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CONSTRUCTION OF A MATHEMATICAL MODEL OF AN IMPACT DEVICE WITH A TWO-ELEMENT STRIKER

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The object of this study is the oscillation process of an impact device with a two-element striker, which makes it possible to increase the efficiency of rock destruction and reduce the recoil on the device body. The task addressed was the construction of a mathematical model that describes the dynamic interaction between the striker elements and the tool while taking into account the resistance of the working medium and impulse loads on the device elements. In the given model, the tool is represented by a rod of variable cross-section, and the striker is represented by two discrete elements with reduced masses. The impact interaction is modeled by the presence of rigid and dissipative links and is described by a system of differential equations with initial and boundary conditions. To solve the initial boundary value problem, a numerical method has been used; the parameters of the method are determined by solving the model problem, which is constructed for a discrete model with three discrete masses. An increase in the co-impact time relative to a device with a solid striker by 1.5...2 times to values of 350...500 μs was established. With a load force of 50 to 500 kN in the time range of 0...1 ms and element speeds of 1...8 m/s, the normal stresses in the tool cross-sections were 200...380 MPa. The combination of discrete and continuous elements in the model made it possible to refine the numerical method, taking into account the essential properties inherent in the impulse interaction of the striker elements with the tool and the transfer of impact energy to the processing environment. The model built can be used in the design of impactors with optimal parameters for assessing the shape and duration of the shock pulse, in mining, construction, and oil production

Keywords: impact device, impact prolongation, co-impact force, discrete-continuous model, boundary value problem

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1. Introduction

Impact devices with a linear drive (hydraulic, electromechanical, etc.) have been widely applied in construction, mining, and oil production technology. The study of processes with pulse characteristics requires the use of mathematical modeling methods. An important task is to increase the efficiency of the impact device by extending the time of impact of the striker with the tool depending on the characteristics of the medium being processed. This is due to the fact that the characteristics of the medium being processed are not constant, that is, they can change during the machining process. Impact devices must include elements of adaptation to variable characteristics of the environment. One of the methods of such adaptation is the use of impactors, in which the characteristics of the striker are selected in accordance with the characteristics of the medium being processed.

The design of impact machines involves the use of mathematical modeling methods due to the short interaction time of the elements of these devices. One of the current tasks in the design of pulse devices is the task of increasing their efficiency while reducing the recoil reaction on the device body. Mathematical models are used to solve the problem of increasing efficiency in interaction with the environment being processed and reducing the recoil reaction on the striker body. Thus, the design of strikers requires the use of mathematical modeling methods, which are based on the construction and study of effective mathematical models. Building the mathematical models of the impulse interaction between the elements of a "striker-tool-environment being processed" system, which ensure an increase in the duration of the shock pulse and its rational form, is necessary in order to design impact devices for effective destruction of rocks. Therefore, research on the construction of mathematical models of an impact device is relevant.

2. Literature review and problem statement

In [1], the interaction of the tool with the working environment is considered. The mathematical model consists of discrete elements connected by elements of rigidity and plasticity. The parameters of low-frequency oscillations have been obtained. The use of only discrete elements does not make it possible to determine the power characteristics of the tool of variable cross-section and evaluate the parameters of interaction between the striker and the tool.

In [2], a dynamic model of the impact-rotation type with two drives is considered. In such models, low and high-frequency oscillations were studied, which were obtained when solving a system of ordinary differential equations by numerical methods. The use of only discrete elements limits the boundaries of the study, does not make it possible to isolate high-frequency oscillations that are characteristic of the impact interaction between the striker and the tool. The problem of increasing the efficiency of the transfer of the shock pulse to the working environment remains important.

In [3], a method of holding the striker is proposed to increase the efficiency of the transfer of the shock pulse to the working environment. The model is built only on discrete elements, which limits its effectiveness.

In [4], Adams software and the energy method are used to optimize parameters. The use of efficient programs can be considered an achievement, but the study of physical processes requires the development of open algorithms and their analysis.

Analysis of the method for improving the efficiency of an impact device with several strikers is carried out in [5]. The efficiency of a two-striker device has been verified experimentally. The use of a two-striker striker makes it possible to increase the degree of energy transfer to the medium being processed by an average of 15 %. Using only experimental data is not sufficient to determine the rational parameters of the pulse process and the factors influencing these parameters. To determine the factors influencing the pulse parameters, it is necessary to use the method of mathematical modeling. In [6], the problem of the influence of the shape of the striker on the parameters of the shock pulse was studied. The shape of the tool was assumed to be simplified, which did not make it possible to determine the distribution of stresses in the tool sections. Experimental studies on the influence of the shape of the striker on the nature of the pulse are reported in [7]. The need to study the factors that influence the nature and efficiency of the pulse transfer to the working medium is confirmed. In [8], an algorithm and a program for determining the rational parameters of the striker are given. The Saint-Venant wave model was chosen to describe the interaction between the striker and the tool. In the presented model, the striker is replaced by a set of cylindrical disks, for which the problem of shock pulse propagation is solved. The numerical method is based on the method of characteristics, which follows from the d'Alembert method. It should be noted that the use of graphical methods does not make it possible to solve the problem of determining the parameters of the nonlinear interaction of the main elements in the "striker-tool-working environment" system.

The listed problems are partially solved in work [9], in which a discrete-continuous model is considered; in it, in the presence of a rod tool, the masses of the striker, and the device body are taken into account when the tool is subjected to an impulse load from the side of the processed medium. The study of such a system was carried out using the finite difference method. The scheme with a striker consisting of two elements is similar under the condition of loading on the external and internal elements during their interaction with the tool, and simulating resistance from the side of the processed medium. In work [10], such a scheme is partially implemented, where a model of a striking device with one striker is considered when an impulse force acts on the striker and simulates the resistance of the working medium using elasticity and plasticity elements.

All this gives grounds to argue that it is advisable to conduct a study aimed at constructing a discrete-continuous mathematical model that describes the dynamic interaction of the striker elements with the tool while taking into account the resistance of the working medium and impulse loads on the device elements.

The general problem is to determine the main parameters when using a striker with two elements under their impulse loads in the presence of variable resistance from the side of the medium being processed. Such a scheme makes it possible to extend the time of impact interaction with the tool. Dividing the striker into two elements on which external forces act made it possible to increase the possibilities of varying the parameters for controlling the period of co-impact and the shape of the pulse transmitted to the medium being processed.

