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DETERMINING THE CHARACTERISTICS OF CONTACT INTERACTION BETWEEN STRUCTURAL ELEMENTS WITH VARIED PROPERTIES OF SURFACE LAYERS

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The object of this study is the stressed-strained state of contacting elements of close-shaped machine-building structures. The presence and flexibility of surface layers and coatings are modeled. There are cases of the matching shape of the contacting surfaces of bodies, as well as the perturbation of the shape of these surfaces. The task being solved relates to the fact that the analysis methods of such bodies contact interaction are not yet sufficiently developed.

It was established that for the case of the matching shape of contacting surfaces, the contact area does not depend on the level of loads. In this case, the contact pressure distribution is proportional to the operating load. Such features of the solution do not depend on the properties of the materials of surface layers. A different case is when the shape of the contacting surfaces of bodies is disturbed. In particular, it was established that the properties of the materials of surface layers exert a strong influence on the shape and dimensions of bodies contact area, as well as on the distribution of contact pressure (the difference is 1.5–2.5 times or more).

The theory of variational inequalities is used to model the stressed-strained state of contacting bodies. As a result, the problem about contact interaction of bodies with surfaces of close shape is reduced to the problem of minimizing the modified energy functionality. The minimization is carried out on a set of distributions of displacements, which describes conditions of bodies not penetrating each other. The finite element method is used to discretize the problem of determining the stressed-strained state of contacting bodies. The parametric model built makes it possible to determine the stressed-strained state of contacting bodies when the disturbance of the nominal shape of the bodies and the properties of their surface layers is varied

Keywords: contact interaction, intermediate layer, contact area, contact pressure, separation die

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1. Introduction

Among the contacting elements of machine-building structures, a significant share is occupied by parts that contact each other on surfaces that are nominally congruent or close in shape. These are, in particular, elements of dies, molds, machine tools, internal combustion engines, gears, hydraulic transmissions, suspension systems, etc.

The effort to approximate the shape of the surfaces of contacting bodies is due to the desire to reduce the level of contact pressure. However, the option with nominally matching surfaces is not the most acceptable. In particular, this is hindered by macro deviations of the shape of real parts from their design (nominal) shape. In this case, the actual properties of the materials of the surface layers or coatings become important [1, 2]. Therefore, studies aimed at analyzing the contact interaction between structural elements with varying properties of surface layers are relevant.

2. Literature review and problem statement

In many structures, elements with nominally matching (congruent) surfaces of contacting parts are used [3, 4]. It is natural that in the calculation model of the stressed-strained state (SSS) of these contacting structural elements, their nominal geometric shape is assumed [2, 3]. In works [3, 5], this case is considered as the contact of bodies of "coherent" shape. Certain regularities are characteristic of this variant of contacting structural elements. In particular, the independence of the contact area on the load level is characteristic of die elements [2, 6].

However, real structures have deviations from the nominal form. This is not fully taken into account in [3, 5]. And then, in contact, these bodies contact in the initial state not along congruent surfaces [6]. In this case, the regularities previously established [2, 6] for bodies with nominally coincident surfaces are violated.

Another important factor, which is particularly important in the analysis of the contact interaction of bodies, is the property of their surface layers (roughness, spraying, coating, etc.) [3]. In this case, an elastic layer is placed between the contacting bodies, which also affects the distribution of contact pressure and contact areas. For example, in [6], the contact area increases while the contact pressure level decreases. However, the model described in [6] needs refinement.

Despite some progress in the construction of mathematical models of the contact interaction of bodies of parts [3, 5], there is currently no unified approach for various cases of contacting bodies.

Thus, there are a number of classical analytical models such as the Hertz model [3, 5] for determining contact zones and contact pressure. In the case considered in this work, such a model is not applicable.

On the other hand [7, 8], direct application of such a universal method as the finite element method is possible. It would seem that this guarantees automatic consideration of any factors. However, it should be noted that, for example, taking into account disturbances in the shape of surfaces, it is possible to accumulate uncontrolled errors of approximation of the gap between contacting bodies when using one or another type of finite elements [7]. This reduces the value of using the finite element method according to works [7, 8].

Generalized problem statements about contact interaction based on the theory of variational inequalities [9–11] or Kalker's variational principle [12] also attract attention. Despite their fairly universal nature, they require adaptation to take into account additional factors. For example, these are the properties of the surface layers of contacting bodies. In works [9, 10], for example, there is no consideration of nonlinear properties of roughness. The same is characteristic of papers [11, 12].

Regarding the properties of surface layers [13], models of this type have the most different character. Thus, the following are taken into account: fractal distribution of roughness micro-uniformities [14], adhesive properties [15], statistical models of micro-uniformities [16, 17], etc. Such models have gained significant development. However, they need a mechanism for integrating such micro-scale models into macro-scale models of the contact interaction of bodies. Thus, in [13], the "plane-ball" interaction model was used, which is not always acceptable. Studies [14, 16] also used a partial model, which is not universal. Works [15, 17] used models that cover all important factors. However, such models are unsuitable for describing the properties of various types of coating materials [18, 19]. In [20, 21], there are no models describing the properties of coating materials. In addition, there are a number of new techniques for strengthening the contacting elements of structures [22, 23], which form surface layers of materials with fundamentally different properties compared to both roughness and coatings. However, in [23, 24], models of surface layers are not sufficiently developed.

Therefore, our review of related area of research requires construction of improved models of the contact interaction of structural elements, which are adapted for the case of taking into account the disturbance of nominally congruent contacting surfaces and the properties of the materials of the surface layers, etc. Moreover, these models are required to take into account these factors not in isolation, as in the considered works, but in their combination and mutual influence. And already with their use, it is possible to study the regularities of SSS of contacting bodies when these factors are varied.

The results of our study have built on the data reported in [1, 2, 6, 22–24].

3. The aim and objectives of the study

The purpose of our work is to devise improved variational statements of problems about the contact interaction of bodies with surfaces of close shape and to study the stressed-strained state of these bodies. This will make it possible to determine the characteristics of contact pressure distributions between structural elements.

To achieve the goal, the following tasks were set:

– to build a mathematical model of the stressed-strained state of contacting bodies with surfaces of close shape;

– to construct parametric numerical models of the stressed-strained state of contacting bodies of close shape;

– to determine the regularities of the impact of the disturbance of the shape of the surfaces of the contacting bodies and the properties of the materials of the surface layers and coatings on their stressed-strained state.

4. The study materials and methods

The object of our study is the SSS of contacting elements of close-shaped machine-building structures. The presence and flexibility of surface layers and coatings are modeled. There are cases of coincident shape of the contacting surfaces of bodies, as well as perturbation of the shape of these surfaces. The problem to be solved is that the methods for analyzing the contact interaction of such bodies are not yet sufficiently developed.

The study assumes the elastic nature of deformation of contacting bodies.

In the work, the theory of variational inequalities for the analysis of SSS of contacting bodies with surfaces of close shape has been supplemented. The finite element method was applied. The finite element model was built under a semi-automated mode taking into account the axisymmetric statement. The LS-DYNA licensed software was used to analyze the stressed-strained state. The reliability of the data obtained using this software has been confirmed in the course of numerous earlier studies.

5. Results of investigating the stressed-strained state of contacting bodies with closely shaped surfaces

5. 1. Mathematical model of the stressed-strained state of contacting bodies with closely shaped surfaces

The conventional classification of problems of contact mechanics [3] provides for the selection of two heterogeneous typical options. The first of them is the contact of bodies of an agreed (coinciding, congruent) form. The second is the contact of bodies of inconsistent shape. Usually, in this case, the normal (along the normal to the contact surfaces) gap between the bodies is given as a quadratic function of the coordinates in the tangent common plane. As already mentioned, various methods of contact interaction analysis have been devised for these cases: boundary integral equations, the Hertz model, the theory of variational inequalities, etc. [3–12]. In many cases, bodies are considered smooth.

At the same time, the real elements of structures, firstly, are rough or with layers of coatings made of materials that have properties that differ significantly from the properties of the main materials. Second, even surfaces of consistent shape have macro deviations from the nominal shape and undulations. Therefore, we are not arguing about the contact of bodies of nominally matching (congruent) shape but about the contact of surfaces of disturbed shape. And, finally, thirdly: the gap between the contacting surfaces can take on a form that cannot be represented in the form of a Taylor series [3].

Thus, the transition to the problem of contact interaction of bodies, which does not follow conventional models, has been made (Fig. 1).

Indeed, between bodies 1 and 2 along the normals n_1, n_2 to the surfaces S_1, S_2 (Fig. 1 shows an extended view of the surfaces), the gap δ is given in the form of a distribution as a function of coordinates x, y of the general form:

$$\delta=\delta(x,y). \quad (1)$$

Taking into account the fact that bodies 1 and 2 have near-surface layers Ω_1' and Ω_2', according to the approach from the theory of variational inequalities [9–11], the problem is reduced to the minimization of the functional of the total internal energy J of this system of elastic bodies:

$$J=\frac{1}{2}a(u,u)-b(u)\rightarrow \min, \quad (2)$$

$$u\in K=\left\{u:u_{n_1}\Big|_{S_1}+u_{n_2}\Big|_{S_2}\leq\delta\right\}. \quad (3)$$

Here, a and b are quadratic and linear forms built on the distributions of elastic displacements $u(x_1, x_2, x_3)$ (Fig. 1). At the same time [25]:

$$2a(u,u)=\int_{(\Omega)}\sigma_{ij}(u)\varepsilon_{ij}(u)\mathrm{d}\Omega, \quad (4)$$

$$b(u)=\int_{(S)}Fu\,\mathrm{d}s, \quad (5)$$

where σ, ε are the components of stress and strain tensors, F are external forces (in Fig. 1, they are represented in the form of compressive forces Q), Ω is the combination of all areas occupied by the elements of the system of bodies, S is the merging of all boundaries.

In expressions (4), (5) there is no binding of components to a specific body. The general appearance of these forms is meant.

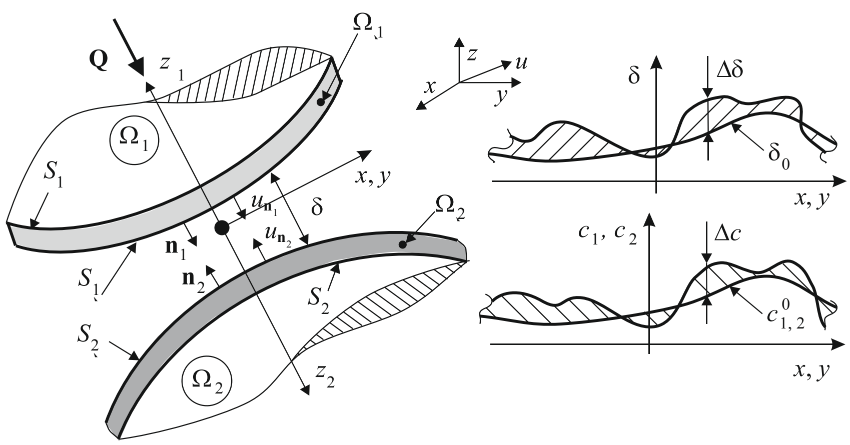


Fig. 1. Contact of bodies Ω_1, Ω_2 in the presence of intermediate layers (coatings) Ω_1', Ω_2', respectively

If we refer to the individual components of functional J in (2), we can write:

$$a = a_1 + a_1' + a_2 + a_2'; \; b = b' + b_2, \tag{6}$$

$$a_1 = \int_{(\Omega_1)} \psi \, d\Omega_1; \; a_1' = \int_{(\Omega_1')} \psi \, d\Omega_1';$$

$$a_2 = \int_{(\Omega_2)} \psi \, d\Omega_2; \; a_2' = \int_{(\Omega_2')} \psi \, d\Omega_2'; \tag{7}$$

$$b_1 = \int_{(S_1)} \varphi \, dS_1; \; b_2 = \int_{(S_2)} \varphi \, dS_2. \tag{8}$$

Here for each of the bodies:

$$\psi = \sigma_{ij} \cdot \varepsilon_{ij}, \;\; \varphi = Fu.$$

Taking into account the small thicknesses of the intermediate layers (t_1 and t_2, Fig. 1), terms a_1' and a_2' can be represented in the form:

$$a_1' = \int_{(S_1)} \lambda_1 \sigma_{n_1}^2 t_1 \, dS_1;$$

$$a_2' = \int_{(S_2)} \lambda_2 \sigma_{n_2}^2 t_2 \, dS_2, \tag{9}$$

where σ_{n_1}, σ_{n_2} are the normal stresses in layers Ω_1', Ω_2';

λ_1, λ_2 are the coefficients of contact compliance of the layers, respectively (inversely proportional to the contact stiffnesses c_1 and c_2).

