The object of this study is an optimization method under conditions when the initial data (parameters of the system or the environment in which the system operates) are not precisely defined. The problem that arises in this case is related to the lack of universal mathematical methods that solve optimization problems under conditions of uncertainty of the initial data. To solve these problems, approaches are proposed based on the transformation of the initial fuzzy problems into clear problems of mathematical programming. In this case, either a solution to the optimization problem "on average" or solutions obtained for extreme values of inaccurately specified parameters of the problem are proposed as the desired result. The error of the resulting solution is unpredictable.

This paper proposes an alternative approach to solving optimization problems under conditions of fuzzy initial data. The method is based on the use of a multiplicative convolution of the objective function of the problem and a set of membership functions of fuzzy parameters. A feature of the method is that it is stable with respect to the possible variety of analytical descriptions of the objective function of the problem and ensures an adequate solution that takes into account the real uncertainty of the initial data. The fundamental feature of the method: the technique of its construction and the computational scheme of its implementation do not depend in any way on the type, nature, and complexity of the analytical description of the objective function of the original problem. At the same time, to implement the proposed optimization procedure, it is sufficient to have the ability to calculate the value of the objective function on any set of its variables. It is shown that in all cases the original problem with fuzzy initial data is transformed into a conventional deterministic optimization problem solved by known methods. An example of an analytical solution to the problem is given

Keywords: optimization method, fuzzy initial data, development of a general approach

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DEVISING A UNIVERSAL OPTIMIZATION METHOD UNDER CONDITIONS OF FUZZY INITIAL DATA

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1. Introduction

The content and meaning of the overwhelming majority of various problems of studying systems involve the sequential solution of two problems. Firstly, the choice and justification of the criterion for assessing the state (efficiency of operation) of the system. Secondly, the search and development of a method that delivers an extreme numerical value to the selected criterion, quality indicator, efficiency of this system (gain, losses, costs, performance level, reliability level, etc.). All these problems belong to the class of optimization problems. To solve these problems, a powerful mathematical tool has been designed that resolves the problem if all parameters of the objective function and constraints are specified accurately. Otherwise, the known optimization apparatus can be used to solve only a very narrow limited class of separable and simultaneously linear problems. Each of the methods developed to solve other real problems in the general case is focused on reducing the original fuzzy problem to a clear one due to the loss in accuracy of the solution obtained in this case.

In this regard, research on developing a universal method for solving system optimization problems under conditions of fuzzy initial data is relevant.