The main parameters are also the characteristics of high and low frequency tool oscillations, the force of interaction, the distribution of force characteristics in variable cross-sections of the tool along the length and over time.

In the model of an impactor with a two-element striker, it is necessary to add one discrete element, the forces acting on the discrete elements, and use a discrete model with three masses as a model problem. Adding an additional discrete element to the model complicates the system of differential equations but makes it possible to use a similar research algorithm and obtain new results.

3. The aim and objectives of the study

The purpose of our study is to construct a discrete-continuous model of an impactor with a two-element striker, which allows for variation of the stiffness and dissipation values of the elements under impulse loading for possible control over impulse parameters. This will make it possible to determine rational parameters of the impactor during design.

To achieve the goal, the following tasks were set:

– to synthesize a calculation scheme: "two consecutive discrete elements-rods of variable cross-section-working medium";

– to state an initial-boundary problem with a wave equation in partial derivatives and two ordinary differential equations with nonlinear stiffness coefficients;

– to determine the approximation of the differential problem by a difference scheme, to state a model problem for determining rational parameters of the scheme, to develop an algorithm for solving the difference problem and its implementation in the Mathcad system (USA);

– to conduct computational experiments and determine the parameters of the influence on the duration of the impact interaction and on the distribution of stresses in variable cross-sections of the tool along the length and over time.

4. The study materials and methods

The object of our study is the process of interaction between sequentially installed elements of the striker and the tool of an impact device under impulse loads and in the presence of the resistance force of the medium being machined. Longitudinal oscillations of the tool under the action of longitudinal impulse forces and given velocities of the striker elements before impact are considered.

The hypothesis of the study is as follows. The pulse-wave process of interaction in the system "two-element striker-tool-processing medium" can be adequately described by the system of ordinary differential equations and a wave equation in partial derivatives, which are related via boundary and nonlinear contact conditions. The parameters of the numerical method of the study are estimated using a model problem. In a combination, this will make it possible to determine the power and dynamic characteristics of the tool, the efficiency of using a two-element striker in relation to pulse prolongation with an increase in the co-impact time, which will affect the efficiency of energy transfer to the processed medium.

The mathematical model consists of two discrete elements and a variable cross-section rod placed in series, which are connected by rigid and dissipative elements. The resistance of the working medium is also simulated by rigid and dissipative links.

The basic assumptions adopted in the work:

a) the cross-section of the rod remains flat under impulse loads;

b) the force of the impact interaction of the discrete elements with each other and the second discrete element with the end of the rod depends on the difference of their displacements according to the power law (similar to the Hertz-Steiermann model).

Basic simplifications:

a) the displacements and deformations of the rod sections depend only on the coordinate x and time t;

b) transverse vibrations of the rod are not taken into account;

c) the action of the pressure of the pneumatic accumulator on the striker elements is described by short-term active forces;

d) the impulse load on the tool is determined by the pre-impact initial velocities of the discrete elements and short-term active forces.

The oscillation equations constitute a system of three differential equations: two ordinary and one partial derivative. The connections between the elements are described by boundary conditions. The initial conditions specify the initial state of the system, that is, the state of the system before the impact interaction. Thus, the initial-boundary-value problem is stated.

To find a solution to the problem, the finite difference method is used in combination with the Euler method. With partial linearization of the boundary condition, the sweep method is used. The parameters of the finite difference method are found using a model problem, which consists of the equations of motion of three discrete elements with elastic and dissipative connections, taking into account the resistance of the working medium.

The implementation of the finite difference method for the initial-boundary-value problem and the Runge-Kutta method for the model problem was performed using the Mathcad software (USA).

Regarding the computational experiments, the following should be noted. The common software contains solutions to the main and model problems. The graphs constructed in the same coordinate system make it possible to compare the results of calculations and choose rational parameters of the numerical method. This approach ensures the stability of the numerical method and the necessary accuracy of solutions on small and large time intervals. The use of the numerical method made it possible to determine the rational parameters of the impact pulse, the distribution of stresses in the tool cross-sections depending on the force loads and pre-impact velocities of the striker elements.

5. Results of investigating the interaction process between the elements of the impact device

5. 1. Construction of the design scheme of the impact device

Fig. 1 shows the structural and design scheme of an adaptive impact device with a striker, which consists of two elements, external and internal.

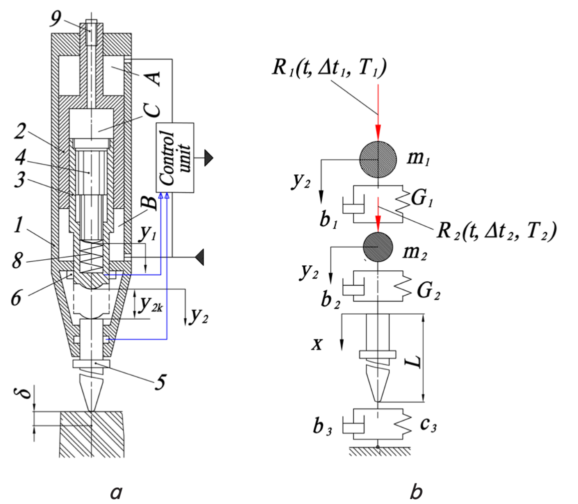

Fig. 1. Impact device: *a* — structural diagram; *b* — calculation diagram; 1 — housing; 2 — element that controls the energy of charging the pneumatic accumulator; 3, 4 — external and internal elements of the striker, respectively; 5 — tool, 6, 7 — sensors for moving the tool and the striker elements, respectively; 8 — elastic element for regulating the interaction of the striker elements; 9 — pneumatic accumulator charger; *A*, *B* — hydraulic chambers for controlling the element (2) and the striker elements (3, 4) when charging or discharging the pneumatic accumulator, respectively; *C* — pneumatic accumulator chamber; m_1, m_2 — masses of the internal and external elements of the striker, respectively; G_1, G_2 — stiffness coefficients; b_1, b_2, b_3 — dissipation coefficients of the connection of the striker elements, the tool, and the rock mass; *L* — length of the tool-rod; — the value of the tool's embedment in the medium being processed, for example, rock

The following notations are introduced: $u(t,x)$ – deviation of the rod cross-section with coordinate x from the neutral position, t – time; $y_1(t)$ – displacement of the center of the first discrete mass element m_1, $y_2(t)$ – displacement of the center of the second mass element m_2; E – modulus of elasticity, ρ – density of the rod material, a – speed of sound in the rod material; $S=S(x)$ – cross-sectional area of the rod.

