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DEVISING A COMBINED METHOD FOR SETTING PI/PID CONTROLLER PARAMETERS FOR OIL AND GAS FACILITIES

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The object of this study is automatic control systems of the first, second, and third orders. The principal task was to ensure the stability of control systems while minimizing overshot and regulation time.

A combined method for determining the tuning parameters of PI/PID controllers has been devised, which combines the s-plane method and the generalized quadratic criterion.

The s-plane method is based on the Vieta theorem, which relates the roots of the characteristic equation of a closed-loop control system to its parameters. They are functions of the tuning parameters of PI/PID controllers. By choosing the left roots of the characteristic equation of a closed-loop system on the s-plane, the desired quality indicators of the control system can be achieved. The roots of the equation are functionally related to the parameters of PI/PID controllers. From the system of algebraic equations that follow from the Vieta theorem, the tuning parameters for PI/PID controllers are found as a solution to such a system.

At the second stage of solving the problem, the roots of the characteristic equation are chosen so that the generalized quadratic criterion is a function only of real part of one of the characteristic equation's roots. As a result, we obtain a one-dimensional minimization problem, the local minimum of which was sought within a predetermined search interval. This interval was chosen on the condition that the parameters for PI/PID controllers would be strictly positive. The roots of the characteristic equation of the closed-loop system would belong to the left half-plane of the s-plane. Such a choice of the search interval guarantees the stability of the closed-loop automatic control system.

It was found that compared to the s-plane method, the overshot and regulation time were reduced by an average of 73.5 % and 66.5 %. This could increase the speed of industrial controllers

Keywords: control system, combined criterion, PI/PID controller, tuning parameters, local minimum

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1. Introduction

One of the main tasks in the design of automatic control systems for technological processes is the calculation of settings for PI/PID controllers. There are two approaches to solving the problem – analytical and graphical-numerical. The first of them uses the minimization of the generalized quadratic criterion or, as an alternative, the s-method to determine the settings of the controllers.

The task of determining the settings of controllers by the generalized quadratic criterion is a nonlinear programming problem. The conducted machine experiments showed that the generalized quadratic criterion as a function of the settings parameters is a multi-extreme function. At different values of the initial starting point, different values of the settings of the controllers were obtained. In addition, in the process of solving the optimization problem, it is difficult to control the stability of the automatic control system.

Graphical-numerical methods are based on the use of frequency characteristics such as Bode and Nyquist diagrams, or diagrams constructed in polar coordinate systems. Their purpose is the synthesis of corrective devices in order to achieve the desired quality indicators of the control process: frequency bandwidth, stability margin in phase and amplitude, etc.

Thus, a relevant scientific task is to devise simple and effective methods for tuning PI/PID controllers, based on the selected criterion of the quality of functioning of the automatic control system.

2. Literature review and problem statement

Most control systems implemented today use PI/PID controllers due to their simple structure and ease of implementation [1]. One of the problems related to modern PI/PID controllers is the need for additional refinement in traditional methods, such as the Ziegler-Nichols method, as well as the lack of a universal approach that would provide high performance for all types of applications. Therefore, according to study [1], future research could be directed to the expanded use of PID controllers, their adaptation to digital control systems, and the development of new approaches for automatic tuning.

When using PI/PID controllers in various industries, the task of tuning their parameters arises [2–6] due to the high complexity of the algorithms used. This makes them unsuitable for scaling in the industry.

The use of a modified firefly algorithm using a chaotic Tinkerbell map for optimizing the tuning of PID controllers [2] shows better accuracy in tuning PID controllers compared to other methods. Specifically, in reducing the objective function and integral absolute error by 10–17 % depending on the metric. The issue with the developed algorithm is that the considered algorithm does not always guarantee global optimization and, subsequently, requires further analysis in tuning controllers for multidimensional systems, including checking the control error and robustness of the systems.

The results reported in [3] show that the PI controller tuned using the particle swarm optimization (PSO) algorithm demonstrates the best results in some parameters. This is a reduction in the rise time of the transient process, overshoot and settling time of the electric motor compared to the traditional Ziegler-Nichols (ZN) and modified Ziegler-Nichols (MZN) methods. PSO provides stable convergence, high computational efficiency, and ease of implementation. However, the issues related to the generalization of this approach to other types of electric motors and taking into account external disturbances under real conditions remain unresolved. The likely reason is objective difficulties associated with the complexity of modeling real systems.

The use of the integral of squared errors (ISE) criterion and the restriction on the slope of the Nyquist curve at the gain frequency formed the basis of study [4]. It is shown that the proposed approach provides improved load disturbance suppression compared to existing methods, in particular the Sree and Chidambaram and SIMC methods. However, due to the high level of oscillation in the responses to the setpoint change, the need to use the Smith predictor for a significant delay, and the lack of experimental verification, there is a need to further improve the proposed method to reduce overshoot and improve the response to the setpoint change.

In [5], the results of a comparative analysis of three approaches to speed control of drive systems with elastically coupled loads are reported: a classical PI controller, a spatial state-based PI controller (PI-SSSC), and a model-based predictive control (MBPC). The higher stability of the PI-SSSC and MBPC controllers compared to the classical PI controller is demonstrated. However, due to the high sensitivity of MBPC to changes in mechanical characteristics and the need for an observer to estimate the mechanical states of the system, there is a need for further research. In [5], optimization of the MBPC algorithm is proposed to reduce the computational load, as well as combining PI-SSSC and MBPC to combine the advantages of both methods.

Nature-inspired methods and machine learning methods are analyzed in [6]. The advantages of these algorithms for achieving high performance and adaptability to faults are demonstrated. The lack of accurate mathematical models, sensitivity to noise, and measurement inaccuracies, as well as the lack of data balance between data on normal system operation and fault data, require further development of autonomous algorithms that could adapt. In addition, optimization of hybrid controllers and improvement of data analysis methods are required.

Modern methods for tuning PI/PID controllers are based on criteria that are functionals of control error. There are several such criteria [7]: integral absolute error (IAE); integral squared error (ISE); integral time absolute error (ITAE); integral time squared error (ITSE); mean squared error (MSE); integral error (IE).

The IE criterion could be applied only for aperiodic transients. The IAE, ITAE, ITSE, MSE criteria require knowledge of the control errors $\varepsilon(t)$ for their calculation. They, in turn, functionally depend on the tuning parameters of PI/PID controllers. This significantly complicates the computational procedures.

The metaheuristic algorithms presented in [7] for tuning PID controllers show significant potential for optimizing the control of complex systems, providing high accuracy, stability, and minimization of errors under dynamic modes. In particular, such approaches as genetic algorithms, swarm optimization, ant colony optimization, the learning-teaching method, etc. are covered. However, there are still unresolved issues related to the high computational complexity and the lack of clear recommendations for choosing an algorithm depending on the type of problem. The difficulties associated with the high complexity of dynamic models and the dependence of the results on the initial conditions and algorithm parameters could be overcome by creating hybrid algorithms, which was done in [7] for the analysis of multifactor processes. However, these unresolved issues remain relevant for further development and research of algorithms.

