This study focuses on the processes occurring within the cooling channels of a liquid rocket engine chamber. It is important to note that, from a design-stage efficiency prediction perspective, the cooling system is the most critical part of the engine chamber. This is because it cannot be tested without costly and labor-intensive fire tests. Therefore, mathematical models of heat transfer and fluid dynamics must describe accurately all processes taking place in the chamber.

The study emphasizes accounting for the change in the density of the propellant component within the cooling channels. To verify the importance of this issue, an analysis was conducted on the changing parameters of the propellant components in the cooling channels of an engine. The assessment revealed that even when using high-boiling propellant components and moderate heating in the cooling channels, density changes can exceed 25 %.

This paper presents the results of developing a model for the flow of propellant in the cooling channels of a liquid rocket engine chamber, taking density changes into account. The model builds on a cooling channel model previously developed by the authors. An equation that accounts for density variations was derived using established principles of fluid mechanics.

Using the developed mathematical model, test calculations were performed, and the simulation results were compared with and without considering density changes. Furthermore, a comparison was conducted with calculated data available in the literature on heat transfer in the RD107 engine chamber, revealing an error margin of no more than 1.5 %.

The resulting mathematical model may be recommended for use in the design of new rocket engine chambers with regenerative cooling

Keywords: liquid propellant rocket engine, mathematical model of cooling channels, propellant density variation

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DEVELOPMENT OF A MATHEMATICAL MODEL FOR THE COOLING CHANNEL OF A LIQUID PROPELLANT ROCKET ENGINE'S CHAMBER WITH RESPECT FOR VARIATIONS IN COOLANT DENSITY

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1. Introduction

Designing a liquid propellant rocket engine (LPRE) presents numerous challenges, many of which stem from the rigid hydraulic and mechanical interconnections between its components. These connections mean that any deviation in the performance of one component can critically impact the entire system's parameters. A notable example of such interdependence is the interaction between the cooling system and the turbopump unit. For instance, if the cooling system operates with insufficient efficiency, it may require increased flow rates, leading to higher pressure losses in the cooling channels. Consequently, this necessitates redesigning the pump to provide greater head. Therefore, obtaining highly accurate calculations of both heat transfer and fluid dynamics in the cooling channels of the LPRE chamber during the design phase is essential. This accuracy not only accelerates the engine development process but also allows for a rational reduction in the design margins typically incorporated by engineers during development.

Accurate modeling of hydrodynamics is crucial for calculating the cooling of the engine chamber, requiring consideration of several factors. Key factors include the surface roughness of the channel walls, the transverse and longitudinal shapes of the channels, deviations in the geometric parameters of the channels during manufacturing, and changes in the coolant properties along the channel. The last factor is particularly significant not only when cryogenic coolants are used but also for engines with high-boiling fuel components. For example, when using the widely used high-boiling propellant pair – nitrogen tetroxide and unsymmetrical dimethylhydrazine – the density change in the cooling channels can reach 15–25 % even with moderate heating of the propellant (100–120 K) [1, 2]. Accounting for this physical phenomenon enables the calculation of the most efficient coolant velocity distribution in the channels, optimizing cooling performance.

It is important to emphasize that a detailed analysis of the processes occurring in the cooling channels of LPRE chambers requires the use of advanced heat transfer models. This necessity arises because traditional approaches, which rely on dividing the computational domain into a finite number of sections, have become outdated and lack the accuracy of more modern methods [3]. Leveraging the significant advancements in computational technology in recent years enables the development of more sophisticated mathematical models. This is supported by studies [4–6], which demonstrate that applying modern methods and approaches to the analysis of processes in rocket engines enhances the quality and reliability of the results.

Thus, developing a mathematical model that accounts for the density changes of the propellant component as it heats up in the cooling channels of the engine chamber is a highly relevant task.