The striker elements are acted upon by forces R_1 and R_2 from the pressure of the pneumatic accumulator during the impact process. The co-impact force between the elements and the external element with the end of the tool is determined by a power law depending on the difference in displacements. The control element (2), the tool displacement sensors, and the striker elements (6, 7) enable the process of adapting the energy characteristics of the impact device relative to the characteristics of the machined medium.

It should be noted that the calculation scheme describes the process of collision between the internal and external elements of the striker and also the external element of the striker and the end of the tool in the presence of resistance of the machined medium. Calculations are carried out under the initial condition $y_{2k}=0$, i.e., the process is simulated under the condition of initial contact of the external element of the striker with the end of the tool.

In impact devices, the tool is the main element for transferring energy to the machined medium; it has a complex geometric shape, which is determined by the design features and characteristics of the machined medium. To determine the stresses and displacements, it is necessary to take into account the variable cross-sectional area of the tool. In this regard, the wave equation for a rod with a variable cross-section was chosen. The study of the influence of the shape of a two-element striker on the oscillation process after impact is less influential, so the striker elements are replaced by discrete reduced masses.

5. 2. Initial boundary value problem

The initial boundary value problem was considered in the form:

$$\frac{\partial^2 u(t,x)}{\partial t^2}=a^2\left[\frac{1}{S(x)}\cdot\frac{dS(x)}{dx}\frac{\partial u(t,x)}{\partial x}+\frac{\partial^2 u(t,x)}{\partial x^2}\right],$$

$$0<t\le T,\ 0\le x\le L, \tag{1}$$

$$m_1\frac{d^2y_1}{dt^2}=R_1(t,\Delta t_1,T_1)+$$
$$+G_1(\Delta y)(y_2-y_1)+b_1\frac{d}{dt}(y_2-y_1),\ 0<t\le T, \tag{2}$$

$$m_2\frac{d^2y_2}{dt^2}=R_2(t,\Delta t_2,T_2)+$$
$$+G_2(\Delta u)\cdot(u(t,0)-y_2)+b_2\frac{d}{dt}(u(t,0)-y_2)+$$
$$+G_1(\Delta y)(y_1-y_2)+b_1\frac{d}{dt}(y_1-y_2). \tag{3}$$

Boundary conditions for the rod:

$$S(0)E\frac{\partial u}{\partial x}(t,0)=-G_2(\Delta u)(y_2(t)-u(t,0))-$$
$$-b_2\left(\frac{dy_2}{dt}-\frac{\partial u(t,0)}{\partial t}\right), \tag{4}$$

$$S(L)E\frac{\partial u}{\partial x}(t,L)=-c_3u(t,L)-b_3\frac{\partial u(t,L)}{\partial t}. \tag{5}$$

Initial conditions for the rod and discrete elements:

$$u(0,x)=0,\ \frac{\partial u}{\partial t}(0,x)=0, \tag{6}$$

$$y_1(0)=0,\ \frac{dy_1}{dt}(0)=W_1,\ y_2(0)=0,\ \frac{dy_2}{dt}(0)=W_2. \tag{7}$$

The dependence of the rigid coupling coefficients of two discrete masses on the difference in displacements and of the second discrete element with the end of the rod on the difference in displacements was determined from the formulas:

$$G_1(\Delta y)=\begin{cases}c_1\cdot\Delta y^{\alpha},\ \text{if}\ \Delta y\ge 0,\\ c_0,\ \text{if}\ \Delta y<0;\end{cases} \tag{8}$$

$$G_2(\Delta u)=\begin{cases}c_2\cdot\Delta u^{\alpha},\ \text{if}\ \Delta u\ge 0,\\ c_0,\ \text{if}\ \Delta u<0.\end{cases} \tag{9}$$

In formulas (8), (9):

$$\Delta u=y_2(t)-u(t,0),\ \Delta y=y_1(t)-y_2(t),\ 0\le\alpha\le 0.5.$$

Formulas (8), (9) simulate the contact interaction of the second discrete element with the end of the rod and the discrete elements among themselves. This interaction corresponds to the Hertz and Steyerman models [11]. The stiffness c_0 simulates the connection due to friction and is taken as a small value. The values of the dissipative coefficients were taken as constant taking into account the experimental data obtained in [5] and which can be specified when designing under different operating environment conditions without the need to change the model. It should be noted that the contact interaction is also determined by the ratio of the initial pre-impact velocities of the discrete elements.

In equations (2), (3), the terms $R_1(t,\Delta t_1,T_1)$ and $R_2(t,\Delta t_2,T_2)$ simulate the external forces acting on the discrete elements. The action of the external force on the discrete element over a short period of time was considered, for the internal element $[T_1, T_1+\Delta t_1]$:

$$R_1(t,\Delta t_1,T_1)=\begin{cases}P_{01},\ \text{if}\ T_1\le t\le T_1+\Delta t_1,\\ 0,\ \text{if}\ t<T_1\vee t>T_1+\Delta t_1.\end{cases} \tag{10}$$

For the external element, the striker force acts during the time $[T_2, T_2+\Delta t_2]$ and is determined from the formula:

$$R_2(t,\Delta t_2,T_2)=\begin{cases}P_{02},\ \text{if}\ T_2\le t\le T_2+\Delta t_2,\\ 0,\ \text{if}\ t<T_2\vee\ t>T_2+\Delta t_2.\end{cases} \tag{11}$$

External forces are a consequence of the pressure action during the discharge of the pneumatic accumulator and affect the strengthening or weakening of the impact interaction and, thus, make it possible to influence the increase or decrease of the amplitude of oscillations of the tool end.