2. Literature review and problem statement

Methods for solving optimization problems under uncertainty are a new and insufficiently developed theoretical section of modern mathematics [1]. Known technologies for solving the corresponding problems do not provide optimal solutions in the general case. Traditional optimization technology under conditions of initial data fuzziness uses a general approach consisting of reducing the original fuzzy problem to a clear one as follows. In [2], the problem of managing cargo transportation from m suppliers, with production volumes a_1, a_2, ..., a_m units, to n consumers, with consumption volumes b_1, b_2, ..., b_n units, is considered. The cost of transporting a unit of cargo from the i-th supplier to the j-th consumer is a fuzzy number. The efficiency criterion of the transportation plan (x_{ij}) is also a fuzzy number. The problem is solved by finding the matrix $X^{(0)}$ that satisfies the cost constraints and minimizes the average total cost of transportation, calculated through a set of modal values of fuzzy numbers that determine the costs of specific transportation c_{ij}. Thus, the membership function of each fuzzy parameter of the problem is represented by a single number. It is noted that the resulting set $X^{(0)}$ can be recommended for use only if the length of the intervals, which carry fuzzy values, is small, i.e., the uncertainty of the data is small, which is unrealistic. In this regard, the possibility of strengthening the method using a tabular representation of the membership functions of fuzzy parameters of the problem is considered in [2]. The idea of a tabular description of the membership functions is implemented differently using the Zadeh expansion principle [3], according to which the membership functions of fuzzy parameters of the problem are described by a set of intervals of fuzzy parameter values. The ends of these intervals are determined by solving the corresponding equations. Another version of the described approach is often used, which is implemented using other clear parameters of the given fuzzy numbers. In this case, in the inventory management problem with fuzzy demand [4], the following two problems are solved independently: to find purchasing plans that minimize and maximize the corresponding cost criteria. The two plans obtained in this case define unrealistic (corresponding to the extreme values of the fuzzy parameters of the problem) pessimistic and optimistic solutions. These solutions are practically not informative. They can be used only for approximate orientation if the fuzzy numbers are compact. If this is not the case in specific problems, the result obtained can be significantly erroneous. In this case, the danger of possibly obtaining an inadmissibly erroneous solution [4] is discussed. It is noted again that there is a single class of fuzzy optimization problems that can be solved exactly. These are problems in which the objective function and constraints satisfy the strict requirement of separability and linearity. It is in this case that, using analytical descriptions of the membership functions of the fuzzy parameters of the problem, analytical descriptions of all elements of the problem can be obtained, which makes it possible to use standard optimization procedures to obtain a solution. In particular, a fuzzy transportation problem is solved in exactly this way in [5]; in [6], the problem of finding the optimal route; in [7], a servicing system is optimized for two fuzzy variables (service probability, service cost); in [8], the problem is solved for a flow servicing system with a fuzzy specified service duration. It is clear that the assumption used in this case about the adequacy of separable and linear models for real systems is far from always justified, which radically reduces the possibility of their useful practical use. Another negative factor that arises here, discussed in [9], is that the computational complexity of implementing the method is proportional to the number of fuzzy parameters, which complicates the possibility of using the method to study multidimensional fuzzy objects, such as servicing systems.

All these circumstances predetermine the advisability of further research on searching for and devising a general universal method for solving optimization problems under conditions of fuzzy initial data.

3. The aim and objectives of the study

This study aims to devise a general method for solving optimization problems under conditions of fuzzy initial data. The generality of the method should be manifested in the independence of its implementation on the type of the objective function of the original problem and the method for describing the uncertainty. Devising a general method for solving optimization problems under these conditions almost completely resolves the problem of studying systems whose parameters are not precisely defined. The method makes it possible to solve such problems in the initial adequate statement without simplifications that unpredictably reduce the accuracy of the result.

To achieve the goal, it is necessary to solve the following problems:

– to develop a structure and technology for implementing a general method for solving optimization problems under conditions of fuzzy initial data based on the requirements for the quality criterion for solving the problem under these conditions;

– to devise an optimality criterion and a method for solving optimization problems under conditions of fuzzy initial data.

4. The study materials and methods

The object of our study is the optimization method under conditions of fuzzy initial data. Our review of the known approaches to solving this problem showed that these methods significantly depend on the specificity and nature of the corresponding mathematical models of the problems to be solved. When implementing them, it is necessary to determine the membership function of the fuzzy value of the objective function of the original problem in each specific case, which leads to insurmountable problems when solving most practical tasks due to the lack of the appropriate mathematical apparatus. Therefore, the fundamental requirement for the devised method for solving optimization problems under fuzzy conditions is that it should not require constructing the membership function of the objective function of the original problem. The assumption is adopted: the objective function of the problem, as well as its fuzzy parameters, are specified by their analytical descriptions. Therefore, they can be calculated on any set of initial data. It follows that any zero-order method can be used to solve the problem. In accordance with this, the computational procedure must be iterative. In this case, at each step, the value of the objective function is calculated, and the stopping criterion is checked. If necessary, the corresponding procedure enables the calculation of the next, better solution. The ideological and structural basis of the proposed method is a specialized rule to form a criterion, which does not depend on the type and nature of the objective function of the original problem and the type of uncertainty of the initial data.

5. Results of devising a general method for solving optimization problems under conditions of fuzzy initial data

5. 1. Construction of a general structure of the optimization method under uncertainty. Requirements for the quality criterion

To solve the problem, an iterative algorithm is proposed that implements a zero-order optimization procedure, at each step of which the value of the quality assessment criterion obtained by this step of the solution is calculated. In accordance with the devised methodology, a new set of sought variables of the problem is determined, improving the obtained solution. The procedure for improving the solution continues until the stop is determined by the corresponding criterion.