The use of a quadratic error model for PID controller adaptation [8] allowed the authors to design a dynamically updated (DUPID) controller. It showed a decrease in the IAE criterion by approximately 62 % for 1D DUPID and 59 % for 2D DUPID compared to a standard PID controller. However, the lack of testing on variable parameters and the lack of focus on sudden changes in parameters make this controller unsuitable for real-world conditions. Although the paper proposes an adaptive algorithm that takes into account the influence of noise and unexpected changes in parameters, however, there is currently no detailed justification for the stability of the algorithm. In modern algorithms, the minimization of a certain criterion makes it possible to determine the tuning parameters for PI/PID controllers. To ensure the stability of the automatic control system, an additional condition is imposed on the minimization process – the tuning parameters of PI/PID controllers must be positive numbers. In such a statement, the problem of finding the optimal tuning parameters of PI/PID controllers is a nonlinear programming problem. Modern optimization algorithms, including stochastic sequential quadratic programming [9], by minimizing a certain criterion, make it possible to solve the problems of tuning control system parameters, which are nonlinear programming problems with constraints in the form of equations and inequalities. Constraints in such problems, as is known, are formed in terms of "not more" and "not less", which introduces difficulties since the tuning parameters for PI/PID controller must be strictly positive. In [9], adaptive approaches to overcoming such difficulties are considered, which ensures global convergence of algorithms even under stochastic conditions. However, the choice of penalties that are adaptive and dependent on noise in the data, and the application of the algorithm to stochastic problems with high-dimensional constraint vectors require further development of stochastic optimization algorithms.

These circumstances explain the presence of a number of minimization methods used in modern PI/PID controller tuning algorithms.

Thus, in [10, 11], a genetic algorithm [12] was used to determine the PID controller tuning parameters. In [10], the results of using genetic algorithms for tuning a PID controller in a mixed-tank chemical reactor (CSTR) process were demonstrated. At the same time, in [11], the results of studies on the use of modern heuristic methods – differential evolution and genetic algorithms – were reported. This allowed for minimizing the composition of the ISE, IAE, and ITAE criteria. However, issues related to the practical implementation of this technique under real conditions remained unresolved. In particular, among the objective reasons, the following are noted: the complexity of calculations with a large number of parameters, which could lead to high costs of time and computing resources; the need to tune the GA for specific process conditions, which requires additional experiments and testing. An option to overcome these difficulties may be the use of hybrid methods that combine classical approaches and modern optimization algorithms, which reduces the need for high computational resources, which is proposed in those works.

In work [13], to solve the problem of calculating the tuning parameters of a PI/PID controller, a criterion is minimized, which is the sum of the partial criteria ISE, IAE, INAE, and ITSE. The minimization is carried out using the particle swarm optimization (PSO) algorithm [14]. A comparison of the performance of a PID controller based on PSO (PSO-PID) and a PID controller tuned using the Ziegler-Nichols method (ZN-PID) is presented. The results demonstrate the advantages of PID tuning using the PSO optimization approach. However, due to the tendency of PSO to get stuck in local optima, the algorithm has limitations in optimization problems. Because of this, the authors propose further comparison and improvement of the particle swarm algorithm, for example, using the differential evolution method.

The advantage of optimization algorithms over classical methods for tuning PID controllers is demonstrated in [15]. Classical methods for tuning PID controllers, such as Cohen-Kuhn, Internal Model Control (IMC), and the Chien-Hrones-Reswick (CHR) method, were compared with genetic algorithms, ant colony algorithms, etc. The authors demonstrate that optimization methods outperform classical approaches in terms of accuracy and speed of regulation. The Shuffled Frog-Leaping algorithm was particularly effective. However, due to the stochastic nature of some algorithms, in particular, Shuffled Frog-Leaping, as well as the loss of accuracy of neural network algorithms, further improvement of identification methods remains relevant, in particular, combining neural networks with optimization methods and extending research to nonlinear control systems, since most industrial processes are not linear.

Improvement of the ant colony algorithm, which is used for tuning PID controllers [15], is described in [16]. The Saltatory Evolution Ant Colony Optimization (SEACO) algorithm described by the authors reduces the number of iterations, which shows a higher convergence rate of the new algorithm compared to the classical ant colony algorithm. However, due to the loss of accuracy after 100 iterations and testing only on the traveling salesman problem, this algorithm requires further research and improvements. As prospects for further development, the authors propose the integration of neural networks into the algorithm and testing the algorithm on graph optimization problems, task scheduling, etc. The bacterial colony optimization algorithm in [17] demonstrates high efficiency on most test functions compared to PSO and GA and, due to communication between "bacteria", reaches the global optimum faster. However, because of the high computational complexity of the communication mechanism and the dependence on parameter settings, such as the chemotaxis step length, the algorithm requires further research. Adaptive parameter optimization and integration with other methods, such as deep learning or neural networks, is one of the proposed options for improving the developed method.

The hybrid of the bat colony and ant colony optimization algorithm demonstrated in [18] provides better classification accuracy compared to other methods, such as Support Vector Machine and its variations. It also provides a significant reduction in the number of features in the sample, which allows for reduced computation time and higher classification accuracy (Ginselman test – 95.93 % and Schiller test – 90.91 %). However, the problem of optimal tuning of the bat colony and ant colony algorithm parameters, as well as the lack of a detailed assessment of the algorithm's performance on large data sets, necessitates further improvements to the method under study. Namely, the use of deep learning is proposed to improve performance. A review of the basic concepts and use of the particle swarm optimization (PSO) algorithm and its modifications in engineering problems [19] demonstrates the ease of using PSO in optimization problems, including PI/PID controllers. Insufficient efficiency in high-dimensional problems and a small number of studies in the field of parameter tuning demonstrate the need for further improvement of the algorithm, both hybridization of PSO with deep learning and automatic parameter tuning.

In [20], the root hodograph method was used, which is graph-analytic. It was used together with the analytical method of parametric synthesis. The graph-analytic method is focused on determining the parameters of PI/PID controllers for placing the roots on a complex plane. Since the method allows one to obtain a graphical image of the trajectory of the roots on the plane, only one parameter of the controller is subject to change. The main disadvantage is the complexity of implementing the two-channel system proposed in [20] due to its inertia and cost. A single-channel robust system requires significant efforts for parametric optimization.

In [21], intelligent methods based on fuzzy logic, artificial neural network (ANN), adaptive neuro-fuzzy logical inference system (ANFIS), and genetic algorithms (GA) for tuning a PID controller are presented. The controller tuned using these methods was used to control the concentration in a continuous stirred tank reactor (CSTR). The simulation results show that intelligent methods provide better performance compared to the traditional Ziegler-Nichols (ZN) method in various control quality indicators. However, the main disadvantage of the proposed approaches is high computational costs and modeling complexity. Therefore, the work indicates the need for further improvement of intelligent methods.