5. 3. Finite difference method algorithm

The initial-boundary value problem (1) to (9) is approximated by a discrete problem. The justification for the choice of a mixed scheme is the results reported in [10] for a model with a solid striker during computational experiments. The mixed scheme with the weight parameter γ had the form:

$$\frac{u_i^{n+1}-2u_i^n+u_i^{n-1}}{\tau^2}=$$
$$=\gamma a^2\left[\frac{1}{S(x_i)}\cdot\frac{S(x_{i+1})-S(x_{i-1})}{2h}\cdot\frac{u_{i+1}^{n+1}-u_{i-1}^{n+1}}{2h}+\frac{u_{i+1}^{n+1}-2u_i^{n+1}+u_{i-1}^{n+1}}{h^2}\right]+$$
$$+(1-\gamma)a^2\left[\frac{S(x_{i+1})-S(x_{i-1})}{S(x_i)2h}\cdot\frac{u_{i+1}^n-u_{i-1}^n}{2h}+\frac{u_{i+1}^n-2u_i^n+u_{i-1}^n}{h^2}\right], \tag{12}$$

$$i=1,\dots,N-1,\ n=1,\dots,M-1.$$

To simplify the implementation of the algorithm, the approximation of boundary conditions with the first order for h was adopted with partial linearization of nonlinear coefficients:

$$S(0)E\frac{u_1^{n+1}-u_0^{n+1}}{h}=-G_2\left(\Delta u^n\right)\cdot\left(y_2^{n+1}-u_0^{n+1}\right)-$$
$$-b_2\left(\frac{y_2^{n+1}-y_2^n}{\tau}-\frac{u_0^{n+1}-u_0^n}{\tau}\right), \tag{13}$$

$$S(L)E\frac{u_N^{n+1}-u_{N-1}^{n+1}}{h}=-c_3u_N^{n+1}-b_3\frac{u_N^{n+1}-u_N^n}{\tau}. \tag{14}$$

The equations of oscillations of discrete elements are approximated according to the implicit Euler scheme:

$$m_1\frac{y_1^{n+1}-2y_1^n+y_1^{n-1}}{\tau^2}=R_1\left(t_n,\Delta t_1,T_1\right)+$$
$$+G_1\left(\Delta y^n\right)\left(y_2^{n+1}-y_1^{n+1}\right)+b_1\left(\frac{y_2^{n+1}-y_2^n}{\tau}-\frac{y_1^{n+1}-y_1^n}{\tau}\right), \tag{15}$$

$$m_2\frac{y_2^{n+1}-2y_2^n+y_2^{n-1}}{\tau^2}=$$
$$=R_2\left(t_n,\Delta t_2,T_2\right)+G_2\left(\Delta u^n\right)\cdot\left(u_0^{n+1}-y_2^{n+1}\right)+$$
$$+b_2\left(\frac{u_0^{n+1}-u_0^n}{\tau}-\frac{y_2^{n+1}-y_2^n}{\tau}\right)+$$
$$+G_1\left(\Delta y^n\right)\left(y_1^{n+1}-y_2^{n+1}\right)+b_1\left(\frac{y_1^{n+1}-y_1^n}{\tau}-\frac{y_2^{n+1}-y_2^n}{\tau}\right). \tag{16}$$

The initial conditions for the rod and discrete elements are approximated with the first order for τ:

$$u_i^0=0,\ \left(u_i^1-u_i^0\right)\tau^{-1}=0,\ x_i=ih,\ i=1,2,...,N. \tag{17}$$

$$y_2^0=0,\ \left(y_2^1-y_2^0\right)\tau^{-1}=W_2,\ y_1^0=0,\ \left(y_1^1-y_1^0\right)\tau^{-1}=W_1. \tag{18}$$

Here $t_n=n\times\tau$, $\tau=T\times M^{-1}$, $x_i=i\times h$, $h=L\times N^{-1}$ are the parameters of the grid region.

The following algorithm for solving the discrete problem (12) to (18) was adopted.

The system of equations (12) to (16) at each time layer $t_{n+1}=(n+1)\tau$ was solved by the sweep method adapted for a mixed system with boundary conditions [12]. Equations (12) were reduced to the form:

$$A_iu_{i+1}^{n+1}-B_iu_i^{n+1}+C_iu_{i-1}^{n+1}=-F_i. \tag{19}$$

By transforming equations (12), formulas were derived for the coefficients A_i, B_i, C_i and F_i, i=1,2,...,N:

$$A_i=-\frac{a^2\gamma\tau^2}{h^2}\left(\frac{S\left(x_{i+1}\right)-S\left(x_{i-1}\right)}{4S\left(x_i\right)}+1\right),$$

$$B_i=-\left(1+\frac{2a^2\tau^2\gamma}{h^2}\right),$$

$$C_i=\frac{a^2\gamma\tau^2}{h^2}\left(\frac{S\left(x_{i+1}\right)-S\left(x_{i-1}\right)}{4S\left(x_i\right)}-1\right),$$

$$F_i=\left[\begin{array}{l}-2u_i^n+u_i^{n-1}-\left(1-\gamma\right)\frac{a^2\tau^2}{h^2}\times\\ \times\left(\begin{array}{l}\frac{S\left(x_{i+1}\right)-S\left(x_{i-1}\right)}{4S\left(x_i\right)}\left(u_{i+1}^n-u_{i-1}^n\right)+\\ +u_{i+1}^n-2u_i^n+u_{i-1}^n\end{array}\right)\end{array}\right],$$

$$i=1,2,...,N-1.$$

The algorithm of the sweep method taking into account the boundary conditions and equations (15), (16) was used in the following form:

1) The boundary condition (14) and the sweep method formulas $u_i=\alpha_iu_{i+1}+\beta_i$ for $i=N-1$ made it possible to find α_{N-1} and β_{N-1}:

$$\begin{cases}u_{N-1}^{n+1}=u_N^{n+1}\cdot\frac{S(L)E+c_3h+b_3h\tau^{-1}}{S(L)E}-\frac{b_3h}{\tau S(L)E}u_N^n\Rightarrow\\ u_{N-1}^{n+1}=\alpha_{N-1}u_N^{n+1}+\beta_{N-1};\end{cases}$$
$$\Rightarrow\alpha_{N-1}=\frac{S(L)E+c_3h+b_3h\tau^{-1}}{S(L)E};$$

$$\beta_{N-1}=-\frac{b_3h}{\tau S(L)E}u_N^n.$$

The coefficients α_i and β_i were found by the "inverse formulas" of the sweep method:

$$\alpha_{i-1}=\left(b_i-\frac{a_i}{\alpha_i}\right)\left(c_i\right)^{-1},\ \beta_{i-1}=\frac{\beta_i\cdot\left(b_i-c_i\alpha_{i-1}\right)-f_i}{c_i};$$

$$i=N-1,N-2,...,2,1.$$

2) Boundary condition (13) made it possible to find the relationship between u_1^{n+1} and y_2^{n+1} using the system of equations:

$$\begin{cases}S(0)E\frac{u_1^{n+1}-u_0^{n+1}}{h}=-G_2\left(\Delta u^n\right)\left(y_2^{n+1}-u_0^{n+1}\right)-\\ -b_2\left(\frac{y_2^{n+1}-y_2^n}{\tau}-\frac{u_0^{n+1}-u_0^{\ n}}{\tau}\right),\\ u_0^{n+1}=\alpha_0u_1^{n+1}+\beta_0.\end{cases} \tag{20}$$