2. Literature review and problem statement

A review of recent research was conducted to assess the current state of development in heat transfer calculation methods. Special attention was given to how these studies address the change in coolant density during heating. Given the ongoing trend of using methane as a highly efficient fuel, an increasing number of researchers are exploring its potential as a coolant and studying its specific characteristics in detail. For example, in studies [7, 8], one-dimensional mathematical models of heat transfer for the analysis of regenerative cooling in chambers using methane are presented. These models rely on empirical equations, such as the Dittus-Bolter correlation for determining the heat transfer coefficient of the coolant and the Bartz correlation for calculating the heat flux into the chamber wall. Both studies apply the well-known fin theory [9], with the calculations performed by dividing the computational domain into a finite number of discrete sections. Since the properties of cryogenic methane vary significantly with temperature and pressure, all the necessary parameters of the coolant in each design section are determined using NIST REFPROP [10]. This approach requires the use of tabular functions or the integration of third-party programs into the mathematical model.

In the work [11], which focuses on the development of an expander-cycle scheme for a rocket engine using an electric battery, the temperature of methane gasified in the cooling channels for the turbine drive, as well as pressure losses in the cooling channels, were calculated using criterion correlations that account for the density of both the liquid and gas phases of methane. However, for a more detailed analysis of processes in channels with significant changes in the density of the component, a mathematical model based on the fundamental laws of fluid mechanics is required.

In the case of using another cryogenic fluid, such as nitrogen, as a coolant, the NIST REFPROP database is also used in [12] to determine the fluid's physical properties, which is particularly important when the fluid transitions to a supercritical state. The study simulated a cooling system with spiral channels using a simple one-dimensional method based on the traditional semi-empirical correlation, adjusted for the flow rate. To validate the proposed one-dimensional model, it was compared with results from a three-dimensional numerical simulation. The method was then applied to analyze the temperature distribution of the inner wall and optimize the channel parameters.

The process of optimizing the cooling channel is also presented in [13]. An iterative search algorithm was employed to determine the optimal geometry of the cooling channel. Two criteria for optimization were considered. In the first case, the goal was to minimize thermal resistance, which resulted in a lower wall temperature for a given pressure drop. In the second case, the objective was to minimize the pressure drop while keeping the wall temperature below the permissible limit. However, due to the use of an iterative algorithm and the division of the channel into a finite number of sections, the calculation process became significantly more complex.

In the case of using a high-boiling propellant component, such as kerosene, as a coolant, the temperature dependence of the component's properties is also considered using NIST SUPERTRAPP [14], as demonstrated in [15]. This paper investigates the possibility of combining conjugate heat transfer and variational calculus to design optimal cooling channels. A simplified model of the cooling channel is used to determine the optimal geometric parameters that minimize the average temperature, temperature inhomogeneity, and pressure drop simultaneously.

In [16], a simplified model of the steady thermal state of liquid propellant engine chambers with regenerative cooling is presented. The model is based on semi-empirical correlations for convective heat transfer from the combustion products side, with coefficients calibrated using experimental data from fire tests of a water-cooled nozzle. The study also employs an original multi-zone approach to model thermal conductivity in the walls and ribs of the engine chamber. The work notes that the NIST REFPROP program was used to determine the fluid properties, necessitating either its integration into the calculation algorithm or the use of tabular functions.

It is worth highlighting the article [17], which describes a model conceptually similar to the one presented in this work. Differential equations in one-dimensional form were used to describe the processes in the engine chamber. The model was implemented in the EcosimPro system, which was then used to obtain numerical solutions. The resulting mathematical model was applied to predict the results of hydraulic and fire tests of the engine chamber, demonstrating high accuracy. While the calculations do not account for changes in the coolant's density, since water was used as the coolant in the fire tests and its density can be considered constant at low heating, no significant calculation errors were observed.