Below are the requirements for the quality assessment criterion obtained at each step of the solution:

1. The quality criterion of the set of variables X obtained at each step of the solution to the problem must depend on all components of this set, as well as on the values of the membership functions of the fuzzy parameters of the problem.

2. The criterion must not depend on any parameters whose numerical value is chosen intuitively, for example, for reasons of simplicity of calculations.

3. The dependence of the numerical value of the criterion on all variables in the problem must be monotonic.

4. The parameters included in the analytical description of the criterion must be built into the design of the criterion in such a way that a change in any of them does not affect the value of others.

5. The efficiency criterion of a fuzzy solution to the problem must be naturally determined through a sequence of clear solutions.

6. The calculation of the ratio for determining the value of the criterion must be performed in accordance with the rules for performing operations on fuzzy numbers.

7. The analytical expression for the criterion must allow the possibility of its optimization by zero-order methods.

5. 2. Devising an analytical description of the criterion and method for optimizing control under fuzzy conditions

In accordance with the statement of the task on devising a general method for solving optimization problems under fuzzy initial data, we introduce the objective function $F=(A, X)$, where $X=(x_1, x_2, ..., x_n)$ is a vector of variables, and $A=(a_1, a_2, ..., a_m)$ is a vector of fuzzy parameters with membership functions $\mu_j(a_j)$. We introduce the structure of a general efficiency criterion that satisfies the stated requirements in the form of a multiplicative convolution:

$$\Psi(X,A)=F(X(A))\prod_{j=1}^{m}\mu_j(\alpha_j). \tag{1}$$

The technology for implementing the proposed optimization method as applied to the standard problem of resource distribution between two consumers is as follows. The objective function takes the form (1), and the membership functions of the fuzzy parameters a_1 and a_2 are defined, for example, by Gaussian functions:

$$\mu_1(\alpha_1)=\begin{cases}\exp\left(-\dfrac{(m_1-\alpha_1)^2}{2\sigma_{11}^2}\right), \ \alpha_1\le m_1;\\ \exp\left(-\dfrac{(\alpha_1-m_1)^2}{2\sigma_{12}^2}\right), \ \alpha_1> m_1;\end{cases} \tag{2}$$

$$\mu_2(\alpha_2)=\begin{cases}\exp\left(-\dfrac{(m_2-\alpha_2)^2}{2\sigma_{21}^2}\right), \ \alpha_2\le m_2;\\ \exp\left(-\dfrac{(\alpha_2-m_2)^2}{2\sigma_{22}^2}\right), \ \alpha_2> m_2.\end{cases} \tag{3}$$

In accordance with the proposed methodology, the criterion of resource allocation efficiency is modified as follows:

$$\Psi(x_1,x_2)=F\left(x_1(\alpha_1,\alpha_2),x_2(\alpha_1,\alpha_2)\mu_1(\alpha_1)\mu_2(\alpha_2)\right). \tag{4}$$

The resulting maximization problem is solved by the corresponding method of mathematical programming, including any zero-order method, for example, Nelder-Mead. In this case, the solution to the problem is achieved iteratively. At the first iteration, some admissible set of variable values is specified. In this case, it is advisable to select as the initial set a set corresponding to the modal values of the fuzzy parameters of the problem. Then, an iterative improvement of the solution obtained at the next step is performed.

Thus, the problem of rational distribution of a limited resource in a system consisting of two subsystems is solved. There is a certain resource that must be distributed between these two subsystems, maximizing some natural criterion.

We introduce:

x_1 – resource allocated for the first subsystem;

x_2 – resource allocated for the second subsystem;

$F(x_1, x_2)$ – criterion of the efficiency of the systems functioning, depending on x_1, x_2.

