

UDC 629.113

DOI: 10.15587/1729-4061.2025.321663

# DETERMINING THE EFFECT OF TRIANGULAR INTERWINDOW OPENINGS IN A BUS BODY ON ITS STRUCTURE, STRENGTH, AND PASSIVE SAFETY

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The object of this study is a spatial model of the body frame for the *Ukrautobusprom 4289* city bus, which is subject to structural optimization of the sidewalls in order to strengthen them because of the increase in equipped weight. The reason for loading is the need to install batteries on the roof, which is the only possible location, given the low-entry layout. Electrification of city buses is associated with the inevitable regulated reduction of CO<sub>2</sub> emissions by 30 % by 2030 (Euro 7) and complete decarbonization by 2050. Making up 30–40 % of the total cost of the bus, the body requires preservation when re-equipping diesel city buses for electric traction. Electrification automatically imposes UNECE R100 requirements for the absorption of 5.5 and 6g accelerations by the battery pack together with the body. To solve the problem, a transition from classic rectangular to triangular interwindow openings of the sidewalls has been proposed. An analytical methodology for simulating full-scale tests is proposed, close to real physical tests. Owing to the higher rigidity of triangular structures, a reduction in maximum stresses by 2.85 and 16.75 % was achieved under the static torsion and bending modes while the structure was maintained within the yield strength of steel  $\sigma_y=252$  MPa. Maximum deformations decreased by 28.71 % in bending and by 50.77 % in torsion. Stresses under R100 conditions decreased by 18.52 and 16.07 % under the 6.6g and 5g modes, respectively. Deformations in the latter case decreased by 46.09 % and amounted to 10.83 mm only. Owing to the proposed approach, it was possible to achieve unification of the body for any type of drive: diesel, hybrid, or electric. Given sufficient technological feasibility of bus body production, this solution could be used in practice

**Keywords:** interwindow openings, bus body, structural optimization, von Mises stress, passive safety

Received 31.08.2024

Received in revised form 23.12.2024

Accepted 20.01.2025

Published 26.02.2025

## 1. Introduction

The EU sets ambitious targets for reducing CO<sub>2</sub> emissions for buses (city and intercity with an equipped weight of more than 3,500 kg) by 15 % by the end of 2025 and by 30 % closer to 2030. To achieve climate neutrality by 2050, buses need to achieve full decarbonization. A possible solution is the introduction of vehicles with a zero emission level (Zero Emission Vehicles – ZEVs), which include electric buses on fuel cells (Fuel Cell Electric Buses – FCEBs) or directly on batteries (Battery Electric Buses – BEBs). The latter significantly increase the equipped mass of city buses (up to 12–18 % depending on the class), creating an additional load on the body. Given the low-entry or low-floor layout, the only

possible place to install batteries is the roof, which in turn requires finding ways to strengthen the sides of the body. Since the body of a bus makes up about 30–40 % of its cost, and the city fleet of the EU as of 2024 has about 900,000 vehicles, there is a global task of adapting bodies to BEBs with minimal structural changes. One of the possible solutions for strength optimization is the transition from conventional rectangular side window openings to more rigid triangular ones. As a result of electrification, which requires mandatory compliance with the requirements of R100, the question of the uniform strength of the body under the conditions of daily operation in the modes of so-called “bending” and “torsion” also arises. Adherence to allowable stresses (relative to yield strength), safety factor, and deformations are among

the requirements that must be checked. Therefore, studies aimed at designing the optimal structure and strength of the bus body by integrating triangular window openings and comparing the stress-strain state (SSS) with the classic rectangular model are relevant. In practice, the research results are equally important both for existing car fleet (local intervention in the body side structure above the window frame) and for new promising models with maximum unification for any type of drive.

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## 2. Literature review and problem statement

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Before starting to analyze available studies, it is advisable to get acquainted with our previous work, which forms the foundation of technological advancements in the field of modeling the stress-strain state of structures in the Ansys environment. In fact, the results of our studies acted as a driver for devising such new measures to optimize strength as the integration of triangular structures to the sidewalls of the body. Thus, the current work is a logical continuation of a series of previously published papers that are more practical for engineers and scientists as a complex. Thus, paper [1] focuses on the features of the low-floor body and the structural differences of city buses, such as the absence of deformation zones. We modeled a frontal impact of a city bus taking into account the UNECE R29 requirements for passenger safety. The research results demonstrate the stress and deformation zones of the body, taking into account the reduced stiffness indicators and lower energy absorption of city buses in the event of a collision. The subject of estimating stresses, strains, and safety factor  $SF$  of a spatial body truss was also investigated by us in [2], however, using a different type of body as an example. Under the conditions of the transition to Euro-7 environmental standards with the change of diesel traction to electric, the question arises of the influence of the installed battery unit on the architecture and strength indicators of the body. The paper simulates the static and dynamic load modes of the truss using the coefficient  $k_d=2.0$ , analyzes the stresses and deformations under the “bending” and “torsion” modes. The research results are important from the point of view of the conditions of daily operation, but the issues related to passive safety, such as UNECE R100, remained unresolved. It is shown that measures to optimize the structure made it possible to reduce the mass of the truss by 4.13 % and reduce the maximum stresses from 381.13 MPa to 270.5 MPa. Our paper [3] also provides the theoretical and applied foundations of Ansys, which are necessary for effective work with the model – Mises stress estimation, deformation zones, and the  $SF$  safety factor. On the basis of the processed materials and taking into account the global trends towards the electrification of buses (increasing the equipped weight), an idea was formed to strengthen the sides of the body by transforming the window openings from a rectangular to a triangular shape. In the history of Ukrainian bus construction, there were already prototypes with a similar “triangular” concept – models of the LAZ-360 series from Ukrbusprom, which is a source of pride for domestic engineering. However, the roof structure remained classical in contrast to the proposed “triangular” one in the current study. The likely reason is objective difficulties associated with the complexity of production and its manufacturability (profile cutting and welding at an angle, etc.).

Modern international publications by other authors consider the strength and passive safety of bus bodies not only on

diesel but also on electric traction (BEB). This is the approach used in work [4], in which the authors conducted static and dynamic tests in order to optimize the body structure of a fully electric bus manufactured by Letenda Inc. To ensure durability in accordance with regulatory requirements, experiments were performed with variable driving conditions. Special attention is paid to the methods of correlating experimental data of vehicle response with known input signals for model validation. Work [5] considers the optimization of the design of the light iron-aluminum body of an electric bus, taking into account static characteristics (strength and stiffness), safety in the event of a side rollover, and possible weight reduction. Finite-element models of static loads and side impact were developed, and the body truss was segmented by the thickness of profile pipes and their contribution to deformation. Three optimization schemes targeting different objectives were compared and the results evaluated through FE simulations. It was shown that the stresses decreased by 34.41 %, and the penetration depth by 4.48 %, while the mass remained almost unchanged; however, the issues of stiffness remained unresolved. Side rollovers as one of the basic certification test modes of electric buses are also covered in [6]. An analysis of variable parameters of the body was carried out and a mathematical model of multi-criteria optimization of the structure was built. A neural network based on radial basis functions was used to approximate the optimization model, and its accuracy was verified using test data. The optimization problem was solved using the Non-dominated Sorting Genetic Algorithm II (NSGA-II). The results showed that the weight of the optimized model was reduced by 240 kg (9.0 %) without changing the body frame materials, while maintaining the required strength and stiffness characteristics. All this gives reason to argue that it is advisable to use the NSGA-II algorithm in this kind of problems. This is a popular evolutionary algorithm used to solve multi-objective optimization problems. This is the approach used in [7]. The work considers the optimization of the bus body structure to reduce mass, taking into account rollover safety (ECE R66). A combination of a neural network (back propagation – BP) and the NSGA-II algorithm was used for multi-criteria optimization. The results showed a weight reduction of 7.7 % without loss of safety, which confirms the effectiveness of the proposed approach. Paper [8] studies the strengthening of the bus frame with composite materials to increase its strength and protect passengers during accidents. The frame of the bus, made of steel, was reinforced with a coating of composite materials. In accordance with the ECE R66 standards for the strength of superstructures of large passenger vehicles, a structural analysis was performed using LS-DYNA simulations. It is shown that the use of a frame made of composite-reinforced steel improves the protection of passengers, which was confirmed by the final rollover simulations. In addition to side rollovers, frontal crash tests remain relevant. For example, work [9] considers the development and implementation of a scheme for optimizing the structure of an electric bus body, taking into account a frontal collision. The authors consider uncertainty factors that could affect the energy-absorbing characteristics of the body during an impact. An approach to strength optimization is proposed, which takes into account the design of deformation zones and makes it possible to analyze the influence of parameter variations on objective functions and constraints. Analytical methods such as entropy weight method, TOPSIS method, and correlation analysis were used to select variables, which

made it possible to choose 15 shape parameters with the greatest influence on body strength. The research results of the work, like all the publications analyzed above, are based on a model with rectangular window openings. None of the models that were subjected to side or frontal crash tests in the cited works had triangular windowsills, which could theoretically improve the rigidity of the body. The likely reason is the objective difficulties associated with the expendable part of the body production with triangular armholes. The lack of real buses with similar sidewalls causes a lack of prototypes for their research and, therefore, related scientific works.

