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DETERMINING THE DEFORMATION OF AN ABSOLUTELY ELASTIC AXIS OF CURVED RODS UNDER BENDING

Serhii Pylypaka
Doctor of Technical Sciences, Professor, Head of Department*
Victor Nesvidomin
Doctor of Technical Sciences, Professor*
Tetiana Volina
Corresponding author
PhD, Associate Professor*
E-mail: volina@nubip.edu.ua
Iryna Hryshchenko
PhD, Associate Professor*
Taras Pylypaka
PhD, Associate Professor
Department of Agricultural Engineering
National University of Water and Environmental Engineering
Soborna str., 11, Rivne, Ukraine, 33017
Yaroslav Kremets
PhD, Researcher
Research Department for the Development of Military Specialist Training Systems at the Research Center
Zhytomyr Military Institute named after S. P. Koroliov
Myru ave., 22, Zhytomyr, Ukraine, 10004
Serhii Andrukh
PhD, Associate Professor
Department of Architecture and Engineering Surveying***
Oleksandr Savoiskyi
PhD, Head of Department**
Larysa Korzh-Usenko
Doctor of Pedagogical Sciences, Professor
Department of Management of Education and Pedagogy of High School
Sumy State Pedagogical University named after A.S. Makarenko
Romenska str., 87, Sumy, Ukraine, 40002
Yuriy Semirnenko
PhD, Associate Professor**
*Department of Descriptive Geometry, Computer Graphics and Design
National University of Life and Environmental Sciences of Ukraine
Heroyiv Oborony str., 15, Kyiv, Ukraine, 03041
Department of Transport Technologies*
***Sumy National Agrarian University
Herasyma Kondratieva str, 160, Sumy, Ukraine, 40021

The object of this study is the deformation of an elastic axis with a large deflection of a cantilever clamped absolutely elastic rod under the action of an applied concentrated force. The rod in the free state can have a rectilinear or curved elastic axis. This fact implies a difference in the analytical description of the bending process. However, there is a factor by which some similarity can be found between the bending of rectilinear and curved rods. This factor is the curvature of the elastic axis of the rod in a free state. According to this feature, they can be divided into rods of constant and variable curvature of the elastic axis. The former include rectilinear rods and those that in the free state have the shape of an arc of a circle, and the latter – rods with a variable curvature of the elastic axis. There is a difference between the bending of these groups of rods: in the first case, the deformation of the elastic axis of the rod during its bending will be the same regardless of which end will be cantilever pinched.

A distinctive feature of the current research is that the bending of rods with variable curvature of the elastic axis was carried out by alternate pinching of their opposite ends. Moreover, the rods of constant and variable curvature were of the same length s=0.314 m, the same cross-section of 0.005×0.02 m. That has made it possible to visually show the difference between the shape of the elastic axis of the bent rod under the action of the same force when the pinch end is changed. When attached to the rods of the working bodies of agricultural machines, pulsating dynamic loads are smoothed out due to their elasticity. It is important for practice to be able to calculate the value of their deviation, which should be within the given limits.

The results are explained by the fact that in the analytical description of the shape of the elastic axis of a curved rod, a technique was proposed in which the length of the axis can start counting both from one end and from the opposite end

Keywords: arc length, concentrated force, pulsating loads, axis curvature, cantilever fastening

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1. Introduction

Absolutely elastic rods are used in various mechanisms and devices. After the termination of the applied force, they acquire their original shape. They can perform the function of switches with sufficient force acting on them, in measuring devices, in the automotive industry in suspension systems, etc. They play a particularly important role in agricultural machines, which generate vibrations and pulsating loads [1, 2]. Owing to elastic elements, the stability of the unit's operation is achieved, its oscillations are reduced, and damping from vibration and shocks that occur during operation is provided [3, 4].

The elastic axis of rods in the free state can be straight or curved. Accordingly, the analytical description of their deformation under the action of the applied force will differ. However, there is a certain similarity between the bending of rods with a rectilinear elastic axis in the free state and a curved one in the form of an arc of a circle. When changing the pinching ends of such rods, their shape after deformation with the same force will also be the same. It will be different for curved rods of variable curvature. This means that when the pinching end is changed under the action of the same force, the shape of the elastic axis will be different, so the deflections will also be of different magnitudes. Bending of absolutely elastic rods does not take this fact into account; however, it is important in the operation of such elements. Therefore, it is a relevant task to consider general approaches to bending absolutely elastic rods and the peculiarities of bending rods of variable curvature.