Next, a natural criterion of rational distribution is introduced, which takes the form:

$$F(x_1,x_2)=\alpha_1 x_1^{\frac{1}{2}}+\alpha_2 x_2^{\frac{1}{2}}, \tag{5}$$

where a_1, a_2 are the parameters of the system.

The desired distribution x_1, x_2 must satisfy the constraints:

$$x_1+x_2=1; \tag{6}$$

$$x_1\ge 0;\ x_2\ge 0. \tag{7}$$

Thus, the mathematical model of the problem is to find a set (x_1, x_2) that maximizes (5) and satisfies constraints (6), (7).

At first, this problem is solved under the assumption that its parameters a_1 and a_2 are clearly defined. To solve this problem, the method of undetermined Lagrange multipliers is used. The Lagrange function takes the form:

$$\Phi(x_1,x_2)=F(x_1,x_2)-\gamma(x_1+x_2-1)=$$
$$=\alpha_1 x_1^{\frac{1}{2}}+\alpha_2 x_2^{\frac{1}{2}}-\gamma(x_1+x_2-1). \tag{8}$$

Now, (8) is differentiated with respect to x_1 and x_2; the derivatives are set equal to zero, and the resulting equations are solved for x_1 and x_2:

$$\frac{d\Phi(x_1,x_2)}{dx_1}=\frac{1}{2}\alpha_1 x_1^{-\frac{1}{2}}-\gamma=0,$$

$$x_1^{-\frac{1}{2}}=\frac{2\gamma}{\alpha_1};\ x_1=\left(\frac{2\gamma}{\alpha_1}\right)^{-2}=\frac{\alpha_1^2}{(2\gamma)^2}; \tag{9}$$

$$\frac{d\Phi(x_1,x_2)}{dx_2}=\frac{1}{2}\alpha_2 x_2^{-\frac{1}{2}}-\gamma=0,$$

$$x_2^{-\frac{1}{2}}=\frac{2\gamma}{\alpha_2};\ x_2=\left(\frac{2\gamma}{\alpha_2}\right)^{-2}=\frac{\alpha_2^2}{(2\gamma)^2}. \tag{10}$$

To find the value of the indefinite multiplier γ, constraint (6) is used. We have:

$$x_1+x_2=\frac{1}{(2\gamma)^2}\left(\alpha_1^2+\alpha_2^2\right)=1,$$

$$\frac{1}{(2\gamma)^2}=\frac{1}{\alpha_1^2+\alpha_2^2}.$$

Substituting $1/(2y)^2$ into (9) and (10), we obtain a solution to the problem:

$$x_1=\frac{\alpha_1^2}{\alpha_1^2+\alpha_2^2},\ x_2=\frac{\alpha_2^2}{\alpha_1^2+\alpha_2^2}. \quad (11)$$

The value of criterion (5) corresponding to this set takes the form:

$$F(x_1,x_2)=\alpha_1\left(\frac{\alpha_1^2}{\alpha_1^2+\alpha_2^2}\right)^{\frac{1}{2}}+\alpha_2\left(\frac{\alpha_2^2}{\alpha_1^2+\alpha_2^2}\right)^{\frac{1}{2}}=$$
$$=\frac{\alpha_1^2}{\sqrt{\alpha_1^2+\alpha_2^2}}+\frac{\alpha_2^2}{\sqrt{\alpha_1^2+\alpha_2^2}}=\frac{\alpha_1^2+\alpha_2^2}{\sqrt{\alpha_1^2+\alpha_2^2}}=\sqrt{\alpha_1^2+\alpha_2^2}.$$

Next, the transition is made to the situation when the parameters a_1 and a_2 are not deterministic values. The technology for solving this problem using the proposed method is considered for the case when a_1 and a_2 are fuzzy numbers with specified membership functions $\mu_1(a_1)$ and $\mu_2(a_2)$. It is assumed, for example, that these functions are triangular and are specified by sets:

$$\alpha_1=<m_1,\sigma_{11},\sigma_{12}>,\ \alpha_2=<m_2,\sigma_{21},\sigma_{22}>.$$

In this case:

$$\alpha_{1\min}=m_1-\sigma_{11},\ \alpha_{1\max}=m_1+\sigma_{12},$$

$$\alpha_{2\min}=m_2-\sigma_{21},\ \alpha_{2\max}=m_2+\sigma_{22}.$$

Now for any set of variables x_1 and x_2, the length of the corresponding interval – the carrier of the criterion value is found.