There are a number of publications whose authors chose an approach to optimizing the body based on the use of aluminum in combination with steel, but without the inclusion of triangles in the body side structure. Thus, work [10] considers reducing the mass of the hybrid body of an electric bus made of steel and aluminum to increase the mileage. A combination of a genetic algorithm (GA) and a radial basis function (RBF) model was used for multicriteria optimization. The proposed approach made it possible to reduce the mass of the body by 38.4 % owing to the use of an aluminum frame, and additional optimization reduced the mass by another 4.28 % without losing rigidity and strength. The research results are correlated with [11], in which the aluminum-steel structure of the body of an electric bus is also analyzed in order to reduce mass and increase mileage. The use of gradient optimizers ensured a reduction in mass while preserving the design characteristics, but the NSGA-II algorithm could have been used. Paper [12] describes the optimization of the bolted T-connection of the steel-aluminum body of an electric bus to increase its durability and reduce deformations. A finite element model was used to analyze fatigue strength, and the Kriging method was used to build a surrogate model. The optimization was carried out using the adaptive simulated annealing method (Adaptive Simulated Annealing – ASA), and to increase reliability, the Monte Carlo optimization approach, and archive-based Micro Genetic Algorithm (AMGA) were used. The research results showed an increase in the minimum resource by 60.80 % and a decrease in the maximum deformation by 4.87 %, which significantly improves the overall efficiency of the structure. All this gives reason to assert that it is expedient to carry out a comprehensive analysis both of the structural optimization of the body and the use of materials alternative to steel.

Separate studies consider the structural optimization of the strength of bus bodies, however, in the absence of triangular armholes in the composition of the body. Thus, the authors of [13] investigated the multi-criteria optimization of the body design of a large electric bus to reduce mass, reduce movement resistance, and increase energy efficiency. They performed topological optimization of the roof, floor, and side panels of the frame using FEA. Through sensitivity analysis, 13 variables affecting weight reduction were identified. Further optimization was performed by the MOGA method, taking into account the minimization of mass and the maximization of bending and torsion frequencies. The results showed that the weight of the frame was reduced by 303 kg (11 %), providing all the requirements for static and dynamic characteristics, but the question of body rigidity remained open. Study [14] improved the conventional bus body frame design by focusing on the cross-sections of the tubular beams instead of just their thickness. The multi-objective approach to optimization includes a radial basis function neural network (RBFNN) and a genetic sorting algorithm II (NSGA-II – non-dominated sorting genet-

ic algorithm). The results show a 5.9 % reduction in structural mass and a significant improvement in stiffness, with prediction errors not exceeding 0.3 %. It is shown that the selection of combinations of cross-sections is an effective solution but the issues of welded joints, the area of which is reduced when using a lighter assortment of pipes, remain unresolved.

Bus drive schemes could be different: not only diesel or electric, but also hydrogen. The body of the bus must be unified, so it is advisable to read, for example, [15], in which the authors present the adaptation of the design for the integration of heavy and bulky components of alternative drives, such as battery or hydrogen systems. Within the framework of the AnWaAS project, structural solutions with the greatest potential for lightness have been identified, in particular, the use of trapezoidal windows, which is close to the topic of current research. The optimization made it possible to reduce the weight of the body by more than 20 %, ensuring compliance with the flow of forces and weight reduction. All this gives reason to assert that it is advisable to conduct research on the selection of alternative rectangular window openings. The authors of paper [16] investigated the safety of the hydrogen city bus frame. The analysis showed that the maximum stresses during emergency turning and torque exceed the yield strength of the material. Optimizing the thickness of the components reduced stress to an acceptable level and reduced the frame weight by 106 kg, meeting the strength and weight requirements, but no deformation information.

Since among the boundary conditions of the current study there is a battery pack on the roof (R100 test), it is important to analyze the strength studies on the battery pack structures. This is the approach used in work [17], in which the design of the battery unit was analyzed with static and modal analysis under various conditions (turning and emergency braking on a rough road). Modal analysis revealed a frequency of 33.69 Hz, indicating the possibility of resonance. The authors of paper [18] proposed a method for optimizing the design of the battery unit housing to reduce weight through the thickness of the housing as an optimization parameter. It is shown that the maximum stress is reduced to an acceptable level, the first-order frequency is increased by 41 %, the resonance is reduced, the maximum deformation is reduced from 2.7 to 1.12 mm, and the total mass is reduced by 26.8 %. Study [19] considers the strength of the attachment of battery packs with corresponding reactions, modal analysis, and the influence of the battery pack on the overall strength of the body. The shortcoming of the research and a possible future direction of its development is the lack of vibration tests, as it is given in work [20]. The authors describe the methodology for modeling and analyzing the vibrations of the bus body in accordance with the AIS-153:2018 standard. An approach to determining the lowest natural frequency and analyzing vibrations in accordance with regulatory requirements has been devised. Paper [21] analyzed the frame of a medium-sized passenger bus and its dynamic and vibrational properties when replacing the materials of auxiliary structural elements with fiberglass. Theoretical and experimental modal analysis confirmed a change in dynamic characteristics of up to 20 % (resonant frequencies) compared to a steel frame. The replacement of materials made it possible to reduce the weight of the frame by 11 %, lower the center of gravity, and maintain the safety of the structure, which improves stability, comfort, and reduces fuel consumption. The research results suggest the expediency of conducting discrete local tests of the battery block as part of the two-mass “body-battery” system.

Summarizing the above publications, it could be concluded that the use of the triangular structure of window openings has not been found in modern literature and has prospects for the development of research. It could be assumed that among the possible reasons for the lack of publications is the more complex process of manufacturing the body and its glazing on real buses. In the case of a curvilinear shape of the sidewall profile, conventional vertical racks have the same radius of rounding, and in one plane (transverse), and allow the use of a common pipe bender. The transition to diagonal racks, which as a result form triangular window openings, requires simultaneous bending and twisting of pipes. The situation is further complicated by the fact that they are of different lengths according to the sections of the bus, and therefore there is no question of a single pipe bender. Cutting diagonal pipes and their welding are also significantly more difficult in comparison with the conventional rectangular shape of the lumbar section of the body.

From our review of the literature, the following could be stated. Works [4–9] focused on strength and passive safety of buses (including BEB), but without tests according to R100; [10–12] compare aluminum-steel structures with conventional steel ones, but only with a rectangular sidewall structure. Publications [13, 14] study the influence of various pipe profiles on the strength of a conventional body configuration; [17–19] analyze the fastening of battery blocks according to R100, but do not consider the entire body as a whole. The authors of [20, 21] study the oscillations, vibrations, and comfort of buses with typical vertical body struts. Thus, among the considered works, no studies on structural optimization of bodies were found, as a way to solve the problem of loading buses in connection with their electrification by installing batteries on the roof. This body part is possible for low-floor urban BEBs, which causes the need to strengthen the body sides, the relevance of which is obvious in connection with the introduction of Euro 7 from 2030.

### 3. The aim and objectives of the study

The purpose of our study is to evaluate the effectiveness of transition from rectangular to triangular window openings of the bus body sidewalls with a comparison of stresses, displacements, and the coefficient of safety margin under different test modes. This will make it possible to partially preserve the existing fleet of city buses by restructuring their sides to match battery loads, as well as design new universal bodies.

To achieve the goal, the following tasks were set:

- to conduct tests under the modes of static bending and torsion of the body;
- to simulate the conditions of the UNECE R100/R110 Rules.

### 4. The study materials and methods

The object of our research is the spatial model of the body frame for the low-entry city bus “Ukravtobusprom – 4289”. The basic hypothesis of the research is to check the increase in strength and rigidity of the body as a result of the integration of triangular windowsills instead of conventional rectangular ones. A triangle is a more rigid figure than a rectangle because of its geometric stability – the impossibility of changing the shape while preserving the area without changing the length of the sides and angles. Instead, a rectangle could easi-

ly deform into a parallelogram if forces are applied to it, even if the side lengths remain the same. Mathematically, this is confirmed by the basic equation of the area of the triangle  $A_T$ , which connects the sides and angles:

$$A_T = \frac{1}{2} a \cdot b \cdot \sin(\theta), \tag{1}$$

where  $a, b$  are the sides of the triangle, mm;  $\sin(\theta)$  is the sine of the angle between them.

When a force  $P$  is applied to the rectangle, its angles could change, and it deforms to form a parallelogram. Formally, this could be described using a shift:

$$\gamma = \frac{\Delta x}{h}, \tag{2}$$

where  $\Delta x$  is the displacement of the upper side, mm;  $h$  is the height of the rectangle, mm.

Consider the deformations of the triangle  $\delta_T$  and the rectangle  $\delta_R$ , represented by rods with moment of inertia  $I$ , length  $L$ , and cross section  $A$ :

$$\delta_T \approx \frac{P \cdot L}{EA}; \delta_R \approx \frac{P \cdot L^3}{48EI}; \delta_R > \delta_T, \tag{3}$$

where  $P$  is force, N;  $E$  is Young’s modulus of the material, MPa.