2. Literature review and problem statement

In [5], a mechanical model of an elastic rod fixed at the base and placed in a viscoelastic medium is proposed. The rod is subjected to an axial force and a local horizontal load distributed along it. This study provides a deep theoretical basis for the analysis of the displacement of elastic rods in a viscoelastic medium. However, in the study, only the rectilinear axis of the rod is taken into account, which limits the application of the model to cases with complex geometry.

Study [6] considered the process of bending a strip with an initial curvature to a spiral shape. Similar to [5], the strip is fixed at the lower point, and a certain force acts on its upper end. The authors note that as the applied force and strip length increase, the amount of deflection increases, and the strip can take on a spiral shape. However, the study focused exclusively on this strip shape, without considering other possible configurations.

Questions related to the complex analysis of the deformation of rods and strips with initial curvature remain unresolved. The likely reason is difficulties associated with the complexity of mathematical modeling of such problems, as well as the expense part, which makes experimental studies impractical. The construction of an integrated model that takes into account the initial curvature of the rod may be an option for overcoming related difficulties.

When bending rods with a large deflection, nonlinear bending theory is used to find the shape of the elastic axis. The action of the forces applied to the rod can be of various nature, accordingly different approaches to solving such problems are devised. For example, work [7] reports analysis of geometrically nonlinear behavior of composite laminated beams with large displacements and post-critical state. Questions related to the adaptation of similar models for practical engineering tasks remain unresolved. The likely reason is the significant computational costs required to solve the nonlinear equations describing such problems, as well as the difficulty of simplifying the models without losing their accuracy. An option to overcome related difficulties may be the construction of simplified models or numerical algorithms that reduce the computational complexity without significantly reducing the accuracy of the results.

Study [8] reports experimental and theoretical analysis of the large deflection of fixed curved beams subjected to bending and tensile loads under various conditions of fixation. Specimens are fixed with different torque values and arranged in vertically concave and convex orientations, which creates different combinations of bending and plane loads. Unsolved questions remain regarding the universalization of models for analyzing the behavior of curved beams under combined loads, especially under different anchoring conditions. The reason for this is the complexity of calculations resulting from geometric nonlinearities and the insufficient consideration of local effects, which can significantly affect the accuracy of calculations. An option to overcome these difficulties may be the construction of models with simplified geometric assumptions or improved numerical modeling algorithms that provide more accurate accounting of local deformations.

Work [9] considers modeling of compliant mechanisms, which, unlike conventional rigid-body mechanisms, transmit motion, force, and energy through the deformation of flexible elements. The paper reviews the modeling methods of such elements, mainly based on the geometrically nonlinear Euler-Bernoulli beam theory, which is a boundary value problem for an ordinary differential equation. Common numerical methods for solving modeling problems of straight and curved beams are presented. The same authors in [10] considered five typical cases of bending of beams and also proposed a new type of mechanisms – pre-deformed bistable mechanisms that combine the properties of rigid and compliant systems. Issues related to the development of effective methods for modeling compliant mechanisms that would take into account complex geometry and material properties while maintaining computational efficiency remain unsolved. The reason for this is the complexity of nonlinear analysis and the significant resources required for the numerical solution of such problems. An option to overcome these difficulties may be the implementation of simplified analytical models for typical bending cases.

Paper [11] investigates large plane deflections of flat curved beams made of functional gradient materials. The exact geometric theory of the beam is used, where the properties of the material change depending on the position on the cross section of the beam. The results presented for different material gradients demonstrate deflections and deformation patterns of beams, including buckling and loss of stability for hinged arches. Unsolved questions remain regarding the universalization of models for the analysis of beams from functional gradient materials, especially for structures with more complex geometry and gradients of material properties. The reason for this is the increased requirements for modeling accuracy and the growing complexity of computational processes when taking into account complex changes in material characteristics. A likely option for overcoming these difficulties is to devise adaptive methods of numerical analysis that would make it possible to take into account greater variability of material properties or the construction of simplified models for typical structures.

Work [12] investigates the dynamics of the trailed harrowing section for soil processing in the longitudinal-vertical plane. The results could be used to calculate the deviation of the section and other elastic elements from the initial position, which is important to ensure the depth of the stroke within the given limits.