In this case:

$$F_{\min}(x_1,x_2)=\alpha_{1\min}x_1^{\frac{1}{2}}+\alpha_{2\min}x_2^{\frac{1}{2}}, \quad (12)$$

$$F_{\max}(x_1,x_2)=\alpha_{1\max}x_1^{\frac{1}{2}}+\alpha_{2\max}x_2^{\frac{1}{2}}. \quad (13)$$

Then the length of the interval of possible values of the criterion is equal to:

$$\Delta=F_{\max}(x_1,x_2)-F_{\min}(x_1,x_2)=$$
$$=\left(\alpha_{1\max}-\alpha_{1\min}\right)x_1^{\frac{1}{2}}+\left(\alpha_{2\max}-\alpha_{2\min}\right)x_2^{\frac{1}{2}}=c_1x_1^{\frac{1}{2}}+c_2x_2^{\frac{1}{2}}. \quad (14)$$

Now the result of solving problem (11) under the assumption that the values of the fuzzy parameters of the problem are equal to the modal ones, i.e., $a_1=m_1$ and $a_2=m_2$ takes the form. The corresponding solution in this case takes the form:

$$x_1=\frac{m_1^2}{m_1^2+m_2^2},\ x_2=\frac{m_2^2}{m_1^2+m_2^2}.$$

The solution of problem (5) to (7) was continued. As a result of its solution earlier, the following set (11) was obtained:

$$x_1=\frac{\alpha_1^2}{\alpha_1^2+\alpha_2^2},\ x_2=\frac{\alpha_2^2}{\alpha_1^2+\alpha_2^2}.$$

The quality criterion of the solution, in accordance with the proposed technology, takes the form (4). To solve the problem (4) to (7), the Nelder-Mead method was used. In accordance with this method, a matrix of sets of coordinates of the initial simplex D was constructed, choosing for the coordinates of the reference point the values of the variables x_1 and x_2 corresponding to the modal values of the fuzzy parameters a_1, a_2, i.e., $a_{11}=m_1$ and $a_{21}=m_2$. In this case:

$$D=\begin{pmatrix}x_{11} & x_{21} & x_{31}\\ x_{12} & x_{22} & x_{32}\end{pmatrix}=\begin{pmatrix}m_1 & m_1+d_1 & m_1+d_2\\ m_2 & m_2+d_2 & m_2+d_1\end{pmatrix},$$

where:

$$d_1=\frac{t}{n\sqrt{2}}\left(\sqrt{n+1}+n-1\right),\ d_2=\frac{t}{n\sqrt{2}}\left(\sqrt{n+1}-1\right),$$

t is a selectable parameter that specifies the size of the simplex; n is the number of variables.

Let $t=2$, $n=2$, $m_1=2$, $m_2=4$, $\sigma_{11}=3$, $\sigma_{12}=1$, $\sigma_{21}=1$, $\sigma_{22}=4$. Then:

$$d_1=0.96,\ d_2=0.26,$$

$$D=\begin{pmatrix}x_{11} & x_{21} & x_{31}\\ x_{12} & x_{22} & x_{32}\end{pmatrix}=\begin{pmatrix}2 & 2.96 & 2.26\\ 4 & 4.26 & 4.96\end{pmatrix}.$$