Under the action of the force  $P$  applied to the top of the triangle, the rods work mainly in tension and compression. In the case of a rectangle, the force  $P$  is distributed along the edge, so individual rods (especially the upper and side ones) are additionally bent. Their moments  $I$  may not be sufficient to counteract the deflection  $\delta_R$ , and the rectangle will show larger deformations compared to the triangle.

Next, the deflection of the systems under the action of a uniformly distributed load  $P=q(x)$  is analyzed (Fig. 1): one consists of triangular elements (triangular lattice – model  $a$  in Fig. 1), and the other – of rectangular elements (rectangular lattice – model  $b$  in Fig. 1).

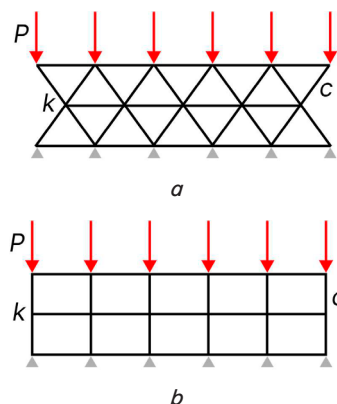


Fig. 1. The loading scheme of the rod lattice:  $a$  – triangular;  $b$  – rectangular

The differential equation for the deformation of a triangular system, based on the principle of stretching and compressing rods (Hooke’s law), taking into account stiffness and load transfer between adjacent elements, could be written in the form:

$$EA \frac{d^2 u_i}{dx^2} = P_i - k(u_i - u_{i+1}) - C \frac{du_i}{dt}, \quad \forall i \in [1, N], \tag{4}$$



where  $u(x)$  is the horizontal movement of the element as a function of the  $x$  coordinate;  $k$  – stiffness coefficient of the connection between adjacent triangles, N/mm;  $u_i - u_{i+1}$  – difference in displacements between adjacent elements, mm.

If the triangle receives vertical loads, its rigid structure compensates for them through the tension or compression of the diagonal rods with a corresponding horizontal displacement of the nodes. This explains the nature of the horizontal displacement  $u(x)$ .

For a rectangular lattice, each element is prone to bending since there are no diagonal elements that would stabilize the structure, as in a triangular system. Thus, one could consider the Euler-Bernoulli bending equation of the beam for each element, taking into account the damping to account for the dynamic behavior and energy loss in the system:

$$EI \frac{d^4 w_i}{dx^4} = q_i(x) - k(w_i - w_{i+1}) - C \frac{dw_i}{dt}, \quad \forall i \in [1, N], \quad (5)$$

where  $w(x)$  – the vertical deflection of the element as a function of the  $x$  coordinate;  $q_i(x)$  – distributed load on rods, N/mm;  $k(w_i - w_{i+1})$  – stiffness of connections between elements (takes into account how the deflection of one element affects the deflection of the adjacent one), N;  $N$  – the number of elements providing  $N$  interconnected equations.

The dynamic damping coefficient  $C$  could generally be written as:

$$C = C_0 (1 - \alpha \cdot \omega^2), \quad (6)$$

where  $C$ ,  $C_0$  are dynamic and basic damping coefficient for a triangular/rectangular system, N·s/mm;  $\alpha$  is a correction factor for taking into account the effect of vibrations (takes into account the influence of the vibration frequency and has units of measurement that compensate for the quadratic influence of the angular frequency (characterizes the frequency with which the system oscillates around its equilibrium state), s<sup>2</sup>;  $\omega$  is the angular speed of system vibrations, rad/s.

The value of  $\omega$  is different for triangular and rectangular systems, so the difference between the damping coefficients could be written as:

$$\Delta C = C_T - C_D = C_0 \left[ \left( 1 - \alpha \cdot \frac{EA}{mL} \right) - \left( 1 - \alpha \cdot \frac{EI}{mL^3} \right) \right], \quad (7)$$

where  $m$  is the mass of the rod, kg.

Prospects for advancing the above equations include taking into account nonlinearity of stiffness and damping, as well as delayed responses to loads. To model nonlinear stiffness, one could use the extended Hooke's law:

$$F = k_1 x + k_2 x^2 + k_3 x^3, \quad (8)$$

where  $F$  – force, N;  $x$  – deformation, m;  $k_1, k_2, k_3$  – stiffness coefficients that describe linear and non-linear components.

Damping could also be non-linear. Higher-order speed-dependent damping models are commonly used, such as:

$$C(u) = C_0 + C_1 \left( \frac{du_i}{dt} \right)^2, \quad (9)$$

where  $C_1$  is a coefficient that takes into account the nonlinear effect, which increases with an increase in the rate of deformation, N·s/mm.

The time delay could be described in terms of additional terms that account for system memory or hysteresis effects. One method is to use models such as the Maxwell model or the Kelvin-Voigt model:

$$\sigma(t) + \tau \frac{d\sigma(t)}{dt} = E \left( \varepsilon(t) + \lambda \frac{d\varepsilon(t)}{dt} \right), \quad (10)$$

where  $\sigma(t)$  – stress in the system, MPa;  $\varepsilon(t)$  – strain;  $\tau, \lambda$  – parameters that describe viscoelasticity (system reaction delay), s.

The complete equation for one element of the structure within the framework of the system, taking into account all nonlinear effects, takes the form:

$$\begin{aligned} EA \frac{d^2 u_i}{dx^2} + \tau \frac{d}{dt} \left( EA \frac{d^2 u_i}{dx^2} \right) = \\ = Pi - \left( k_1 (u_i - u_{i+1}) + k_2 (u_i - u_{i+1})^2 \right) - \\ - \left( C_0 + C_1 \left( \frac{du_i}{dt} \right)^2 \right) \frac{du_i}{dt}. \end{aligned} \quad (11)$$

Let's turn to the equation that describes the behavior of the entire system, taking into account both linear and nonlinear effects of stiffness and damping. For a system of  $N$  triangles, equation (11) will look like a system of differential equations, which could also be represented in the form of a matrix form for compact notation:

$$J\ddot{u} + C_0 \dot{u} + C_1 (\dot{u})^2 + K_1 u + K_2 (u)^2 = Y, \quad (12)$$

where  $J$  is the diagonal mass matrix of each element (moment of inertia or rod mass);  $C(u')$  is a damping matrix that takes into account both linear and non-linear damping effects (may include dynamic coefficients  $C_0$  and  $C_1$ );  $K(u)$  is the stiffness matrix, which includes linear and nonlinear elastic properties (coefficients  $k_1$  and  $k_2$ ).  $C_0$  is the basic damping matrix, and  $C_1$  is the matrix of coefficients that takes into account nonlinear damping (dependence on displacement speed); where  $K_1$  is a linear stiffness matrix (for linear connections between elements), and  $K_2$  is a matrix for taking into account nonlinear effects (for example, for cases where the displacement between elements is significant).

Numerical methods such as Newton-Kant methods or direct integration using explicit and implicit schemes could be used to solve the system.

In this study, an analytical technique for simulating natural tests is proposed, which is as close as possible to real physical tests without conducting them. This approach makes it possible to carry out multi-iteration strength optimization of the body structure in the FEA environment and precedes laboratory certification with the destruction of elements of full-scale prototypes of the bus.

The above mathematical modeling is an effective way of pre-predicting the optimal configuration of the sides of the bus body. The next step is FEM calculation based on an iterative approach. We shall form the unified boundary conditions for the analysis of the object of research – the beam model of the body frame of the bus “Ukrautobusprom – 4289” in the Ansys Static Structural environment in a rectangular (initial) and triangular configuration. On the example of the T-T (Triangular-Torsion) mode, let's consider the peculiarities of the problem statement (Fig. 2):

- a model with triangular (Triangular) window openings of the side walls and roof slope;
- a “Fixed Support” tie (tag A, Fig. 2) is attached to the right front wheel (the mounting flange of the air cylinders of the suspension), and the left wheel is suspended without a support, which simulates the torsion mode;
- the suspension of the rear wheels is limited in vertical movement – a yoke of the “Displacement” type (tags B and C, Fig. 2);
- distributed masses (spheres in Fig. 2) according to the technical documentation: Powertrain (1315 kg), Front (220 kg), Driver (75 kg), Standing passengers (3808 kg), Seated passengers (1972 kg), Rest (3055 kg), and the body weight (2400 kg);
- Standard Earth gravity ( $9.806 \text{ m/s}^2$ ) with the coefficient of dynamism  $k_d=1.0$ ;
- Material – steel S235 (yield strength  $\sigma_y=252 \text{ MPa}$ ; Young’s modulus= $2.099 \times 10^5 \text{ MPa}$ , tensile ultimate strength= $428 \text{ MPa}$ , tangent modulus= $1180 \text{ MPa}$ ) with “Bilinear Isotropic Hardening” (stress-strain curve). Material data was imported from Granta EduPack into Ansys.