In works [13, 14] it is stated that it is possible to increase the reliability of the working elements of machines under the conditions of typical use by applying a special coating, without significantly changing their geometric parameters. The issues of integration of different approaches to increase the reliability of machine elements remain unresolved as the use of only one method may not be sufficient. The reason is the difficulty of ensuring an optimal balance between material properties, element geometry, and coating technology, which requires a complex analysis. An option to overcome these difficulties may be the implementation of a combined approach, which combines the optimization of the geometry of elements at the design stage with the selection of special coatings adapted to operating conditions.

Based on the above, related studies consider different approaches to modeling the deformation of elastic rods and beams and their calculation with possible errors. Various approaches are used to this end but there is no emphasis on bending rods with variable curvature. To solve this problem, it is proposed to apply the classical approach of finding the elastic axis of the rod, which in the free state has an initial constant or variable curvature. This makes it possible to find the shape of the elastic axis of a curved rod after applying a tracking force to one or the opposite end of it. For curved rods with variable curvature of the elastic axis, this shape will be different.

3. The aim and objectives of the study

The purpose of our study is to analytically describe the bending of cantilever-fixed absolutely elastic rods of constant and variable curvature by a concentrated tracking force. This will make it possible to smooth out pulsating loads when they are attached to the working bodies of agricultural machines.

To achieve the goal, the following tasks were set:

– using an example of a perfectly elastic strip of rectangular cross section, find the shape of its elastic axis after the action of the applied force, the curvature of which in the free state has a constant value;

– using an example of a similar strip with an elastic axis of variable curvature and the same length, show the differences in its deformation under the action of a similar applied force.

4. The study materials and methods

The object of our study is the deformation of an elastic axis with a large deflection of the cantilever clamped absolutely elastic rod under the action of the applied concentrated force. The hypothesis of the study was to apply the reverse countdown of the length of the arc of the elastic axis of curved rods with variable curvature for further calculations. The accepted assumption is the constant stiffness of the rod.

Research methods are based on the provisions of the theory of resistance of materials when the elastic axis of the rod has significant deflections. In this case, a non-linear theory of bending is used, in contrast to construction mechanics with a linear theory, in which the deflection of beams is insignificant compared to their length. According to the theory of materials resistance, the curvature $k=k(s)$ of the elastic axis of a rod or strip is directly proportional to the applied moment $M=M(s)$ and inversely proportional to the stiffness $E \cdot I$ of the rod (strip):

$$k(s)=\frac{M(s)}{EI}, \tag{1}$$

where the independent variable s is the arc length of the elastic axis. The stiffness $E \cdot I$ of the strip is the product of the moment of inertia I of the cross section of the strip by the Young's modulus E. For a strip with a rectangular cross section, the moment of inertia is determined from formula $I=a^3 \cdot b/12$, where a and b are the sides of the rectangle, and the smaller side is the side a since we bend the strip in such a way that it offers less resistance to bending (using the example of a metal ruler). Considering the fact that the moment of inertia I and the Young's modulus are constant values, the stiffness of the strip will also be a constant value. Curvature k is a variable value and is defined as $k=d\alpha/ds$, where $d\alpha$ is the increment of the angle by which the arc ds of the elastic axis was bent due to the action of the applied moment. Thus, the angle between the tangents at the initial and current point of the elastic axis can be determined by the following integral:

$$\alpha=\int k(s)\mathrm{d}s. \tag{2}$$

If dependence (2) is known, then the curve of the elastic axis can be found using the well-known formulas from differential geometry:

$$x=\int \cos\alpha(s)\mathrm{d}s; \tag{3}$$

$$y=\int \sin\alpha(s)\mathrm{d}s.$$

Even for the simplest cases of bending (for example, the bending of a straight strip by a concentrated force), equations (3) require numerical methods of integration.

5. Results of investigating the elastic axis of a fixed strip under the action of a concentrated tracking force

5. 1. Bending of a cantilevered strip, the elastic axis of which in the free state has a constant curvature

Curvature is the inverse of the radius of curvature of the curve at the current point. If it is constant, then the radius is also constant and the same at all points of the curve, that is, the curve is an arc of a circle. A cantilevered strip was considered, the elastic axis of which in the free state is an arc of a circle. The applied concentrated force P is considered tracking, that is, one that remains perpendicular to the elastic axis during its deformation. Depending on the direction in which its action is directed, the curvature of the elastic axis can increase (Fig. 1, *a*) or decrease (Fig. 1, *b*).