The u_1^{n+1} values were expressed in terms of the remaining quantities:

$$u_1^{n+1}=$$
$$=\frac{h\left[\left(G_2\left(\Delta u^n\right)+\frac{b_2}{\tau}\right)\right]y_2^{n+1}-\frac{b_2h}{\tau}\left(y_2^n-u_0^n\right)-\beta_0d_0\left(\Delta u^n\right)}{\alpha_0d_0\left(\Delta u^n\right)-S(0)E}, \tag{21}$$

where:

$$d_0\left(\Delta u^n\right)=G_2\left(\Delta u^n\right)h+b_2h\tau^{-1}+S(0)E,$$

$$\Delta u^n=y_2^n-u_0^n.$$

The resulting system of equations relative to u_1^{n+1}, y_1^{n+1}, y_2^{n+1} took the form:

$$\begin{cases} u_1^{n+1} = \dfrac{y_2^{n+1}\left(hG_2\left(\Delta u^n\right)+\dfrac{hb_2}{\tau}\right)-\dfrac{b_2 h}{\tau}\left(u_0^n - y_2^n\right)-\beta_0 d_0}{\alpha_0 d_0 - S(0)E}, \\ m_1 \dfrac{y_1^{n+1}-2y_1^n+y_1^{n-1}}{\tau^2} = R_1\left(t_n,\Delta t_1,T_1\right)+ \\ +G_1\left(\Delta y^n\right)\cdot\left(y_1^{n+1}-y_2^{n+1}\right)+\dfrac{b_1}{\tau}\left(y_2^{n+1}-y_2^n-y_1^{n+1}+y_1^n\right), \\ m_2 \dfrac{y_2^{n+1}-2y_2^n+y_2^{n-1}}{\tau^2} = R_2\left(t_n,\Delta t_2,T_2\right)+ \\ +G_2\left(\Delta u^n\right)\left(u_0^{n+1}-y_2^{n+1}\right)+\dfrac{b_2}{\tau}\left(u_0^{n+1}-u_0^n-y_2^{n+1}+y_2^n\right)+ \\ +G_1\left(\Delta y^n\right)\left(y_1^{n+1}-y_2^{n+1}\right)++\dfrac{b_1}{\tau}\left(y_1^{n+1}-y_1^n-y_2^{n+1}+y_2^n\right). \end{cases} \quad (22)$$

The second equation of system (22) made it possible to express y_2^{n+1} in terms of the remaining variables, including u_N^{n+1}. Taking into account the notations:

$$d_1\left(\Delta y^n,\Delta u^n\right)=\tau^2\left(G_1\left(\Delta y^n\right)+G_2\left(\Delta u^n\right)\right)m_2^{-1}+ +\tau\left(b_1+b_2\right)m_2^{-1},$$

$$d_2\left(\Delta y^n\right)=\tau\left(\tau G_1\left(\Delta y^n\right)+b_1\right)m_1^{-1},$$

$$d_4\left(\Delta u^n\right)=\tau\left(\tau G_2\left(\Delta u^n\right)+b_2\right)m_2^{-1},$$

the following formula is derived:

$$y_2^{n+1}=\left(RY_1+RY_2+RY_3\right)\cdot Rz^{-1},$$

$$d_3\left(\Delta y^n\right)=\tau\left(\tau G_1\left(\Delta y^n\right)+b_1\right)m_2^{-1}, \quad (23)$$

where:

$$RY_1=\frac{\dfrac{\tau^2 R_1\left(t_n,\Delta t_1,T_1\right)}{m_1}+\left(y_1^n-y_2^n\right)\dfrac{b_1\tau}{m_1}+2y_1^n-y_1^{n-1}}{1+d_2}\cdot d_3,$$

$$RY_2=\frac{\alpha_0 d_4\left[\dfrac{hb_2}{\tau}\left(u_0^n-y_2^n\right)-\beta_0 d_0\right]}{\alpha_0 d_0-S(0)E}+\beta_0 d_4,$$

$$Rz=1+d_1-\frac{d_2 d_3}{1+d_2}-\frac{d_4\alpha_0 h\cdot\left(G_2\left(\Delta u^n\right)+\dfrac{b_2}{\tau}\right)}{\alpha_0 d_0-S(0)E},$$

$$RY_3=\frac{\tau^2 R_2\left(t_n,\Delta t_2,T_2\right)}{m_2}+\left(y_2^n-y_1^n\right)\frac{b_1\tau}{m_2}+ +\left(y_2^n-u_0^n\right)\frac{b_2\tau}{m_2}+2y_2^n-y_2^{n-1}.$$

Then we found the value:

$$y_1^{n+1}=\begin{pmatrix} d_2 y_2^{n+1}+2y_1^n-y_1^{n-1}+ \\ +\dfrac{\tau^2 R_1\left(t_n,\Delta t_1,T_1\right)}{m_1}+\dfrac{b_1\tau}{m_1}\left(y_1^n-y_2^n\right)\end{pmatrix}\left(1+d_2\right)^{-1}. \quad (24)$$

Substituting the y_2^{n+1} values into the first formula of system (22) led to equations for u_1^{n+1} and u_0^{n+1}:

$$u_1^{n+1}= =\frac{h\left[\left(G_2\left(\Delta u^n\right)+\dfrac{b_2}{\tau}\right)\right]y_2^{n+1}-\dfrac{hb_2}{\tau}\left(y_2^n-u_0^n\right)-\beta_0 d_0}{\alpha_0 d_0-S(0)E},$$

$$u_0^{n+1}=\alpha_0 u_1^{n+1}+\beta_0, \quad (25)$$

next, we determined:

$$u_i^{n+1}=\left(u_{i-1}^{n+1}-\beta_{i-1}\right)\cdot\alpha_{i-1}^{-1},\ i=2,3,\dots,N. \quad (26)$$

The general algorithm for examining the mathematical model is shown in Fig. 2; the Mathcad system was chosen as a scheme for implementing the research algorithm. In this system, functional autonomous modules were developed that solve special tasks.