Due to proximity, when defining expressions (8), the surfaces S_1 and S_1' (as well as S_2 and S_2') are not distinguished.

Considering the peculiarities of the structure of quadratic and linear forms (4) to (9), problem (2), (3) is reduced to a problem of quadratic programming. If we apply the approximation of the sought SSS components u, ε and σ in the form of a partial sum of the series according to certain basic functions [23, 24], then a discrete variant of such a problem is obtained. For example, when applying the finite element method [8, 25], the approximate distributions of the sought SSS components u, ε and σ are determined, as well as the actual areas of contact S_1^C, S_2^C and contact pressure $q(x,y) \equiv q_1 \equiv q_2$.

When solving the resulting problem, it is possible to consider the variations of the shapes of the surfaces S_1 and S_2, as well as the contact stiffnesses c_1 and c_2. This leads to disturbances in the nominal distributions of the gap δ_0, c_1^0 and c_2^0 (Fig. 1):

$$\delta(x,y) = \delta_0(x,y) + \Delta\delta(x,y), \tag{10}$$

$$c_1(x,y) = c_1^0(x,y) + \Delta c_1(x,y);$$

$$c_2(x,y) = c_2^0(x,y) + \Delta c_2(x,y). \tag{11}$$

Therefore, it is possible to determine the dependence of solutions to the contact interaction problem on the shape disturbances of the contacting bodies and the contact stiffness of the near-surface layers. For this purpose, problem (1) to (9) should be solved (for example, using the finite element numerical method [8, 25]) taking into account (10), (11).

5. 2. Parametric numerical models of the stressed-strained state of contacting bodies of close shape

The described methodology has been applied to solving the given problem using an example of the contact interaction of the elements of separation dies (namely, punches, matrices, and the material being stamped).

Fig. 2 shows a scheme of the studied system of bodies of rotation with axisymmetric SSS (A – punch, B – stamped material, C – matrix).

Punching force Q acts on the punch, which leads to displacement of the loaded face by amount W. That is, one can set force load Q or kinematic load W:

$$Q = Q(\tau); \;\; W = W(\tau), \tag{12}$$

where $\tau \epsilon [0; 1]$ is some parameter.

The nominal version of the structure under study is shown in Fig. 3. The finite-element model consists of eight-nodal finite elements (120,000 degrees of freedom in total).

Materials of all bodies are steel. The modulus of elasticity of steel E=200 GPa, Poisson's ratio ν=0.3. Force Q=$Q(\tau)$. Maximum force $Q(\tau=1)$=4 kN, radius R=0.5 m, r_1=15·10^{-3} m, r_2=25·10^{-3} m, H_1=10^{-2} m, H=2·10^{-2} m, t=10^{-3} m, h=10^{-4} m, R_1=5·10^{-4} m.

Loads and boundary conditions are shown in Fig. 4. The bottom edge is fixed. The upper one is loaded with force Q.

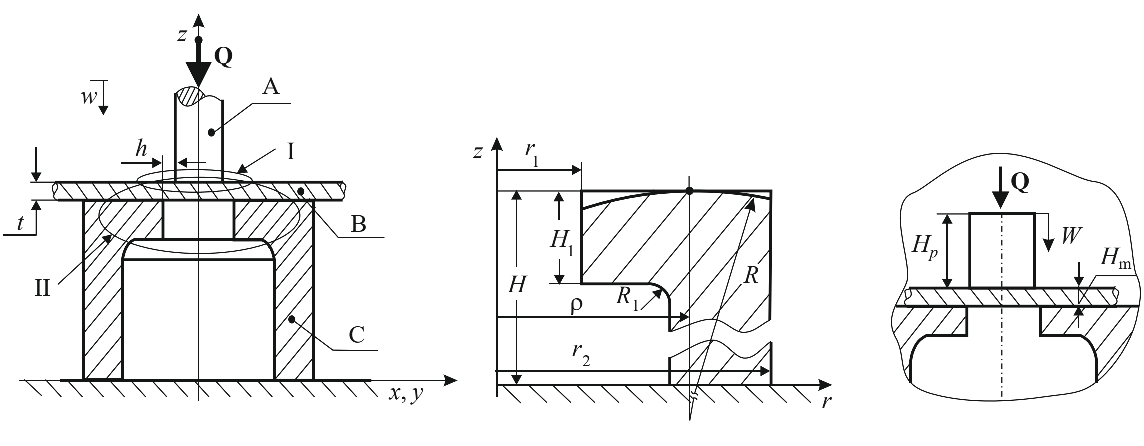
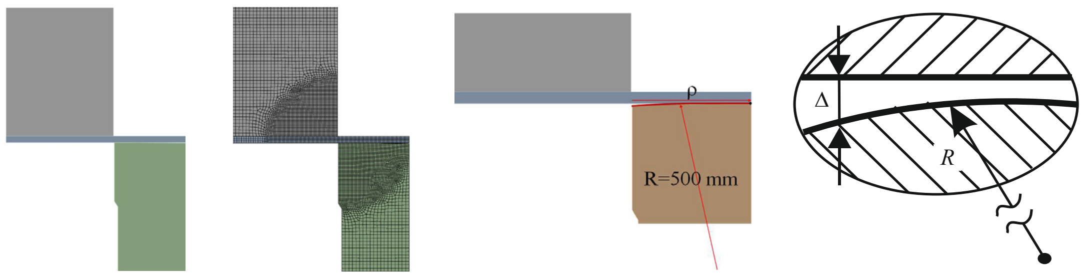
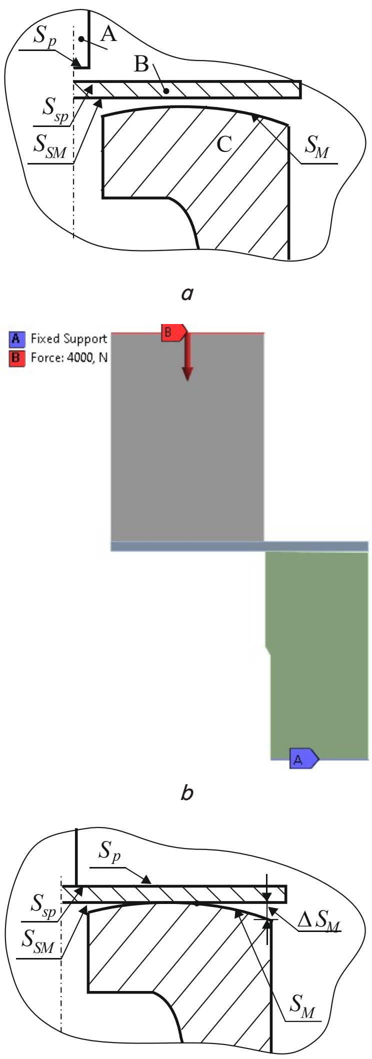


Fig. 2. Calculation scheme of the body system "punch-workpiece-matrix"



a *b* *c* *d*

Fig. 3. Geometrical and finite-element models of the studied system of bodies "punch – workpiece – matrix": *a* – geometric model; *b* – finite element model; *c* – disturbed profile of the matrix surface; *d* – a fragment of the disturbed surface



c

Fig. 4. Boundary conditions and loads of the studied system of bodies "punch – workpiece – matrix": *a* – spaced elements; *b* – applied force and fixation; *c* – non-spaced view

In the case under consideration, the boundary of the contacting surfaces of the bodies between the punch and the workpiece are nominal:

$$S_p = S_{sp} = \text{const.} \tag{13}$$

On the other hand, shape S_M of the surface varies in the coupling of the workpiece and the matrix (Fig. 2–4):

$$S_{MS} = \text{const}; \;\; S_M = S_M^0 + \Delta S_M, \tag{14}$$

where ΔS_M is the disturbance (variation) of the surface shape (in the axial cross section – Δ, Fig. 2–4).

This perturbation is defined in this particular case by the arc of a circle of radius R with radial coordinate ρ:

$$\Delta = \Delta(r, \alpha, \beta), \tag{15}$$

where r is the radial coordinate, α and β are dimensionless parameters:

$$\alpha = \frac{(\rho - r_1)}{(r_2 - r_1)}; \;\; \beta = \frac{R}{(r_2 - r_1)}. \tag{16}$$

As for contact rigidity, in this case:

$$c_{sp} = c_0; \;\; c_{SM} = c_o \cdot 10^{\gamma}, \tag{17}$$

where $c_0 = 10^{12}$ N/m^3 – some nominal stiffness.

5. 3. Patterns of influence of the disturbance of the shape of surfaces of contacting bodies and the properties of the materials of the surface layers and coatings on their stressed-strained state

In the course of research, the influence of parameters α, β, γ on the contact interaction and the stressed-strained state of the studied body system "punch – workpiece – matrix" is determined.

In particular, when determining the dependence of characteristics of the contact interaction and SSS of this system on parameters α, γ, R=0.5 m (i.e., β=50) is accepted. At the same time, ρ takes a number of values: ρ=[10,4; 12.5; 15; 17.5; 19.6] mm (i.e., α=[$4\cdot10^{-2}$; 0.25; 0.5; 0.75; 0.96]), and the contact stiffness varies by a set of values c=[10^{11}; 10^{12}; 10^{13}; 10^{14}; 10^{15}] N/m^3, i.e., γ=[–1; 0; 1; 2; 3].

Fig. 5–13 show some characteristic distributions of equivalent Mises stresses for different ρ (i.e., α) and c (i.e., γ). It should be noted that here and further in the upper part of the figure, the stress distributions in the zone near the junction of three contacting bodies are given, and in the lower part of the figure – in the upper part of the matrix.

Fig. 14–16 show similar dependences obtained when varying R=[0.25; 0.5; 1.0] m at ρ=$15\cdot10^{-3}$ m (i.e., α=0.5) and at γ=0.

Fig. 17 shows distributions of the same controlled values for the case of matching (congruent) surfaces of the matrix and material and at stiffness c=10^{12} N/m^3 (γ=0).

Fig. 18–21 show some integral dependences for the components of the stressed-strained state of the studied system of bodies on varied parameters α, β, γ.