Our review of the literature reveals that the calculation of PI/PID controller tuning parameters is usually based on criteria that characterize the quality of the control process. Minimization of the selected criterion by the parameters of the PI/PID controllers gives rise to a nonlinear programming problem, the solution to which, under the constraints imposed by the requirements for the stability of the system, gives rise to significant computational difficulties. In addition, such a nonlinear programming problem is multi-extreme, and its solution depends on the initial conditions. Modern minimization algorithms, based on the analogy with the behavior of living organisms that form colonies, are complex and their implementation presents difficulties with assessing the stability of the system at the stages of calculations.

3. The aim and objectives of the study

The aim of our work is to devise a method for determining the optimal parameters of PI/PID controllers as a combination of the s-plane method and the generalized quadratic criterion method. This could make it possible to simplify the computational process, reducing it to one-dimensional optimization of the generalized quadratic criterion.

To achieve the set goal, it is necessary to solve the following problems:

– using a combination of the s-plane methods and the generalized quadratic criterion, reduce the function of the PI/PID controller of the first, second, and third-order automatic control system to a one-dimensional minimization problem and compare the obtained values of the tuning parameters and control quality with the corresponding values of the s-plane method;

– to synthesize an algorithm for determining the tuning parameters of the PI/PID controller.

4. The study materials and methods

The object of our study is the automatic control systems of the first, second and third orders. The subject of the study is the automatic control system, which is depicted in Fig. 1.

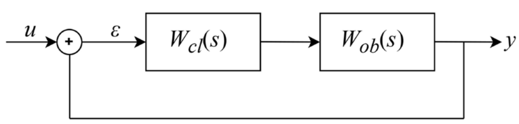

Fig. 1. Single-loop automatic control system

In [22], the task to determine the tuning parameters of PID controllers based on the location of the roots of the characteristic equation of an automatic control system on the complex root plane (s-plane) was posed and solved. The essence of the problem is that according to Vieta's theorem, there are certain relationships between the roots and coefficients of the characteristic equation. They are represented in the form of a system of linear algebraic equations, in which the unknown quantities are the tuning parameters for PI/PID controllers. Such a system of equations has a unique solution only for the characteristic equations of the first and second orders. In the case when the order of the characteristic equation is more than two, the system of equations will be overdetermined, and its unique solution could be found when the conditions of the Kronecker-Capelli theorem are met [23]. If the conditions of the Kronecker-Capelli theorem are not met, then a pseudo-solution to the overdetermined system of algebraic equations is sought.

Our paper proposes a method that is a combination of two methods, namely the method of placing the poles of the transfer function of the system on the s-plane (s-plane method) and the method based on minimizing the generalized quadratic criterion of the quality of the control process (generalized quadratic criterion method).

Effectiveness of the devised method was assessed by comparing it with similar results reported in [22].

We used software developed in the MATLAB environment (USA).

5. Results of devising a combined method for determining the parameters of PI and PID controllers

5. 1. Determining tuning parameters and control quality indicators for automatic control systems of different orders

5. 1. 1. First-order object

It is assumed that the system includes a PI controller with a transfer function:

$$W_{cl}(s)=\frac{C_0 s+C_1}{s}, \qquad (1)$$

and the transfer function of the object is as follows:

$$W_{ob}(s)=\frac{k}{Ts+1}. \qquad (2)$$

Since the transfer function of a closed-loop system is as follows (Fig. 1):

$$W_{yu}(s)=\frac{W(s)}{1+W(s)}, \qquad (3)$$

then, taking into account formulas (1) and (2), the characteristic equation of the closed system is obtained:

$$a_0 s^2+a_1 s+a_2=0, \qquad (4)$$

where $W(s)=W_{cl}(s)W_{ob}(s)$ is the transfer function of the open system; $a_1=1+kC_0$; $a_2=kC_1$.

Let s_1 and s_2 be the roots of characteristic equation (4). Then, by Vieta's theorem, we obtain the following system of equations:

$$s_1+s_2=-\frac{a_1}{a_0},$$

$$s_1 s_2=\frac{a_2}{a_0}.$$

In the general case, the roots of characteristic equation (4) are complex conjugate: $s_1=-\alpha+j\zeta$, $s_2=-\alpha-j\zeta$.

For the stability of a closed-loop automatic control system, the following condition must be met:

$$\alpha>0,\ \ \zeta>0. \qquad (5)$$

An indicator $\mu=\frac{\zeta}{\alpha}$, is introduced that characterizes the degree of oscillation of the system. Taking into account the value $\zeta=\alpha\mu$, the roots of the characteristic equation will be as follows:

$$s_1=-\alpha(1-j\mu),\ \ s_1=-\alpha(1+j\mu). \qquad (6)$$

If we now take into account the values of the coefficients of the characteristic equation a_0–a_2 and the roots of the characteristic equation, which are determined from formula (6), we obtain the following relations:

$$2\alpha=\frac{1+kC_0}{T},\ \ \alpha^2\left(1+\mu^2\right)=\frac{kC_1}{T},$$

hence, we derive:

$$C_0=\frac{2\alpha T-1}{k}, \qquad (7)$$

$$C_1 = \frac{\alpha^2\left(1+\mu^2\right)T}{k}. \quad (8)$$

The generalized quadratic criterion was defined as follows [24]:

$$J = \int_0^\infty \left(\varepsilon^2(t) + \tau^2\dot{\varepsilon}^2(t)\right)\mathrm{d}t,$$

which is represented as:

$$J = J_0 + \tau^2 J_1, \quad (9)$$

where:

$$J_0 = \int_0^\infty \varepsilon^2(t)\mathrm{d}t; \quad J_1 = \int_0^\infty \dot{\varepsilon}^2(t)\mathrm{d}t.$$

There are tables, first published in [25], from which the values of J_0 and $J1$ could be determined. This is possible when the transfer functions $w_{\varepsilon u}(s)$, $w_{\dot{\varepsilon}u}(s)$ of the control error $\varepsilon(s)$ and its derivative $\dot{\varepsilon}(s)$ with respect to the control action $u(t)$ are known. In this case, the condition must be met that the order of the polynomial in the numerator is one less than the order of the polynomial in the denominator of the transfer functions $w_{\varepsilon u}(s)$ and $w_{\dot{\varepsilon}u}(s)$.

Since:

$$W_{\varepsilon u}(s) = \frac{1}{1+W(s)}, \quad (10)$$

then after taking into account the value of $W(s)$, we obtain:

$$W_{\varepsilon u}(s) = \frac{(Ts+1)s}{a_0 s^2 + a_1 s + a_2}.$$

For a single jump-like input value $u(t)$, the Laplace representation of the control error $\varepsilon(t)$ will be as follows:

$$\mathrm{E}(s) = \frac{b_0 s + b_1}{a_0 s^2 + a_1 s + a_2}, \quad (11)$$

where $b_0=a_0=T$; $b_1=1$.

The Laplace representation of the function $\dot{\varepsilon}(t)$ is as follows:

$$L\left[\dot{\varepsilon}(t)\right] = \mathrm{E}_1(s) = s\mathrm{E}(s) - \varepsilon(0),$$

where $\varepsilon(0) = \lim_{s\to\infty} s\mathrm{E}(s)$.