List of tasks: $G_1(\Delta y)$, $G_2(\Delta U)$ determine the dependence of stiffness coefficients on the difference in element displacements; *DN*(*N*, *T*, *M*) – connection between modules; *trdag* (*N*, *T*, *M*) – sweep method; *S* (*x*, *X*, *Y*) – tool profile.

The discrete model with three masses is used to compare the results obtained for the discrete-continuous model by the numerical method (the difference method is related to the Euler method) and with the solution for the discrete model with reduced masses, which was obtained by the more accurate Runge-Kutta method.

The system of equations of motion of three discrete elements connected by rigid and dissipative links took the form:

$$m_1\frac{d^2x_1}{dt^2}=R_1\left(t,\Delta t_1,T_1\right)- -P_1\left(x_1,x_2\right)+b_1\left(\frac{dx_2}{dt}-\frac{dx_1}{dt}\right), \quad (27)$$

$$m_2\frac{d^2x_2}{dt^2}=R_2\left(t,\Delta t_2,T_2\right)+P_1\left(x_1,x_2\right)- -P_2\left(x_2,x_3\right)+b_1\left(\frac{dx_1}{dt}-\frac{dx_2}{dt}\right)+b_2\left(\frac{dx_3}{dt}-\frac{dx_2}{dt}\right), \quad (28)$$

$$m_3\frac{d^2x_3}{dt^2}=P_2\left(x_2,x_3\right)- -b_2\left(\frac{dx_2}{dt}-\frac{dx_3}{dt}\right)-c_3x_3-b_3\frac{dx_3}{dt},\ t\in\left[0,T\right]. \quad (29)$$

Initial conditions:

$$\frac{dx_1}{dt}(0)=W_1,\ \frac{dx_2}{dt}(0)=W_2,\ \frac{dx_3}{dt}(0)=W_3, \quad (30)$$

$$x_1(0)=0,\ x_2(0)=0,\ x_3(0)=0. \quad (31)$$

The force of the contact co-impact was determined from the formulas:

$$P_1\left(x_1,x_2\right)=\begin{cases} c_1\left(x_1-x_2\right)^{\alpha+1}, \text{if } x_1\ge x_2, \\ 0, \text{if } x_1<x_2, \end{cases} \quad (32)$$

$$P_2\left(x_2,x_3\right)=\begin{cases} c_2\left(x_2-x_3\right)^{\alpha+1}, \text{if } x_2\ge x_3, \\ 0, \text{if } x_2<x_3. \end{cases} \quad (33)$$

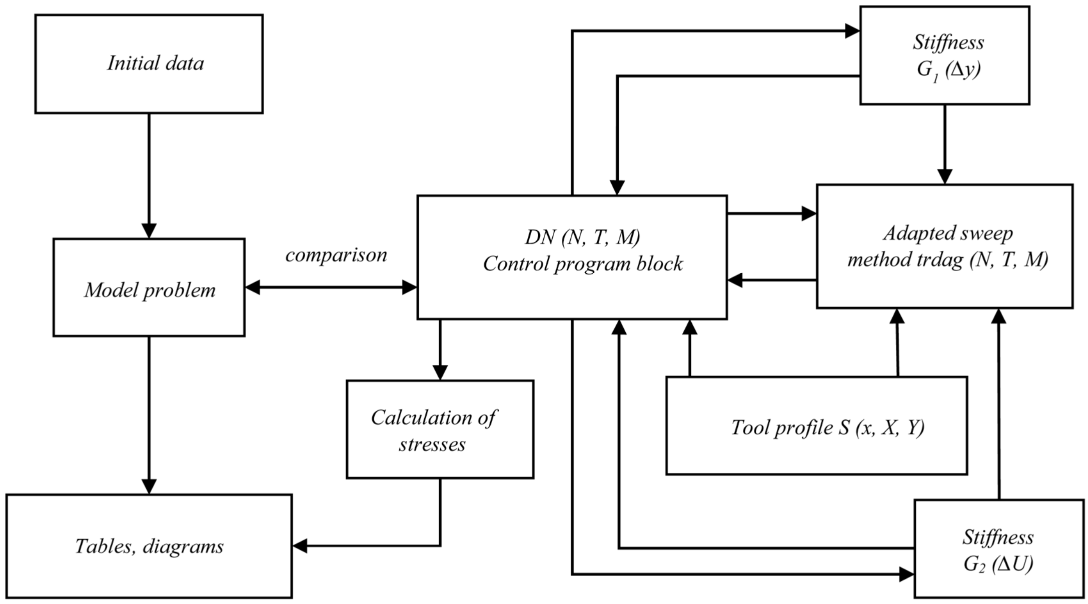

Fig. 2. Mathematical model identification scheme

In equation (29), the mass of the discrete element m_3 was taken equal to the mass of the rod of variable cross-section and was determined from the formula:

$$m_3 = \rho \int_0^L S(x)\,\mathrm{d}x. \qquad (34)$$

Comparison of the solutions to the model problem and problem (12) to (18) was the basis for the correct choice of the algorithm for solving the difference problem and its parameters.

5. 4. Results of computational experiments

During the calculations, attention was mainly paid to determining the parameters of the collision process of discrete elements with the elements of the rod – tool. Due to the short process time, a fine mesh was used. The reliability of results is of great importance. To estimate the parameters of the difference method, which ensure acceptable accuracy of the solution to the problem, a model problem was used. The solution to the model problem (27) to (31) was obtained by the Runge-Kutta method. To estimate the rational parameters of the difference scheme, the derived solution to the model problem was compared with the solution to problem (12) to (18) for large sizes of the rod of constant cross-section, which is shown in Fig. 3, *b*. The main parameters for such solutions:

- $m_1=m_2=5$ kg;
- $m_3=17150$ kg;
- $L=11.2$ m;
- $c_1=2\times10^{11}$ N/m$^{\alpha+1}$;
- $c_2=2\times10^{11}$ N/m$^{\alpha+1}$;
- $c_3=2\times10^{8}$ N/m;
- $c_0=2\times10^{4}$ N/m;
- $W_1=5$ m/s;
- $W_2=7$ m/s;
- $b_1=9\times10^{3}$ Ns/m;
- $b_2=1.2\times10^{3}$ Ns/m;
- $b_3=3\times10^{3}$ Ns/m;
- $S(x)=0.196$ m^2.