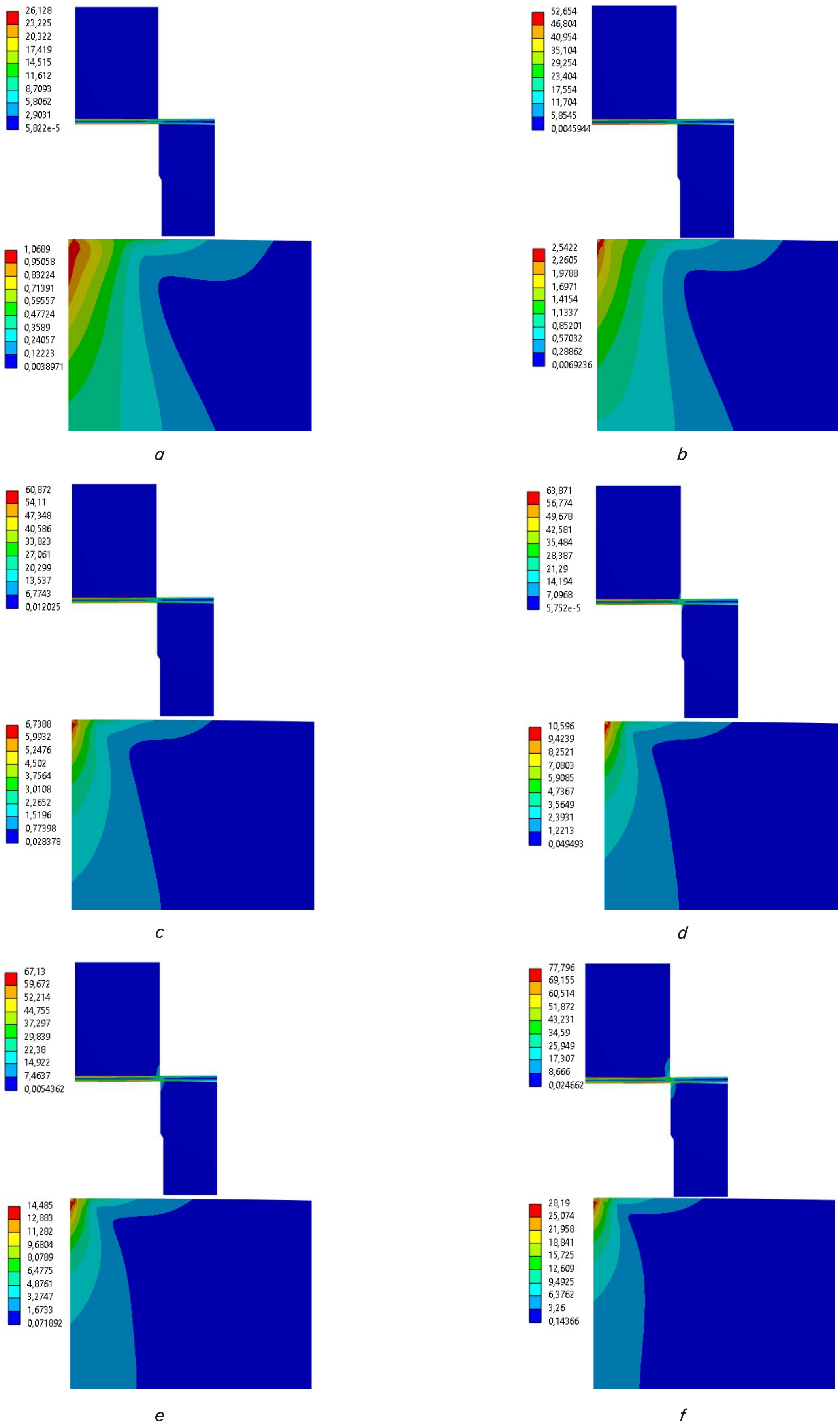


Fig. 5. Distributions of equivalent stresses according to Mises, MPa, at ρ=10.4 mm; c=10^{11} N/m^3: a – τ=0.05, Q=200 N; b – τ=0.1, Q=400 N; c – τ=0.15, Q=600 N; d – τ=0.2, Q=800 N; e – τ=0.25, Q=1000 N; f – τ=0.4, Q=1600 N

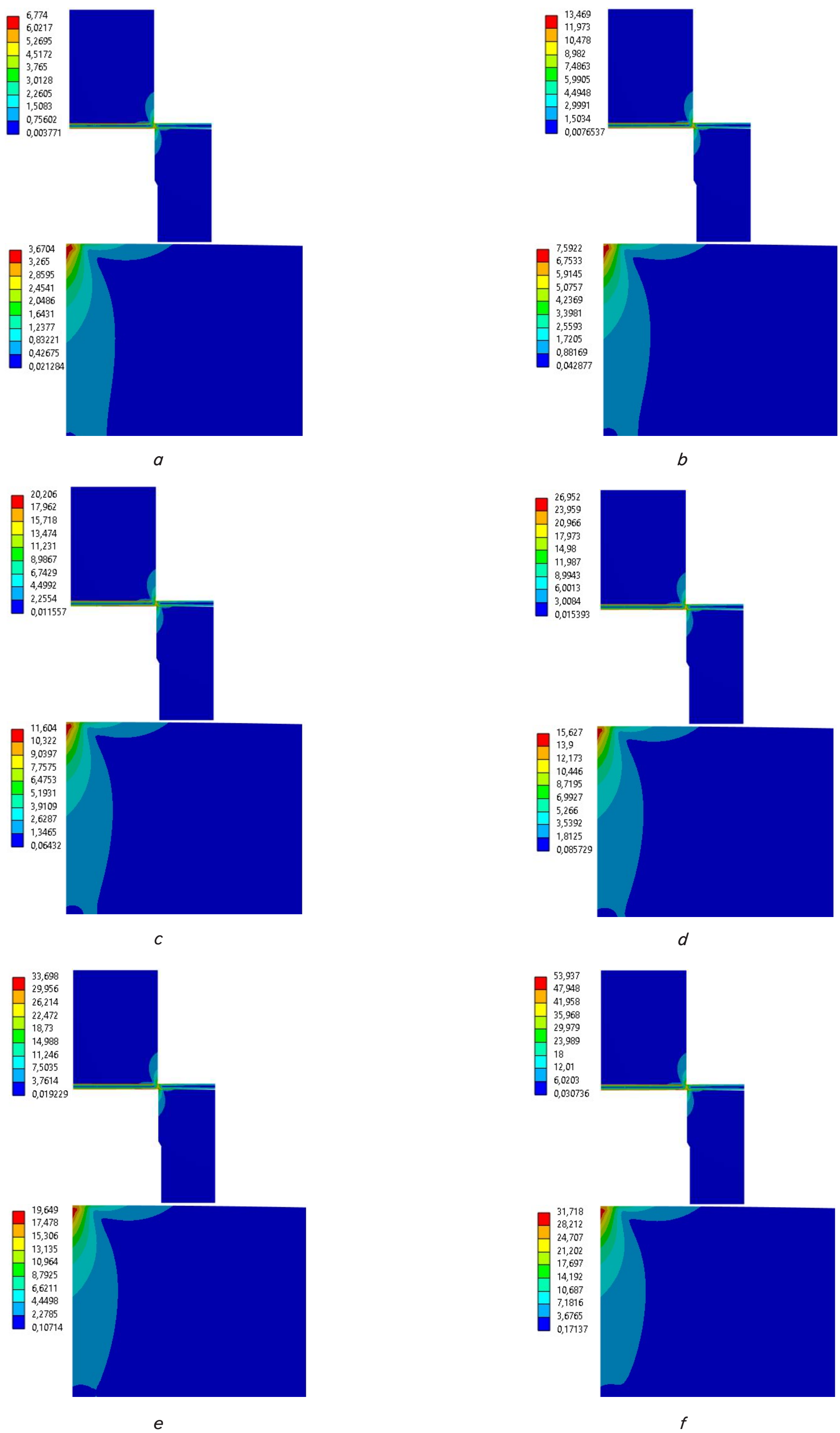


Fig. 6. Distributions of equivalent stresses according to Mises, MPa, at ρ=10.4 mm; c=10^{13} N/m^3: *a* – τ=0.05; *b* – τ=0.1; *c* – τ=0.15; *d* – τ=0.2; *e* – τ=0.25; *f* – τ=0.4

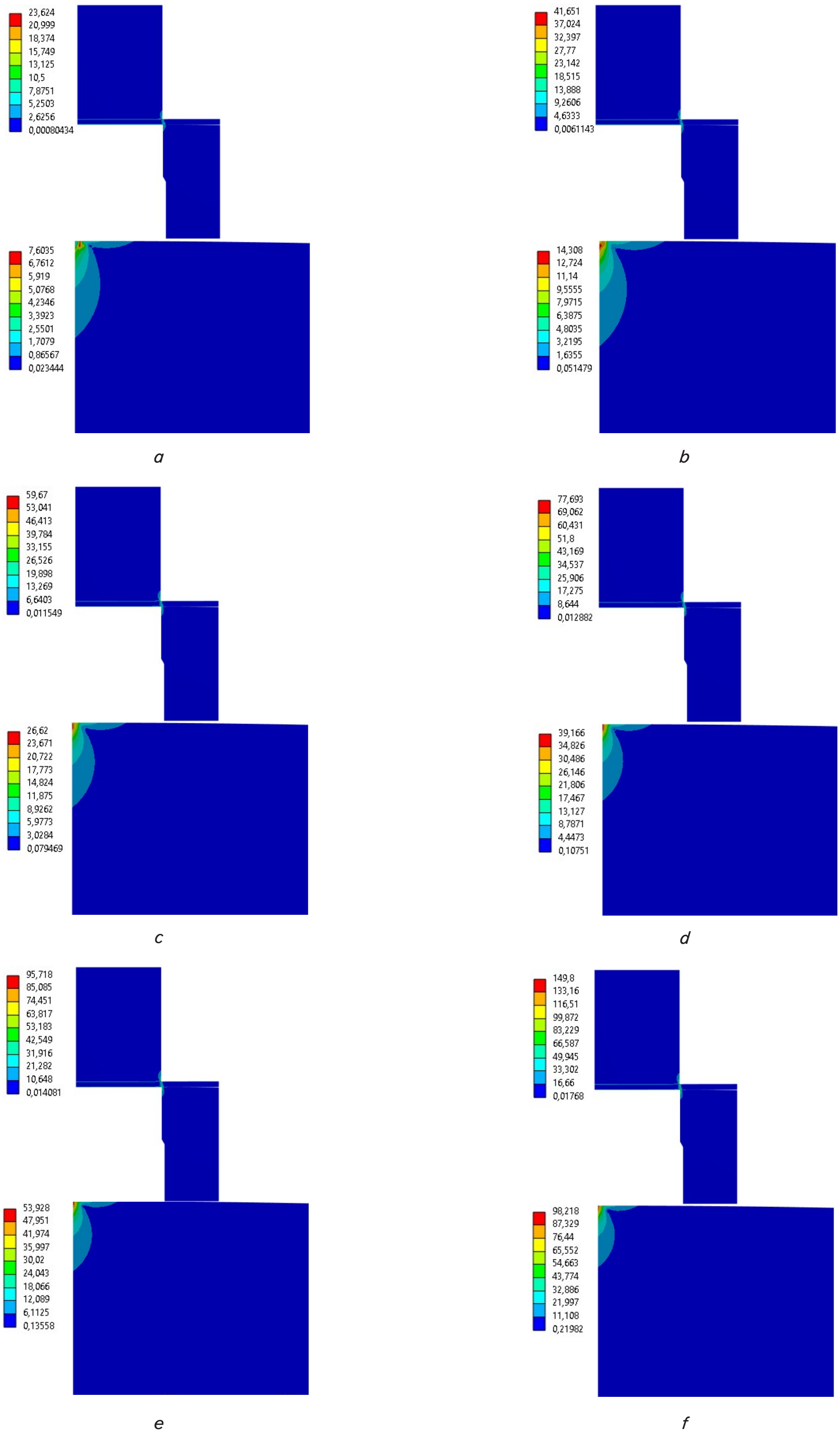


Fig. 7. Distributions of equivalent stresses according to Mises, MPa, at ρ=10.4 mm; c=10^{15} N/m^3: a – τ=0.05; b – τ=0.1; c – τ=0.15; d – τ=0.2; e – τ=0.25; f – τ=0.4