Another approach to calculating heat transfer in LPRE chambers is CFD modeling. For example, in article [18], a unified framework based on the Navier-Stokes equations was developed to simulate the flow of combustion products within the engine chamber. Empirical Nusselt correlations were used to calculate the heat transfer of the fluid, while the coolant properties across its entire range of thermodynamic states were determined using NIST SUPERTRAPP. This, in turn, necessitates the use of tabular functions or the integration of external software solutions into the mathematical model.

In [19], the possibility of calculating conjugate heat transfer using open-source tools such as OpenFOAM, CalculiX, and preCICE is explored. The study focused on a helium heat exchanger for tank pressurization located on the exhaust pipe of a liquid propellant turbine, but the calculation algorithm presented can also be applied to the engine chamber. The properties of the coolants were provided as tabular functions. Unlike [18], where empirical models were used, the flow of both fluids in this study was modeled using CFD, without relying on empirical models.

A similar approach is used in [20], where CFD modeling is employed to describe both the processes occurring in the combustion chamber and the coolant flow in the engine's cooling channels. However, to simplify the calculations, the coolant is treated as an incompressible fluid in this study.

Commercial computational software, such as Ansys Fluent, are also used for cooling calculations. For example, in [21], the simulation of kerosene flow in a rectangular cooling channel of a ramjet engine is presented. The modified Benedict-Webb-Rubin equation of state was used to determine the coolant's density. Special attention is given to kerosene pyrolysis, which is also relevant for liquid propellant rocket engines.

Based on the results of the literature review [7, 8, 11–13, 15–21], it can be concluded that modern heat transfer calculation methods, in addition to CFD modeling, often employ an approach that divides the one-dimensional calculation domain into a finite number of sections. It was also found that tabular functions obtained using the NIST REFPROP software package are used to account for changes in density. However, this approach to modeling engine chamber cooling is inconvenient to implement and can significantly lose accuracy, especially if the number of calculation sections is not properly selected. Additionally, it is difficult to standardize this approach for modeling processes in different units with varying working environments.

3. The aim and objectives of the study

The aim of this study is to develop a mathematical model for the cooling channels of a liquid rocket engine chamber, accounting for changes in coolant density during heating. This model will enable the simultaneous solution of heat transfer and hydrodynamics problems, thereby simplifying the process of determining the most efficient geometric parameters for cooling channels during parametric heat transfer calculations. Additionally, the developed model should be easy to implement and should not require integration with external programs.

To achieve the study's objective, the following tasks were set:

– to derive a system of equations for the mathematical model of the fluid in the rocket engine's cooling channels, accounting for changes in fluid density;

– to validate the obtained mathematical model.

4. The study materials and methods

This work focuses on the processes occurring in the cooling channels of liquid rocket engine chambers. For their modeling, a mathematical model was previously developed [3], which is represented by the following system of equations:

$$\begin{cases} \dfrac{d(\rho u f)}{dx}=0; \\ \dfrac{d\left(f\left(p+\rho u^2\right)\right)}{dx}=p\dfrac{df}{dx}-\dfrac{\lambda}{8}\rho u^2\sqrt{P^2-\left(\dfrac{df}{dx}\right)^2}; \\ \dfrac{d\left(uf\left(p+\rho cT+\dfrac{\rho u^2}{2}\right)\right)}{dx}+(pu)\dfrac{df}{dx}= \\ =\dfrac{\lambda}{8}\rho u^3\sqrt{P^2-\left(\dfrac{df}{dx}\right)^2}+\alpha(T_w-T)P; \end{cases} \quad (1)$$

where x – longitudinal coordinate along the channel axis, with the origin located at the coolant supply point; ρ – coolant density; f – channel cross-sectional area; u – coolant velocity; p – static pressure; λ – empirical coefficient of friction loss; P – channel perimeter; c – heat capacity of the coolant; T – coolant temperature; α – heat transfer coefficient; T_w – wall temperature. In this version of the mathematical model u, p, T – functions of the x-coordinate, while the component density ρ is constant.

The model describes the processes in the cooling channel under the following constraints and assumptions:

– the length of the channel is much greater than its transverse dimension;

– the channel area may vary;

– the radius of curvature of the channel is much larger than its transverse dimension.