Between the tangents at the ends of the elastic axis in the free state (marked by number 1) there is an angle α. Depending on the direction of force P, it can increase to $\alpha+\alpha_1$ (Fig. 1, *a*) or decrease to $\alpha-\alpha_2$ (Fig. 1, *b*) in the process of axis deformation. For the first case, the curvature k of the elastic axis after its bending can be written as follows:

$$k(s)=\frac{d}{ds}\big(\alpha(s)+\alpha_1(s)\big)=k_0(s)+k_1(s), \tag{4}$$

where $k_o(s)$ is the curvature of an elastic axis in the free state;

$k_1(s)$ is the additional curvature of an elastic axis caused by the action of moment M from the applied force P.

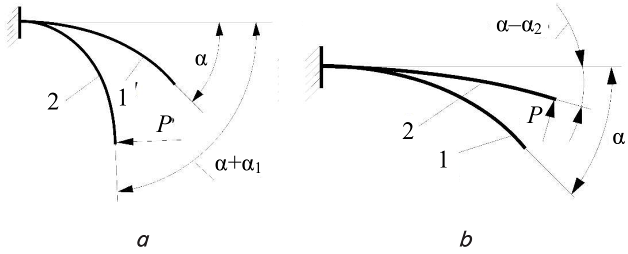


Fig. 1. Elastic axis of the strip before the action of the applied force — 1, and after the action of the applied force *P* — 2: *a* — moment from the applied force *P* increases the curvature of the elastic axis; *b* — moment from the applied force *P* reduces the curvature of the elastic axis

Accordingly, in the second case (Fig. 1, *b*), it is possible to write: $k=k_o-k_2$. Since the elastic axis of the strip in the free state is an arc of a circle, then $k_o=1/r$, where r is the radius of this arc. Additional curvatures k_1 and k_2 are caused by the force P and are determined from formula (1).

The moment of force $M(s)$ is defined as the product of the concentrated force P by the length of the arc s from the point of applied force P to the current point of the elastic axis: $M=Ps$. When substituting expression (1) into expression (4), it should be borne in mind that the curvature component $k_1(s)$ is caused by the action of the applied moment, i.e., instead of $k_1(s)$, expression (1) with the moment $M=Ps$ should be substituted into expression (4). Then the equations of the curvature of an elastic axis for the first case (Fig. 1, *a*) take the form:

$$k(s)=k_0+\frac{Ps}{EI}, \tag{5}$$

or:

$$\frac{d\alpha}{ds}=k_0+\frac{Ps}{EI}.$$

By integrating expression (5), the angle α from the point of force application to the current point of the elastic axis:

$$\alpha=k_0 s+\frac{Ps^2}{2EI}. \tag{6}$$

Substitution (6) in (3) gives the equation of the elastic axis of the strip for the first case when its curvature increases. For the second case, the direction of force P changes to the opposite, which means changing the sign of "+" to "–" in expressions (5), (6). This corresponds to the previously derived expression $k=k_o-k_2$.

Example. Let the curved strip in the form of a quarter arc of a circle have radius $r=0.2$ m, which corresponds to the change in the length of the elastic axis in the range $s=0...0.314$ m. The rectangle of the cross-section of the strip has dimensions of 0.005×0.02 m. The strip material is spring steel, for which Young's modulus is $E=2.2\cdot10^{11}$ N/m^2. Thus, the stiffness of the strip is $E\cdot I=45.83$ N·m^2. The curvature of the strip in the free state is $k_o=1/r=5$ m^{-1}.

In Fig. 2, *a*, the results of numerical integration of equations (3) for the first case are shown (Fig. 1, *a*). It should be borne in mind that the moment increases as the arc s grows, that is, the construction of the elastic axis is carried out from the point of applied force to the fixed end. Because of that, the curves in Fig. 2, *a* are depicted so that the free end is common, which does not correspond to the physical model, for which the fixed end of the strip is common. In Fig. 2, *b*, elastic axes for a given force P in newtons are depicted according to the physical model. To this end, formula (6) was used to determine the required angle of rotation of the elastic axis and parallel transfer along the axes was carried out.