Taking into account formula (11), it is found that $\varepsilon(0)=1$. Therefore:

$$\mathrm{E}_1(s) = \frac{\beta_0 s + \beta_1}{a_0 s^2 + a_1 s + a_2}, \quad (12)$$

where $\beta_0=-kC_0$; $\beta_1=-kC_1$.

Since in formulas (11) and (12) the order of the denominator polynomial is n=2, and the numerator has the order n–1, then according to tables [25] we find:

$$J_0 = \frac{b_0^2 a_2 + b_1^2 a_0}{2a_0 a_1 a_2},$$

$$J_1 = \frac{\beta_0^2 a_2 + \beta_1^2 a_0}{2a_0 a_1 a_2}.$$

The following designations have been introduced:

$$J_{N,1} = b_0^2 a_2 + b_1^2 a_0, \quad J_{N,2} = \beta_0^2 a_2 + \beta_1^2 a_0, \quad J_D = 2a_0 a_1 a_2.$$

Then formula (9) will take the following form:

$$J = \frac{1}{J_D}\left(J_{N,1} + \tau^2 J_{N,2}\right). \quad (13)$$

After substituting the values of C_0 i C_1, which are determined from formulas (7) and (8), into formula (13), taking into account the values of $J_{N,1}$, $J_{N,2}$, and J_D, the generalized quadratic criterion (13) will be a function of two quantities α and μ.

The magnitude of the overshoot σ depends on the degree of oscillation μ [25]; by choosing the value of μ, it is possible to achieve the desired value of quantity σ. Therefore, with the selected value of μ, the generalized quadratic criterion (13) will be a function of one variable α.

To compare the results of tuning the parameters of the PI controller using the devised method with the results reported in work [22], the parameters of the transfer function (2) of the object from work [22] were taken. That is, k=2.5; T=12 s. The following values of the degree of oscillation were chosen: $\mu\in[0.2; 0.4; 0.6; 0.8]$ and $\tau=4$.

As a result, plots of the function $J(\alpha)$ were constructed (Fig. 2), from which it is clear that the dependence $J(\alpha)$ has a clearly expressed minimum. The minimum value of the function $J(\alpha)$ belongs to the interval $\alpha\in[0.1; 0.2]$. Using the built-in file function fminbnd, the one-dimensional optimization problem was solved:

$$\min_{\alpha_1 \le \alpha \le \alpha_2} : J(\alpha), \quad (14)$$

where α_1=0.1 and α_2=0.2 (Fig. 2).

The result of solving (13) is the value of α*. By substituting α* into formulas (7) and (8), the optimal parameters for setting the PI controller were calculated.

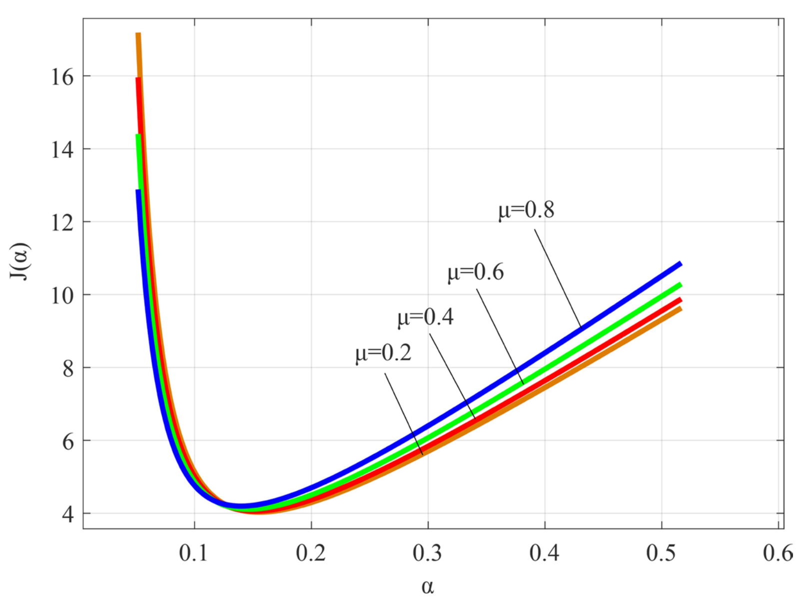

Fig. 2. Plots of function $J(\alpha)$

The transient characteristics of the automatic control system (Fig. 1) at the optimal values of C_0 and C_1 are shown in Fig. 3.

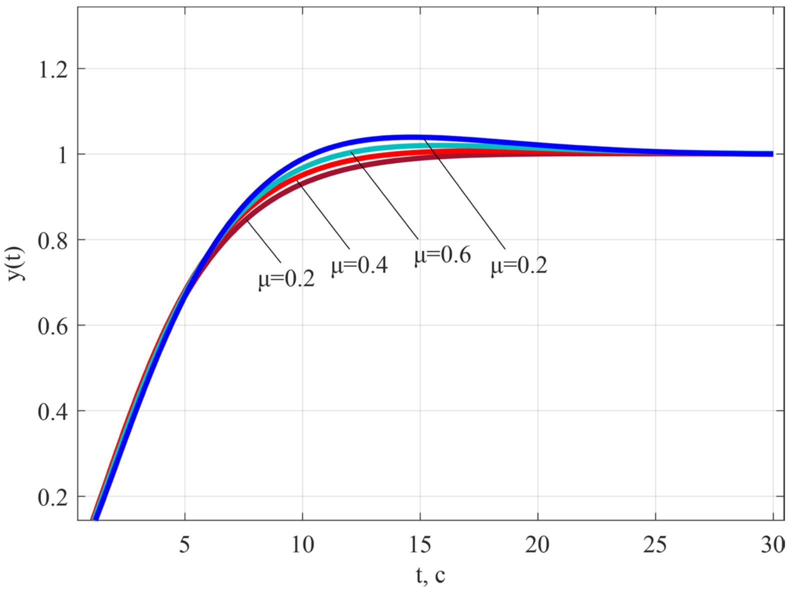

Fig. 3. Plots of the transient characteristics of the system at different values of μ

The quality of control process was evaluated by two indicators: overshoot σ % and control time t_c.

The results of our calculations are given in Table 1, in which the results reported in [22] are given for comparison.

Table 1

Comparing the results of determining the PI controller tuning parameters with the results reported in [22]

μ	Tuning parameters				Control quality indicators			
	1	2	1	2	1	2	1	2
	C_0		C_1, s^{-1}		σ, %		t_c, s	
0.2	1.09	0.35	0.1202	3.00	1.1885	2.64	10.5	28.32
0.4	1.05	0.19	0.1277	1.50	2.0593	4.04	9.9	29.12
0.6	1.00	0.15	0.1392	1.00	3.6942	6.30	9.3	28.64
0.8	0.94	0.15	0.1536	0.75	6.1782	9.08	19.5	26.40

Note: 1 – combined method; 2 – s-plane method [22].

For the first-order object, it is possible to observe a change in the control quality indicators compared to the s-plane method, namely, the overshoot changed by 122.13–46.97 % and the control time by 169.7–35.4 %, depending on the degree of oscillation, in favor of the combined method.