The equation that accounts for the change in the component's density was derived from the definition of the volumetric thermal expansion coefficient [22]:

$$\beta=\frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_p, \quad (2)$$

where v – the specific volume of the liquid.

The coolants considered in this work are droplet liquids, whose density changes only slightly under the influence of pressure forces. This allows the assumption of constant pressure to be neglected in the equation.

The component density calculated using the differential model was compared with density data obtained from the NIST REFPROP software package. To obtain a numerical solution for the differential model, the fourth-order Runge-Kutta method was employed, using the *NDSolve* function from the Wolfram Mathematica software package.

It is anticipated that accounting for density changes during the heating of the component will improve the accuracy of heat transfer calculations in engine chambers, compared to models that assume a constant coolant density.

5. Results of the development of the cooling channels mathematical model

5. 1. Derivation of the system of equations for the cooling channels mathematical model

To account for the change in density with temperature in the existing differential model, it is necessary to supplement it with an equation of the following form:

$$\rho=f(T).$$

To achieve this, in expression (2), both sides of the equation were multiplied by the derivative of temperature with respect to the coordinate. After some simplifications, the equation takes the form:

$$\beta\frac{dT}{dx}=\frac{1}{v}\frac{dv}{dx}.$$

By applying the differentiation rules and using the equation of state, an equation of the following form was obtained:

$$\frac{d\left(\ln\left(\rho^{-1}\right)\right)}{dx}=\beta\frac{dT}{dx}.$$

Using the properties of logarithms, the negative exponent can be eliminated:

$$\frac{d(\ln(\rho))}{dx}=-\beta\frac{dT}{dx}. \quad (3)$$

In this form, the equation for the change in the density of a component due to heating will be used in the differential model. With a constant or slightly varying volumetric expansion coefficient, equation (3) can be integrated as follows:

$$\int\frac{d(\ln(\rho))}{dx}=-\beta\int\frac{dT}{dx},$$

this leads to the result:

$$\ln(\rho) = -\beta T + C.$$

By applying the properties of logarithms:

$$\rho = C\exp(-\beta T).$$

Considering the initial conditions:

$$x = 0 \to \rho = \rho_0, T = T_0,$$

finally, the equation can be written as:

$$\rho = \rho_0 \exp(\beta(T_0 - T)). \quad (4)$$

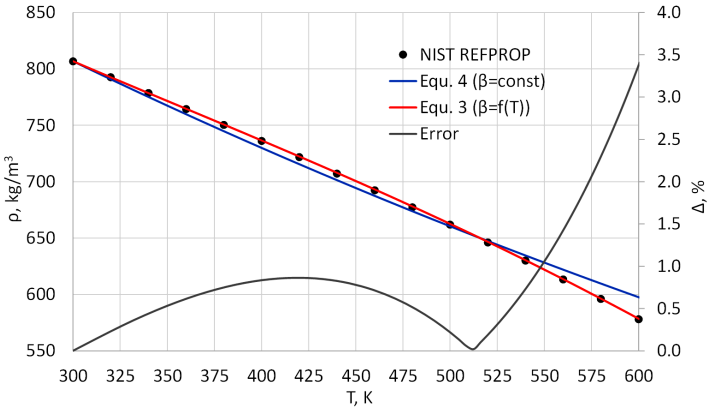

Fig. 1. Kerosene density determined using different methods

Using equations (3) and (4), it is possible to determine the change in density with temperature and compare the results with the density data obtained using the NIST program. The comparison was made using the example of the density change of RP-1 kerosene, one of the most commonly used coolants in LRPE. In equation (3), the volumetric expansion coefficient was expressed as a function of temperature. To achieve this, the values of the coefficient at different temperatures, obtained from the NIST program, were approximated using a second-degree polynomial:

$$\beta = 1.58\cdot10^{-3} - 4.58\cdot10^{-6}T + 7.57\cdot10^{-9}T^2. \quad (5)$$

In equation (4), the coefficient of volumetric expansion was assumed to be constant. It was obtained by averaging equation (5) over a specified range of liquid temperature changes and is equal to β=0.00111 in this work.

