The object of this study is the processes of operational control and correction of data errors in non-positional code structures (NCS). Based on a critical analysis of the existing data control method based on the use of the projection of a number in RCS, limited control efficiency and the ability to detect only single errors have been established.

The study improves methods for rapid control and data correction of a real-time computer system (CS) operating in a non-positional number system, in the so-called residual class system (RCS). A comprehensive approach to control and eliminate errors in RCS is built on the basis of non-positional coding, underlying which is the Chinese residual theorem. This theorem proves that NSC is the next stage in the development of the theory of information control using arithmetic control by modulus. The use of the property of complete arithmetic of NSC has made it possible to improve the method and increase the efficiency of data control due to information processing in RCS without controlling each intermediate result obtained. Comparison with the most efficient existing method has made it possible to establish that the devised method provides an increase in the speed of data control by 1.2–1.3 times.

An effective process of operational and accurate error detection based on an improved method of data control in RCS, which is based on the use of the corrective properties of NCS, has been proposed. Parallel error correction in NCS increases the efficiency of error correction by 2 times, due to a decrease in the number of intermediate operations in the improved method. At the same time, with an increase in the bit grid of the operands being processed, the efficiency of the application of the considered error correction process improves

Keywords: data processing speed, non-positional code structure, residue class system, control efficiency, data correction

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IMPROVING THE PROCESS OF CONTROL AND CORRECTION OF ERRORS IN NON-POSITIONAL CODE STRUCTURES

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1. Introduction

Modern information processing and management systems are based on computer systems (CSs), in which the execution time of operations is a critical factor [1]. Increasing the efficiency of their functioning under real-time data processing conditions is achieved by optimizing such key characteristics of CS as productivity (speed) and reliability (reliability and fault tolerance) [2]. The use of the residual class system (RCS), with its characteristic features such as independence, equality, and small-bit processing of numerical values of residues $\{x_q\}$ is a tool for optimizing computational processes. Due to the non-positional code structure (NCS) of data formation in the form $X=(x_1||x_2||...||x_{q-1}||x_q||x_{q+1}||...||x_p)$, RCS expands the possibilities for high-performance calculations. It is this structure that makes it possible to effectively perform calculations based on modular arithmetic operations [3]. The use of RCS demonstrates the greatest efficiency when executing algorithms that include arithmetic operations of subtraction, addition, and multiplication [4–6]. However, to guarantee the fault tolerance of CS operating in RCS, specialized approaches to control, diagnostics, and data correction are required. This is due to the fact that conventional methods used in binary CSs based on the positional number system (PNS) are inefficient [7].

Existing methods of control, diagnostics, and error correction in RCS are based on positional arithmetic. The greatest time and technical costs in RCS are associated with the implementation of arithmetic positional operations. The long time of the data control and error correction process in RCS negatively affects the efficiency of using non-positional coding in general. Thus, the relevance and importance of research aimed at devising and optimizing fast methods for monitoring, diagnosing, and correcting data errors in real-time CSs (RTCSs) operating in RCS is obvious. Under modern conditions of continuous growth of data volumes in the information space, increasing the reliability and speed of processing large data sets through effective methods for detecting and correcting errors is rightfully considered an objective necessity [8].

The relevance of the issue of increasing the reliability and efficiency of data processing, which directly affects the main indicators of productivity, reliability, and fault tolerance of CS, is confirmed by intensive scientific research. The scientific community is actively working to solve these problems, emphasizing the importance of continuing such research under modern conditions. The share of control (diagnostic) equipment in modern RTCSs is about 18–25 % of the total equipment. Therefore, devising effective mathematical (software) control and diagnostic methods for detecting errors in RTCSs is of strategic importance for enabling stable and safe operation of data processing systems.

2. Literature review and problem statement

In the context of the constantly growing volumes of data circulating in information and communication systems, the need to ensure a high level of efficiency (speed) and reliability of data processing is an urgent task.

In work [9], the authors analyzed methods and algorithms of noise-tolerant coding, which provide the ability to detect and correct errors caused by the influence of interference in information transmission channels. The main attention is paid to the study of the characteristics of nested convolutional codes with variable rate under the conditions of using adaptive coding and decoding in information and communication systems. At the same time, the paper is mainly of a review nature, focusing on the analysis of existing methods, without making proposals for their further improvement. The likely reason is objective difficulties associated with the complexity of implementing adaptive algorithms, the fundamental impossibility of ensuring the universality of such methods for different types of communication channels, etc. Accordingly, there is a need to conduct research on devising new approaches to adaptive noise-tolerant coding.

In [10, 11], the issue of increasing the security of CS is considered through the use of VPN gateways that provide protection of transmitted data. The authors analyzed the parameters of the transmission channel that contribute to an increase in the level of information security, in particular, such characteristics as the delay time and throughput of data packets through the protected channel. However, the studies did not pay due attention to such important aspects as reliability, robustness, and fault tolerance, which are of critical importance for a comprehensive assessment of the functional characteristics of computer systems.

The author of study [12] proposed a scheme for more secure and faster message transmission over a network based on DNA cryptography. The experiment conducted in the work confirmed that the proposed encryption system provides low computing time regardless of the length of the characters of the text being encrypted. However, the system was developed using the MATLAB programming language, which limits its use. In [13, 14], to ensure the security of the digital environment and the confidentiality, integrity, authenticity, and availability of data, the authors propose to use DNA cryptography using a residue class system. The authors' main attention is focused on reducing the data encryption time, which is a key factor in increasing the performance of cryptographic systems. However, issues related to the need to ensure the promptness of error detection and correction in RTCS, which is an important aspect for increasing the reliability and resilience of systems to failures, remain unresolved. This may be due to technical problems associated with the adaptation of error detection methods in the context of DNA cryptography, as well as the high costs of developing integrated solutions that combine encryption and error correction. All this suggests the need for research on improving the error correction process.

In [15], a study was conducted on the procedures for developing error-correcting codes intended for use in various telecommunication technologies. The authors' attention is focused on the assessment and identification of mechanisms for introducing redundancy into signal-code structures in protected information channels. At the same time, the work does not take into account the specificity of the nature of information redundancy, which limits the possibilities of developing optimal solutions to increase the efficiency of telecommunication systems. The likely reason is objective difficulties associated with the complexity of formalizing information redundancy in telecommunication systems, as well as the lack of unified approaches to its modeling.

The issue of using error-correcting codes to improve the efficiency of RTCSs is considered in detail in [16, 17]. The studies report the results of experimental tests, which confirm that the algorithms developed by the authors demonstrate significantly higher performance in the context of functional error correction, compared to conventional approaches. The results indicate the prospects of the proposed solutions for ensuring the reliability of the functioning of RTCSs. However, the issues related to the adaptation of these algorithms to work under conditions of high load intensity, which are characteristic of modern RTCSs, remain unresolved. Therefore, the feasibility of conducting research aimed at developing effective error correction algorithms that meet the specificity of RTCS operation under various operating conditions is objectively justified. The authors of [18, 19] proposed new error control algorithms for redundant residue number systems. At the same time, the constantly growing data sets that need to be processed in real time require the search for new solutions to increase the level of reliability and speed of their control, diagnostics, and correction. Considering the scientific and practical significance of available research [9–19] on devising effective methods for data processing in RTCS, there remain a number of unresolved issues that require further study. In particular, the aspects related to increasing the efficiency of control, diagnostics, and error correction methods in RTCS are relevant. Systems operating under complex dynamic conditions require adaptation to an unstable and rapidly changing operating environment [20]. This includes the need to ensure continuous monitoring of the data state, timely detection and correction of errors.

Optimization of these processes could significantly increase the overall level of reliability of information systems. Improving methods for monitoring, diagnosing, and correcting data errors in RTCS would increase the reliability and speed of data processing. In addition, the implementation of new mathematical and software solutions would contribute to improving the reliability of systems, reducing the time for identifying and eliminating vulnerabilities, as well as increasing the overall efficiency of information systems.

This confirms the need for further research aimed at devising new methods and technologies that could make it possible to adapt systems to new conditions and enable a high level of speed and accuracy of data processing circulating in information and communication systems.

3. The aim and objectives of the study

The purpose of our study is to optimize the speed of implementing data control and correction methods in RTCSs operating in RCSs. This will make it possible to increase the level of efficiency in detecting and correcting data errors, while effectively using the computing resources of RTCS.