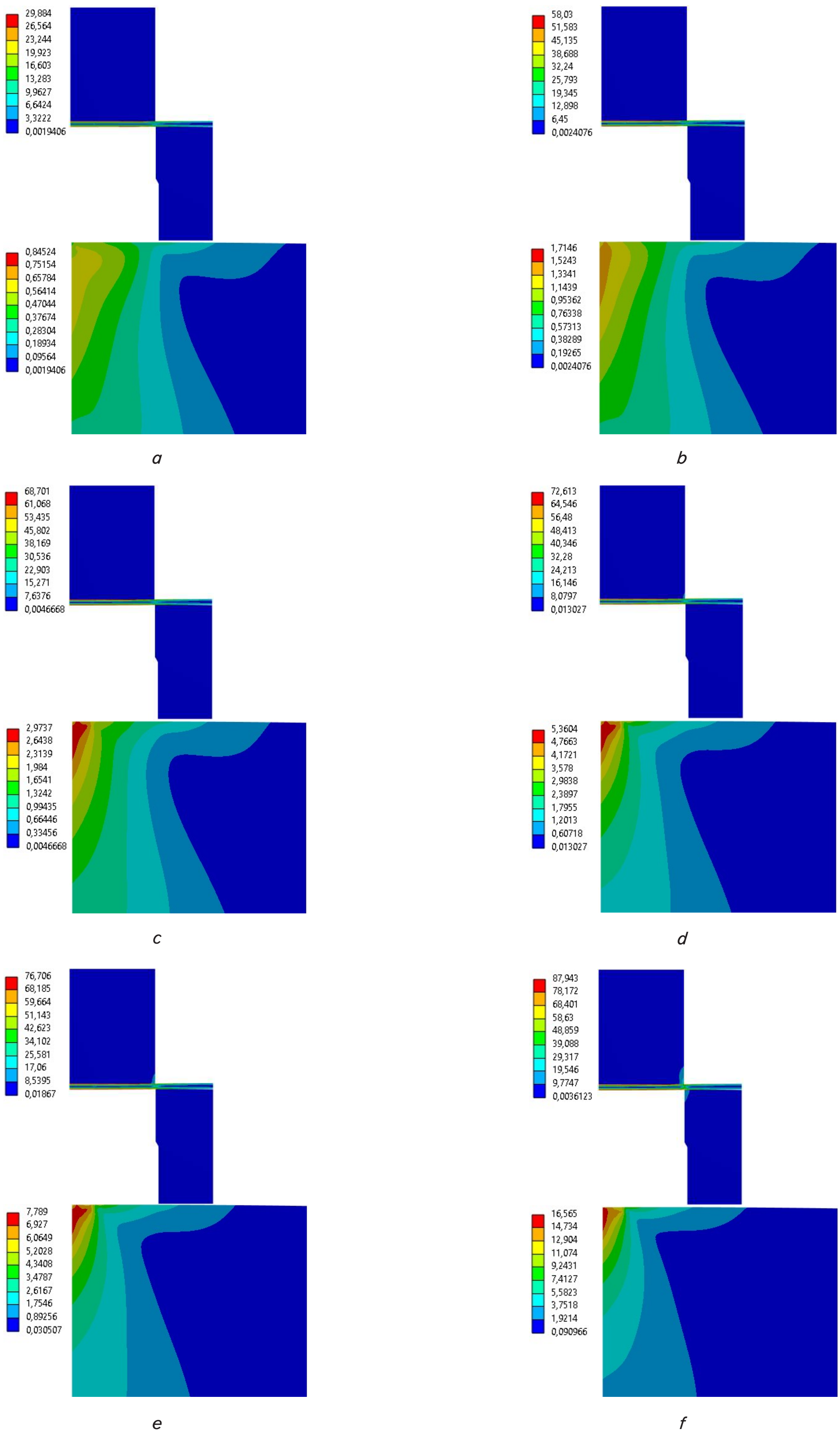


Fig. 8. Distributions of equivalent stresses according to Mises, MPa, at ρ=12.5 mm; c=10^{11} N/m^3:
a – τ=0.05; *b* – τ=0.1; *c* – τ=0.15; *d* – τ=0.2; *e* – τ=0.25; *f* – τ=0.4

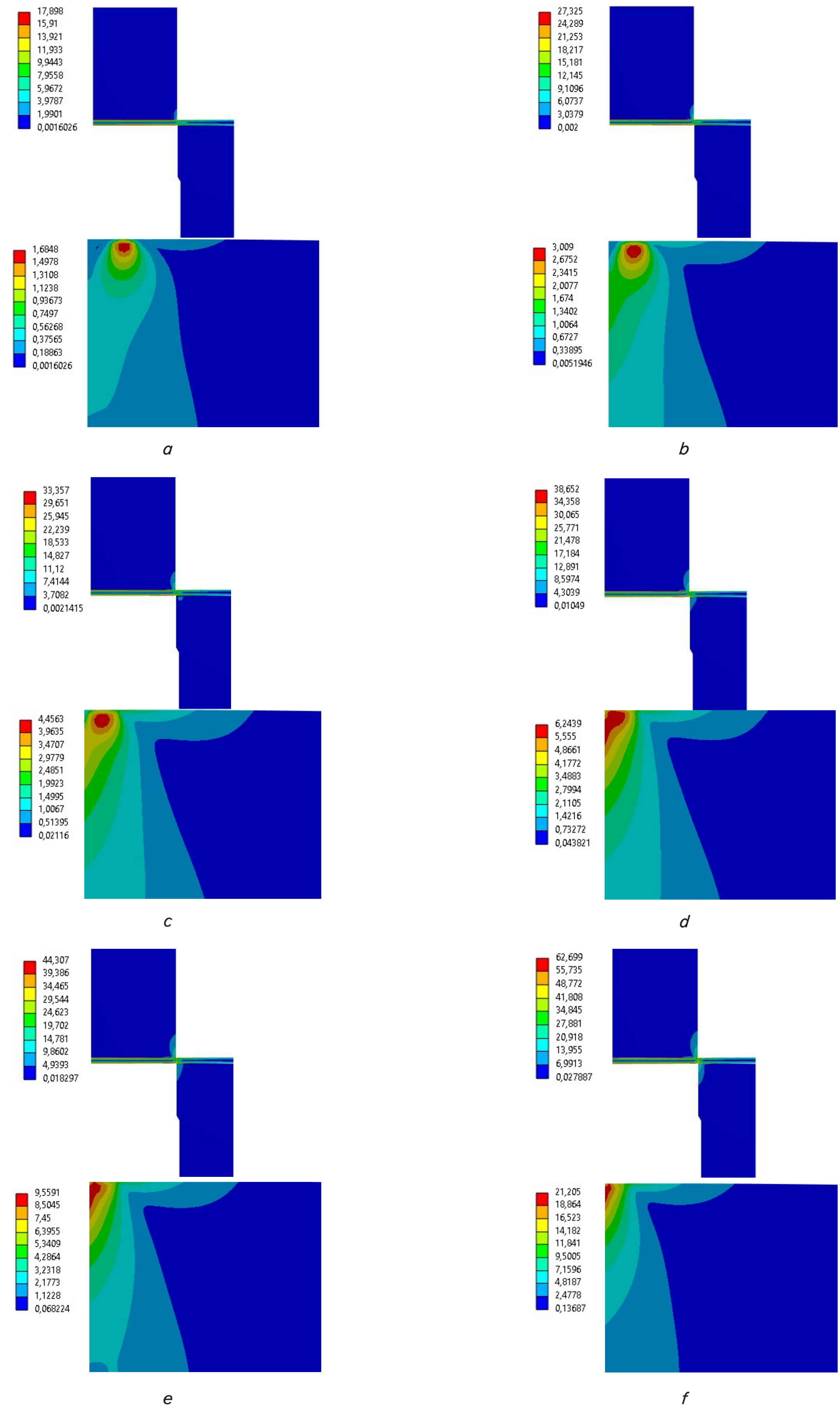


Fig. 9. Distributions of equivalent stresses according to Mises, MPa, at ρ=12.5 mm; c=10^{13} N/m^3: *a* – τ=0.05; *b* – τ=0.1; *c* – τ=0.15; *d* – τ=0.2; *e* – τ=0.25; *f* – τ=0.4

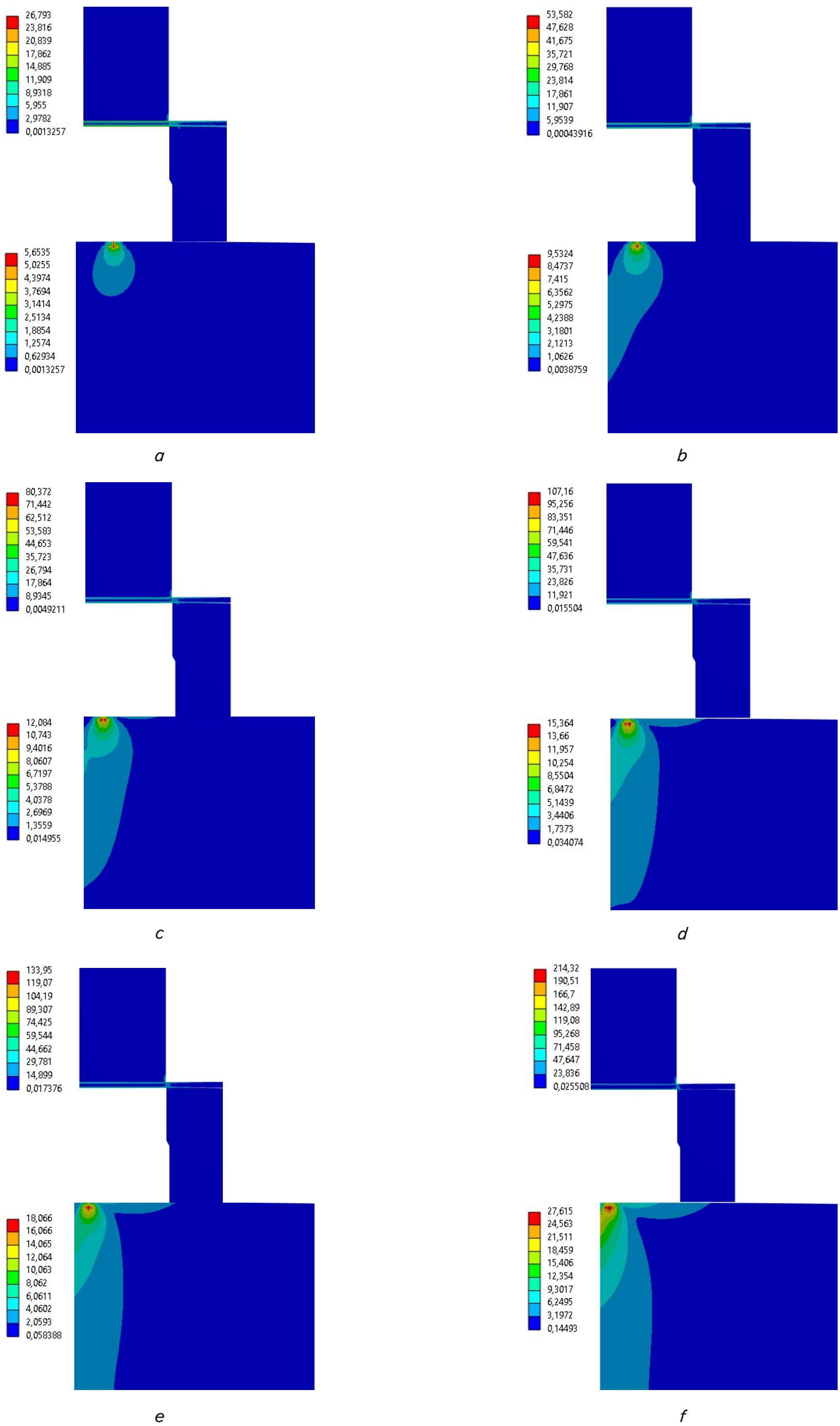


Fig. 10. Distributions of equivalent stresses according to Mises, MPa, at ρ=12.5 mm; c=10^{15} N/m^3: *a* – τ=0.05; *b* – τ=0.1; *c* – τ=0.15; *d* – τ=0.2; *e* – τ=0.25; *f* – τ=0.4