For the results obtained using equation (4), the relative deviation (Δ) was calculated. The comparison results are shown in Fig. 1.

After confirming the correctness of the obtained equation, it was integrated into the existing system of equations (1). The new system of equations takes the following form:

$$\begin{cases} \dfrac{d(\rho u f)}{dx} = 0; \\ \dfrac{d\left(f\left(p+\rho u^2\right)\right)}{dx} = p\dfrac{df}{dx} - \dfrac{\lambda}{8}\rho u^2\sqrt{P^2 - \left(\dfrac{df}{dx}\right)^2}; \\ \dfrac{d\left(uf\left(p+\rho cT+\dfrac{\rho u^2}{2}\right)\right)}{dx} + (pu)\dfrac{df}{dx} = \\ = \dfrac{\lambda}{8}\rho u^3\sqrt{P^2 - \left(\dfrac{df}{dx}\right)^2} + \alpha(T_w - T)P; \\ \dfrac{d(\ln(\rho))}{dx} = -\beta\dfrac{dT}{dx}; \end{cases} \quad (6)$$

and is closed with the following boundary conditions:

$$p(0) = p_0; T(0) = T_0; u(0) = u_0; \rho(0) = \rho_0.$$

A key distinction of the newly obtained mathematical model from the system of equations (1) is that, in system (6), the density is the unknown variable and a function of the coordinate x.

5. 2. Verification of the developed mathematical model

The obtained differential model was verified by comparing the simulation results with the calculated data for the RD107 engine [23] presented in the literature. The entire engine chamber cooling system was considered, excluding the final section of the nozzle.

To achieve this, some modifications were made to system (6): the variable f in the energy conservation and momentum conservation equations was replaced by the expression (f/n) (where n is the number of cooling channels) for simulation the multi-channel system.

Additionally, it is necessary to account for the change in the coefficient of friction losses for correct simulation the pressure losses in the channels with variable geometric parameters. The Colebrook-White correlation [24] is typically used in the analysis of pressure losses, including in channels similar to the LPRE chamber channels [25]. However, based on the results of a previous analysis, it was found that, for this particular case, the flow regime is self-similar with respect to the Reynolds number. This finding allowed for simplification of the equation and the derivation of an explicit expression for the dependence of the coefficient of friction losses:

$$\lambda = \frac{1}{\left(2\lg\left(\dfrac{3.7}{\overline{\Delta}}\right)\right)^2},$$

where $\overline{\Delta}$ – relative roughness. Since the absolute roughness of the channel walls (Δ) can be considered constant along the length of the chamber, it is possible to write:

$$\lambda = f(\overline{\Delta}) = f\left(\frac{\Delta}{d_h}\right) = f(d_h),$$

where d_h – the hydraulic diameter of the channel.

Also, to determine the heat input, the term in the energy conservation equation $\alpha(T_w - T)P$ was replaced by the expression:

$$\frac{qP_{ch}}{n},$$

where q – the specific heat flux given in the literature, and P_{ch} – the internal perimeter of the engine chamber.

Equation (5) was used to determine the volumetric expansion coefficient. The system of equations used to perform the calculation is as follows:

$$\begin{cases} \dfrac{d(\rho u f)}{dx}=0; \\ \dfrac{d\left((f/n)\left(p+\rho u^2\right)\right)}{dx}= \\ =p\dfrac{d(f/n)}{dx}-\dfrac{\lambda(d_h)}{8}\rho u^2\sqrt{P^2-\left(\dfrac{d(f/n)}{dx}\right)^2}; \\ \dfrac{d\left(u(f/n)\left(p+\rho cT+\dfrac{\rho u^2}{2}\right)\right)}{dx}+(pu)\dfrac{d(f/n)}{dx}= \\ =\dfrac{\lambda(d_h)}{8}\rho u^3\sqrt{P^2-\left(\dfrac{d(f/n)}{dx}\right)^2}+\dfrac{qP_{ch}}{n}; \\ \dfrac{d(\ln(\rho))}{dx}=-\beta\dfrac{dT}{dx}; \end{cases}$$