5. 1. 2. Second-order object

The method for determining the tuning parameters of PI controllers is applied to a more complex case when the transfer function of the object is as follows:

$$W_{ob}(s)=\frac{b_0 s+b_1}{a_0 s^2+a_1 s+a_2}. \qquad (15)$$

For the transfer function of the object (15), the transfer function of the closed system was found from formula (3):

$$W_{yu}(s)=\frac{\beta_0 s^2+\beta_1 s+\beta_2}{\pi_0 s^3+\pi_1 s^2+\pi_2 s+\pi_3}, \qquad (16)$$

where:

$$\beta_0=b_0C_0;\;\; \beta_1=b_0C_1+b_1C_0;\;\; \beta_2=b_1C_1;$$

$$\pi_0=a_0;\;\; \pi_1=b_0C_0+a_1;$$

$$\pi_2=b_0C_1+b_1C_0+a_2;\;\; \pi_3=b_1C_1.$$

According to formula (11), we determined the transfer function of the discrepancy value ε(t) relative to the controller task u(t):

$$W_{\varepsilon u}(s)=\frac{\left(a_0 s^2+a_1 s+a_2\right)s}{\pi_0 s^3+\pi_1 s^2+\pi_2 s+\pi_3}. \qquad (17)$$

Analysis of formulas (16) and (17) reveals that the transfer functions have identical characteristic equations:

$$\pi_0 s^3+\pi_1 s^2+\pi_2 s+\pi_3=0. \qquad (18)$$

Since equation (18) is a third-order polynomial, we have three roots s_1, s_2 and s_3. Between them and the coefficients of characteristic equation (18), there are relations that follow from Vieta's theorem, i. e.:

$$s_1+s_2+s_3=-\frac{\pi_1}{\pi_0}; \qquad (19)$$

$$s_1s_2+s_1s_3+s_2s_3=\frac{\pi_2}{\pi_0}; \qquad (20)$$

$$s_1s_2s_3=-\frac{\pi_3}{\pi_0}. \qquad (21)$$

The following notation is introduced: $s_1=-\alpha+j\zeta$ and $s_2=-\alpha-j\zeta$. From equation (19), the value of the root is determined. Taking into account the roots s_1 and s_2, we find $s_3=2\alpha-\frac{\pi_1}{\pi_0}$.

Since the automatic control system (Fig. 1) must be stable, the roots s_1, s_2, and s_3 of characteristic equation (18) must belong to the left half-plane of the root plane (s-plane). This condition will be fulfilled with the following relations: α>0, ζ>0, and $s_3<0$ or $\alpha<\frac{\pi_1}{2\pi_0}$. The latter condition determines the range of change of α in optimization problem (14), i. e.:

$$0<\alpha<\frac{\pi_1}{2\pi_0}. \qquad (22)$$

The degree of oscillation of the system in the transient process is denoted as follows: $\mu=\frac{\zeta}{\alpha}$. Then $s_1=-\alpha(1-j\mu)$ and $s_2=-\alpha(1+j\mu)$.

Substituting the values of s_1, s_2 and s_3 into equations (20) and (21), the relationship between the values of α and μ and the tuning parameters of the PI controller is found, that is:

$$\left(2\alpha b_0-b_1\right)C_0-b_0C_1=a_2+\pi_0\alpha^2\left(4-\left(1+\mu^2\right)\right)-2\alpha a_1,$$

$$-b_0\alpha^2\left(1+\mu^2\right)C_0+b_1C_1=\alpha^2\left(1+\mu^2\right)\left(a_1-2\pi_0\alpha\right).$$

After entering the notation:

$$a_{11}=2\alpha b_0-b_1;\;\; a_{12}=-b_0;\;\; a_{21}=-b_0\alpha^2\left(1+\mu^2\right);$$

$$a_{22}=b_1;\;\; q_1=a_2+\pi_0\alpha^2\left(4-\left(1+\mu^2\right)\right)-2\alpha a_1;$$

$$q_2=\alpha^2\left(1+\mu^2\right)\left(a_1-2\pi_0\alpha\right),$$

we obtained systems of linear equations with two unknowns C_0 and C_1:

$$a_{11}C_0+a_{12}C_1=q_1,$$

$$a_{21}C_0+a_{22}C_1=q_2,$$

whose solution is:

$$C_0=\frac{a_{22}q_1-a_{12}q_2}{a_{11}a_{22}-a_{12}a_{21}}, \quad (23)$$

$$C_1=\frac{a_{11}q_2-a_{21}q_1}{a_{11}a_{22}-a_{12}a_{21}}. \quad (24)$$

The generalized quadratic criterion (13) for n=3 is as follows: [25]:

$$J_{N,1}=a_0^2\pi_2\pi_3+\left(a_1^2-2a_0a_2\right)\pi_0\pi_3+a_2^2\pi_0\pi_1, \quad (25)$$

$$J_{N,2}=\gamma_0^2\pi_2\pi_3+\left(\gamma_1^2-2\gamma_0\gamma_2\right)\pi_0\pi_3+\gamma_2^2\pi_0\pi_1, \quad (26)$$

$$J_D=2\pi_0\pi_3\left(\pi_1\pi_2-\pi_0\pi_3\right). \quad (27)$$

Analysis of formulas (25) to (27) reveals that the generalized quadratic criterion (13) is a function of the variable α. Minimization of the criterion is reduced to solving problem (14), in which the search interval is determined by the conditions C_0>0 and C_1>0 and condition (22).

In order to compare the results of calculating the parameters of the controller settings with the corresponding results of work [22], the values for the coefficients of the transfer function of the object (15) are taken from [22]. Namely: a_0=20, a_1=6, a_2=4; b_0=4, b_1=7. The coefficient τ, which determines the influence of the rate of change of the control error on the value of the generalized criterion (13), is as follows: τ=4.

As a result, plots of the dependence $J(\alpha)$ for the values μ∈[0.2; 0.4; 0.6; 0.8] have been constructed (Fig. 4).

Analysis of Fig. 4 revealed that the minimum values of the function $J(\alpha)$ (for given values of μ) belong to the interval α∈[0.15; 0.34]. The result of solving problem (14) is given in Table 2.

Fig. 5 shows the transient characteristics of the closed-loop control system (Fig. 1), obtained at different values of the oscillation coefficient μ (Table 2).

For the second-order object, a change in control quality indicators is observed, namely, the overshoot changed by 3304.63–608.90 %, and the control time by 402.22–824.14 % depending on the degree of oscillation in favor of the combined method.

Table 2

Comparing the results of calculating the controller tuning parameters using two methods

μ	Tuning parameters		Control quality indicators			
	C_0	C_1, s^{-1}	σ, %		t_c, s	
			1	2	1	2
0.2	1.5122·10^{-1}	3.3474·10^{-2}	0.0605	2.0598	10.8000	54.2400
0.4	1.5000·10^{-1}	3.5742·10^{-2}	0.6345	12.6662	9.9000	54.2400
0.6	1.4012·10^{-1}	3.7220·10^{-2}	1.8946	20.9134	9.3000	74.6400
0.8	1.2771·10^{-1}	3.8967·10^{-2}	3.9324	27.8766	8.7000	80.4000

Note: 1 – combined method; 2 – s-plane method [22].