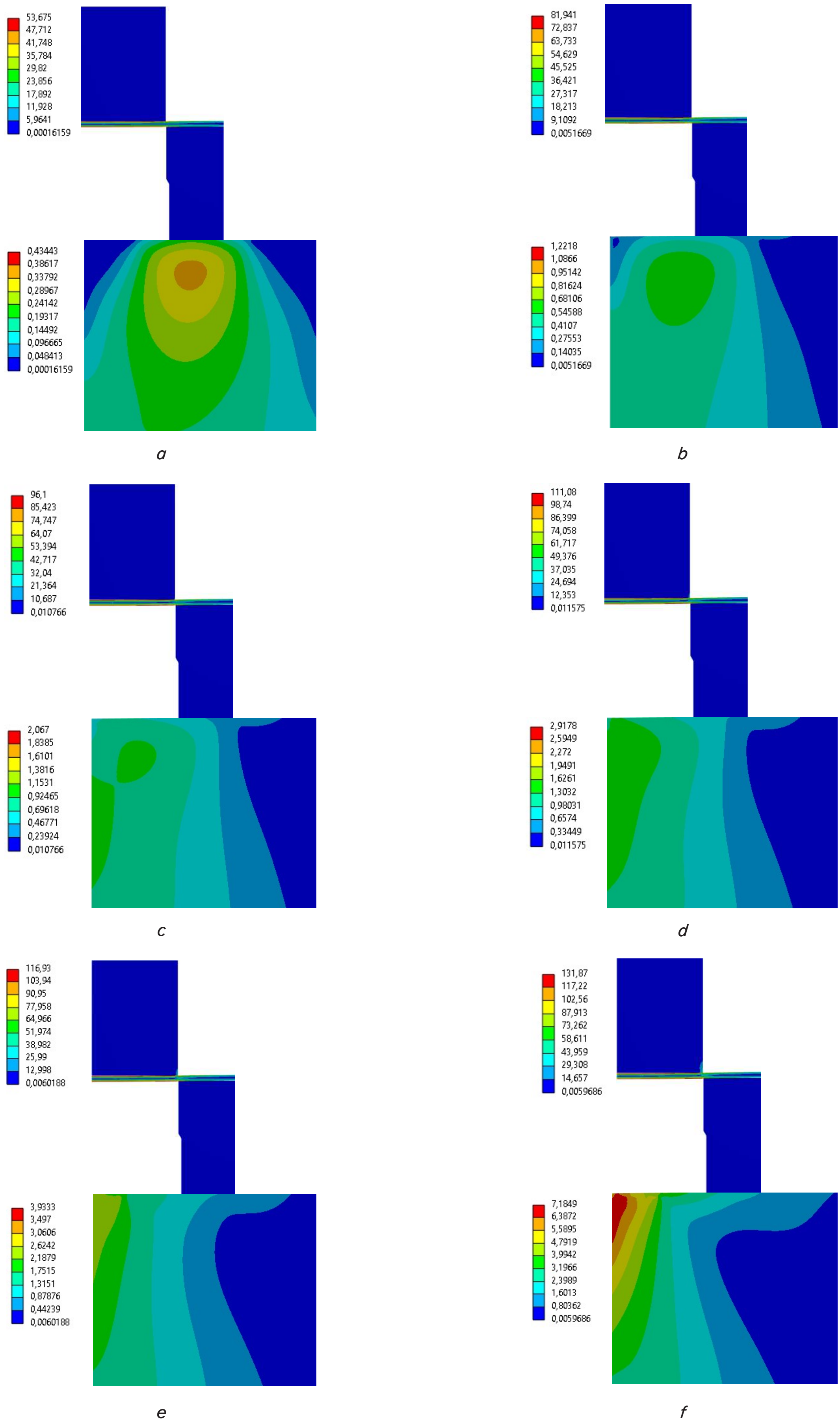


Fig. 11. Distributions of equivalent stresses according to Mises, MPa, at ρ=17.5 mm; $c=10^{11}$ N/m^3: *a* – τ=0.05; *b* – τ=0.1; *c* – τ=0.15; *d* – τ=0.2; *e* – τ=0.25; *f* – τ=0.4

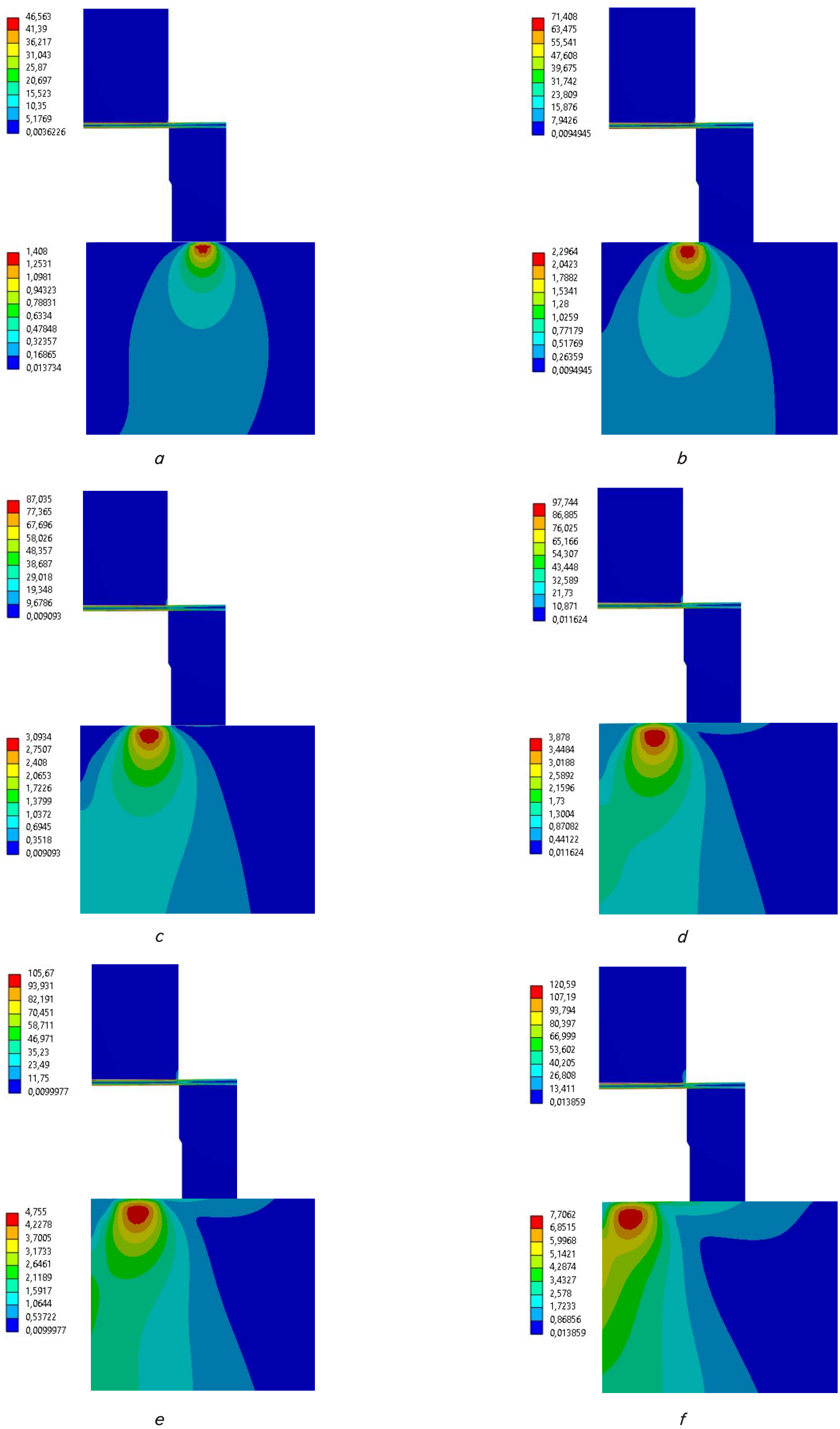


Fig. 12. Distributions of equivalent stresses according to Mises, MPa, at ρ=17.5 mm; c=10^{13} N/m^3:
a – τ=0.05; b – τ=0.1; c – τ=0.15; d – τ=0.2; e – τ=0.25; f – τ=0.4

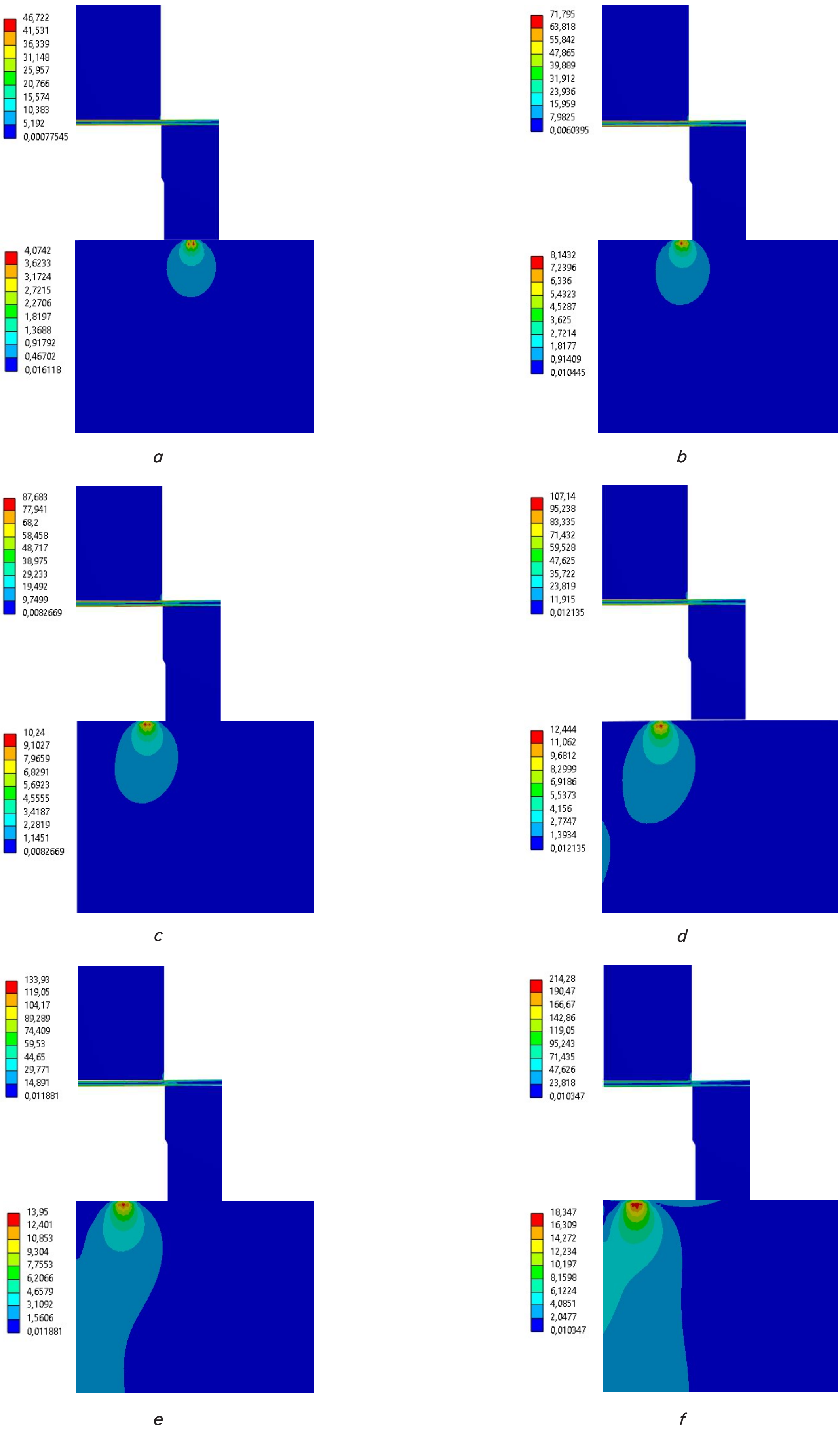


Fig. 13. Distributions of equivalent stresses according to Mises, MPa, at ρ=17.5 mm; c=10^{15} N/m^3: *a* – τ=0.05; *b* – τ=0.1; *c* – τ=0.15; *d* – τ=0.2; *e* – τ=0.25; *f* – τ=0.4