In total, 8 test modes were analyzed (Table 1), among which 4 relate to static load on bending (Bending) and torsion (Torsion) and another 4 simulate UNECE

R100/R110 conditions. Static bending modes assume a value of  $k_d=2.0$ , i.e., the loads double, but the bus does not have overhanging wheels (it rests on all 4 wheels). Modes for simulating R100/110 are focused on the strength of the roof and are based on the dynamic coefficient acting in the horizontal direction:  $k_{hd}=5.0$  transversely or  $k_{hd}=6.6$  longitudinally. The acceleration is applied to the air conditioning system (1250 kg) and to the electric batteries on the roof (1500 kg) or to the gas cylinder equipment (Fig. 3). The batteries apply to buses with hybrid or fully electric traction (UNECE R100).

For example, under the T-R-6.6 torsion mode with triangular window openings (conditionally, they could be called “triangular”), the load from the battery is 9900 N, and the conditioning is 8250 N, taking into account  $k_{hd}=6.6$ .

The mesh of the beam model with triangular window openings (Fig. 4) consists of 13194 elements (Elements) and 21,250 nodes (Nodes).

The model with rectangular window openings has 12,726 elements and 20,338 nodes. Both values are about 2–3 mathematical orders lower than the similar Solid model, which significantly simplifies the process of iterative optimization in the early stages of body design.

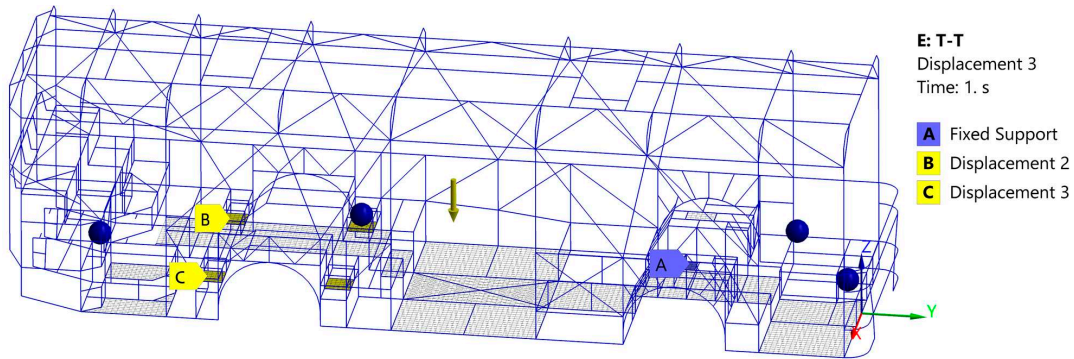


Fig. 2. Boundary conditions of the Ansys model under T-T mode

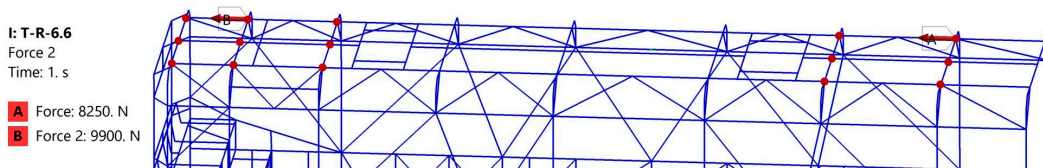


Fig. 3. UNECE R100/R110 modeling boundary conditions (mode T-R-6.6)

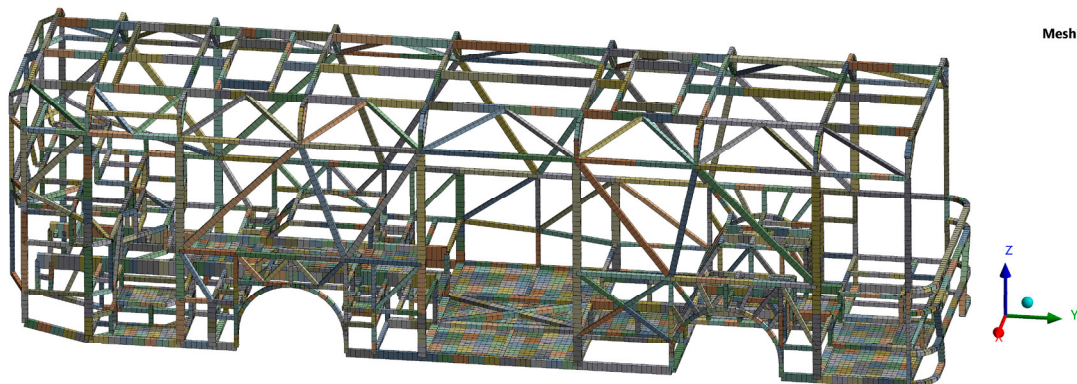


Fig. 4. Grid of finite elements of the model under T-T mode

Bus body frame test modes in Ansys

Mode	Frame	Type	$k_d$	$k_{hd}$	Conditioning, kg	Batteries, kg
R-B	rectangular	bending	2	1	–	–
R-T	rectangular	torsion	1	1	–	–
T-B	triangular	bending	2	1	–	–
T-T	triangular	torsion	1	1	–	–
R-R-6.6	rectangular	R100/110	1	6.6	1,250	1,500
T-R-6.6	triangular	R100/110	1	6.6	1,250	1,500
R-R-5	rectangular	R100/110	1	5	1,250	1,500
T-R-5	triangular	R100/110	1	5	1,250	1,500

**5. Results regarding the effect of triangular window openings on body strength**

**5.1. Tests under the modes of static bending and torsion of the body**

A comparison of the Mises stress results of the classic model with rectangular (map *a* in Fig. 5) and triangular (map *b* in Fig. 5) openings showed that the maxi-

imum stresses  $\sigma_{max}$  decreased from 276.43 to 230.13 MPa, and the average  $\sigma_{ave}$  from 22.34 MPa to 18.88 MPa. The location of  $\sigma_{max}$  has also changed – the max tag (Fig. 5) has migrated from the floor spar to the roof frame (rear door opening). Consolidated data are given in Table 2.

We shall compare the R-T and T-T simulation modes of hanging one of the wheels (static torsion) on the basis of the maximum displacement maps (Fig. 6). When switching from R-T to T-T, the maximum values halved: from 18.77 mm to 9.24 mm. This indicates a significant increase in the rigidity of the body.

The theory states that the material begins to plastically deform in locations where the Mises stress ( $\sigma_{mis}$ ) becomes equal to the ultimate stress, that is, the yield strength ( $\sigma_y$ ). Thus, the safety factor ( $SF_{min}=\sigma_y/\sigma_{mi}$ ) should be greater than 1. So, the value of  $SF_{min}$  left the dangerous zone (below 1.0) in both cases and increased from 0.92 to 1.10 when transitioning from the R-B to T-B mode (+19.57%), and from 0.99 to 1.03 for the case from R-T to T-T (+4.04%).