The calculation scheme that formed the model problem is shown in Fig. 3, *a*.

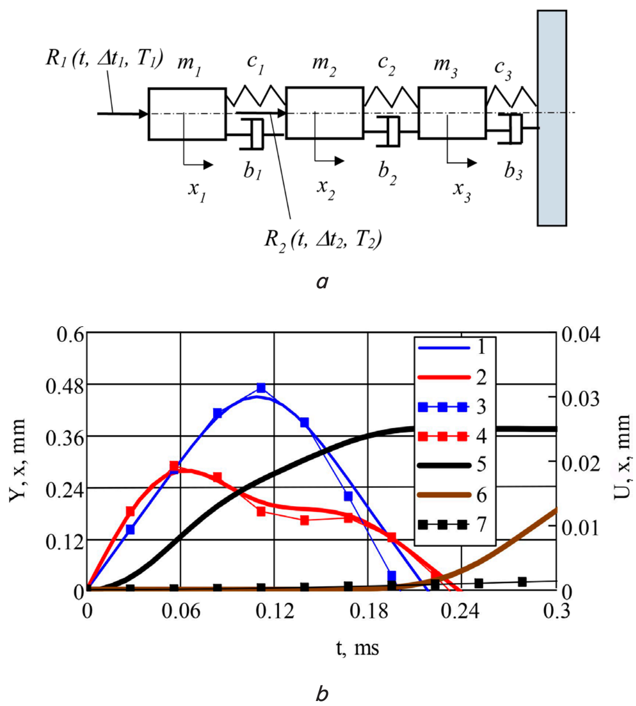

Fig. 3. Model problem and problem solutions: *a* — calculation scheme of the model problem; *b* — displacement of discrete elements: 1 — y_1; 2 — y_2 — solution to the initial-boundary problem for large rod sizes; 3 — x_1; 4 — x_2; 7 — x_3 — solution to the model problem (Runge-Kutta method in Mathcad); 5, 6 — U_0, U_N — displacement of the tool ends

The solution to problem (12) to (18) over a long period of time under the condition of loading due by external force P_{01}, which acted over a short period of time, is shown in Fig. 4. The basic parameters:

- $P_{01}=5\times10^{4}$ N;
- $\Delta t_1=1$ ms;
- $T_1=0$;
- $W_1=W_2=0$;
- $P_{02}=0$.

The plot in Fig. 4 shows the high-frequency oscillations characteristic of an impact against a background of low-frequency oscillations. A test over a short time interval, when only the impulse load $R_1(t,0.001,0)$ is present, is shown in Fig. 5, *a*.

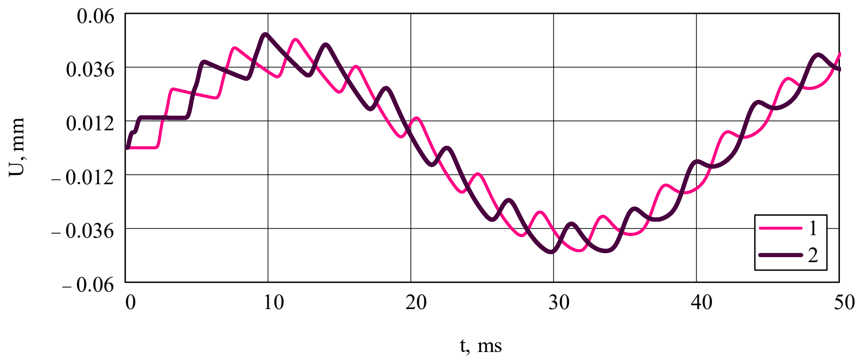

Fig. 4. Oscillation of the ends of the tool: 1 – U_N; 2 – U_0

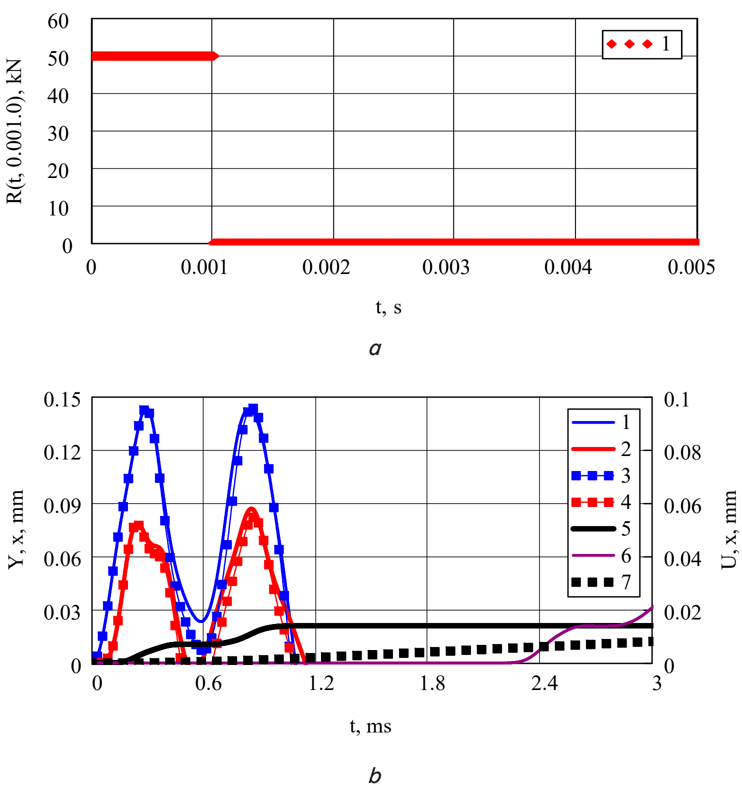

Fig. 5. Comparison of solutions in the presence of only the impulse force $P_{01}=5\times10^4$ *N*, $W_1=W_2=0$: *a* – impulse force in a small time interval; *b* – displacement of discrete elements and ends of the rod: 1 – y_1; 2 – y_2 – solution to the initial-boundary value problem for large rod sizes; 3 – x_1; 4 – x_2; 7 – x_3 – solution to the model problem; 5, 6 – U_0, U_N – displacement of the tool ends

The practical coincidence of the solutions to the model and basic problems confirms the correctness of the chosen algorithm. The solution on a fine grid makes it possible to determine the difference in displacements of the rod ends after the impact (propagation of the wave of displacements of the rod sections). The results for the nonlinear interaction of the second discrete element with the end of the rod of constant cross-section are shown in Fig. 6, 7. The dependences $G_1(y_1,y_2)$ and $G_2(y_2,u_0)$ on the difference in displacements, respectively, between the discrete elements and the second discrete element and the end of the rod were determined from formulas (8), (9). With stiffness $G_1(y_1,y_2)$ and $G_2(y_2,u_0)$ N/m$^{\alpha+1}$ and equal speeds $W_1=W_2=5$ m/s, the effect of double impact was not detected according to the results of calculations. The effect of double impact interaction was obtained with increased values of the coefficients c_1 and c_2 and the masses of discrete elements (Fig. 6). The stress distribution in the cross-sections of the rod along its length also demonstrates the presence of double collision.