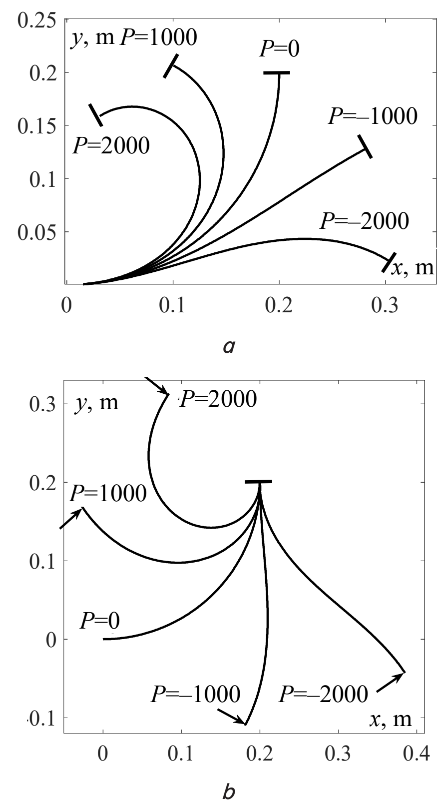


Fig. 2. Deformation of the elastic axis of the strip, which in the free state has a constant curvature, under the action of the applied tracking force *P* of different magnitudes: *a* — points of application of the force *P* are common, the points of attachment of the strip are different; *b* — points of application of the force *P* are different, the points of attachment of the strip are common

If $k_o=0$ is given in equations (4) to (6), then the elastic axis of the strip in the free state will be rectilinear. Fig. 3 shows the corresponding results of calculations of the deformation of the elastic axis at $k_o=0$, and the dimensions of the strip, including its length, remained unchanged.

When the direction of action of the tracking force P changes, the elastic axes of the strip will be symmetrical relative to its initial position in the free state. A peculiarity of the bending of perfectly elastic strips of constant curvature is that the shape of their elastic axis will not change under the action of the same force P if the free end and the end of the attachment are interchanged. In this sense, strips of variable curvature have significant differences.

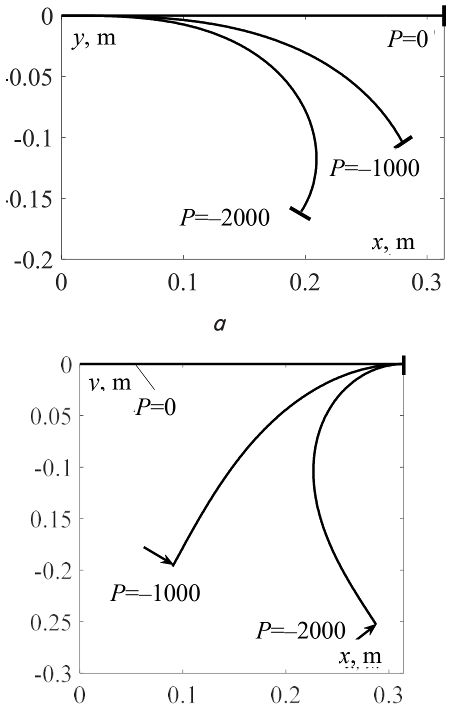


Fig. 3. Deformation of the elastic axis of the strip, which is straight in the free state, under the action of the applied tracking force *P* of different magnitudes: *a* – points of application of the force *P* are common, the points of attachment of the strip are different; *b* – points of application of the force *P* are different, the points of attachment of the strip are common

5. 2. Comparative bending of a cantilevered strip, the elastic axis of which in the free state has a variable curvature and the same length

The elastic axis of the strip of variable curvature is given by dependence $\alpha=\alpha(s)$. With a linear dependence according to $k=d\alpha/ds$, the curvature is a constant value, which was considered in the previous subsection. A strip whose elastic axis has a variable curvature is known as a curve – a logarithmic spiral. The dependence $\alpha=\alpha(s)$ for it takes the following form:

$$\alpha = a \ln s, \tag{7}$$

where a is a constant value.

Integrating expressions (3) after substituting dependence (7) into them makes it possible to obtain the equation of the elastic axis of the strip in the free state:

$$x = \frac{s}{1+a^2}\left[\cos\left(a\ln s\right)+a\sin\left(a\ln s\right)\right]; \tag{8}$$

$$y = \frac{s}{1+a^2}\left[\sin\left(a\ln s\right)-a\cos\left(a\ln s\right)\right].$$

For comparison with the arc of a quarter circle, it is necessary to take the section of the curve when s changes within s=0.1...0.414, that is, as in the previous cases, the length of the elastic axis is equal to 0.314 m. By turning and by parallel transfer of this part of the curve, it is brought to the position where its beginning (point A) is tangent to the x-axis and coincides with the beginning of the arc of the circle (Fig. 4, *a*).