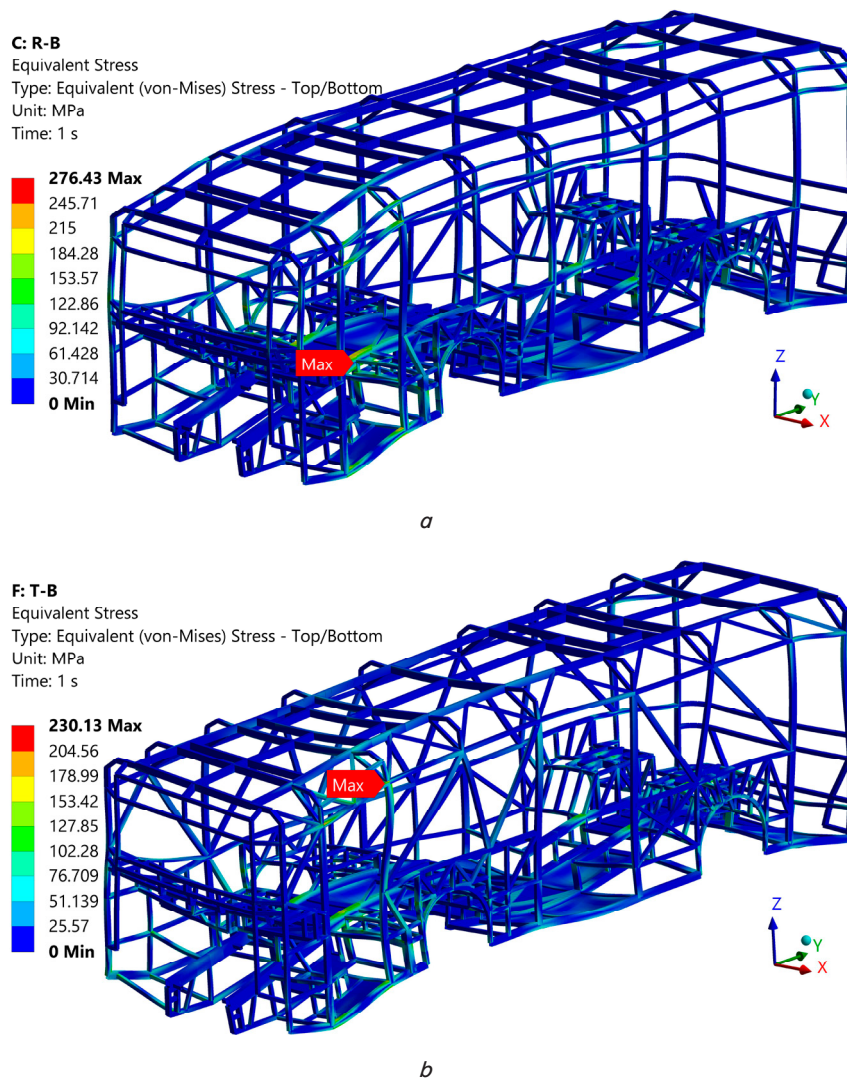


Fig. 5. Mises stress maps under the following mode: *a* – R-B; *b* – T-B



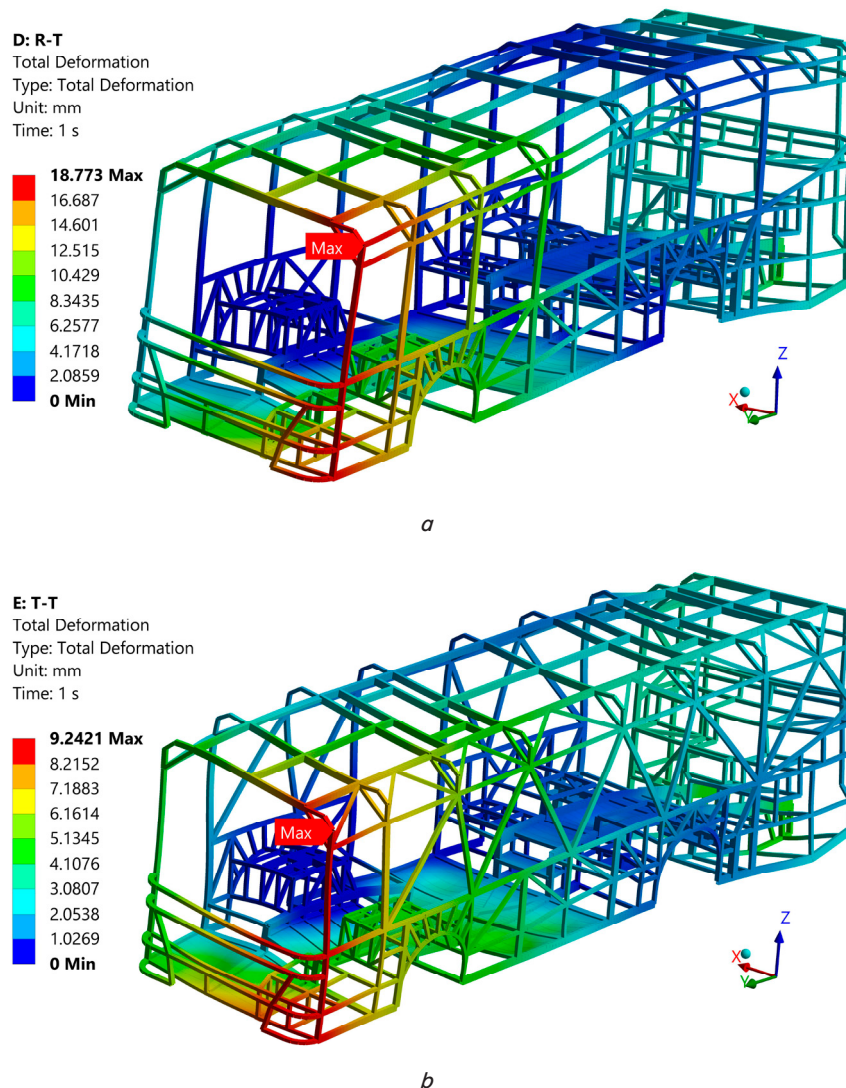


Fig. 6. Maps of movements under the following mode: *a* – R-T; *b* – T-T

### 5.2. Modeling the conditions of UNECE R100/R110 Rules

Simulation of tests for compliance with UNECE R100/R110 certification requirements began with checking the body’s ability to withstand longitudinal loads from the air conditioner and batteries on the roof (Fig. 3) taking into account  $k_{hd}=6.6$ . The R-R-6.6 model with a rectangular design of the window openings is much more pliable – the loss of shape is well observed relative to the initial undeformed state (*a* in Fig. 7). The stress  $\sigma_{max}$  under the R-R-6.6 regime is almost 33 MPa higher than under T-R-6.6 (*b* in Fig. 7) with triangular window openings (177.72 vs. 144.8 MPa). Average stresses  $\sigma_{ave}$  decreased from 13.49 to 9.74 MPa.

The assessment of the maximum stresses  $\sigma_{max}$  of the key pillars of the right side shows a reduction from 115.44 MPa (*a* in Fig. 8) to 89.63 MPa (*b* in Fig. 8). Average stresses decreased from 26.52 to 11.33 MPa.

The longitudinal stiffness of the structure could be judged on the basis of displacement maps – the maximum values of  $\delta$  are 12.91 mm under the R-R-6.6 mode (*a* in Fig. 9) and 6.96 mm under T-R-6.6 (*b* in Fig. 9). To understand: in the Ansys environment, the “Deformations” indicator, which is measured in mm, is responsible for displacement. In the clas-

sical sense, deformations are a relative quantity in mm/mm, but in Ansys they are termed “Strain”. In the following, the term “deformations” is used in accordance with Ansys terminology ( $\delta$ , mm).

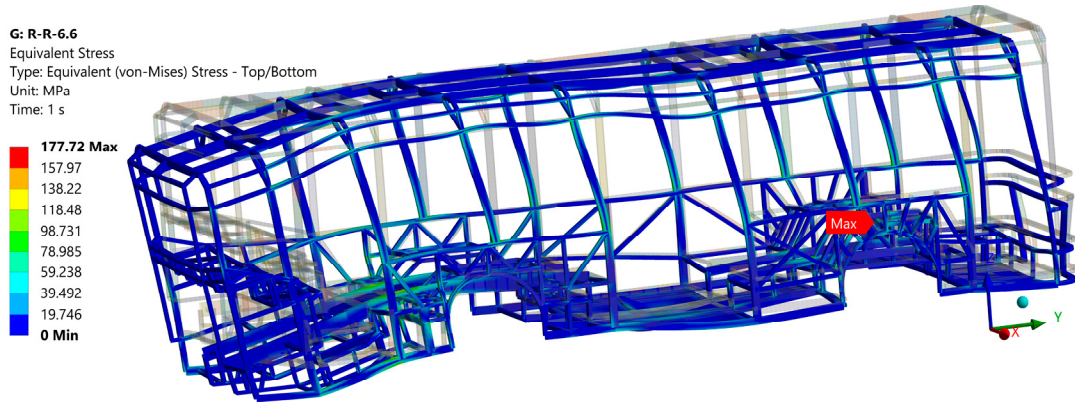
The transverse stiffness of the body is proposed to be evaluated based on the results of deformations  $\delta$  under the R-R-5 and T-R-5 modes: 13.05 versus 10.83 mm (Fig. 10). The maximum stresses  $\sigma_{max}$  decreased from 187.3 to 157.2 MPa.

The frontal view of the body frame shows transverse deformations, which are important in view of the lateral safety of passengers – the trapezoid of the remaining interior space (Fig. 11).

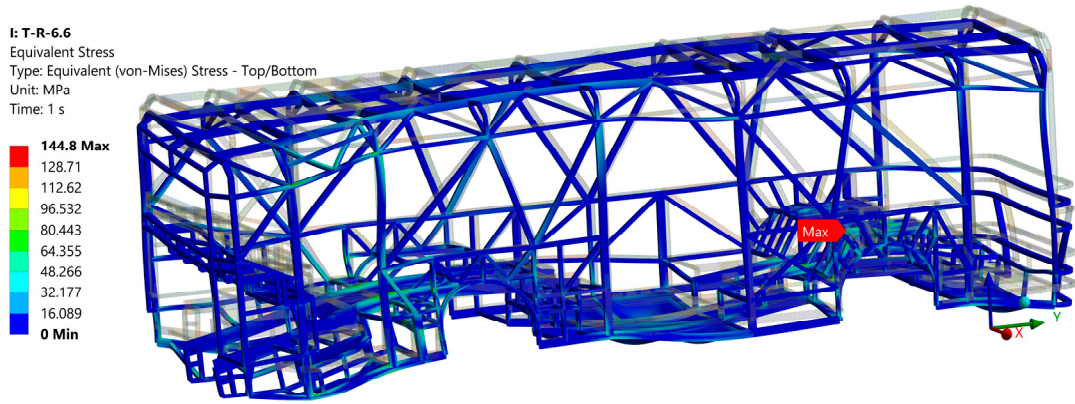
The given maps of *SF* relative to the yield strength  $\sigma_y=252$  MPa for the selected regimes (Fig. 12) are an effective applied method for assessing the uniform strength of the structure, primarily for engineers. Uniform strength is directly related to the material consumption, which means the cost of the bus, the body of which makes up at least 30 % of the total cost of the vehicle.