The values of criterion (4) at the points corresponding to the vertices of the simplex are equal to:

$$\psi_1(x_{11},x_{12})=F_1(x_{11},x_{12})\mu_1(2)\mu_2(4)=$$
$$=\left(\frac{4}{\sqrt{4+16}}+\frac{16}{\sqrt{4+16}}\right)\mu_1(2)\mu_2(4)=\frac{20}{\sqrt{20}}=4.47;$$

$$\psi_2(x_{21},x_{22})=F_2(x_{21},x_{22})\mu_1(x_{21})\mu_2(x_{22})=$$
$$=\left[\begin{array}{l}\dfrac{(2.96)^2}{(2.96)^2+(4.26)^2}+\\ +\dfrac{(4.26)^2}{(2.96)^2+(4.26)^2}\end{array}\right]\mu_1(2.96)\mu_2(4.26)=$$
$$=\left(\frac{8.76}{\sqrt{8.76+18.15}}+\frac{18.15}{\sqrt{8.76+18.15}}\right)\exp\left\{-\frac{(0.96)^2}{2\times 1}\right\}\times$$
$$\times\exp\left\{-\frac{(0.26)^2}{2\times 16}\right\}=\sqrt{26.91}\exp\{-0.46\}\exp\{-0.002\}=$$
$$=5.19\times 0.63\times 0.0997=3.26;$$

$$\psi_3(x_{31},x_{32})=F_3(x_{31},x_{32})\mu_1(x_{31})\mu_2(x_{32})=$$
$$=\left[\frac{(2.26)^2}{\sqrt{(2.26)^2+(4.96)^2}}+\frac{(4.96)^2}{\sqrt{(2.26)^2+(4.96)^2}}\right]\exp\left\{-\frac{(0.26)^2}{2}\right\}\times$$
$$\times\exp\left\{-\frac{(0.96)^2}{2\times 16}\right\}=\left(\frac{5.11}{\sqrt{5.11+24.6}}+\frac{24.6}{\sqrt{5.11+24.6}}\right)\times$$
$$\times\exp\{-0.0338\}\exp\{-0.028\}=\sqrt{29.71}\times 0.967\times 0.972=5.12.$$

The obtained set of values of criterion (4) at the vertices of the simplex ($\psi_1(x_{12}, x_{12})$=4.47, ($\psi_2(x_{21}, x_{22})$=3.26, ($\psi_3(x_{31}, x_{32})$=5.12,) are ordered in ascending order: {3.26; 4.47, 5.12}={$\psi_1(x_{21}, x_{22})$, $\psi_2(x_{11}, x_{12})$, $\psi_3(x_{31}, x_{32})$}. The worst is the vertex $X_2=(x_{21}, x_{22})$. In accordance with the Nelder-Mead method, the operation of reflecting this point relative to the center of gravity of the figure obtained by removing the worst vertex $X_2=(x_{21}, x_{22})$ from the simplex is carried out.

A new point is obtained:

$$U = C + \gamma_1(C - X_2),$$

C is the center of gravity of the truncated simplex after removing X_2:

$$C = \begin{pmatrix} x_{21} \\ x_{22} \end{pmatrix} = \frac{1}{2}\left[\begin{matrix} x_{11} \\ x_{12} \end{matrix} + \begin{matrix} x_{31} \\ x_{32} \end{matrix}\right] = \frac{1}{2}\left[\begin{pmatrix} 2 \\ 4 \end{pmatrix} + \begin{pmatrix} 2.26 \\ 4.96 \end{pmatrix}\right] = \begin{pmatrix} 2.13 \\ 4.48 \end{pmatrix}.$$

Then, assuming $\gamma_1=1$, we obtain:

$$U = \begin{pmatrix} 2.13 \\ 4.48 \end{pmatrix} + \left[\begin{pmatrix} 2.13 \\ 4.48 \end{pmatrix} - \begin{pmatrix} 2.96 \\ 4.26 \end{pmatrix}\right] = \left[\begin{pmatrix} 2.13 \\ 4.48 \end{pmatrix} + \begin{pmatrix} -0.83 \\ 0.22 \end{pmatrix}\right] = \begin{pmatrix} 1.3 \\ 4.7 \end{pmatrix} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}.$$