To achieve this goal, the following tasks were defined:

– to devise an effective method of data control in RCS, based on the principle of comparison and provide examples of the implementation of the method;

– to carry out a critical analysis of the existing method of data control and diagnostics, based on the use of number projection in RCS;

– to improve the method of data control and diagnostics, based on the use of number projection in RCS, and provide examples of specific implementation of the proposed method;

– to implement the process of data correction in RCS based on the devised methods.

4. The study materials and methods

The object of our study is the processes of operational control and data error correction in non-positional code structures of RCS.

The subject of the study is methods and algorithms of operational control (diagnostics) and error correction in non-positional code structures.

The basic hypothesis of the study assumes the introduction of new methods of control, diagnostics, and error correction in NCS, based on the principles of parallelism of data processing and noise-resistant coding, would make it possible to increase the speed and accuracy of error detection. This could enable increased reliability and efficiency of data processing circulating in RTCS.

Taking into account the hypothesis put forward, the study is based on the theoretical foundations of arithmetic and corrective properties of NCS, as well as on the key properties of RCS.

It is worth noting that the control methods in RCS are a further development of numerical or digital methods of module control in PSC, using arithmetic AN codes. In the context of information redundancy [21], NCS in RCS can be considered as an improved version of the multi-residue arithmetic codes used in PSC. These codes in PSC are characterized by a representation in the following form:

$$X_c^{'}=\begin{bmatrix} X_c \,||\, X_c\left(\mathrm{mod}\, w_1\right) || \, X_c\left(\mathrm{mod}\, w_2\right) || \ldots \\ \ldots || \, X_c\left(\mathrm{mod}\, w_q\right) || \ldots \\ \ldots || \, X_c\left(\mathrm{mod}\, w_{p-1}\right) || \, X_c\left(\mathrm{mod}\, w_p\right) \end{bmatrix}, \qquad (1)$$

that is $X_c^{'}=\left(X_c \,||\, x_1 \,||\, x_2 \,||\ldots||\, x_p\right)$.

Where the value of the residual of a number in RCS is determined as follows [22]:

$$x_q=X_c-[X_c/w_q]\, w_q. \qquad (2)$$

In this case, the value of $X_c\,(\mathrm{mod} w_q)$ is the residual of dividing the original number X_c by the w_q modulus. When the conditions $\prod_{\substack{c=1,\\ c\neq g.}}^{p+1} w_c \geq X_c$, are met: the set (2) of residues $\{x_q\}$ uniquely determines the NCS $X_{\tilde{n}}^{'}$ and the numerical value of X_c becomes unnecessary at all. The multi-residue code in PSC takes the form of NCS $X_c^{'}=\left(x_1 \,||\, x_2 \,||\ldots||\, x_p\right)$ in RCS, which makes it possible to implement modular operations on separate independent paths that work only with residues $\{x_q\}$. Based on the use of the independence and equality properties of RCS, it is possible to design RTCS, in which the processing of all residues $\{x_q\}$ of the number $X=(x_1||\ldots||x_q||\ldots||x_p)$ is carried out independently of each other, but, at the same time, in parallel in time [23].

The simplified structural diagram of RTCS in RCS (Fig. 1) is a set of independent computational paths. Each path processes data according to its modulus w_q and works in parallel with the others. This organization enables a high level of system performance, in particular data processing speed.

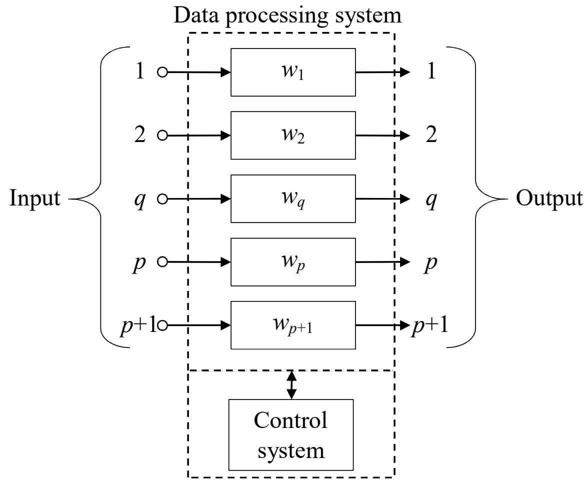

Fig. 1. Simplified block diagram of a real-time computer system in a residual class system

The given structure of RTCS operating in RCS guarantees parallelism of data processing at all stages, including control, diagnostics, comparison operations, etc. In this case, the computational process is limited to the use of only integers, and the results of all operations belong to the numerical interval *T*, which is defined as the product of the bases of specific RCS. Such an organization of calculations warrants the efficiency and accuracy of the system due to the parallel execution of operations in a certain interval.

5. Results of research on improved methods of data control and correction in the residual class system

5. 1. Data control method in the residual class system, based on the principle of comparison

To devise a data control method in RCS, based on the principle of comparison [24], the results of proving a previously established scientific fact (SF) are used.

SF 1. It is advisable to introduce RCS with information $\{w_q\}\left(q=\overline{1,p}\right)$ and one control $w_c=w_{p+1}$ bases, which is ordered ($w_q<w_{q+1}$), and the result of the obtained number $X=(x_1||x_2||\ldots||x_{q-1}||x_q||x_{q+1}||\ldots||x_p||x_{p+1})$ is correct [23]; respectively, the condition $X<T\left(T=\prod_{q=1}^{p} w_q\right)$. is fulfilled. Therefore, it is assumed that one of the residuals $\tilde{x}_q \neq x_q$ of the number $\tilde{X}=\left(x_1 \,||\, x_2 \,||\ldots||\, x_{q-1} \,||\, \tilde{x}_q \,||\, x_{q+1} \,||\ldots||\, x_p \,||\, x_{p+1}\right)$ has undergone distortion in the base w_q, i. e., the number $\tilde{X}$ is incorrect $\left(\tilde{X}\geq T\right)$. It should be proved that the residual $\tilde{x}_q$ is incorrect (distorted as a result of the operation). The correctness of any number *X* is established by comparing it with the value obtained from the relation $X<T=T_0/w_{p+1}\left(T_0=\prod_{q=1}^{p+1} w_q\right)$ [23]. On the other hand, it is evident that the conditions $T_0/w_{p+1}<T_0/w_q$ and $X<T_0/w_q$ are met for a given RCS within the framework of the considered SF 1 at $q=\overline{1,p}$. It should be noted that the residual $x_q\equiv X\,(\mathrm{mod} w_q)$ of the number *X* modulo w_q can only take the value $x_q=\overline{0,w_q-1}$. In accordance with SF 1 that $\tilde{x}_q\neq x_q$, and provided that the values of the other residuals

$x_g\left(g=\overline{1,p+1}\right)$, $q \neq g$ of the improper number $\tilde{X}$ do not change, then in the numerical interval $[0, T_0/w_q)$ the numbers X and $\tilde{X}$ cannot be present simultaneously. Since the number $X<T_0/w_{p+1}$ is correct, i. e., belongs to the interval $[0, T)$, respectively, the number $\tilde{X}$ is outside the interval $[0, T_0/w_q)$, and, as a consequence, also outside the interval $[0, T)$ (Fig. 2, 3).

Fig. 2. Bounds of numerical intervals for different modules of the residual class system

Fig. 3. Bounds of numerical intervals for the value $w_c=w_{p+1}$

In this case, the number:

$$\tilde{X}=\left(x_1 \,||\, x_2 \,||\ldots||\, x_{q-1} \,||\, \tilde{x}_q \,||\, x_{q+1} \,||\ldots||\, x_p \,||\, x_{p+1}\right),$$

which meets the condition $\tilde{X} \geq T$, is distorted.

It is worth focusing on the method of data control in RCS, based on the results and conclusions of the considered SF 1. The control procedure is based on comparing the obtained result X of the operation with the number $T=T_0/w_{p+1}$; for this purpose, the values $X=(x_1||x_2||\ldots||x_p||x_{p+1})$ and T must be converted to PNS. In this case, an effective approach is to use orthogonal bases $E_q\left(q=\overline{1,p+1}\right)$, which are represented in the form: $E_1=(1||0||\ldots||0||\ldots||0||0)$, ..., $E_q=(0||0||\ldots||1||\ldots||0||0)$, ..., $E_{p+1}=(0||0||\ldots||0||\ldots||0||1)$ [23].