Calculations were performed both with and without considering the change in density. The comparison results are shown in Fig. 2–4. For easier comparison with the available heat transfer data for the RD-107 engine chamber [25], the curves in the graphs were inverted.

The obtained results show satisfactory agreement with data from the literature (Table 1) and highlight the importance of accounting for changes in the density of the liquid in the cooling channels.

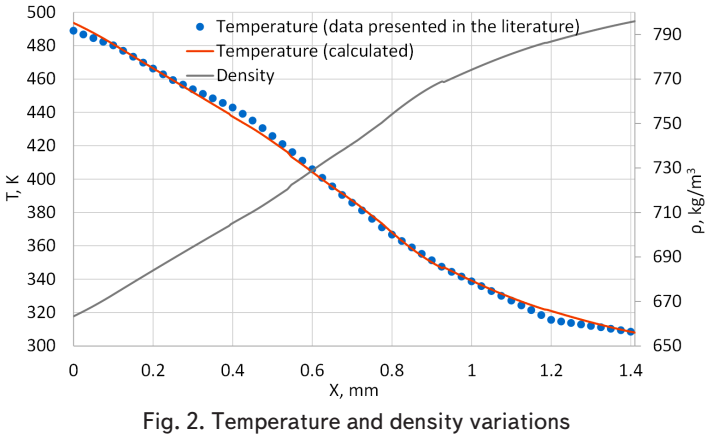

Fig. 2. Temperature and density variations in the cooling channels

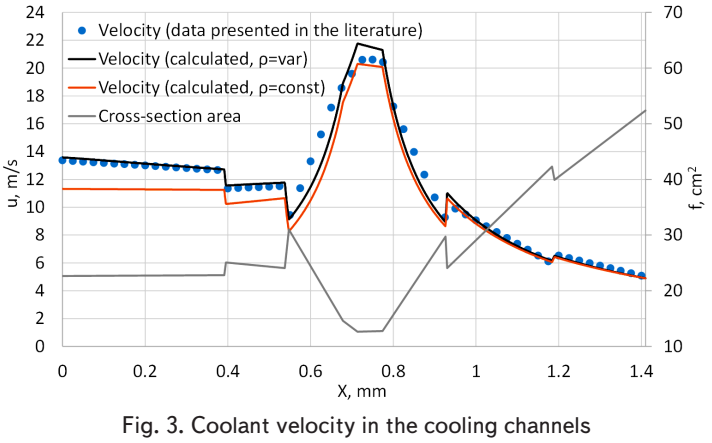

Fig. 3. Coolant velocity in the cooling channels

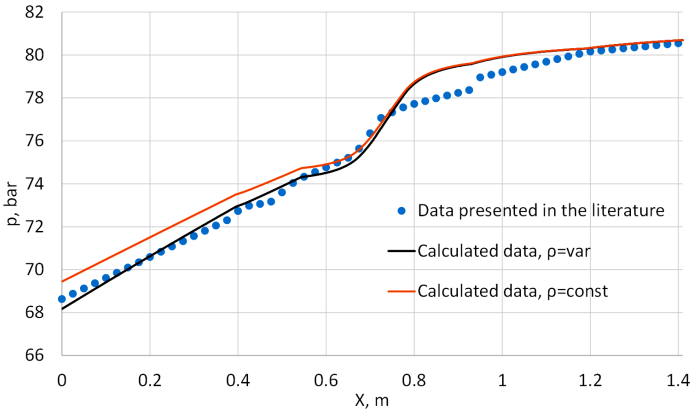

Fig. 4. Total pressure in the cooling channels

Table 1

Comparison of the obtained results with data from the literature

Parameter	Result deviation	
	Without density change	With density change
Outlet cooler velocity	–15 %	+1.5 %
Outlet total pressure	+1.2 %	–0.7 %
Outlet cooler temperature	+1.0 %	

6. Discussion of the results from the development of the cooling system mathematical model

This study focuses on enhancing the existing mathematical model of the cooling channels by incorporating the effects of fuel component density variation.