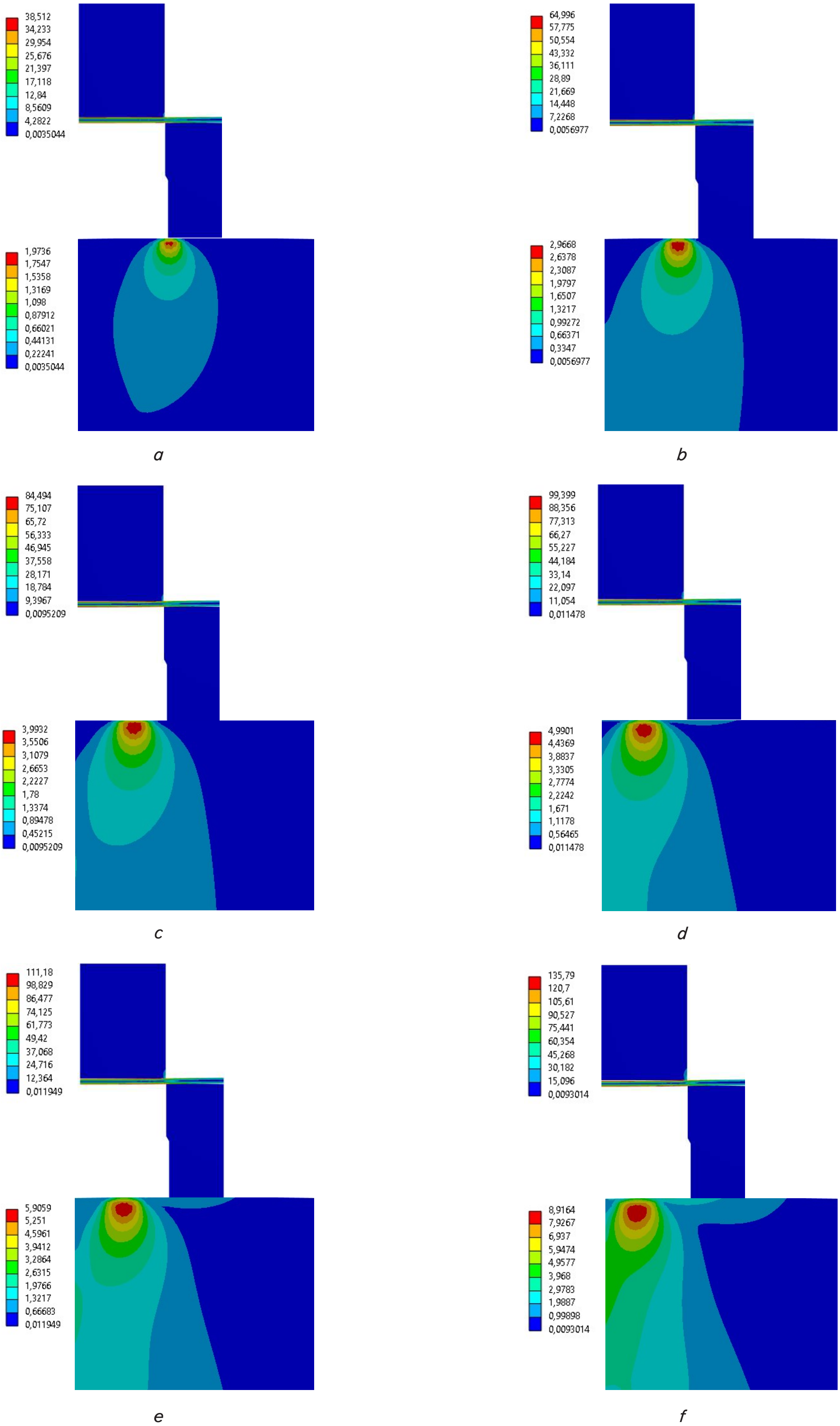


Fig. 14. Distributions of equivalent stresses according to Mises at R=0.25 m:
a – τ=0.05; b – τ=0.1; c – τ=0.15; d – τ=0.2; e – τ=0.25; f – τ=0.4

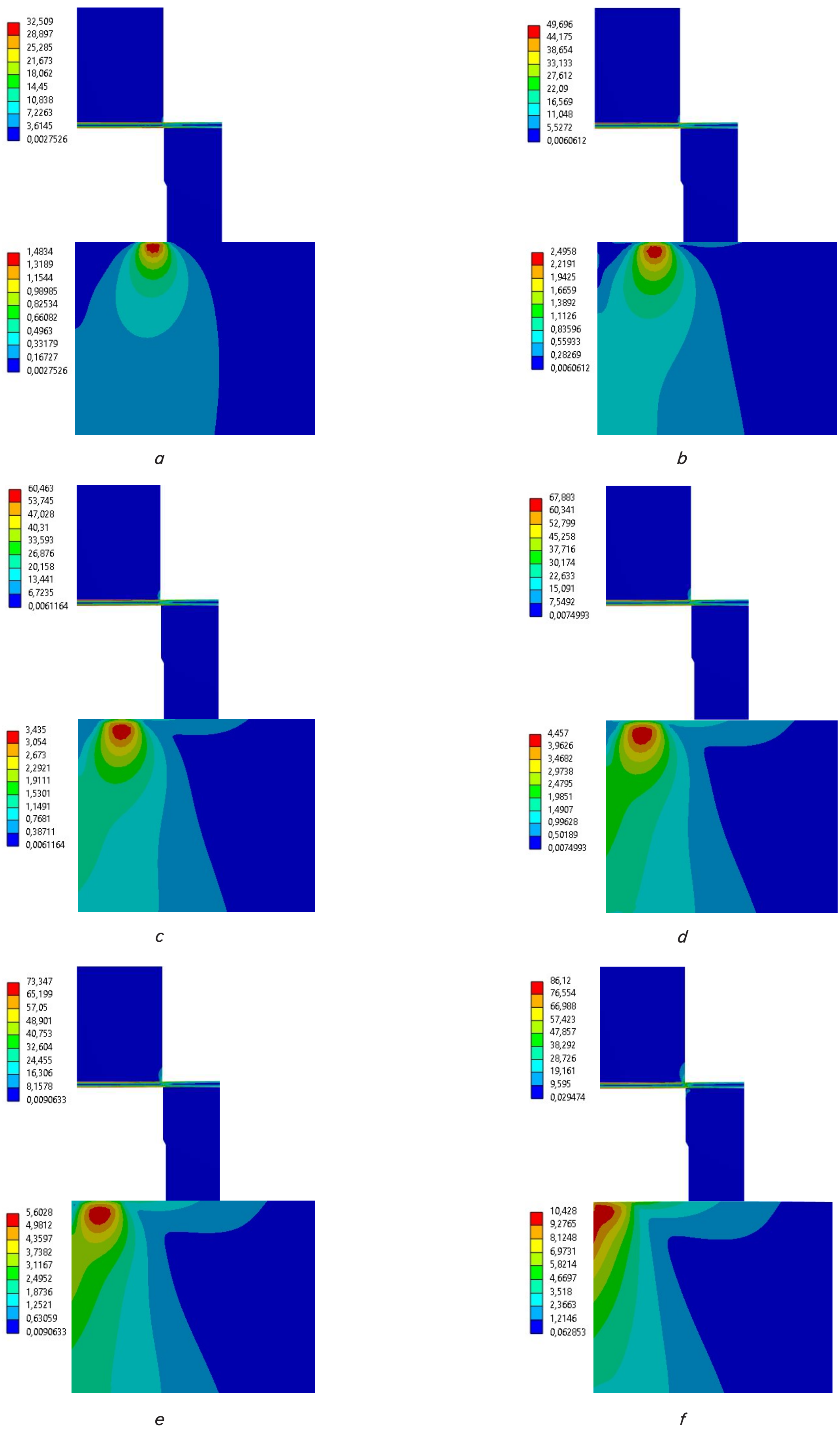


Fig. 15. Distributions of equivalent stresses according to Mises at R=0.5 m:
a – τ=0.05; b – τ=0.1; c – τ=0.15; d – τ=0.2; e – τ=0.25; f – τ=0.4

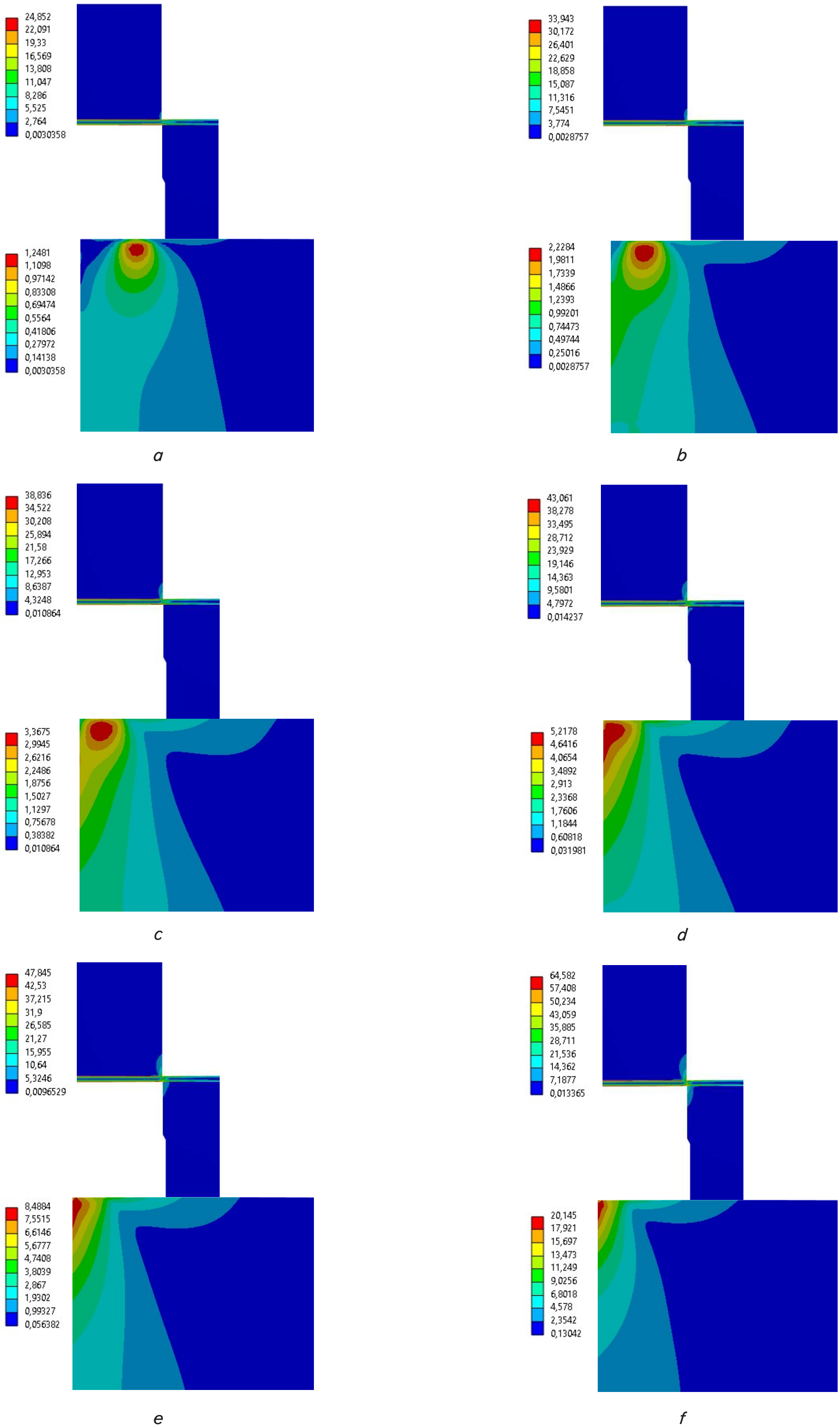


Fig. 16. Distributions of equivalent stresses according to Mises at R=1.0 m:
a – τ=0.05; *b* – τ=0.1; *c* – τ=0.15; *d* – τ=0.2; *e* – τ=0.25; *f* – τ=0.4