The structure of orthogonal bases E_q for each RCS is described by the following expression:

$$E_q=\bar{e}_q \cdot T_0 / w_q=1\left(\operatorname{mod} w_q\right). \tag{3}$$

The value of weight $\bar{e}_q$ of the orthogonal basis E_q is defined as one of the solutions to the system of equations:

$$\begin{cases}\bar{e}_q=1,\ 1\cdot T_q \equiv \rho_1\left(\operatorname{mod} w_q\right),\\ \bar{e}_q=2,\ 2\cdot T_q \equiv \rho_2\left(\operatorname{mod} w_q\right),\\ \qquad\ldots\\ \bar{e}_q=w_q-2,\ (w_q-2)\cdot T_q \equiv \rho_{w_q-2}\left(\operatorname{mod} w_q\right),\\ \bar{e}_q=w_q-1,\ (w_q-1)\cdot T_q \equiv \rho_{w_q-1}\left(\operatorname{mod} w_q\right).\end{cases} \tag{4}$$

The $\bar{e}_q$, value for which condition (3) is satisfied is determined by establishing possible values $\bar{e}_q=\overline{1,w_q-1}$ using the exhaustive search algorithm. The value of the number X in PNS is calculated based on the mathematical expression:

$$X_{PNS}=\left(\sum_{q=1}^{p+1} x_q \cdot E_q\right) \operatorname{mod} T_0. \tag{5}$$

In general, the algorithm for implementing the control method in RCS is shown in Fig. 4.

Therefore, the control of NCS is carried out on the basis of comparing the positional value of the number X_{PNS} with the value of the numerical interval T. The considered method of data control in RCS requires the performance of a positional comparison operation, which slows down the time of implementation of the control process. However, the proposed algorithm of data control (Fig. 4) does not provide for determining additional parameters (zeroing constant, number projections, etc.).

It is advisable to illustrate the operation of the data control method using the example of a single-byte (l=1) CS represented in RCS. For example, RCS is represented by informational w_1=3, w_2=4, w_3=5, w_4=7 and control $w_c=w_5$=11 bases. In the context of a single-byte CS, the considered RCS provides the representation of numbers in the interval $[0, T)$, where $T=\prod_{q=1}^{4} w_q=420.$ The limits of the representation of numbers in RCS are set as follows $[0, T_0)$, where $T_0=T\cdot w_{p+1}=4620$ (Fig. 5).

To illustrate the method of data control in RCS, worth considering are two examples of its application.

Let the correct (X=400<T=420) T=(1||0||0||1||4) number be given in RCS.

1.	Selection of bases (modules) w_q $(q=\overline{1,p+c})$ in RCS 1. A set of $\{w_q\}$ possible bases is determined. 2. The set of bases of RCS is being optimized.
2.	Determining interval values T_q $(q=\overline{1,p+c})$ $T_0=w_1\cdot w_2\cdot\ldots\cdot w_{q-1}\cdot w_q\cdot w_{q+1}\cdot\ldots\cdot w_{p+c-1}\cdot w_{p+c}$, … $T_q=w_1\cdot w_2\cdot\ldots\cdot w_{q-1}\cdot w_{q+1}\cdot\ldots\cdot w_{p+c-1}\cdot w_{p+c}$, … $T_{p+c}=T_0/w_{p+c}=w_1\cdot w_2\cdot\ldots\cdot w_q\cdot\ldots\cdot w_{p+c-1}$.
3.	Determining weights $\bar{e}_q$ $(q=\overline{1,p+c})$ of orthogonal bases E_q $\bar{e}_q=1,\ 1\cdot T_q \equiv \rho_1(\operatorname{mod} w_q)$, … $\bar{e}_q=w_q-1,\ (w_q-1)\cdot T_q \equiv \rho_{w_q-1}(\operatorname{mod} w_q)$.
4.	Determining orthogonal bases E_q in RCS $E_q=\bar{e}_q\cdot T_0/w_q=\bar{e}_q\cdot T_q=1(\operatorname{mod} w_q)$.
5.	Data control $X=(x_1 \,\|\, x_2 \,\|\ldots\|\, x_{p+c})$ in RCS 1. Determining the value of a number in PNS: $X_{PNS}=\left(\sum_{q=1}^{p+1} x_q\cdot E_q\right)\operatorname{mod} T_0$. 2. Comparison of quantities X_{PNS} and $T=\prod_{q=1}^{p} w_q$. 3. If $X_{PNS}<T$, then the number X is correct. 4. If $X_{PNS}\geq T$, then the number X is incorrect (found in one of the residues x_q $(q=\overline{1,p+c})$ of number X).

Fig. 4. Data control algorithm in the residual class system based on the comparison principle

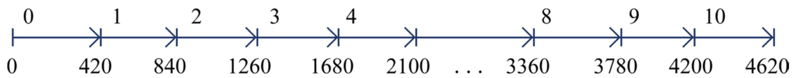

Fig. 5. Bounds of complete numerical intervals for l=1 (w_c=w_5=11)

Example 1. Determine the correctness of the obtained number $\tilde{X}=(\tilde{0}||0||0||1||4)$, distorted by the base w_1=3 $(\tilde{x}_1=0)$.

It is necessary to control this number $\tilde{X}$. To do this, convert the number $\tilde{X}$ to PNS and compare it with the value T=420. The result is the following expression:

$$\tilde{X}_{PNS}=\left(\sum_{q=1}^{p+1} x_q\cdot E_q\right)\bmod T_0=\left(\sum_{q=1}^{5} x_q\cdot E_q\right)\bmod 4{,}620=$$
$$=\left(\begin{array}{l}1{,}540\cdot 0+3{,}465\cdot 0+3{,}696\cdot 0+\\ +2{,}640\cdot 1+2{,}520\cdot 4\end{array}\right)\bmod 4{,}620=$$
$$=12{,}720\bmod(4{,}620)=3{,}480>420.$$

Conclusion. So, it has been established that the value of the number $\tilde{X}$ is incorrect, and there is an error in one of the five residuals of the number.

Example 2. Let the number X=(1||0||0||1||4) be undistorted. In this case:

$$\tilde{X}_{PNS}=\left(\begin{array}{l}1{,}540\cdot 1+3{,}465\cdot 0+\\ +3{,}696\cdot 0+2{,}640\cdot 1+\\ +2{,}520\cdot 4\end{array}\right)\bmod 4{,}620=$$
$$=14{,}260\bmod\left(4{,}620\right)=400<420.$$

Conclusion. The value of the number X is correct; respectively, it is within the numerical interval [0, 420).

From the given examples of application of the proposed method of control (diagnostics) of data in RCS based on comparison, it can be concluded that this method is quite simple to implement in practice with minimal computational costs and without restrictions on the multiplicity of errors.

5. 2. Analysis of the existing data control method based on the use of the projection of a number in the system of residual classes

It is advisable to consider a number of well-known SFs. The results of their proofs serve as the basis for the data control methods represented in RCS [3, 23], and on their basis it is legit to carry out a critical analysis of the considered method. It is worth noting that subsequently only a single error is assumed (in one residue $x_q\left(q=\overline{1,p+1}\right)$, in NCS X=(x_1||x_2||...||x_p||x_{p+1}) represented in RCS). The existing data control method based on the use of the projection of a number in RCS is effective only in the presence of single errors.

The projection of a number X=(x_1||x_2||...||x_{q-1}||x_q||x_{q+1}||... ...||x_p||x_{p+1}) is called such an NCS X_q=(x_1||x_2||...||x_{q-1}||x_{q+1}||... ...||x_p||x_{p+1}) that can represent the number X; when removing one of the bases w_q, the value of X will not change.

SF 2. Let for RCS with ordered RCS bases all possible projections X_q=(x_1||x_2||...||x_{q-1}||x_{q+1}||...||x_p||x_{p+1}), $X_q\left(q=\overline{1,p+1}\right)$ numbers X=(x_1||x_2||...||x_{q-1}||x_q||x_{q+1}||...||x_p||x_{p+1}) coincide, then X is a correct number.