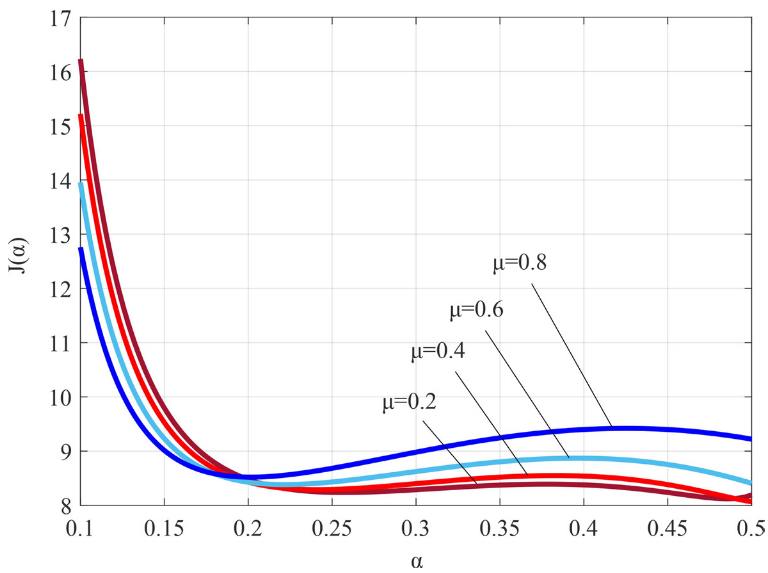

Fig. 4. Dependence of the generalized integral quadratic criterion on the variable α

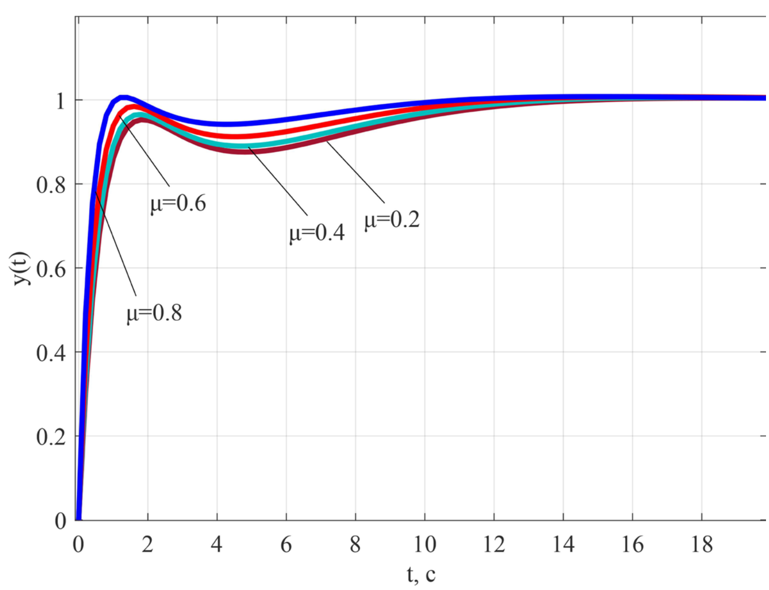

Fig. 5. Transient characteristics of the system at different values of the oscillation coefficient μ

5. 1. 3. Third-order object

The dynamic properties of the controlled object are characterized by a third-order transfer function, i.e.:

$$W_{ob}(s)=\frac{b_0s+b_1}{a_0s^3+a_1s^2+a_2s+a_3}, \quad (28)$$

and the transfer function of the controller is as follows:

$$W_{cl}(s)=\frac{C_0s^2+C_1s+C_2}{s}. \quad (29)$$

The choice of PID control algorithm ensures the correct statement of the problem when implementing the combined method for determining the optimal tuning parameters C_0, C_1, and C_2.

For the automatic control system (Fig. 1), the transfer function of the control error ε(*t*) relative to the control action *u*(*t*) is determined. Therefore:

$$W_{\varepsilon u}(s)=\frac{\left(a_0s^3+a_1s^2+a_2s+a_3\right)s}{\pi_0s^4+\pi_1s^3+\pi_2s^2+\pi_3s+\pi_4},$$

where:

$$\pi_0=a_0,\ \ \pi_1=a_1+C_0b_0,\ \ \pi_2=a_2+C_0b_1+C_1b_0,$$

$$\pi_3=a_3+C_2b_0+C_1b_1,\ \ \pi_4=b_1C_2.$$

According to the algorithm for determining the parameters of the controller settings, the value of the generalized quadratic criterion (13) has been calculated. To that end, we find the Laplace representation of the control error *E*(*s*) and the derivative of the control error $E_1(s)$. So:

$$\mathrm{E}(s)=\frac{a_0s^3+a_1s^2+a_2s+a_3}{\pi_0s^4+\pi_1s^3+\pi_2s^2+\pi_3s+\pi_4}, \quad (30)$$

and:

$$\mathrm{E}_1(s)=\frac{\mu_0s^3+\mu_1s^2+\mu_2s+\mu_3}{\pi_0s^4+\pi_1s^3+\pi_2s^2+\pi_3s+\pi_4}, \quad (31)$$

where:

$$\mu_0=a_1-\pi_1,\ \ \mu_1=a_2-\pi_2,\ \ \mu_2=a_3-\pi_3,\ \ \mu_3=-\pi_4.$$

Analysis of formulas (30) and (31) reveals that they are identical in structure (the difference is only in the coefficients of the numerator) and the orders of the numerators are one less than the orders of their denominators. This allows us to calculate the generalized quadratic criterion (13) according to the formulas given in [25]. For n=4, we obtain:

$$J_{N1}=a_0^2\left(\pi_2\pi_3\pi_4-\pi_1\pi_4^2\right)+\left(a_1^2-2a_0a_2\right)\pi_0\pi_3\pi_4+$$
$$+\left(a_2^2-2a_1a_3\right)\pi_0\pi_1\pi_4+a_3^2\left(\pi_0\pi_1\pi_2-\pi_0^2\pi_3\right), \quad (32)$$

$$J_{N2}=\mu_0^2\left(\pi_2\pi_3\pi_4-\pi_1\pi_4^2\right)+\left(\mu_1^2-2\mu_0\mu_2\right)\pi_0\pi_3\pi_4+$$
$$+\left(\mu_2^2-2\mu_1\mu_3\right)\pi_0\pi_1\pi_4+\mu_3^2\left(\pi_0\pi_1\pi_2-\pi_0^2\pi_3\right), \quad (33)$$

$$J_D=2\pi_0\pi_4\left(\pi_1\pi_2\pi_3-\pi_1^2\pi_4-\pi_0\pi_3^2\right), \quad (34)$$

From formulas (32) to (34), the quadratic criterion (13) at given parameters of the transfer function of the object (28) is a function of the parameters C_0, C_1, and C_2 of the PID controller settings.