Based on the presented *SF* maps, a significantly larger area of green and orange color (*SF* below 5) is clearly observed under the R-R-6.6 regime (*a* in Fig. 12) than under T-R-6.6 (*b* in Fig. 12).



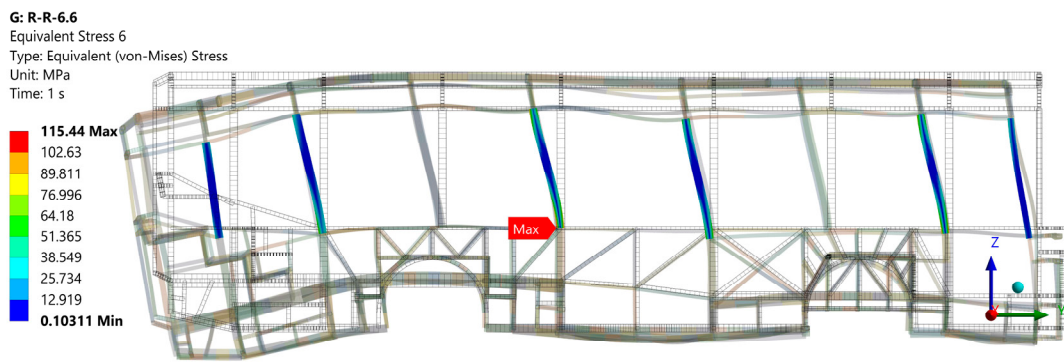


*a*

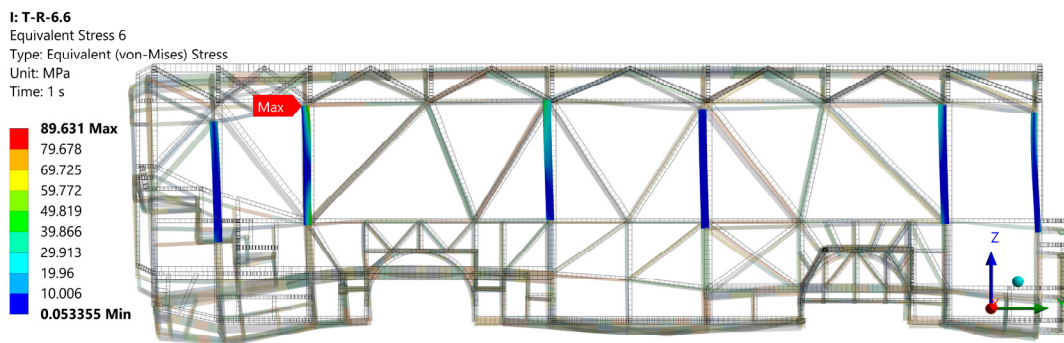


*b*

Fig. 7. Mises stress maps under the following mode: *a* – R-R-6.6; *b* – T-R-6.6



*a*



*b*

Fig. 8. Mises stress maps of the key pillars of the right side under the following mode: *a* – R-R-6.6; *b* – T-R-6.6

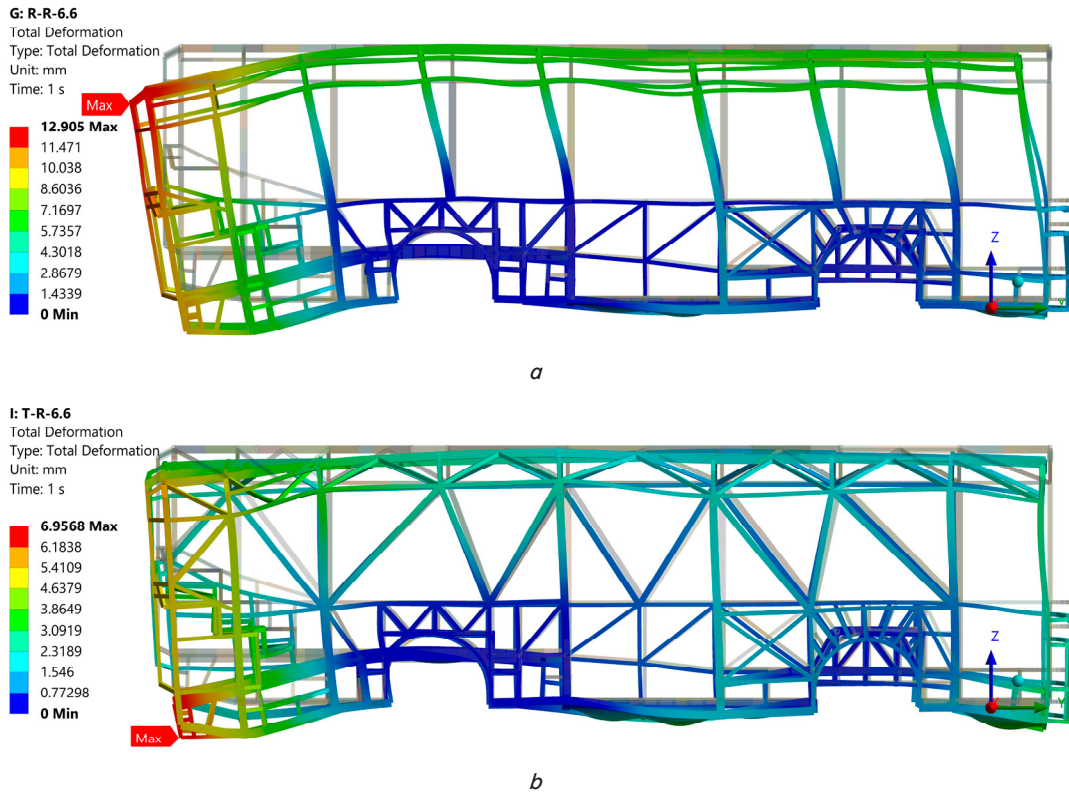


Fig. 9. Maps of deformations under the following mode: *a* – R-R-6.6; *b* – T-R-6.6

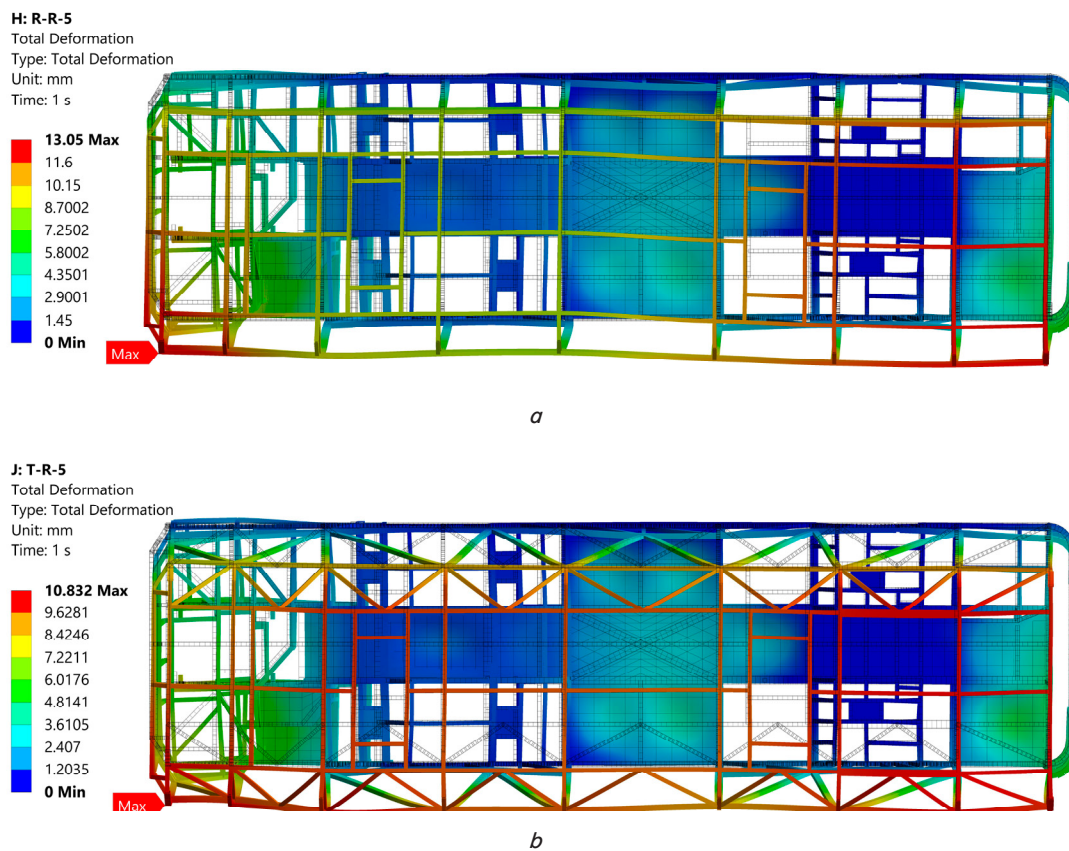


Fig. 10. Maps of deformations under the following mode: *a* – R-R-5; *b* – T-R-5

The uniqueness and practical benefit of the *SF* tool in Ansys is the ability to set the extreme stress threshold (Stress

Tool>Stress Limit Type>Custom Value) relative to which the value of the safety factor would be calculated. For example, let's



set a threshold equal to 50 MPa. If it is exceeded, the elements of the model acquire a red color. The effect is well observed when comparing the modes (Fig. 13): R-R-6.6 has a larger red area (Fig. 13, a), and the value  $SF_{min}=0.28$ , which is lower than T-R-6.6 (Fig. 13, b), where  $SF_{min}=0.35$ . The visual availability of the stress state simplifies the selection of alternative materials with a different yield strength for individual elements of the model (for example, reinforcement using S275 steel).