Proof of SF 2. It is worth assuming that the number X=(x_1||x_2||...||x_{q-1}||x_q||x_{q+1}||...||x_p||x_{p+1}) is incorrect due to the distorted residual x_q modulo w_q. In the case of replacing the distorted residual $\tilde{x}_q$ in the number X with the correct x_q, the correct number $\tilde{X}=\left(x_1\,||\,x_2\,||\ldots||\,x_{q-1}\,||\,\tilde{x}_q\,||\,x_{q+1}\,||\ldots||\,x_p\,||\,x_{p+1}\right)$ is obtained. Then, in accordance with the equality of projections, we have $\tilde{X}_1=\tilde{X}_2=\ldots=\tilde{X}_q=\ldots=\tilde{X}_p=\tilde{X}_{p+1}$. On the other hand, X_q=(x_1||x_2||...||x_{q-1}||x_{q+1}||...||x_p||x_{p+1}) and at the same time:

$$\tilde{X}_q=\left(x_1\,||\,x_2\,||\ldots||\,x_{q-1}\,||\,x_{q+1}\,||\ldots||\,x_p\,||\,x_{p+1}\right),$$

i. e. $X_q=\tilde{X}_q$. In this case, the following relation must hold $X=X_1=\tilde{X}_1=X_2=\tilde{X}_2=\ldots=X_p=\tilde{X}_p=X_{p+1}=\tilde{X}_{p+1}$. However, according to the condition of SF 2, the projection X_g ($q\neq g$) of the number X differs from the projection X_q by the value of the residual x_q in the base w_q. As a result, $X_q\neq X_g$, which contradicts the condition of the incorrectness of the number X [23].

SF 3. Under the condition of single errors, if the projection X_q of the number X satisfies the conditions $X_q\geq T_0/w_{p+1}$, then it is considered that the residual x_q of the number X modulo w_q is not distorted [25].

The existing method of data control and diagnostics, based on the use of number projection in RCS, is effective under the condition of single errors [23]. This circumstance limits the application of this method in real CSs, where the probability of multiple errors is quite high. At the same time, the process of determining number projections requires the involvement of additional computational paths of RCS, which leads to a decrease in the speed of the control process and an increase in productive computational costs.

5. 3. Improving the method of data control and diagnostics based on the use of number projection in the residual class system

Control and diagnostics of NCS in RCS are also carried out sequentially in two stages:

Stage I. The values of partial orthogonal bases E_{qg} are determined for each partial set of p bases of the full RCS. Partial sets of working bases are formed by sequentially removing one of the modules (basis) from the full $w_1\div w_{p+1}$ RCS. In this case, the value of the q-th orthogonal basis for the g-th partial set of RCS modules $\{w_q\}$ $\left(q=\overline{1,p+1}\right)$, ($q\neq g$) is denoted as E_{qg}. The code numbers in the gth such sets of RCS bases belong to the set $\{X_q\}$ of possible projections of the number X. Thus, g is the number of partial working bases of RCS, which is given by the (p+1)-th bases $\left(g=\overline{1,p}\right)$; and q is the number of possible projections $\tilde{X}_q$ of the number X, represented in RCS $\left(q=\overline{1,p+1}\right)$.

In this case, if the complete set of bases w_1||w_2||... ...||w_{q-1}||w_q||w_{q+1}||...||w_p||w_{p+1} in RCS is given, then the scheme of formation of g $\left(g=\overline{1,p+1}\right)$ sets from $q\left(q=\overline{1,p}\right)$ partial working bases is represented by the expression:

$$\left\{\begin{array}{l}w_1^{(1)}=w_2,\\ w_2^{(1)}=w_3,\\ \quad\ldots\\ w_g^{(1)}=w_{g+1},\\ w_p^{(1)}=w_{p+1};\end{array}\right.\ \ldots\ \left\{\begin{array}{l}w_1^{(p)}=w_1,\\ w_2^{(p)}=w_2,\\ \quad\ldots\\ w_g^{(p)}=w_g,\\ w_p^{(p)}=w_{p+1};\end{array}\right.\ \ldots\ \left\{\begin{array}{l}w_1^{(p+1)}=w_1,\\ w_2^{(p+1)}=w_2,\\ \quad\ldots\\ w_g^{(p+1)}=w_g,\\ w_p^{(p+1)}=w_{p+1}.\end{array}\right. \quad (6)$$

Specific sets of partial bases of RCS are given in Table 1.

Stage II. The correctness of each residual $x_q\left(q=\overline{1,p+1}\right)$ of the number $\tilde{X}$, being checked is sequentially determined. For this purpose, all projections $\tilde{X}_q$ of the number $\tilde{X}$. are calculated. After that, all $\tilde{X}_{g\,PNS}$, values are determined using formula (5) where:

$$T_g = T_0 / w_g = \prod_{\substack{c=1,\\ c\neq g.}}^{p+1} w_c,\ (T_{p+1}=T).$$

The general formula for calculating the projections $\tilde{X}_{g\ PNS}$ of a number $\tilde{X}$ in RCS is defined as:

$$\tilde{X}_{g\ PNS} = \left(\sum_{\substack{q=1\\ g=1,p+1}}^{p} x_q \cdot E_{qg} \right) \bmod T_g =$$
$$= \left(x_1 \cdot E_{1g} + x_2 \cdot E_{2g} + \ldots + x_p \cdot E_{pg} \right) \bmod T_g. \quad (7)$$

Table 1

A set of partial working bases of RCS

q / g	1	2	3	…	$p-2$	$p-1$	p	T_g
1	w_2	w_3	w_4	…	w_{p-1}	w_p	w_{p+1}	T_1
2	w_1	w_3	w_4	…	w_{p-1}	w_p	w_{p+1}	T_2
	…			…		…		
p	w_1	w_2	w_3	…	w_{p-2}	w_{p-1}	w_{p+1}	T_p
$p+1$	w_1	w_2	w_3	…	w_{p-2}	w_{p-1}	w_p	T_{p+1}

Then the $(p+1)$th comparison of values $\tilde{X}_{q\ PNS}$ and $T=T_0/w_{p+1}$ is performed. In the case when some projections $\tilde{X}_q$ do not belong to the numerical interval $[0,\ T)$ (i. e., $\tilde{X}_q \geq T$), which contains correct numbers, then the corresponding residues of the number X are considered to be undistorted. Errors can occur only among $[(p+1)-c]$ residues of the number $\tilde{X}$. Determining the actually distorted residues requires separate additional studies. Fig. 6 shows an algorithm for implementing the improved method of data control and diagnostics in RCS based on number projections.

1. Representation of an ordered $(w_q<w_{q+1})$ RCS of a set $\{w_q\}$ $(q=\overline{1,p})$ of bases (modules) $w_1 \| w_2 \| \ldots \| w_{q-1} \| w_q \| w_{q+1} \| \ldots \| w_p \| w_{p+1}$, where the greatest common divisor is $(w_q, w_g=1)$, $q \neq g$.
2. From the bases $\{w_q\}$ of RCS, determine the partial values of $T_q = T_0 / w_q = \prod_{\substack{c=1,\\ c\neq q.}}^{p+1} w_c (T_{p+1}=T)$ in the form:
   $T_1 = w_1 \cdot w_2 \cdot \ldots \cdot w_{q-1} \cdot w_q \cdot w_{q+1} \cdot \ldots \cdot w_p \cdot w_{p+1}$,
   …
   $T_q = w_1 \cdot w_2 \cdot \ldots \cdot w_{q-1} \cdot w_{q+1} \cdot \ldots \cdot w_p \cdot w_{p+1}$,
   …
   $T_{p+1} = T = w_1 \cdot w_2 \cdot \ldots \cdot w_{q-1} \cdot w_q \cdot w_{q+1} \cdot \ldots \cdot w_{p-1}$.
3. Determine orthogonal E_q $(q=\overline{1,p+1})$ bases for a given RCS, in accordance with relation $E_q = \overline{e}_q \cdot T_q = 1 (\bmod w_q)$.
4. Determine the value of T_q for which the relation $T=T_0/w_{p+1}=T_{p+1}<X<T_q$. holds. Detect non-distorted residues x_z $(z=\overline{1,q})$ of number X modulo w_z.

Fig. 6. Algorithm for implementing the improved method for controlling non-positional code structures based on number projections in the residual class system

Worth investigating is the method of data control and diagnostics in RCS, intended for simultaneous verification of the correctness (integrity) of a group (two or more) of the residues of the number $X=(x_1||x_2||\ldots||x_{q-1}||x_q||x_{q+1}||\ldots||x_p||x_{p+1})$. This is based on the result obtained from proving the following SF.