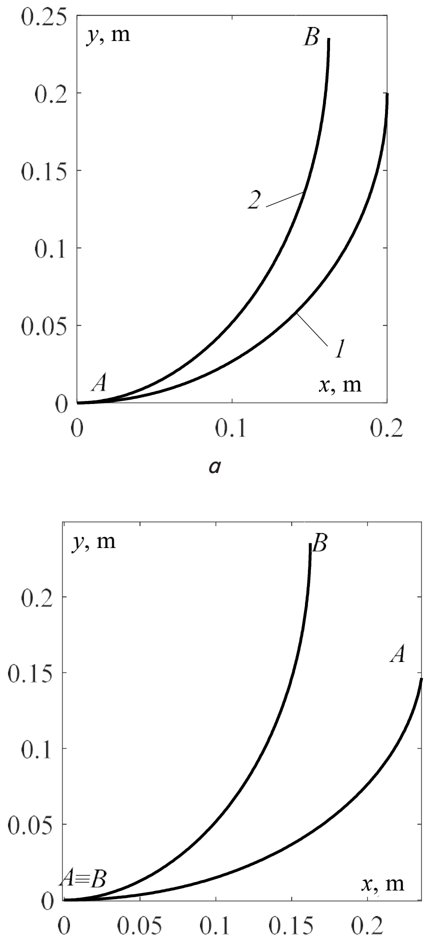


Fig. 4. Curves of the same length s_o=0.314 m in the role of the elastic axis of the strip in the free state: *a* – arc of a circle – 1, and arc of a logarithmic spiral – 2; *b* – equal arcs of a logarithmic spiral when opposite ends are combined

By differentiating dependence (7), the expression of the curvature $k_o=k_o(s)$ can be derived:

$$k_0 = \frac{a}{s}. \tag{9}$$

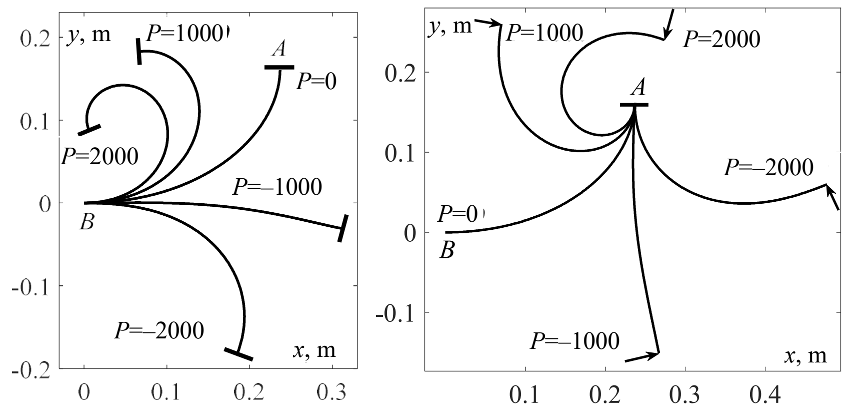
With the accepted value of the constant a=1.1, the curvature of the curve at point A (that is, at s=0.1) has a value of k_o=11 and at point B (that is, at s=0.414) – k_o=2.66. If in the curves (Fig. 4, *a*) the starting and ending points are swapped, the curves will coincide for the arc of a circle, but not for the involute of a circle (Fig. 4, *b*). Accordingly, when bending the strip, in which the elastic axis has a variable curvature, its deformation when the cantilever fixing of the opposite ends will differ. The moment $M=P\cdot s$ as the length of the arc s increases from the initial value s_1 to the final value s_2. The length of the arc of the elastic axis of the strip will be $s_o=s_2-s_1$. For a constant band of curvature (a straight line and an arc of a circle), the start of the reference s_1 is irrelevant. If the end of the fastening of the strip with variable curvature of the elastic axis is changed, then it is necessary to ensure the increase of the moment by increasing the length of the arc, starting from the value of s_2. To this end, in equations (7) to (9), it is necessary to write "s_1+s_2-s" instead of the symbol "s". For the example in question, this would be "0.514-s". The limits of changing the parameter s remain the same: $s=s_1...s_2$, i.e., $s=0.1...0.414$. The constant $a=1.1$ and the limits of change of the parameter s are selected in such a way that the angle between the tangents at the end points, as for the arc of a circle, is a straight line.