The value of the criterion at point U is:

$$\Psi(u_1,u_2) = F(u_1,u_2)\mu_1(u_1)\mu_2(u_2) = \left[\begin{matrix} \dfrac{(1.3)^2}{\sqrt{(1.3)^2+(4.7)^2}} + \\ + \dfrac{(4.7)^2}{\sqrt{(1.3)^2+(4.7)^2}} \end{matrix}\right]\mu_1(1.3)\mu_2(4.7) =$$

$$= \left[\frac{1.69}{\sqrt{1.69+22.09}} + \frac{22.09}{\sqrt{1.69+22.09}}\right]\exp\left\{-\frac{(0.7)^2}{2\times 9}\right\}\times$$

$$\times\exp\left\{-\frac{(0.7)^2}{2\times 16}\right\} = \left(\frac{5.11}{\sqrt{5.11+24.6}} + \frac{24.6}{\sqrt{5.11+24.6}}\right)\times$$

$$\times\exp\{-0.0338\}\exp\{-0.028\} =$$

$$= \sqrt{23.78}\exp\{-0.027\}\exp\{-0.015\} =$$

$$= 4.87\times 0.973\times 0.0985 = 4.67.$$

The values of $\psi(U)$, $\psi_1(x_{12}, x_{12})$, $\psi_2(x_{21}, x_{22})$, $\psi_3(x_{31}, x_{32})$ are written in ascending order:

$$\{3.26;4.47;4.67;5.12\} = \{\psi_2(x_{21},x_{22}),\psi_1(x_{11},x_{12}),\Psi(U),\psi_3(x_{31},x_{32})\}.$$

Since:

$$\psi_1(x_{11},x_{12}) < \Psi(U) < \psi_3(x_{31},x_{32}),$$

then, in accordance with the Nelder-Mead algorithm, the compression operation is performed.

In this case, $\gamma_2=1/2\gamma_1=0.5$.

In this case, point V is obtained:

$$V = C + \gamma_2(C - X_2) = \begin{pmatrix} 2.13 \\ 4.48 \end{pmatrix} + 0.5\left[\begin{pmatrix} 2.13 \\ 4.48 \end{pmatrix} - \begin{pmatrix} 2.96 \\ 4.26 \end{pmatrix}\right] = \left[\begin{pmatrix} 2.13 \\ 4.48 \end{pmatrix} - \begin{pmatrix} -0.415 \\ 0.11 \end{pmatrix}\right] = \begin{pmatrix} 1.715 \\ 4.59 \end{pmatrix}.$$

Next, we determine the value of $\psi(V)$:

$$\Psi(V) = \Psi(v_1,v_2) = F(v_1,v_2)\mu_1(v_1)\mu_2(v_2) = \left[\begin{matrix} \dfrac{(1.715)^2}{\sqrt{(1.715)^2+(4.59)^2}} + \\ + \dfrac{(4.59)^2}{\sqrt{(1.715)^2+(4.59)^2}} \end{matrix}\right]\mu_1(0.285)\mu_2(0.59) =$$

$$= \left[\frac{2.94}{\sqrt{2.94+21.07}} + \frac{21.07}{\sqrt{2.94+21.07}}\right]\exp\left\{-\frac{(0.85)^2}{2}\right\}\times$$

$$\times\exp\left\{-\frac{(0.59)^2}{32}\right\} = \sqrt{24.01}\exp\{-0.36\}\exp\{-0.11\} =$$

$$= 4.9\times 0.697\times 0.99 = 3.396.$$

Since:

$$\psi_2(x_{21},x_{22}) < \Psi(V) < \psi_1(x_{11},x_{12}),$$

then the next operation will again be a reflection.