The consolidated values of displacements  $\delta$  and stresses (maximum  $\sigma_{max}$  and average  $\sigma_{ave}$ ) characterizing the effectiveness of the proposed design solution (triangular openings) are summarized in Table 2.

Under all analyzed modes (Table 2), the relative indicators  $\sigma_{max}$ ,  $\Delta\sigma_{ave}$  and  $\Delta\delta$  have a minus sign, which confirms the corresponding reduction of stresses and deformations when switching to a triangular system of window openings.

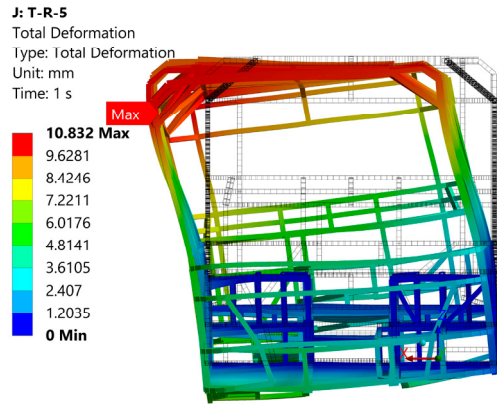
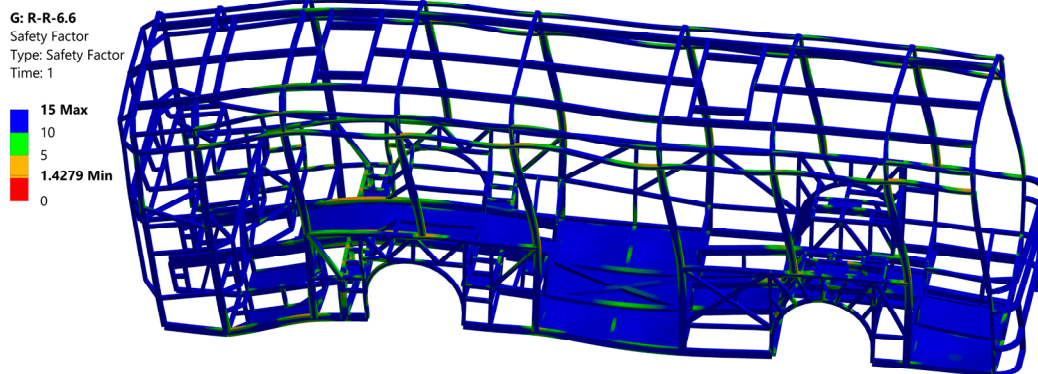


Fig. 11. Frontal deformation map under T-R-5 mode

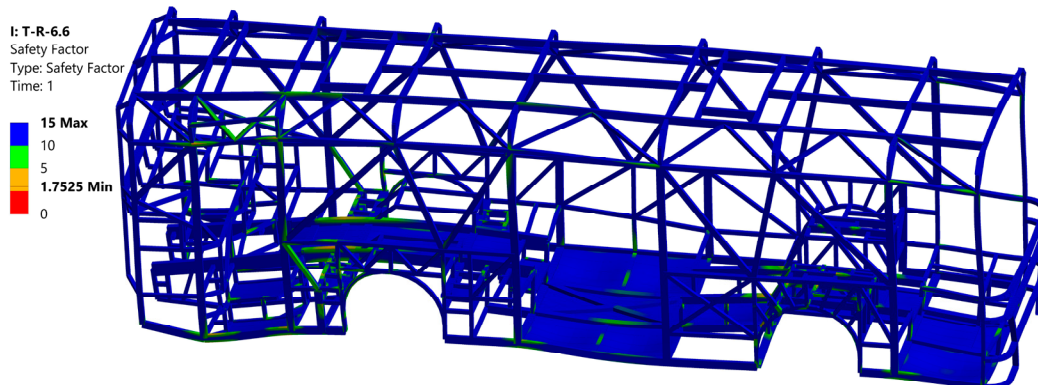
Table 2

Consolidated results of structural optimization of the bus body frame

Mode	R-B	T-B	R-T	T-T	R-R-6.6	T-R-6.6	R-R-5	T-R-5
$\sigma_{max}$ , MPa	276.43	230.13	254.44	247.18	177.72	144.8	187.3	157.2
$\Delta\sigma_{max}$ , %	-16.75		-2.85		-18.52		-16.07	
$\sigma_{ave}$ , MPa	22.34	18.88	14.62	11.54	13.49	9.74	14.09	12.71
$\Delta\sigma_{ave}$ , %	-15.49		-21.07		-27.80		-9.79	
$\delta$ , mm	18.91	13.48	18.77	9.24	12.91	6.96	13.05	10.83
$\Delta\delta$ , %	-28.71		-50.77		-46.09		-17.01	
$SF_{min}$	0.92	1.10	0.99	1.03	1.43	1.75	1.35	1.61
$\Delta SF_{min}$ , %	+19.57		+4.04		+22.38		+19.26	
$SF_{ave}$	12.02	12.59	13.41	14.02	13.51	14.29	13.51	13.79
$\Delta SF_{ave}$ , %	+4.74		+4.55		+5.77		+2.07	



a



b

Fig. 12. SF maps under the following mode: a – R-R-6.6; b – T-R-6.6

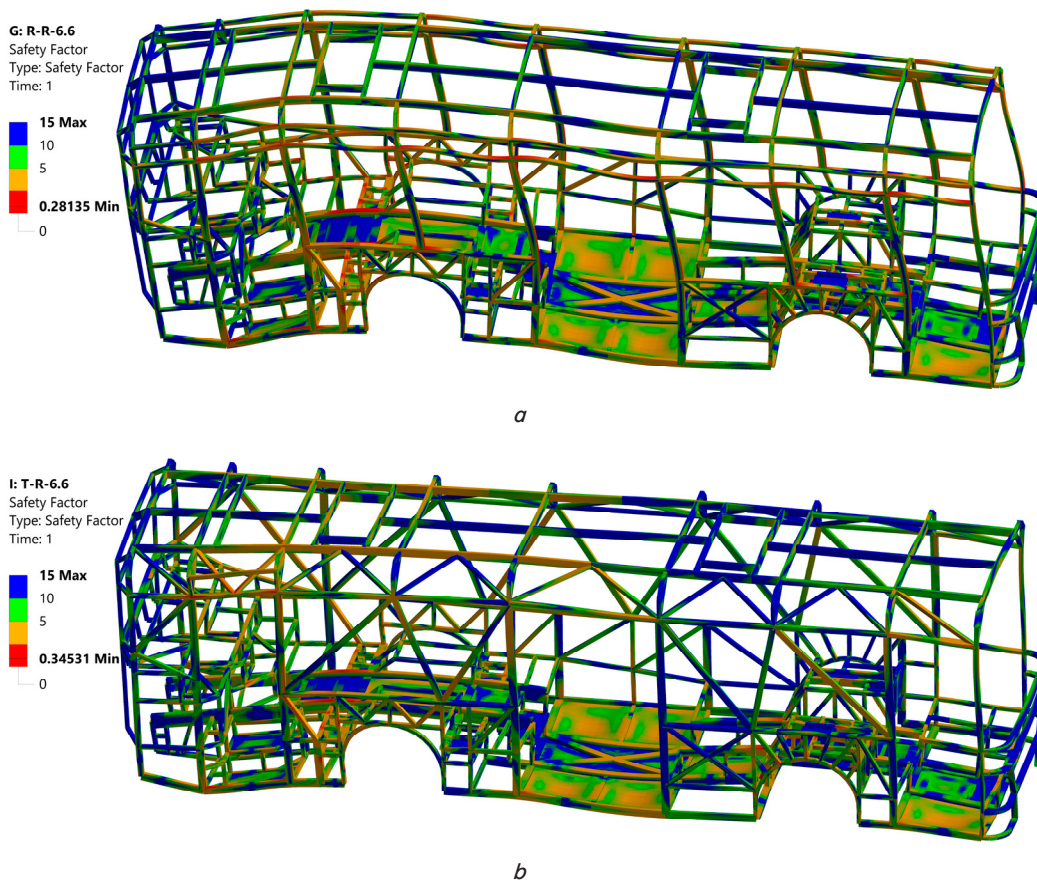


Fig. 13. Maps of  $SF$  relative to  $\sigma=50$  MPa under the following mode:  $a - R-R-6.6$ ;  $b - T-R-6.6$

**6. Discussion of results based on investigating the influence of the shape of window openings on the body strength**

Since the stress-strain indicators of specific models of buses are a trade secret of their manufacturers, and no direct analogs with triangular structures were found among the analyzed contemporary works, it is not possible to compare our results with actual buses. The above-mentioned LAZ-360 model, in fact, like vehicles of other manufacturers, had a different layout, dimensions, center of mass characteristics from the object of research, so any comparison with them would be incompatible. In general, a single approach to the assessment of body SSS is adopted in the industry: measurement of deformations and stresses relative to the yield point, assessment of SF, and material consumption.