Let the strip be fixed at point B, and the counting of the arc starts from point A. According to (5), we write:

$$k(s)=\frac{d\alpha}{ds}=\frac{a}{s}+\frac{Ps}{EI}. \qquad (10)$$

To find dependence $\alpha=\alpha(s)$, expression (10) must be integrated:

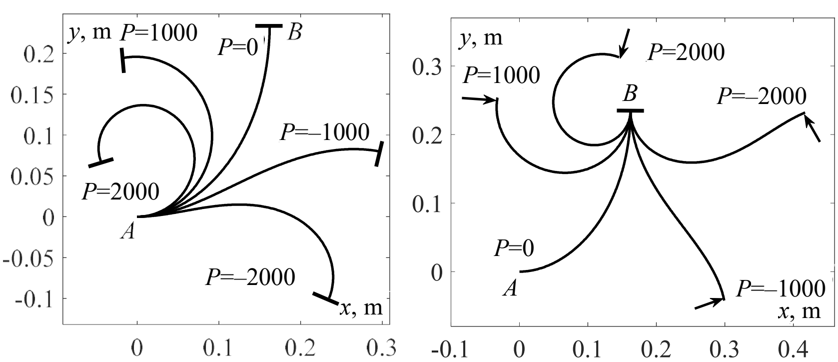
$$\alpha=a\ln s+\frac{Ps^2}{2EI}. \qquad (11)$$



a *b*

Fig. 6. Deformation of the elastic axis of the strip, which in its free state takes the form of a logarithmic spiral and is cantilever fixed at point *A*: *a* — points of application of the force *P* are common, the points of attachment of the strip are different; *b* — points of application of the force *P* are different, the points of attachment of the strip are common

Further construction of the elastic axes of the strip, which bends under the action of the tracking force P, is carried out by numerical integration of equations (3) while substituting expression (11) into them. Fig. 5, *a* shows results of integration with a common reference point of the arc from the free end, and Fig. 5, *b* – according to the physical model with a common attachment point.



a *b*

Fig. 5. Deformation of the elastic axis of the strip, which in its free state takes the form of a logarithmic spiral and is cantilever fixed at point *B*: *a* — points of application of the force *P* are common, the points of attachment of the strip are different; *b* — points of application of the force *P* are different, the points of attachment of the strip are common

Let the strip be fixed at point A, and the counting of the arc starts from point B. In this case, equation (6) takes the form:

$$\alpha=a\ln(0.514-s)+\frac{Ps^2}{2EI}. \qquad (12)$$

Further construction occurs similarly with the substitution of dependence (12) in equation (3). The length of the elastic axis varies within the same limits: $s=0.1...0.414$. Fig. 6 shows the curves of the elastic axes plotted when the strip is fixed at the opposite end, that is, at point A.

Fig. 5, 6 make it possible to compare the shape of the elastic axis of a strip of variable curvature in the form of an arc of a logarithmic spiral when loaded with the same forces if the cantilever attachment points of the ends are interchanged.

6. Discussion of results based on bending a cantilevered strip with an elastic axis of constant and variable curvature

Our results based on determining the shape of an elastic axis during bending of the rods are explained by the fact that the length of the axis was taken as an independent variable when calculating the deformation of the axis. The curvature k of the elastic axis of the cantilevered strip (5) is the sum of two curvatures: the initial curvature k of the elastic axis of the strip in the free state and the additional curvature due to the bending of the strip by the applied force P. If the force P is positive, then the curvature of the elastic axis increases and, accordingly, the angle α between tangent at the extreme points of the axis (Fig. 1, *a*). If the force P is taken with a negative sign, then the curvature of the elastic axis of the strip decreases and the angle α decreases accordingly (Fig. 1, *b*). This makes it possible to use formula (6) to find the dependence of angle $\alpha=\alpha(s)$ for both cases. Finding the shape of the elastic axis is carried out by numerical integration of equations (3), while the parameter s varies within the length of the elastic axis of the strip. This approach makes it possible to find the shape of the elastic axis of the strip using the same algorithm for partial cases:

1) $k_o=0$, that is, the elastic axis in the free state is rectilinear (Fig. 3, *b*);

2) k_o=const, that is, the elastic axis in the free state is an arc of a circle (Fig. 2, *b*);

3) $k_o=k_o(s)$ is a variable value, which is shown on the example of a logarithmic spiral (Fig. 5, *b*). In addition, for a strip with a variable curvature of the elastic axis, its shape during bending depends on which end it is attached to. When changing the fixing end, the growth of the arc s occurs in the opposite direction, which is predicted by formula (12). The result of bending the elastic axis for this case is shown in Fig. 6, *b*.