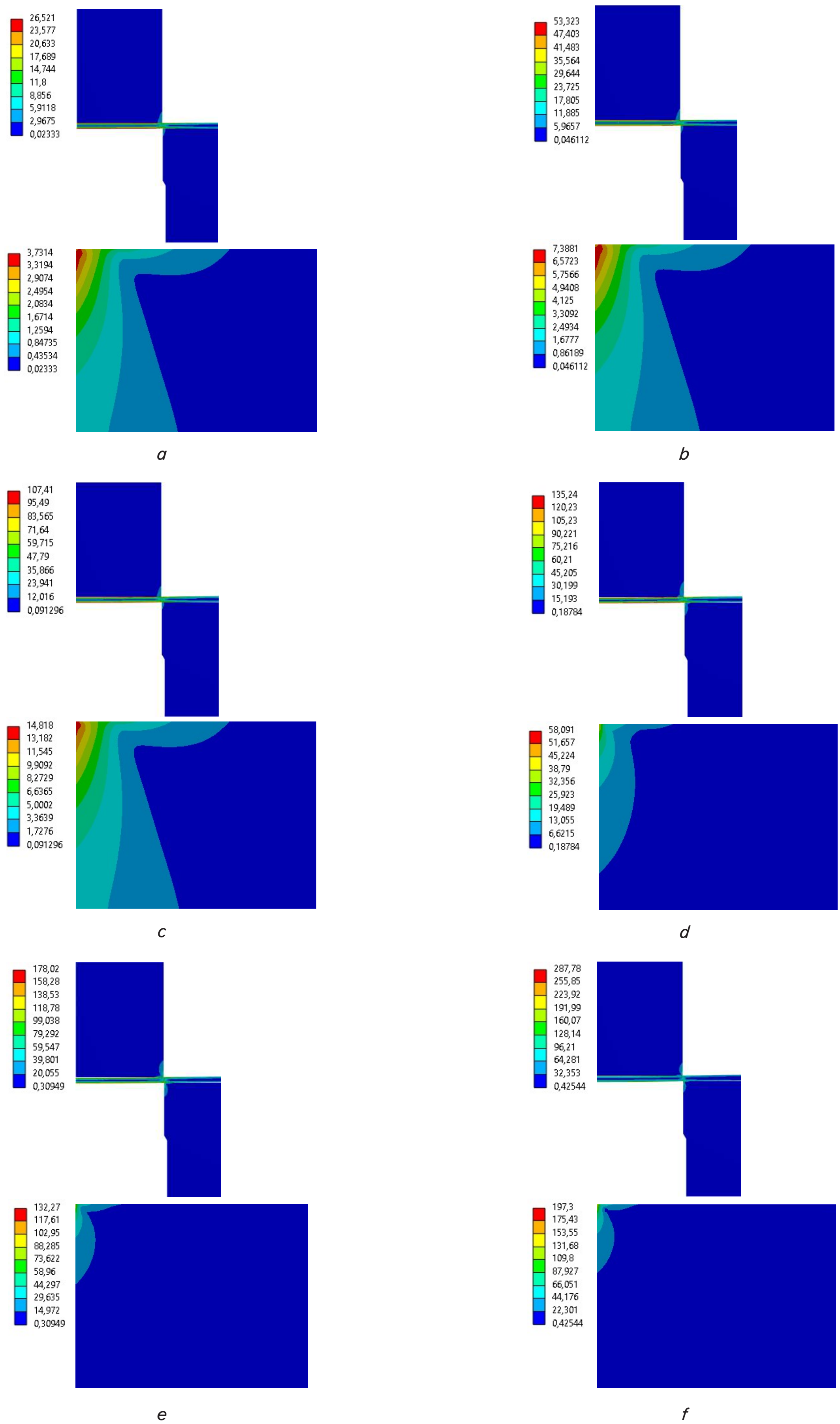


Fig. 17. Distributions of equivalent stresses according to Mises, MPa, the case of coincident surfaces: *a* – τ=0.1; *b* – τ=0.2; *c* – τ=0.4; *d* – τ=0.6; *e* – τ=0.8; *f* – τ=1

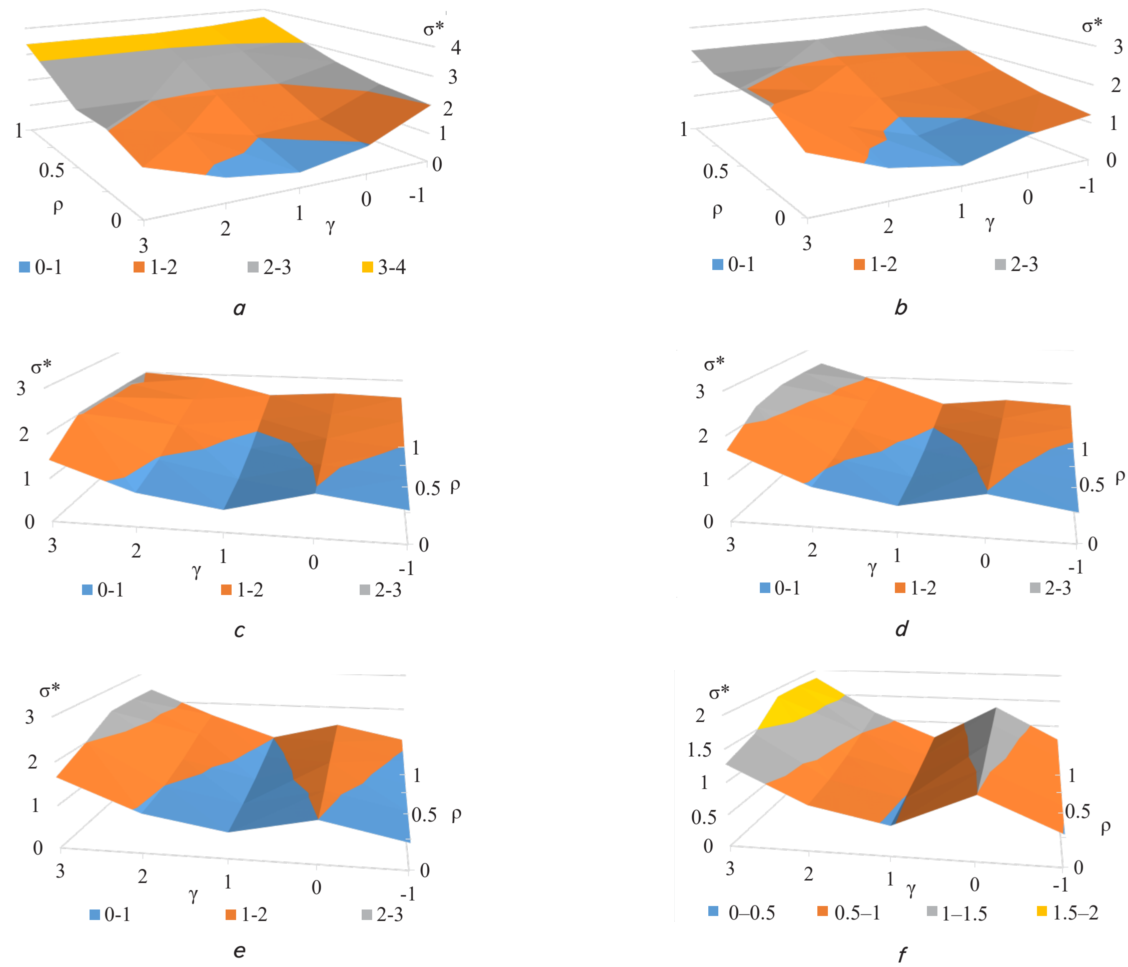


Fig. 18. Maximum equivalent stresses in the entire die (axes: $\rho \in [0; 1]$, $\gamma \in [-1; 3]$, equivalent stresses normalized to the variant of coincident surfaces at $\gamma=0$): *a* – $\tau=0.1$; *b* – $\tau=0.2$; *c* – $\tau=0.4$; *d* – $\tau=0.6$; *e* – $\tau=0.8$; *f* – $\tau=1$

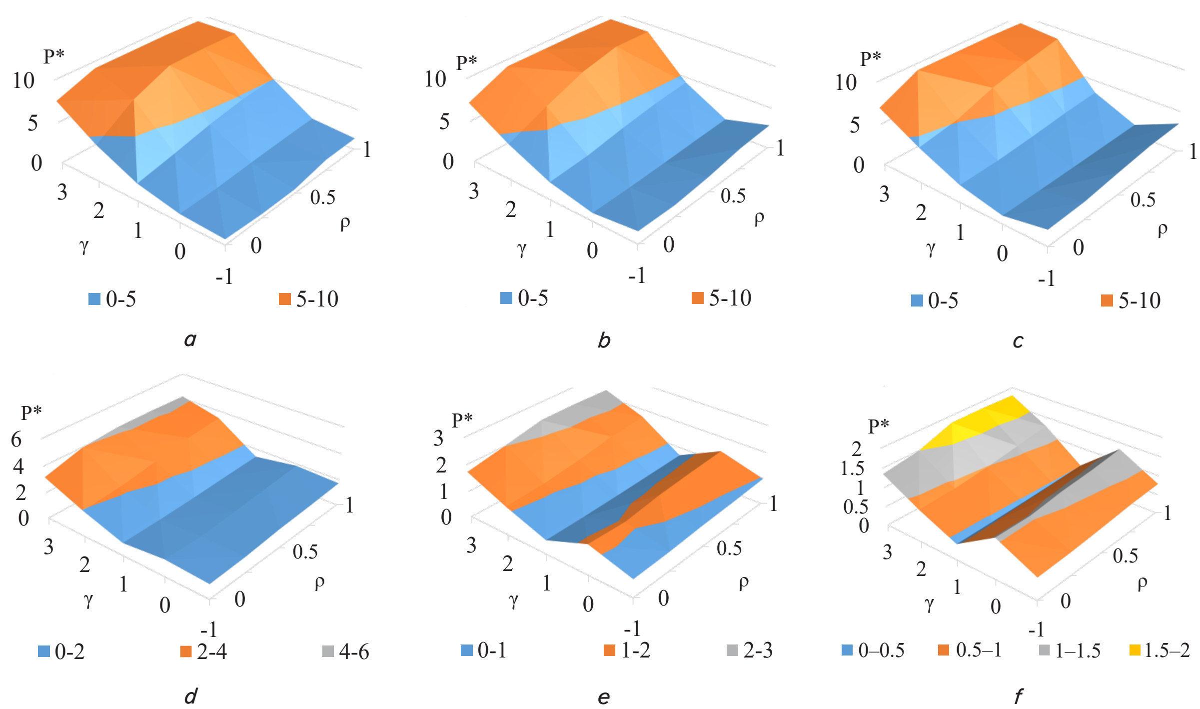


Fig. 19. The maximum contact pressure between the punch and the material being stamped (axes: $\rho \in [0; 1]$, $\gamma \in [-1; 3]$, the contact pressure is normalized to the version of coincident surfaces at $\gamma=0$):
a – $\tau=0.1$; *b* – $\tau=0.2$; *c* – $\tau=0.4$; *d* – $\tau=0.6$; *e* – $\tau=0.8$; *f* – $\tau=1$

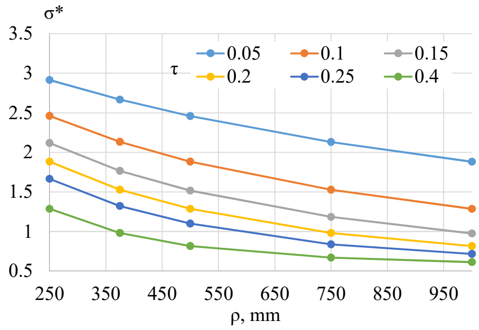


Fig. 20. Dependence of the relative maximum of equivalent stresses in the entire die (ordinate axis) on the radius of rounding of the matrix surface (10^{-3} m) for different values of the load parameter τ

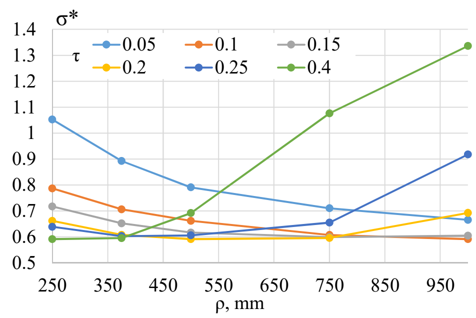


Fig. 21. Dependence of the relative maximum of equivalent stresses in the die matrix (ordinate axis) on the radius of rounding of the matrix surface (10^{-3} m) for different values of the load parameter τ

Shown in Fig. 18–21, the results can be formed into specialized databases. However, even in this form, the given information serves as a basis for establishing certain patterns.