SF 4. Let in an ordered RCS with p informational and w_{p+1} control bases NCS $X=(x_1||x_2||\ldots||x_{q-1}||x_q||x_{q+1}||\ldots||x_p||x_{p+1})$ satisfy the condition $T_{p+1}<X<T_q$, where:

$$T_{p+1} = \prod_{q=1}^{p+1} w_{q+1}.$$

In this case, the residues $x_1, x_2, \ldots, x_q$ are undistorted, provided that there is a single error (in one residue of the number X) [23].

To prove the correctness of the residue x_q of the number X, it is advisable to use the method from the opposite. The assumption is accepted about the distortion of the residual x_q. Then the number X will be incorrect, accordingly $X \geq T$. The undistorted residual is defined as $\overline{x}_q$, a correct number since $X<T$, while assuming that the error ΔX is additive [23]. Then the number X can be represented as:

$$\tilde{X} = \left(x_1 || \ldots || \overline{x}_q || \ldots || x_p || x_{p+1} \right) +$$
$$+ \left(0 || \ldots || \Delta x_q || \ldots || 0 || 0 \right) =$$
$$= \left(x_1 || \ldots || x_q || \ldots || x_p || x_{p+1} \right),$$

where $\ddot{A}x_q = \left(x_q + \overline{x}_q \right) \bmod w_q$. Due to the additivity of the error, the quantitative value of the distorted number can be estimated [22]:

$$\tilde{X} = X + \left[\left(\overline{x}_q - x_q \right) \bmod w_q \right] \cdot \overline{e}_q \cdot T_q. \quad (8)$$

The largest permissible value of the quantity $\left\{ \left[\left(\overline{x}_q - x_q \right) \bmod w_q \right] \cdot \overline{e}_q \cdot T_q \right\} \bmod T_0$ of expression (8) is determined as follows:

$$\max \left(\left\{ \left[\left(\overline{x}_q - x_q \right) \bmod w_q \right] \cdot e_q T_q \right\} \bmod T_0 \right) =$$
$$= \left[\left(w_q - 1 \right) / w_q \right] \cdot T_0.$$

In fact, the expression $\left(\overline{x}_q - x_q \right) \bmod w_q$ can only take values in the range from 0 to w_{q-1}. In this case, $\max \left[\left(\overline{x}_q - x_q \right) \bmod w_q \right] = w_q - 1$, and $T_q=T_0/w_q$. The relation (8) will be represented as:

$$\tilde{X} = X + \left[\left(w_q - 1 \right) / w_q \right] \cdot T_0. \quad (9)$$

In accordance with the condition of SF 4, $X<T_q$. In this case, expression (9) can be written in the form $\tilde{X} < T_0 / w_q + \left[\left(w_q - 1 \right) / w_q \right] \cdot T_0$, or $\tilde{X} < T_0 \cdot \left[1 / w_q + \left(w_q - 1 \right) / w_q \right]$, therefore reduces to the inequality:

$$\tilde{X} < T_0. \quad (10)$$

The number $\tilde{X}$ will be correct $\left(\tilde{X} < T \right)$, provided that when adding the value Δx_q, this number will exceed the value T_0. From inequality (10) it is clear that this cannot be implemented by correcting the incorrect residual x_q, which does not correspond to the assumption that the residual x_q of the number X is distorted. Thus, the residual x_q is not distorted, and the number X is correct. Since $X<T_q$, then even more so $X<T_q<T_{q-1}<T_2<T_1$, from which the correctness of the residuals $x_1, x_2, \ldots, x_{q-1}$ follows. Note that when $q=p$, i. e., $T_{p+1}<X<T_p$, the incorrect residual will be x_{p+1}. The procedure for data control and diagnostics in RCS is implemented on the basis of the improved method, which is illustrated in Fig. 6.

Example 3. It is advisable to consider an example of implementing the process and data diagnostics based on the application of the improved data control method. Suppose it is necessary to control and diagnose data represented in the form of NCS $X=(1||1||0||1||1)$ in RCS.

Stage I. The correctness of the number X is checked. The number of values of X in PNS:

$$X_{PNS}=\left(\sum_{q=1}^{p+1} x_q \cdot E_q\right) \bmod T_0 =$$
$$=\begin{pmatrix} 1\cdot 1{,}540+1\cdot 3{,}465+ \\ +0\cdot 3{,}696+1\cdot 2{,}640+1\cdot 2{,}520 \end{pmatrix} \bmod 4{,}620=$$
$$=10{,}165 \bmod 4{,}620=925.$$

The operation of comparing the value of X_{PNS} and the value of T is performed: $X_{PNS}=925>T=420$.

Conclusion: the number X being checked is incorrect, i. e., $X=\tilde{X}$, one of the residuals $x_q\left(q=\overline{1,5}\right)$ of the numbers $\tilde{X}=\left(1||1||0||1||1\right)$ is distorted.

Stage II. Data $\tilde{X}=\left(1||1||0||1||1\right)$, diagnostics is performed, i.e., the residuals x_q are determined, in which distortions are possible. The $\tilde{X}_{PNS}=925$ location in the following numerical series is determined:

$$T_5=420<T_4=660<T_3=924<T_2=1{,}155<T_1=1{,}540.$$

It is evident:

$$T_3=924<\tilde{X}_{PNS}=925<T_2=1{,}155<T_1=1{,}540.$$

The correct residuals of the number $\tilde{X}$. are determined. Accordingly, the residuals $x_1=1$ and $x_2=1$ of the number $\tilde{X}=\left(1||1||0||1||1\right)$ are reliably undistorted (correct).

Conclusion: the error that led to the distortion of the correct number is possible only in one of the residuals x_3, x_4 and x_5 of the incorrect number $\tilde{X}$.

To check and compare the reliability of the result obtained by the improved method of control and diagnostics based on projections, it is advisable to carry out the procedure of control and diagnostics of NCS $\tilde{X}=\left(1||1||0||1||1\right)$ by existing method and compare the results.

Stage 1. All possible projections $\tilde{X}_g\left(g=\overline{1,5}\right)$ of numbers $\tilde{X}$: are compiled:

$$\tilde{X}_1=\left(1||0||1||1\right),\ \tilde{X}_2=\left(1||0||1||1\right),$$

$$\tilde{X}_3=\left(1||1||1||1\right),$$

$$\tilde{X}_4=\left(1||1||0||1\right),$$

$$\tilde{X}_5=\left(1||1||0||1\right).$$

Stage II. The numerical value of the projection of the number $\tilde{X}$: is limited:

$$\tilde{X}_{1\,PNS}=\left(\sum_{q=1}^{4} x_q \cdot E_{q1}\right) \bmod T_1=$$
$$=\left(1\cdot 385+0\cdot 616+1\cdot 1{,}100+1\cdot 980\right) \bmod 1{,}540=$$
$$=925>420;$$

$$\tilde{X}_{2\,PNS}=\left(\sum_{q=1}^{4} x_q \cdot E_{q2}\right) \bmod T_2=$$
$$=\left(1\cdot 385+0\cdot 213+1\cdot 330+1\cdot 210\right) \bmod 1{,}155=$$
$$=925>420;$$

$$\tilde{X}_{3\,PNS}=\left(\sum_{q=1}^{4} x_q \cdot E_{q3}\right) \bmod T_3=$$
$$=\left(1\cdot 616+1\cdot 693+1\cdot 792+1\cdot 672\right) \bmod 924=$$
$$=1<420;$$

$$\tilde{X}_{4\,PNS}=\left(\sum_{q=1}^{4} x_q \cdot E_{q4}\right) \bmod T_4=$$
$$=\left(1\cdot 220+1\cdot 165+0\cdot 336+1\cdot 540\right) \bmod 660=$$
$$=265<420;$$

$$\tilde{X}_{5\,PNS}=\left(\sum_{q=1}^{4} x_q \cdot E_{q5}\right) \bmod T_5=$$
$$=\left(1\cdot 280+1\cdot 105+0\cdot 336+1\cdot 120\right) \bmod 420=$$
$$=85<420.$$

Conclusion. From our calculations it is clear that the residuals x_1 and x_2 of the number $\tilde{X}$ are correct, and the residuals x_3, x_4 and x_5 can be distorted. This confirms the reliability of the diagnostics of the non-positional code structure $\tilde{X}$ based on the improved method.