In work [21] it is proposed to calculate the values of C_0, C_1, and C_2 by placing the poles of the closed-loop control system (Fig. 1) on the s-plane. Since the characteristic equation:

$$\pi_0s^4+\pi_1s^3+\pi_2s^2+\pi_3s+\pi_4=0, \quad (35)$$

of the closed system has four roots, then their choice is as follows: $s_1=-\alpha+j\zeta$, $s_2=-\alpha-j\zeta$, $s_3=-k_1\alpha$, and the fourth root was defined below.

Between the roots of characteristic equation (35) and its coefficients for n=4, there are the following relations:

$$s_1+s_2+s_3+s_4=-\frac{\pi_1}{\pi_0}, \quad (36)$$

$$s_1s_2+s_1s_3+s_1s_4+s_2s_3+s_2s_4+s_3s_4=\frac{\pi_2}{\pi_0}, \quad (37)$$

$$s_1s_2s_3+s_1s_2s_4+s_1s_3s_4+s_2s_3s_4=-\frac{\pi_3}{\pi_0}, \quad (38)$$

$$s_1s_2s_3s_4=\frac{\pi_4}{\pi_0}. \quad (39)$$

We defined from equation (36):

$$s_4=-\frac{\pi_1}{\pi_0}-\left(s_1+s_2+s_3\right).$$

After taking into account the values of the poles s_1, s_2 and s_3, we obtain:

$$s_4=\left(2+k_1\right)\alpha-\frac{\pi_1}{\pi_0}.$$

To ensure the stability of the closed-loop system (Fig. 1), the poles of characteristic equation (35) must be left-handed. This means that the following relations must hold: $\alpha>0$, $\zeta>0$, $k_1>0$ and $s_4<0$, or:

$$0<\alpha<\frac{\pi_1}{\pi_0\left(k_1+2\right)}. \quad (40)$$

The latter relation imposes certain restrictions on the choice of the value of C_0. Since $\pi_1=a_1+C_0b_0$, then for the chosen value of k_1, C_0 must be such that condition (40) is fulfilled.

The degree of oscillation of the system is denoted by $\mu=\frac{\zeta}{\alpha}$. Then $s_1=-\alpha(1-j\mu)$ i $s_2=-\alpha(1+j\mu)$.

The values of the poles s_1–s_4 were substituted into formulas (37) to (39) and after a series of algebraic transformations, the following result was obtained:

$$\alpha k_g\pi_1-\pi_2=\pi_0\alpha^2\left(k_gk_1+4-r\right),$$

$$\left(r+2k_1\right)\alpha^2\pi_1-\pi_3=2\alpha^3\pi_0\left(k_gk_1+r\right),$$

$$\alpha^3rk_1\pi_1-\pi_4=\pi_0k_1k_gr\alpha^4,$$

where $k_g=k_1+2,\ \ r=\mu^2+1.$

If we take into account the values of the coefficients π_1–π_4 of characteristic equation (35), the system of equations will be as follows:

$$a_{11}C_0+a_{12}C_1+a_{13}C_2=q_1, \quad (41)$$

$$a_{21}C_0+a_{22}C_1+a_{23}C_2=q_2, \quad (42)$$

$$a_{31}C_0+a_{32}C_1+a_{33}C_2=q_3, \quad (43)$$

where:

$$a_{11}=\alpha b_0k_g-b_1,\ a_{12}=-b_0,\ a_{13}=0;$$

$$a_{21}=b_0\alpha^2(r+2k_1),\ a_{22}=-b_1,\ a_{23}=-b_0,$$

$$a_{31}=\alpha^3rb_0k_1,\ a_{32}=0,\ a_{33}=-b_1,$$

$$q_1=\pi_0\alpha^2(4+k_gk_1-r)-k_g\alpha a_1+a_2,$$

$$q_2=2\alpha^3\pi_0(k_1k_g+r)-\alpha^2(r+2k_1)a_1+a_3,$$

$$q_3=\alpha^3rk_1(\pi_0\alpha k_g-a_1).$$

The system of linear algebraic equations (41) to (43) is represented in matrix-vector form:

$$A\bar{C}=\bar{q}, \quad (44)$$

where:

$$A=\begin{bmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{bmatrix},\ \bar{q}=\begin{bmatrix}q_1\\ q_2\\ q_3\end{bmatrix},\ \bar{\tilde{N}}=\begin{bmatrix}\tilde{N}_0\\ \tilde{N}_1\\ \tilde{N}_2\end{bmatrix}.$$

We find from equation (44):

$$\bar{C}=A^{-1}\bar{q}. \quad (45)$$

After performing the corresponding actions on the matrices on the right side of expression (45), we obtain:

$$C_0=\frac{1}{\Delta}\left(a_{12}a_{23}q_3\ -\ a_{12}a_{33}q_2\ +\ a_{22}a_{33}q_1\right), \quad (46)$$

$$C_1=\frac{1}{\Delta}\begin{pmatrix}a_{11}a_{33}q_2\ -\ a_{11}a_{23}q_3\ -\\ -\ q_1\left(a_{21}a_{33}\ -\ a_{23}a_{31}\right)\end{pmatrix}, \quad (47)$$

$$C_2=\frac{1}{\Delta}\begin{pmatrix}q_3\left(a_{11}a_{22}-a_{12}a_{21}\right)+\\ +a_{12}a_{31}q_2-a_{22}a_{31}q_1\end{pmatrix}, \quad (48)$$

where $\Delta=a_{11}a_{22}a_{33}-a_{12}a_{21}a_{33}+a_{12}a_{23}a_{31}$.

According to the values of C_0, C_1, and C_2, which are determined from formulas (46) to (48), it became possible to calculate the coefficients π_1–π_4 from equation (35). Accordingly, the generalized criterion is a quadratic criterion according to formula (13).

Analysis of formulas (32) to (34) reveals that with the given values of the parameters of the transfer function of the object (28) and the selected values of k_1 and μ, the generalized quadratic criterion (13) will be a function of only one variable α.

For the transfer function (28) of the object, the following coefficients were selected: a_0=6, a_1=7, a_2=5, a_3=1; b_0=2, b_1=1. Other parameters: k_1=1,2; τ=4, μ∈[0,2; 0,4; 0,6; 0,8].

Fig. 6 shows plots of the dependences $J(\alpha)$, from which it can be seen that there are minima of the function $J(\alpha)$ in the range of values α∈[0.5;1.0].

The calculation results are given in Table 3. For comparison, Table 3 shows the corresponding values of σ and t_c calculated using the s-plane method.

The plots of transient processes at given values of the degree of oscillation μ are reproduced in Fig. 7.