Various approaches are used in corresponding studies on this topic. Work [12], in which the angular movement of the harrow link is considered, is related in terms of topic. At the same time, it is assumed that the angle of its rotation is small (up to 8°); the correspondingly constructed mathematical model works correctly in this range. In contrast to that work, there are no restrictions on the size of the angle in our study. In [15], a power law model is used to describe the properties of the material in the axial direction. In our research, the physical and mechanical properties are constant. In contrast to works [16, 17], in which the deflection of a beam of round cross-section, which consists of two parts of different diameters is considered, a strip of rectangular cross-section is considered. In the above studies, the curvature of the elastic axis is not investigated, and the main attention is paid to the determination of beam deflections. However, strips with elastic axes of variable curvature in the free state have their own bending characteristics. The curvature of their ends takes a different value, and it is this fact that affects the formation of the elastic axis from the action of the same force when changing the ends of the cantilever fastening of the strip. Our approach has made it possible to obtain the shape of the elastic axes of a strip of variable curvature when changing the ends of its fixation. This is the scientific novelty of the current work. The results could be used to determine the deviation of the loaded end of the rod under the action of a given force, or vice versa – to determine the force by a given deviation. The latter may apply to measuring devices.

The proposed approach has certain limitations. While any dependence $\alpha=\alpha(s)$ can be assigned, then this dependence cannot be derived in reverse order for some known curves. For example, it does not exist for an ellipse since it is impossible to find an expression for the length of the arc in the final form. The disadvantage is that the proposed approach assumes only a constant stiffness of the rod, that is, the shape of the elastic axis cannot be calculated for a two-step strip. Further development of our research may involve finding the shape of an elastic axis of the strip due to the action of variable forces depending on the length of the elastic axis.

7. Conclusions

1. The curvature of the elastic axis of the strip is the sum of the initial curvature of its axis in the free state and the additional curvature under the action of the applied force. The additional curvature is equal to the moment from the applied concentrated force to the free end of the strip. The moment itself is defined as the product of force by the arc length of the elastic axis from the point of applied force to the current point of the axis. The shape of the elastic axis is determined by the means of differential geometry according to the well-known formulas of transition from the dependence of the curvature of the curve to its parametric equations. The shape of the elastic axis depends on the sign before the value of the force, that is, on the direction of its action. In one case, as a result of the action of the force, the curvature of the elastic axis increases, in the other, it decreases. If the initial curvature is zero, then the formulas are valid for bending a straight strip. With a constant value of the initial curvature of the elastic axis (a straight line or an arc of a circle), the shape of the elastic axis does not change when the ends of the strip's cantilever attachment are changed under the action of the same applied forces. A gradual deformation of the elastic axis of the rod in the form of a segment of a straight line or a circle as the applied force increases was found.

2. If the elastic axis of the strip in the free state has a variable curvature, then the shape of the elastic axis of the bent strip under the action of force depends on the end of the cantilever mount. This is explained by the fact that the ends of the elastic axis of the strip in the free state have different values of curvature. Accordingly, the total curvature at its ends due to the bending of the strip will also be different. All the dependences by which the elastic axis of the strip of constant curvature was found are also valid for the strip of variable curvature. However, when changing the cantilever fastening of the end of the strip, it is necessary to ensure that the arc length increases in the opposite direction. To this end, in all dependences, the symbol "s" must be replaced with "s_1+s_2-s", where s_1 is the initial value of the arc length, s_2 is the final value of the arc length. The limits of changing the parameter s remain the same as in the previous case: $s=s_1...s_2$. The difference $s_o=s_2-s_1$ is the length of the elastic axis of the strip. The choice of s_1 and s_2 affects the location of the arc of a given length on the curve and, accordingly, the value of the curvature at its ends. A gradual deformation of the elastic axis of the rod in the form of an arc of a logarithmic spiral as the applied force increases has been found.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study, as well as the results reported in this paper.

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Data availability

All data are available, either in numerical or graphical form, in the main text of the manuscript.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

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