The results obtained using the two control and diagnostic methods confirm their reliability as they demonstrate a significant level of convergence. It is especially important to note that the improved method provides a significant reduction in the number of calculations. Due to the certainty of the correctness of the group of residues at the same time the efficiency of the data control process is significantly increased. This makes the improved method more attractive for practical application under conditions of limited computing resources and high requirements for data processing efficiency.

5. 4. Implementation of the data correction process in the residual class system

A distinctive feature of RCS is the significant manifestation of the primary information redundancy in NCS only if the NCS has additional secondary $Q(l)$ information redundancy, due to the use of control bases [26]. It can be demonstrated that the noise-resistant code in RCS is able to detect a larger number of errors of higher multiplicity, provided by the general theory of coding, i.e., the value of $d_{\min}$ [3, 27]. Let the minimum code value for RCS be determined by the value of $d_{\min}$. It is worth assuming that there are bases in RCS, the number of which is $ll\geq d_{\min}$ while:

$$Q(l)=\prod_{g=1}^{l} w_{z_g}<R=T_0/T.$$

Then the error vector $\Delta X=\tilde{X}-X$ must contain at least $(r-l)$ zero components. The vector ΔX is expedient to represent in the form $\Delta X=\left(0||0||\ldots||\Delta x_{z_1}||\ldots||0||\Delta x_{z_l}||\ldots||0\right)$.

In PNS, the value of ΔX is defined as:

$$\Delta X = E_{z_1} \cdot x_{z_1} + ... + E_{z_l} \cdot x_{z_l}.$$

Given that $E_{z_l} = \overline{e}_{z_l} \cdot T_0 / w_{z_l}$, where $\overline{e}_{z_l}$ is the weight of the l-orthogonal basis, ΔX will take the following form:

$$\Delta X = \frac{\overline{e}_{z_1} \cdot T_0}{w_{z_1}} x_{z_1} + ... + \frac{\overline{e}_{z_l} \cdot T_0}{w_{z_l}} x_{z_l} = R \cdot \Delta R \cdot z. \quad (11)$$

In [23] it is shown that according to the results of analysis of detection (11) it can be shown that in some cases in RCS there is an inequality:

$$\tilde{X} = X + \Delta X \geq T. \quad (12)$$

Inequality (12) shows that the sum of any numbers X and the number of the corresponding error vector ΔX cannot belong to the set T, which indicates the possibility of detecting such an error. It should be noted that even in cases where $Q(l)>R$, there are errors ΔX that correspond to inequality (12). This possibility is provided by secondary information redundancy. The features of data representation in RCS allow us to detect errors in NCS [28]. In some cases, it is even possible to determine the place of occurrence of the error (diagnose the error) and send it, limiting ourselves to using one control basis. This cannot be implemented using existing methods of control, diagnostics, and error correction in PNS, for example, when controlling by modulus [29]. In RCS, error correction can be executed for the value $d_{\min}=2$ either by the projection method or by the concept of an alternative set of numbers. The study considers the method of number projections.

Let the error $\tilde{x}_q = (x_q + \Delta x_q) \bmod w_q$ in an incorrect number $\tilde{X} = (x_1 || x_2 || ... || x_{q-1} || \tilde{x}_q || x_{q+1} || ... || x_p || x_{p+1})$, $\tilde{X} \geq T$ be reliably contained in the residual x_q modulo w_q. Let's consider the ratio by which we can correct the error in this residual $\tilde{x}_q$. It is evident that:

$$\tilde{X} = (X + \Delta X) \bmod T_0. \quad (13)$$

Given that the error can be represented as $\Delta X=(0||0||...$ $...||0||\Delta x_q||0||...||0||0)$, then the correct ($X<T$) number X can be defined in the following form:

$$X = (\tilde{X} - \Delta X) \bmod T_0 =$$
$$= \begin{bmatrix} (x_1 || x_2 || ... || \tilde{x}_q || ... || x_p || x_{p+1}) - \\ -(0 || 0 || ... || \Delta x_q || ... || 0 || 0) \end{bmatrix} \bmod T_0 =$$
$$= \left[(x_1 || x_2 || ... || (\tilde{x}_q - \Delta x_q) \bmod w_q || ... || x_p || x_{p+1}) \right] \bmod T_0.$$

One should perform a quantitative assessment of the value of X. Since the number X is correct, that is, it is in the numerical interval $[0, T)$, then the following inequality must hold:

$$X = (\tilde{X} - \Delta X) \bmod T_0 < T. \quad (14)$$

Taking into account that $\Delta X=\Delta x_q \cdot E_q$, inequality (14) will take the following form:

$$\tilde{X} - E_q \cdot \Delta x_q - r \cdot T_0 < T, \quad (r = 1, 2, 3...),$$

$$\tilde{X} - E_q \cdot \Delta x_q - r \cdot T_0 < T_0 / w_{p+1},$$

$$\tilde{X} - E_q \cdot (\tilde{x}_q - x_q) - r \cdot T_0 < T_0 / w_{p+1},$$

$$\tilde{X} + E_q \cdot (x_q - \tilde{x}_q) - r \cdot T_0 < T_0 / w_{p+1},$$

$$E_q \cdot (x_q - \tilde{x}_q) < T_0 / w_{p+1} - \tilde{X} + r \cdot T_0,$$

$$x_q - \tilde{x}_q < (T_0 / w_{p+1}) / E_q - \tilde{X} / E_q + r \cdot T_0 / E_q,$$

$$x_q < \tilde{x}_q + (T_0 / w_{p+1}) / E_q - \tilde{X} / E_q + r \cdot T_0 / E_q. \quad (15)$$

Taking into account the fact that the orthogonal basis in RCS is represented in the form $E_q = \overline{e}_q \cdot T_0 / w_q$, then expression (15) will take the form:

$$x_q < \tilde{x}_q + (w_q + r \cdot w_q \cdot w_{p+1}) / \overline{e}_q \cdot w_{p+1} - \tilde{X} / E_q,$$

or:

$$x_q < \tilde{x}_q + w_q \cdot (1 + r \cdot w_{p+1}) / \overline{e}_q \cdot w_{p+1} - \tilde{X} / E_q. \quad (16)$$

Since the value of the residual x_q is a natural number, the value of the expression $w_q (1 + r \cdot w_{p+1}) / \overline{e}_q \cdot w_{p+1} - \tilde{X} / E_q$ in inequality (16) must be an integer. Selecting the integer part of this expression allows us to derive a formula for correcting errors in the residual $\tilde{x}_q$ of the number $\tilde{X}$ in the form:

$$x_q = \left(\tilde{x}_q + \left] w_q \cdot (1 + r \cdot w_{p+1}) / \overline{e}_q \cdot w_{p+1} - \tilde{X} / E_q \right[\right) \bmod w_q, \quad (17)$$

where the value $]y[$ is taken equal to the integer part of the number y not exceeding the value of y.

Example 4. It is advisable to consider an example of data error correction in NCS RCS. Let during data transmission or processing the result obtained is represented by NCS of the form $\tilde{X} = (\tilde{0} || 0 || 0 || 2 || 1)$. Since the number $\tilde{X}_{PNS} = 3{,}180 > T = 420$ is incorrect, the last residual x_{p+1} according to the control module w_c is always included in the group of residues being checked. In the example under consideration, this is the residual $x_5=5$ according to the control basis $w_c=w_5=11$ of RCS. First, the value of the residual $\tilde{x}_1 = 0$ is corrected modulo $w_1=3$ according to formula (17). The result is the expression:

$$x_1 = \left(\tilde{x}_1 + \left] \frac{w_1 \cdot (1 + r \cdot w_{p+1})}{\overline{e}_1 \cdot w_{p+1}} - \frac{\tilde{X}}{E_1} \right[\right) \bmod w_1 =$$
$$= \left(0 + \left] \frac{3 \cdot (1 + 1 \cdot 11)}{1 \cdot 11} - \frac{3{,}180}{1{,}540} \right[\right) \bmod 3 =$$
$$= (0 +]3.27 - 2.06[) \bmod 3 =$$
$$= (0 +]1.21[) \bmod 3 = (0 + 1) \bmod 3 = 1 \bmod 3 = 1.$$

The r (r=1, 2, 3...) value is chosen so that the result of the operation in parentheses $]y[$ is a positive number. In this example, r=1. Thus x_1=1.