The derived equations (3) and (4) were validated through comparison with data obtained using the NIST REFPROP program (Fig. 1). The comparison results confirm the accuracy of the derived equations. Additionally, it is demonstrated that using equation (4) with a constant thermal expansion coefficient enables density estimation with an error of no more than 4 %, even under significant temperature variations in the liquid.

The test calculations demonstrate acceptable convergence for the developed model. The correctness of the proposed approach and the importance of accounting for density variation are illustrated in the graphs showing parameter changes along the length of the chamber (Fig. 2–4). This is particularly evident when comparing the velocity in the cooling channels of the cylindrical section of the engine chamber (0–0.4 m, Fig. 3). The velocity predicted by the new model aligns closely with both the values and trends reported in the literature. The uniform increase in velocity within a constant cross-sectional area is attributed to the decrease in coolant density, as confirmed by the new model. In contrast, calculations that neglect density changes during heating significantly deviate from the literature values. Specifically, Fig. 4 reveals that the pressure drop in the cooling channels, calculated without considering

density variation, is underestimated by over 10 % compared to the results obtained using the new model.

Unlike studies [7, 8, 11–13, 15, 16], where the calculation domain was divided into a finite number of sections, this work presents a mathematical model of the cooling channels as a system of differential equations. This approach reduces discretization errors and simplifies calculations due to the availability of existing numerical methods for implementation.

Furthermore, in contrast to works [7, 8, 12, 15, 18, 19], which rely on pre-existing tabular data to determine density or involve integration with external programs, the developed mathematical model incorporates a functional dependency. This allows for a more streamlined cooling calculation process and improves its accuracy.

Thus, the developed mathematical model offers an alternative to existing methods for calculating engine chamber cooling. It enables efficient and comprehensive heat transfer and hydraulic calculations without the need for significant computational resources. Moreover, accounting for density changes enhances calculation accuracy without complicating the algorithm. This, in turn, facilitates the development of more advanced cooling channels designs during the design phase, ultimately reducing the need for expensive fire tests.

Since the density of cryogenic fuel components is significantly affected by pressure even with slight heating, the practical application of this model is limited to high-boiling coolants. A drawback of the proposed method is the assumption of uniform parameter distribution across the channel. A significant temperature gradient (and consequently, a density gradient) within the channel cross-section could lead to inaccuracies in the heat transfer and hydraulics calculations.

In the future, the presented mathematical model will be modified to account for pressure losses resulting from overcoming local resistances in the engine chamber's cooling channels.

7. Conclusions

1. In this study, a new differential model for the cooling system of the rocket engine chamber was developed, incorporating the effects of density variation in the coolant. To achieve this, equations enabling the calculation of density changes with varying liquid temperatures were derived using the volumetric expansion coefficient. The derived equations were validated by comparing the results with data from NIST REFPROP.

2. Test calculations were performed using the developed mathematical model. Several modifications were made to adapt the model for calculations based on the initial data of the RD-107 engine chamber. These included accounting for the multi-channel design of the cooling system, a variable friction loss coefficient, and a specified heat flux into the chamber wall. A comparison of the results with existing data shows that the calculation error does not exceed 1.5 %.

Conflict of interest

The authors declare that they have no conflicts of interest, in relation to the current study, including financial, personal, authorship, or any other, that could affect the study and the results reported in this paper.

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Data availability

All data are available in the main text of the manuscript.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the presented work.

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