The biggest advantage of the transition from the “rectangular” R-B mode to the “triangular” T-B mode is the removal of the body truss from the zone of plastic deformations ( $\sigma_y=252$  MPa) to the elastic limit (the range of Hook’s law). The maximum stresses  $\sigma_{max}$  decreased from 276.5 to 230.1 MPa (Fig. 5), and the safety factor  $SF_{min}$  increased from 0.92 to 1.10 (Table 2), which may indicate the safety of the structure from the point of view of passive safety and resource tests. The location  $\sigma_{max}$  also changed (Fig. 5) – the extremum of stresses migrated from the floor spar to the roof frame (rear door opening). This behavior is extremely positive because the main function of the specified spar is to hold the engine with the transmission, where there are additional loads in the form of vibrations, inertia, etc.

The R-T and T-T modes of simulating suspension of one of the wheels (static torsion) demonstrate a reduction of the maximum displacements by half: from 18.77 mm to 9.24 mm (Fig. 6). Both values correspond to the angle of the glued windshield, which directly determines the safety of its operation: moving the window opening by more than 10–15 mm could cause cracks in it. The value of  $SF_{min}$  left the dangerous zone (below 1.0) and increased from 0.99 to 1.03 during the transition from R-T to T-T mode (Table 2).

In order to comply with the new EURO 7 environmental regulations from July 1, 2027, it is economically more profitable for diesel buses to switch to electric traction. Systems such as exhaust aftertreatment (SCR, DPF), emissions monitoring (RDE), low-emission braking are more expensive than electric power and drive. Conversion of the existing fleet of city low-floor buses to electric traction involves certification according to UNECE R100 with verification of the strength of the upper part of the body. The peak stresses  $\sigma_{max}$  of the key struts of the right side (less rigid due to the presence of doors) decreased from 115.44 MPa to 89.63 MPa (Fig. 8). The maximum values of deformations  $\delta$  decreased from 12.91 mm (mode R-R-6.6) to 6.96 mm (T-R-6.6) (Fig. 9) and also moved from the roof to the floor frame (rear overhang). This is extremely positive from the point of view of passive safety. This could also be seen from the results of the  $SF_{min}$  maps (Fig. 12): the area of orange and green spots has significantly narrowed, which indicates the uniform strength of the frame.

The transverse stiffness of the body is estimated by the results of deformations  $\delta$  under the R-R-5 and T-R-5



modes: 13.05 vs. 10.83 mm (Fig. 11), recorded at the level of the over-window bar of the side walls (the upper corner of the cross-section of the bus body). Such a tendency to decrease is positive because the trapezoid of the remaining space of the cabin, determined by the deformations of the inter-window struts of the sidewalls, dictates the level of passive safety (room space at head level). The same trapezoid is the basis of UNECE R66 regarding the assessment of passive safety under side rollover conditions of a bus. The maximum stresses of the body frame  $\sigma_{\max}$  decreased from 187.3 to 157.2 MPa (Table 2) when switching to the T-R-5 regime, that is, the structure is completely within the limits of the elasticity of the material (the scope of Hooke's law).

Speaking about the effectiveness of the optimization measures (transition from the "rectangular" to the "triangular" structure of the body sides), one should note the modes T-T and T-R-6.6, under which deformations were reduced by 50.77 and 46.09 %, respectively (Table 2). Taking into account the weight reduction of the optimized body relative to the initial one (-37 kg), it is possible to unequivocally state the expediency of the carried-out structural optimization. The above results of a reduction in stress values, deformations, and an increase in the safety factor in all the analyzed modes could be explained by the fact that a triangle is a more rigid shape than a rectangle. This is determined by its geometric stability – the impossibility of changing the shape while preserving the area without changing the length of the sides and angles.

Our solutions resolve the task to unify bus bodies for the installation of conventional diesel power plants or adaptation to the BEB and FCEB schemes, which significantly add mass due to batteries. The triangular structure of the sidewalls is able to compensate for additional loads under the modes of daily operation or certification according to UNECE R100 without increasing the material consumption of the body.

The reviewed literature is entirely based on classic "rectangular" bodies and mainly consider solutions for optimizing the selection of alternative cross-sections or manufacturing materials. Unlike [4–12], which consider typical rectangular structures, the advantage of the current work is the use of triangular elements. Work [15] describes the use of trapezoidal windows, which is close to the topic of current research; however, the shape of a trapezoid, as a quadrilateral, is less rigid than a triangular one.

The limitations of the proposed structural solution regarding the replacement of rectangular window openings with triangular ones are the increased complexity of the body manufacturing technology. In the case of a curvilinear shape of the sidewall profile, conventional vertical racks have the same radius of rounding, and in one plane (transverse), and allow the use of a common pipe bender. The transition to diagonal racks, which as a result form triangular window openings, requires simultaneous bending and twisting of pipes. The situation is further complicated by the fact that they are of different lengths according to the sections of the bus, and therefore there is no question of a single pipe bender. Cutting diagonal tubes and welding them is also significantly more difficult compared to the conventional rectangular configuration of the sidewalls of the body. The problem could be easily solved by switching to a rectangular cross-section of the bus, which is a modern

trend, taking into account the need to minimize production costs and reduce the cost of the final product.

The shortcomings of the study include the lack of analogs – any results of studying the stress-strain state of other models of buses with triangular structures. The objectivity of the reasons for this was substantiated by us above but it should be noted that, under the conditions of the appearance of modern similar scientific publications, one of the priority directions of the development of the current work will be the comparison of our results regarding the SSS of bodies. More applied shortcomings of the current study include the absence of results of forced body oscillations under the considered bending and torsion modes, as well as data on the vibration resistance of the roof truss. In addition, there is a need to check the impact of side glued windows on the final rigidity of the body under different approaches to their installation, which, in fact, is a limitation of this study. In one version, the windows could be triangular (occupying the area defined by the window bars), and in the other – rectangular. In the latter case, they will be glued on top of the triangular window openings with a flywheel so that the inclined racks will cross the windows diagonally from the inside of the cabin. Thus, visually, the bus will retain its manufacturability and at the same time look quite conventional – with classic rectangular windows but will remain innovative with its "triangular" body structure. In addition to modeling of glazing, the simulation of side and frontal impact tests (UNECE R66 and R29) are possible avenues to advance our research that, at the same time, are its limitations.

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## 7. Conclusions

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1. The most common modes of static bending ( $k_d=2.0$ ) and torsion ( $k_d=1.0$  if one of the wheels hangs out) during daily operation showed a reduction in stress by 16.75 % when going from R-B to T-B and by 2.85 % – from R-T to T-T. The most significant achievement is to remove the maximum stresses  $\sigma_{\max}$  from the zone of values of plastic deformations (yield point  $\sigma_y=252$  MPa), that is, the work of the body frame in the zone of elasticity (Hooke's law). The value of the safety factor  $SF_{\min}$  also exceeded 1 under both modes of the "triangular" model (1.10 in T-B and 1.03 in T-T). The stiffness of the frame could be estimated by the deformation index  $\delta$ , which decreased by 28.71 % under the bending mode (T-B) and by 50.77 % under the torsion mode (T-T). The obtained values of  $\delta$  in the optimized model do not exceed 10 mm, which ensures normal operation of glued windows when the body is skewed ( $\delta=9.24$  mm under the T-T mode).

2. Due to the necessity of the inevitable compliance with the environmental requirements of EURO 7, existing city fleet during electrification is automatically subject to certification according to those UNECE Regulations that were not initially foreseen. Being originally diesel, buses when converted to electric traction (installation of batteries) must comply with regulatory requirements R100. Reduction of  $\sigma_{\max}$  occurred by 18.52 and 16.07 % when the body structure was changed to "triangular" under the T-R-6.6 and T-R-5 modes, respectively. The average stresses  $\sigma_{ave}$  managed to be reduced by 27.80 % (to 9.74 MPa) when switching to the T-R-6.6 mode, which allows us to judge the possibility of improving the material consumption by selecting alterna-

tive profiles of smaller cross-section. The total mass of the “triangular” body frame has become even lower than the “rectangular” version by 37 kg. Under the T-R-6.6 mode, deformations  $\delta$  were reduced by 46.09 %, and under T-R-5, they amounted to only 10.83 mm, which is very positive in terms of passive safety – the preservation of the trapezoid of the remaining cabin space.

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#### Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study, as well as the results reported in this paper.

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#### Funding

The study was conducted without financial support.

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#### Data availability

All data are available, either in numerical or graphical form, in the main text of the manuscript.

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#### Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

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