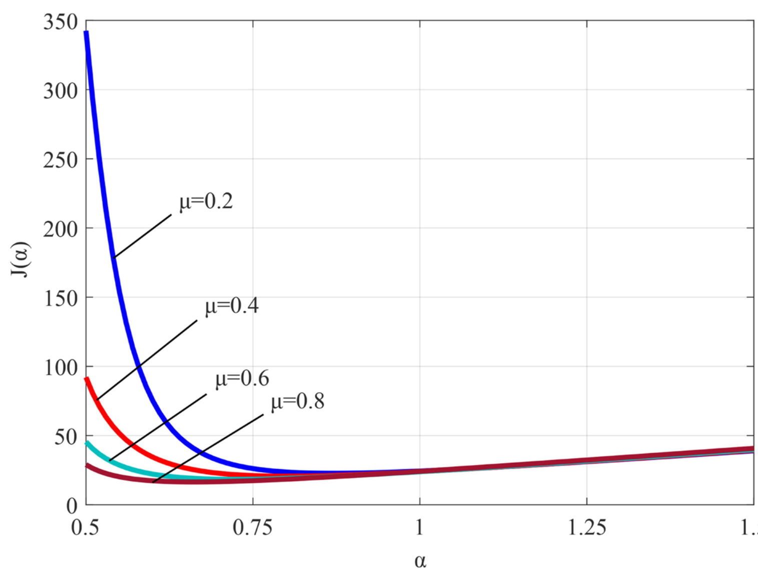

Fig. 6. Plots of the functional dependence $J(\alpha)$ at different values of μ

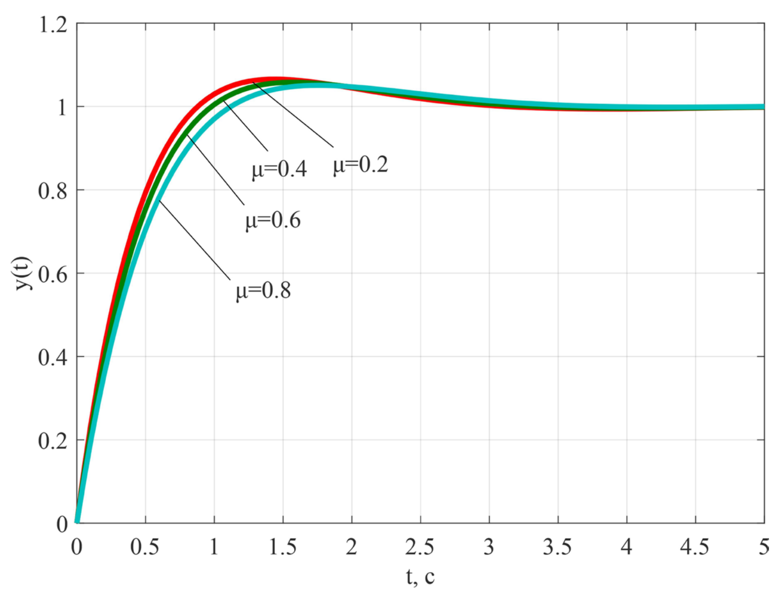

Fig. 7. Plots of the transient processes in a control system for different values of μ

Table 3

Comparing the results of calculating the controller tuning parameters using two methods

μ	Tuning parameters			Control quality indicators			
	C_0, c	C_1	C_2, c^{-1}	σ, %		t_c, c	
				1	2	1	2
0.2	7.7265	9.4505	4.7245	6.3832	17.5124	2.65	4.50
0.4	7.0927	8.2439	4.1503	5.5935	17.4961	2.80	4.51
0.6	6.3703	6.9881	3.6130	4.8242	17.4683	3.05	4.55
0.8	5.7403	6.0503	3.2922	4.6949	17.4281	3.45	4.61

Note: 1 – combined method; 2 – s-plane method [22].

For the first-order object, a change in control quality indicators is observed. Overshoot increased within 174.35–271.21 %, and the control time changed by 69.81–33.62 %, depending on the degree of oscillation, in favor of the combined method.

5. 2. Algorithm for setting the parameters of the PI/PID controller

Therefore, the algorithm for determining the parameters of the PI/PID controllers includes the following sequence of steps:

K1. Search for the interval of change in the value α. Set the interval of values $\alpha \in [\alpha_{min}; \alpha_{max}]$ and calculate the values of C_0, C_1, and C_2 on this interval according to formulas (46) to (48). Fix the values of α_1 and α_2, which will determine the new interval $\alpha \in [\alpha_1; \alpha_2]$. On this interval, the following conditions must be simultaneously met: $C_0>0$, $C_1>0$, $C_2>0$ і $s_4<0$.

K2. Determine the interval of the minimum value of the generalized quadratic criterion (13). According to formulas (32) to (34), calculate the value of criterion (13) on the interval of values $\alpha \in [\alpha_1; \alpha_2]$. Construct a plot of the dependence $J(\alpha)$. Choose the interval $\alpha \in [\alpha_s; \alpha_f]$, which contains the minimum of the function $J(\alpha)$.

K3. Minimization of the generalized quadratic criterion (13). Solve the one-dimensional minimization problem $\min\limits_{\alpha_s \le \alpha \le \alpha_f} : J(\alpha)$ and determine α^*.

K4. Calculate the PID controller tuning parameters and assess the quality of control process. Using the found value of α^*, calculate C_0, C_1, and C_2 according to formulas (46) to (48). Construct a plot of the transient process in an automatic control system and evaluate the quality indicators of control process – overshoot σ and control time t_c.

6. Discussion of results related to devising a method for determining the optimal parameters of PI/PID controller settings

The s-plane method is characterized by the researcher's subjectivity when choosing the placement of roots on the complex root plane. The obtained parameters of PI and PID controller settings guarantee the stability of the system, but they are not optimal. The combination of two s-plane methods and the generalized quadratic criterion allowed us to reduce the multidimensional minimization problem to a one-dimensional one and obtain the optimal parameters for PI and PID controller settings (Fig. 2, 4, 6), provided that the stability of the automatic control system is guaranteed.

The synthesized algorithm for PI/PID controller settings, which is based on the devised method, allows us to determine the optimal parameters for PI/PID controller settings under an iterative mode. The effectiveness of the proposed method is proven by conducting computer experiments for first-, second-, and third-order objects.

The combined method could be used in the design of automatic control systems that include PI/PID controllers and first-, second-, or third-order objects.

The limitation of this study is that the devised method was tested for objects of the first-, second-, and third-orders. Extending the method devised to objects of higher order faces the problem of overdetermination of linear algebraic equations. In the case when the conditions of the Kronecker-Capelli theorem are met, the problem has a unique solution. Otherwise, a pseudo-solution to the overdetermined system of equations is sought, and the question of optimality regarding the tuning parameters of PI and PID controllers requires further research.

7. Conclusions

1. The combination of the *s*-plane methods and the generalized quadratic criterion has made it possible to reduce the problem of determining the optimal parameters for PI/PID controller settings to a one-dimensional problem of minimizing the generalized quadratic criterion.

2. The developed algorithm has made it possible to determine the optimal parameters for PI/PID controller settings. Our calculations showed that the overshoot and the control time decreased on average by 73.5 % and 66.5 %, respectively, compared to the s-method.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study, as well as the results reported in this paper.

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The study was conducted without financial support.

Data availability

All data are available, either in numerical or graphical form, in the main text of the manuscript.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

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