algorithm for calculating the parameters of the cavitation apparatus has been developed. It is built on the possibility of variation of the initial parameters to ensure the given value of the determined efficiency criterion, which is the energy of the process. The peculiarity of the proposed algorithm is the structured approach to forming blocks that determine the choice of physical model depending on the stages of cavitation processing. Establishing on this basis the limit values of frequency-dependent or frequency-independent dissipative coefficients of the model is determined by the viscosity scale of the medium, as a parameter of the classification feature. This testifies to the development of a generalized algorithm and the breadth of its application not only within the framework of the cavitation process but also its use in other, structurally similar, dynamic processes.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study, as well as the results reported in this paper.

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Data availability

All data are available, either in numerical or graphical form, in the main text of the manuscript.

Use of artificial intelligence

The authors used artificial intelligence technologies within acceptable limits to provide their own verified data, which is described in the research methodology section.

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