Next, the residual $\tilde{x}_3 = 0$ is corrected modulo w_3=5:

$$x_3 = \left(\tilde{x}_3 + \left] \frac{w_3 \cdot (1 + r \cdot w_{p+1})}{\overline{e}_3 \cdot w_{p+1}} - \frac{\tilde{X}}{E_3} \right[\right) \bmod w_3 =$$
$$= \left(0 + \left] \frac{5 \cdot (1 + 1 \cdot 11)}{4 \cdot 11} - \frac{3{,}180}{3{,}696} \right[\right) \bmod 5 =$$
$$= (0 +]1.36 - 0.86[) \bmod 5 =$$
$$= (0 +]0.5[) \bmod 5 = (0 + 0) \bmod 5 = 0 \bmod 5 = 0.$$

Thus, we can conclude that the residual $x_3=\tilde{x}_3=0$ is undistorted, which really had to be proven to confirm the validity of the derived mathematical expression (17).

Next, the residual $\tilde{x}_5=1$ is corrected by the control modulo $w_c=w_5=11$ of this RCS:

$$x_5=\left(\tilde{x}_5+\left]\frac{w_5\cdot(1+r\cdot w_{p+1})}{\overline{e}_5\cdot w_{p+1}}-\frac{\tilde{X}}{E_5}\right[\right)\mathrm{mod}\,w_5=$$
$$=\left(1+\left]\frac{11\cdot(1+1\cdot 11)}{6\cdot 11}-\frac{3{,}180}{2{,}520}\right[\right)\mathrm{mod}\,11=$$
$$=(1+]2-1.26[)\mathrm{mod}\,11=$$
$$=(1+]0.74[)\mathrm{mod}\,11=(1+0)\mathrm{mod}\,11=1\,\mathrm{mod}\,11=1.$$

Thus, we can conclude that the residual $x_5=\tilde{x}_5=1$ is undistorted, which actually had to be proven.

In accordance with our calculation results, i. e., $x_1=1$, $x_3=0$ and $x_5=1$, it is worth correcting the distorted result $\tilde{X}=(\tilde{0}||0||0||2||1)$ of data processing in the following way. It is necessary to replace the residuals of the numbers modulo w_1, w_3 and w_5 with the obtained values: $\tilde{x}_1=0\rightarrow x_1=1$; $\tilde{x}_3=0\rightarrow x_3=0$ and $\tilde{x}_5=1\rightarrow x_5=1$. It is evident that the corrected NCS X will take the following form: $\tilde{X}=(\tilde{0}||0||0||2||1)\rightarrow X=(1||0||0||2||1)$. This is confirmed by the fact that $X=(1||0||0||2||1)=X_{PNS}=100<T=420$. In this case, the distorted residual modulo $w_1=3$, i. e.:

$$\tilde{x}_1=(x_1+\Delta x_1)\mathrm{mod}\,w_1=(1+2)\mathrm{mod}\,3=$$
$$=3\,\mathrm{mod}\,3=0\,(\Delta x_1=2).$$

The scheme of the data correction procedure is given in Table 2.

Based on the improved methods of control and diagnostics of data in RCS (Fig. 6), it is advisable to consider an example of correction of NCS in RCS.

Example 5. Suppose that during data processing in RCS a number is obtained in the form $\tilde{X}=(0||0||\tilde{0}||2||2)$. It is necessary to control the number $\tilde{X}$, and, if necessary, conduct diagnostics of NCS, find the error (detect the distorted residual of the number $\tilde{X}$) and eliminate it (correct the incorrect residual). The stages of correcting the specified number are as follows:

I. Control of number $\tilde{X}=(0||0||\tilde{0}||2||2)$.

The operation of finding the value $\tilde{X}$ in PNS is performed:

$$\tilde{X}_{PNS}=\left(\sum_{q=1}^{p+1}x_q\cdot E_q\right)\mathrm{mod}\,T_0=\left(\sum_{q=1}^{5}x_q\cdot E_q\right)\mathrm{mod}\,4{,}620=$$
$$=\begin{pmatrix}0\cdot 1{,}540+0\cdot 3{,}465+\\+0\cdot 3{,}696+2\cdot 2{,}640+2\cdot 2{,}520\end{pmatrix}\mathrm{mod}\,4{,}620=$$
$$=10{,}320\,\mathrm{mod}\,4{,}620=1{,}080>T=420.$$

Conclusion. Since the value $\tilde{X}_{PNS}=1{,}080>T=420$, the number $\tilde{X}$ is incorrect, that is, there is an error in one of the residuals $x_q\,(q=\overline{1,5})$ of the number $\tilde{X}$.

II. Diagnostics of the residuals of the number $\tilde{X}=(0||0||\tilde{0}||2||2)$.

The number $\tilde{X}_{PNS}=1{,}080$ is determined in a series of values $T_g\,(g=\overline{1,5})$ (Table 2):

$$T_5=420<T_4=660<T_3=924<\tilde{X}=$$
$$=1{,}080<T_2=1{,}150<T_1=1{,}540.$$

Since the condition $T_5<\tilde{X}<T_2$, is met, it is concluded that the residuals x_1 and x_2 of the number $\tilde{X}$ are correct (undistorted). An error is possible only in one of the three residuals: x_3, x_4 and x_5.

Conclusions:

– the residuals x_1 and x_2 of the number $\tilde{X}=(0||0||\tilde{0}||2||2)$ are undistorted (correct);

– an error is possible in one of the residuals x_3, x_4 and x_5 of the number $\tilde{X}$.

III. Correction of residuals of the number $\tilde{X}=(0||0||\tilde{0}||2||2)$. As an example, it is advisable to correct the residual $\tilde{x}_3=0$ modulo $w_3=5$ in RCS:

$$x_3=\left(\tilde{x}_3+\left]\frac{w_3\cdot(1+r\cdot w_{p+1})}{\overline{e}_3\cdot w_{p+1}}-\frac{\tilde{X}}{E_3}\right[\right)\mathrm{mod}\,w_3=$$
$$=\left(0+\left]\frac{5\cdot(1+1\cdot 11)}{4\cdot 11}-\frac{1{,}080}{3{,}696}\right[\right)\mathrm{mod}\,5=$$
$$=(0+]1.36-0.29[)\mathrm{mod}\,5=$$
$$=(0+]1.07[)\mathrm{mod}\,5=(0+1)\mathrm{mod}\,5=1\,\mathrm{mod}\,5=1.$$

Thus, the error $\tilde{x}_3=0\rightarrow x_3=1$ in the residual x_3 modulo $w_3=5$ of the number $\tilde{X}$. is corrected. In this case, the corrected NCS X will take the following form: $\tilde{X}=(0||0||\tilde{0}||2||2)\rightarrow X=(0||0||1||2||2)$. The procedure for correcting the number $\tilde{X}=(0||0||\tilde{0}||2||2)$ is schematically given in Table 3.

Verification:

$X=(0||0||1||2||2)=X_{PNS}=156<T=420$.

The option of using the devised methods of operational control and data diagnostics (i.e., error detection) for the implementation of an error correction process in real time has been considered. The method is based on the noise-resistant functions of NCS and the principle of parallel independent processing of the residuals of numbers in RCS.

Table 2

Procedure for correcting data $\tilde{X}=(\tilde{0}||0||0||2||1)$ in RCS

Residual of number $\tilde{X}$	Residuals of number $\tilde{X}$, where errors are likely	Value Δx_q of error	Results of correcting residuals x_q	Result $X=(1\|\|0\|\|0\|\|2\|\|1)$ of RCS correction
$x_1=0$	x_1	$\Delta x_1=2$	$x_q=(\tilde{x}_q-\Delta x_q)\mathrm{mod}\,w_q$, $x_1=(\tilde{x}_1-\Delta x_1)\mathrm{mod}\,w_1=$ $=(0-2)\mathrm{mod}\,3=1$	$x_1=1$
$x_2=0$	–	–	–	$x_2=0$
$x_3=0$	x_3	$\Delta x_3=0$	–	$x_3=0$
$x_4=2$	–	–	–	$x_4=2$
$x_5=1$	x_5	$\Delta x_5=0$	–	$x_5=1$

Table 3

Procedure for correcting data $\tilde{X}=\left(0\|0\|\tilde{0}\|2\|2\right)$ in RCS

Residual of number $\tilde{X}$	Residuals of number $\tilde{X}$, where errors are likely	Value Δx_q of error	Results of correcting residuals x_q	Result X=(0‖0‖1‖2‖2) of correction in RCS
x_1=0	–	–	–	x_1=0
x_2=0	–	–	–	x_2=0
x_3=0	x_3	Δx_3=4	$x_q=\left(\tilde{x}_q-\Delta x_q\right)\bmod w_q$, $x_3=\left(\tilde{x}_3-\Delta x_3\right)\bmod w_3=$ $=(0-4)\bmod 5=1$	x_3=1
x_4=2	x_4	Δx_4=0	–	x_4=2
x_5=2	x_5	Δx_5=0	–	x_5=2

6. Discussion of results of investigating the process of error control and correction in non-positional code structures

Based on our analysis of the properties of NCS, namely arithmetic, the suitability of using RCS codes for fast and reliable data processing has been proven. The structure of codes in NCS includes specialized control bases that ensure verification of the integrity of information. The control bases of NCS play the role of protective mechanisms designed to detect errors. At the same time, the residuals, by means of which numbers are represented in NCS according to the information and control bases, function simultaneously and autonomously in the data processing process. This circumstance is due to the modularity of the structure of CS functioning in NCS (Fig. 1), due to which data processing in NCS is carried out in parallel in time and independently (each residue according to its own base).