As a result of the subsequent execution of operations in accordance with the Nelder-Mead algorithm, after several steps, the solution Z=(1.98, 5.61) is obtained. The value of the criterion at this point is equal to:

$$\Psi(Z) = \Psi(z_1,z_2) = F(z_1,z_2)\mu_1(z_1)\mu_2(z_2) = \left[\begin{matrix} \dfrac{(1.98)^2}{\sqrt{(1.98)^2+(5.61)^2}} + \\ + \dfrac{(5.61)^2}{\sqrt{(1.98)^2+(5.61)^2}} \end{matrix}\right]\mu_1(1.98)\mu_2(5.61) =$$

$$= \left[\frac{3.92}{\sqrt{3.92+31.47}} + \frac{31.47}{\sqrt{3.92+31.47}}\right]\exp\{-2\times 10^{-5}\}\times$$

$$\times\exp\left\{-\frac{(1.61)^2}{32}\right\} = \sqrt{35.39}\exp\{-0.081\} = 5.949\times 0.92 = 5.47.$$

The solution to the problem obtained as a result of implementing the proposed methodology is 1.224 times better than the modal one, which for the two-dimensional problem under consideration quite convincingly demonstrates the advantages of the method.

6. Discussion of results based on devising a general method for solving optimization problems under conditions of fuzzy initial data

The problem of solving optimization problems under conditions of fuzzy initial data considered in this paper is very important and relevant since the hypothesis of perfect accuracy of measurements under real conditions is unacceptable. Known methods for solving problems under conditions of fuzzy initial data provide an exact solution only for a very limited set of separable linear problems. The remaining problems, the vast majority of them, are solved approximately using the transition from the original fuzzy problem to an ordinary mathematical programming problem. Known approaches to implementing this idea use the following simplest techniques. The first of them is based on replacing all uncertain parameters of the problem with their average values. When using the second approach, the original problem is divided into two, in which first the uncertain parameters are specified by their minimum values, and then by their maximum values. The solution to these two problems determines the pessimistic and optimistic variants of the results. Both approaches are acceptable if the variability of parameter uncertainty is small. However, if this is not the case, their practical application may lead to unacceptable losses in the quality of the solution.

Our paper proposes a new approach to solving the optimization problem under uncertainty. The method is based on the formation and use of a special multiplicative convolution (1) of the objective function of the problem and the system of membership functions of all its fuzzy parameters. The fundamental advantages of the method are as follows. Using (1) as a criterion provides the ability to organize an iterative improvement of the naturally chosen initial solution. This criterion takes into account all available information about the uncertainty of the problem parameters, ensuring that its adequate solution is obtained. The most important thing is that the method makes it possible to obtain an exact solution to any fuzzy optimization problem regardless of the type of uncertainty of its parameters. The proposed method reduces the original problem, regardless of its statement, to a conventional mathematical programming problem with the objective function (1). Therefore, any zero-order method, for example, Nelder-Mead, can be successfully applied to solve the problem in all cases. The computational scheme for implementing the method is determined by relations (5) to (14). The proposed method is stable with respect to the structure and type of the objective function of the original problem and the nature of the uncertainty of the initial data. The proper adequacy of the method is ensured by the most correct consideration of the analytical description of the uncertainty of the initial data.

Limitations: the method cannot be used directly if the uncertainty of the initial data is not described analytically or if it is hierarchical.

Thus, the proposed method actually resolves the task of solving any optimization problems under conditions of fuzzy initial data, which determines the high theoretical and practical significance of our result, the importance of which is difficult to overestimate.

A possible area of further research is to extend the proposed method to the case of hierarchical (bine-fuzzy) uncertainty of the initial data [10].

7. Conclusions

1. The general structure and technology for solving the stated problem have been developed. An easy-to-implement method for solving optimization problems under conditions of fuzzy initial data has been proposed, which reduces the initial problem to a clear one. The desired solution to the problem is achieved using known optimization methods of the appropriate order.

2. In accordance with the method, a clearly and naturally interpretable criterion is formed to solve the problem. The proposed computational procedure ensures optimization of the criterion by iteratively improving the convolution of the numerical value of the objective function of the problem and the membership level of its resulting fuzzy value. The effectiveness of the method is demonstrated by an example. Moreover, the result obtained by the proposed method is more than 20 % better than the modal one.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study, as well as the results reported in this paper.

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Data availability

The data will be provided upon reasonable request.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

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