6. Discussion of results of investigating the contact interaction and the stressed-strained state of the system of contacting bodies

The results obtained and described in our work is explained by two circumstances. First, a new and improved model of the stressed-strained state of contacting bodies has been built. And secondly, in the course of numerical modeling, new factors that had not been taken into account in previous models have been considered.

Thus, the energy functional (2) is supplemented with new terms of the type (9), which reproduce the properties of the materials of the surface layers of the contacting bodies. In addition, not only the nominal shape of the contacting surfaces (1) is taken into account but also the perturbation of this shape, which leads to the appearance of the corresponding terms in (10). The perturbation and properties of the materials of the surface layers of contacting bodies are also considered in passing (11).

As a result of our studies on the stressed-strained state of contacting bodies using the example of die elements (punch-matrix-workpiece), the distributions of contact pressure and equivalent stresses according to Mises (Fig. 5–16) were obtained, which are significantly different in nature from the case of contact of bodies with nominal coincident surfaces of contacting bodies (works [2, 6] and Fig. 17).

Thus, in conclusion, we have:

1) an improved model of contact interaction, which takes into account, unlike the known ones [9–12], additional factors (roughness and disturbance of the shape of contacting surfaces); at the same time, unlike single-factor models [13–21], both factors are taken into account, and in aggregate;

2) by building on works [2, 6, 22–24], regularities of the stress-deformed state of contacting bodies have been established; thereby establishing not only qualitative features (Fig. 5–17) but also quantitative dependences (Fig. 18–21).

If we analyze the results, the following features can be noted:

1. At $\Delta\equiv0$, that is, at coincident conjugate surfaces of contacting bodies, and $\lambda\equiv0$, that is, at zero contact compliance, dependences are observed that correspond to the known ones [2, 6]. Thus, the areas of the contact area, stress levels, and contact pressure remain at approximately the same level when the loads change. At the same time, the contact pressure and stress are concentrated in the area of the cutting edges.

2. At $\Delta\not\equiv0$, dependences of the area of the contact area, stress levels, and contact pressure differ from those specified in p. 1. In particular, it can be seen that the area of the contact region, stress levels, and contact pressure are characterized by an increasing tendency with decreasing γ. For $\overline{q}$ and $\overline{\sigma}$, this trend is the opposite.

3. Peculiarities of the contact interaction for $\alpha\neq0$ are the displacement of the contact areas towards the cutting edge as τ increases. At the same time, the trends specified in p. 1 are disrupted.

4. When determining the effect of the rounding radius R on the controlled values, it can be noted that when it increases for low load levels, the nature of the contact pressure distribution is dramatically different from that described in p. 1. However, as τ increases, the differences between these distributions smooth out. And the larger the β parameter, the faster this smoothing is felt.

It can also be seen that an increase in R, i.e., the parameter β, leads to a decrease in contact pressure levels, equivalent to Mises stresses, and an increase in the size of the contact area.

In addition, the fact that the case of contact of axisymmetric bodies was considered is one of the shortcomings of our research. In the future, it is planned to eliminate this shortcoming by expanding the statement to a general spatial statement about the stress-deformed state of non-contacting bodies.

Thus, as can be seen from the results of our research, the varied parameters α, β, γ exert a significant influence on the stressed-strained state and the contact interaction of the system of contacting bodies "punch – workpiece – matrix".

In particular, with nominally matching surfaces and zero contact stiffness, the contact area does not depend on the level of loads, while the contact pressure distribution is directly proportional to the operating load. The components of the stressed-strained state are also directly proportional to the load level.

The noted regularity is preserved when the surfaces of the contacting bodies coincide but with non-zero contact compliance. At the same time, the contact area increases with a decrease in contact stiffness, and the level of contact pressure and the component of the stressed-strained state of the bodies decreases.

If the shape of the conjugate surfaces of the contacting bodies is disturbed, then the above-mentioned property of direct proportionality of the contact pressure and the component of the stressed-strained state of the load level is violated. At the same time, the tendency of the growth of the contact zone and the decrease of the level of contact pressure and the components of the stressed-strained state is preserved.

The results of our work could be applied in mechanics of contact interaction, mechanical science, and mechanical engineering.

The limitations of the study are as follows: the applied axisymmetric statement is partial relative to the full spatial statement; the plastic deformation of the materials of the body system, especially the workpiece, is not taken into account; the process of separation of the workpiece material during stamping is not simulated.

The current research will be extended by taking into account the physically nonlinear properties of the surface layers of contacting bodies.

7. Conclusions

1. An improved mathematical model of the stressed-strained state of contacting bodies with closely shaped surfaces has been built. This model differs, firstly, by taking into account the stiffness factors of the surface layers of the materials of the contacting bodies, as well as the disturbance of the shape of the contacting bodies, and in the aggregate. Secondly, not a local but a variational statement of the problem of contact interaction of bodies is applied. Such features and differences create advantages compared to known models in the study of the contact interaction of elements of real structures.

2. Parametric models of the stressed-strained state of contacting bodies of close shape have been constructed using an example of the elements of separation dies (punch – workpiece – matrix). Such models make it possible to quickly analyze the influence of varied parameters on the contact interaction of these bodies.

3. The regularities in the influence of the disturbance of the shape of the surfaces of contacting bodies and the properties of the materials of the surface layers and coatings on their stressed-strained state have been established. In particular, the perturbation of the shape of the contacting surfaces dramatically changes the character of the contact pressure distribution and the component of the stressed-strained state. This indicates the impossibility of using conventional models using contact models of undisturbed surfaces and additional correction factors. It was also established that taking into account the material properties of the surface layers has a smoothing effect. The contact area expands, and the level of contact pressure and equivalent stresses decreases: the difference is 1.5–2.5 times or more.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study, as well as the results reported in this paper.

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Data availability

The data will be provided upon reasonable request.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

References

1. Atroshenko, O., Tkachuk, M. A., Martynenko, O., Tkachuk, M. M., Saverska, M., Hrechka, I., Khovanskyi, S. (2019). The study of multicomponent loading effect on thinwalled structures with bolted connections. Eastern-European Journal of Enterprise Technologies, 1 (7 (97)), 15–25. https://doi.org/10.15587/1729-4061.2019.154378
2. Tkachuk, M. M., Grabovskiy, A., Tkachuk A., M., Hrechka, I., Ishchenko, O., Domina, N. (2019). Investigation of multiple contact interaction of elements of shearing dies. Eastern-European Journal of Enterprise Technologies, 4 (7 (100)), 6–15. https://doi.org/10.15587/1729-4061.2019.174086
3. Johnson, K. L. (1985). Contact Mechanics. Cambridge University Press, 462. https://doi.org/10.1017/cbo9781139171731
4. Popov, V. L. (2017) Contact Mechanics and Friction. Heidelberg: Springer Berlin, 391. https://doi.org/10.1007/978-3-662-53081-8
5. Barber, J. R. (2018). Contact Mechanics. Springer, 585. https://doi.org/10.1007/978-3-319-70939-0
6. Grabovskiy, A., Tkachuk, M. A., Domina, N., Tkachuk, G., Ishchenko, O., Tkachuk, M. M. et al. (2021). Numercnal analysis of contact interaction of bodies with nearly form surfaces. Bulletin of the National Technical University «KhPI» Series: Engineering and CAD, 2, 29–38. https://doi.org/10.20998/2079-0775.2021.2.05
7. Popp, A., Wriggers, P. (Eds.) (2018). Contact Modeling for Solids and Particles. Cham: Springer, 228. https://doi.org/10.1007/978-3-319-90155-8
8. Ciarlet, P., Luneville, E. (2023). The Finite Element Method: From Theory to Practice. Wiley-ISTE, 400. https://doi.org/10.1002/9781394229758
9. Martynyak, R. M., Prokopyshyn, I. A., Prokopyshyn, I. I. (2015). Contact of Elastic Bodies with Nonlinear Winkler Surface Layers. Journal of Mathematical Sciences, 205 (4), 535–553. https://doi.org/10.1007/s10958-015-2265-0

10. Huang, K., Zhang, S. (2023). Beyond Monotone Variational Inequalities: Solution Methods and Iteration Complexities. Pacific Journal of Optimization. https://doi.org/10.48550/arXiv.2304.04153
11. Trémolières, R., Lions, J.-L., Glowinski, R. (2011). Numerical Analysis of Variational Inequalities. Amsterdam: Elsevier. 775.
12. Kalker, J. J. (1977). Variational Principles of Contact Elastostatics. IMA Journal of Applied Mathematics, 20 (2), 199–219. https://doi.org/10.1093/imamat/20.2.199
13. Ciavarella, M., Papangelo, A. (2021). On the Interaction of Viscoelasticity and Waviness in Enhancing the Pull-Off Force in Sphere/Flat Contacts. Tribology Letters, 69 (4). https://doi.org/10.1007/s11249-021-01488-w
14. Pohrt, R., Popov, V. L. (2013). Contact Mechanics of Rough Spheres: Crossover from Fractal to Hertzian Behavior. Advances in Tribology, 2013, 1–4. https://doi.org/10.1155/2013/974178
15. Ciavarella, M. (2015). Adhesive rough contacts near complete contact. International Journal of Mechanical Sciences, 104, 104–111. https://doi.org/10.1016/j.ijmecsci.2015.10.005
16. Pohrt, R., Li, Q. (2014). Complete boundary element formulation for normal and tangential contact problems. Physical Mesomechanics, 17 (4), 334–340. https://doi.org/10.1134/s1029959914040109
17. Persson, B. N. J. (2001). Elastoplastic Contact between Randomly Rough Surfaces. Physical Review Letters, 87 (11). https://doi.org/10.1103/physrevlett.87.116101
18. Subbotin, O., Bilozerov, V., Volkov, O., Subbotina, V., Shevtsov, V. (2022). Friction properties of mao coatings on aluminum alloys. Bulletin of the National Technical University «KhPI» Series: Engineering and CAD, 2, 59–63. https://doi.org/10.20998/2079-0775.2022.2.07
19. Agureev, L., Savushkina, S., Ashmarin, A., Borisov, A., Apelfeld, A., Anikin, K. (2018). Study of Plasma Electrolytic Oxidation Coatings on Aluminum Composites. Metals, 8 (6), 459. https://doi.org/10.3390/met8060459
20. Subbotina, V., Al-Qawabeha, U. F., Belozerov, V., Sobol, O., Subbotin, A., Tabaza, T. A., Al-Qawabah, S. M. (2019). Determination of influence of electrolyte composition and impurities on the content of α-AL_2O_3 phase in MAO-coatings on aluminum. Eastern-European Journal of Enterprise Technologies, 6 (12 (102)), 6–13. https://doi.org/10.15587/1729-4061.2019.185674
21. Belozerov, V., Sobol, O., Mahatilova, A., Subbotina, V., Tabaza, T. A., Al-Qawabeha, U. F., Al-Qawabah, S. M. (2018). Effect of electrolysis regimes on the structure and properties of coatings on aluminum alloys formed by anodecathode micro arc oxidation. Eastern-European Journal of Enterprise Technologies, 1 (12 (91)), 43–47. https://doi.org/10.15587/1729-4061.2018.121744
22. Tkachuk, N. A., Diachenko, S. S., Posviatenko, E. K., Kravchenko, S. A. et al. (2015). Kontynualnaia y dyskretno-kontynualnaia modyfykatsyia poverkhnostei detalei. Kharkiv: Shchedra sadyba plius, 259.
23. Tkachuk, M. M. (2023). Kontaktna mekhanika til iz urakhuvanniam neliniinykh vlastyvostei poverkhnevykh ta promizhnykh shariv. Dnipro: Vydavets Obdymko Olha Stanislavivna, 255.
24. Grabovskiy, A., Tkachuk, M. A., Kokhanovska, O., Tkachuk, M. M., Domina, N., Tkachuk, G. et al. (2022). Contact interaction of similar shape bodies at small disturbance of initial gap distribution. Bulletin of the National Technical University «KhPI» Series: Engineering and CAD, 2, 23–34. https://doi.org/10.20998/2079-0775.2022.2.03
25. Washizu, K. (1975). Variational Methods in Elasticity and Plasticity. Pergamon Press, 420.