Control of calculation results in RCS can be implemented either by step-by-step verification, or upon completion of all calculations, since the error that occurred in some residual x_q of the number $X=(x_1\|x_2\|...\|x_{q-1}\|x_q\|x_{q+1}\|...\|x_p\|x_{p+1})$,does not spread to other residuals x_g $\left(g=\overline{1,p+1}\right)$, $(q\neq g)$ of numbers X (does not spread to neighboring data processing paths of CS). Therefore, based on the complete arithmetic of NCS, the applied bases (modules) are an integrated part of the general system of bases (modules) of RCS. The numerical values of NCS for all both the main and control digits (bases) are directly involved in all operations. At the same time, the processing of the main and additional (control) digits is carried out absolutely identically, without any difference. As a result, RCS allows calculations to be performed without constant verification of intermediate results. The intervals between checks are set dynamically, based on an assessment of the probability of errors. The correctness of the operations at each stage of the calculations can be confirmed by checking the final result. It should be noted that only one control basis is an effective means of detecting both any single error and most double errors.

Based on the consideration and proof of scientific fact 1 and based on the results of mathematical relations (2) to (5), a method of data control in RCS has been devised, which is based on the principle of comparison. The use of this control method is characterized by a minimum number of elementary operations and control modules of RCS, which leads to the minimization of computational costs and the absence of restrictions on the multiplicity of errors. This is in contrast to [18], which considers a scheme for detecting and correcting only single errors, and the possibility of localizing multiple errors after and/or during calculation/transmission under the condition of redundancy of check modules. Accordingly, in [18], positive results of the process of control and error correction for a redundant system of residual classes (Redundant Residue Number System) were obtained. That is, the achievement of these results is accompanied by an increase in the number of computational resources required to increase the number of modules (implementation of redundancy). This should be taken into account when designing real systems, where the cost of hardware and power consumption are critical factors.

Therefore, the key advantage of the devised control method is the ability to detect multiple errors without a significant increase in computational complexity, hardware costs, and power consumption. This ensures more efficient use of system resources. This opens up prospects for the application of the devised method in real-time CSs. The practical implementation of the proposed method using examples 1 and 2 has demonstrated its functionality and efficiency.

On the basis of scientific facts 2 and 3, a critical analysis of the existing data control method based on the projection of a number in RCS was conducted. The results of our analysis became the basis for improving the data control method in RCS in order to eliminate the identified shortcomings. The essence of the improvement is to use orthogonal bases E_{qg} of partial sets of bases (modules). Orthogonal bases E_{qg} are formed from the complete system of bases $\{w_q\}$, $\left(q=\overline{1,p+1}\right)$, and their use makes it possible to organize the process of parallel processing of projections $X_q=(x_1\|x_2\|...\|x_{q-1}\|x_{q+1}\|...$ $...\|x_p\|x_{p+1})$ of the number $X=(x_1\|x_2\|...\|x_{q-1}\|x_q\|x_{q+1}\|...$ $...\|x_p\|x_{p+1})$ of NCS in RCS. This arrangement reduces the time for determining the projections of the number and accordingly improves the efficiency of data control. The results of our study significantly complement work [23], which considered a method for diagnosing errors in data represented by a non-positional code using projections onto orthogonal bases. However, calculating number projections within the framework of the method proposed in [23] involves the involvement of additional computational paths of RCS, which negatively affects the speed of the data control process.

In addition to the efficiency of data control and diagnostics (error detection), an important characteristic of any CS is the correction of detected errors. To implement the error correction process, it is necessary that the applied code structure has certain corrective functions [30]. This is achieved by introducing a certain information redundancy into the basic code structure. In NCS in RCS, this is implemented by introducing additional control ones into the information bases. Based on the theoretical principles of noise-resistant data coding (11) to (17), a detailed study of corrective capabilities was conducted. Based on these properties, a procedure for varying (changing) the corrective capabilities of NCS in the dynamics of the computational process was implemented.

The proposed method of error correction in NCS makes it possible, due to parallel error correction, to increase by 2 times the speed of data correction when two residues of RCS are distorted. That is, the method is able to localize and correct double errors. At the same time, with an increase in the bit grid of the operands being processed, the efficiency of the application of the considered error correction process increases. This contrasts with [26], where an error correction scheme is proposed based on residue number arithmetic, which is designed to correct only a single soft error.

The main limitation of our study is the insignificant time costs for the operations of converting the controlled number X from RCS to PSN for comparing the numbers X in PSN with the value of the time interval T. But this limitation is not significant since such elementary operations of CS operating in RCS are performed in one cycle.

Further prospect of our research is to design an effective, in terms of reliability, system for detecting and correcting errors in RTCS based on the non-positional number system in RCS. The presence of effective (operational and reliable) methods and means of control, diagnostics, and correction of data in RCS could provide further impetus to this promising number system since the absence of such methods restrains the broad potential opportunities for highly reliable and fast data processing, which are inherent in the properties of RCS.

7. Conclusions

1. The devised method is quite simple to implement in practice with minimal computational costs. When using the devised control method, there are no restrictions on the multiplicity of errors (single or multiple). Even when a block of errors occurs, the control method is effective, provided that the bit length of the error does not exceed the value of $\log_2(w_q-1)+1$ within the residual x_q of RCS. The devised method of data control in RCS, which is based on the principle of comparison, is most effective for highly reliable CSs with limited computing resources.

2. Based on our critical analysis of the existing method of data control based on projections, the main shortcomings were identified, which were taken into account when improving the method. It is proven that the existing method of data control in RCS has a significant drawback, namely, low control efficiency and the ability to detect only single errors. The main drawback is due to the significant time costs of finding the projections of the number. The standard procedure for determining number projections requires significant intermediate operations and additional computational resources. This circumstance necessitates the acceleration of the control process in RTCS operating in RCS by minimizing the time for determining number projections.

3. The improvement of the method of control and diagnostics in RCS involves the use of partial orthogonal bases. Their use provides effective parallel processing of projections of numbers. This circumstance allows us to increase the efficiency of data diagnostics in CS. The proposed approach to data control and diagnostics, based on the use of partial orthogonal bases, which provides for synchronous analysis of projections of check numbers, is more efficient in comparison with the existing method. A comparative analysis of examples of the implementation of the existing and improved methods allows us to conclude that the improvement of the method has made it possible to increase the efficiency of data control by 20–30 %. The efficiency of data control depends on the bit length of the operand (number) processed in CS. At the same time, the control system in RCS, based on the improved method, provides high reliability of error detection (distorted numbers). Examples of the implementation of the improved method are given to demonstrate its practical use and the reliability of our data control results (error detection).

4. The process of single error correction in RCS has been improved, which is based on the analysis of the magnitude and localization of the error in the residual of the number and implements parallel error correction of the group of residuals of the controlled number. This makes it possible to increase the efficiency of correcting CS errors in RCS. The improvement of the data correction methodology in RCS is achieved by integrating mechanisms that use natural and artificial information redundancy of NCS. This approach makes it possible to correct single data errors using minimal code information redundancy by introducing additional control bases of RCS.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study, as well as the results reported in this paper.

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Data availability

All data are available, either in numerical or graphical form, in the main text of the manuscript.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

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