It should be noted that Fig. 5 shows the case when two consecutive collisions of the second element of the striker with the tool practically occur. Fig. 6, *b* shows the dependence of stress in the cross-sections close to the ends of the rod on time. The second peak of the stress wave appears more clearly when the initial velocities of the first and second discrete elements differ (Fig. 6, *b*).

Basic parameters:
- $m_1=12$ kg;
- $m_2=5$ kg;
- $L=1.2$ m;
- $c_1=2\times10^{11}$ N/m$^{\alpha+1}$;
- $c_2=2\times10^{11}$ N/m$^{\alpha+1}$;
- $c_3=2\times10^{8}$ N/m;
- $c_0=2\times10^{4}$ N/m;
- $W_1=5$ m/s;
- $W_2=8$ m/s;
- $P_{01}=P_{02}=0$,;
- $S(x)=0.014$ m^2.

Comparison of the time of co-impact of the second discrete element and the rod in the absence and presence of the movement of the first discrete element is shown in Fig. 7. The time of co-impact was determined under the condition $y_2\geq U_0$ and was $t_1=0.18$ ms, $t_2=0.42$ ms, respectively.

Basic parameters:
- $c_1=2\times10^{10}$ N/m$^{\alpha+1}$;
- $c_2=8\times10^{11}$ N/m$^{\alpha+1}$;
- $c_3=2\times10^{8}$ N/m;
- $c_0=2\times10^{4}$ N/m;
- $L=1.5$ m.

Our calculations demonstrate the effectiveness of using a shock device with a striker, which is divided into two elements. It should be noted that the condition $y_2\geq U_0$, which determines the duration of the impact interaction, allows for a contact of the elements through a rigid and dissipative link. The latter is only a simulation of physical contact to determine the interaction time in a fast-moving process.

Comparison of stresses in the tool cross-sections in the presence and absence of a double impact is shown in Fig. 8. Due to the short-acting force on the first element of the striker, the nature of the stress distribution along the length of the tool has changed (Fig. 8, *b*).

Parameters:
- $L=1.5$ m;
- $S=0.014$ m^2;
- $W_1=0$;
- $W_2=7$ m/s;
- $m_1=5$ kg;
- $m_2=5$ kg, $m_3=167.5$ kg;

– P_{01}=4.5×10^5 N;
– T_1=0;
– Δt=0.1 ms;
– $c_1=c_2$=2×10^{11} N/m$^{\alpha+1}$;
– c_3=2×10^8 N/m;
– b_1=9×10^3 Ns/m;
– b_2=1.2×10^3 Ns/m;
– b_3=3×10^3 Ns/m.

Calculations were performed for a variable cross-section of the rod-tool with a conical working part. The change in the reduced radius of the cross-section of the tool along the length is shown in Fig. 9, *a*. The stress distribution for a rod of variable profile is shown in Fig. 9, *b*. The stresses were determined from the formula:

$$\sigma_i^n = E\cdot\frac{u_{i+1}^n - u_{i-1}^n}{2h}, \quad n=0,...,M, \quad i=1,...,N-1. \tag{35}$$

Parameters:
– L=0.96 m;
– W_1=5 m/s;
– W_2=8 m/s;
– m_1=6 kg;
– m_2=8 kg;
– m_3=95 kg;
– E=2.1×10^{11} Pa;
– P_{01}=6×10^5 N;
– T_1=0, Δt_1=0.1 ms;
– $c_1=c_2$=2×10^{11} N/m$^{\alpha+1}$;
– c_3=2×10^8 N/m;
– b_1=0, b_2=0, b_3=0.

Note that the distribution of normal stresses along the length of the tool corresponds to the distribution of the cross-sectional area. The highest values of stresses occur in the conical part of the tool.

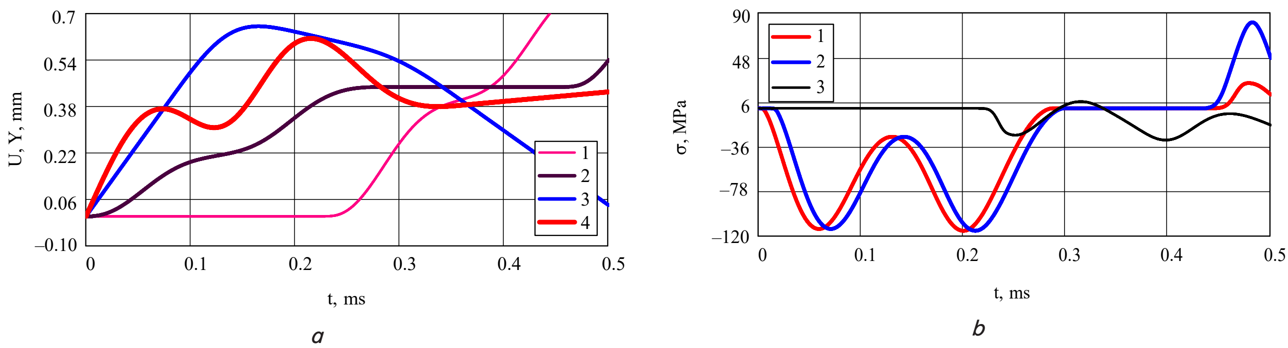

Fig. 6. Dynamics of displacements and stresses: *a* – displacements of elements in the process of interaction: 1 – U_N; 2 – U_0, 3 – Y_1, 4 – Y_2; *b* – dependence of stresses in the cross-sections of the tool: 1 – x=0.024 m; 2 – x=0.17 m; 3 – x=1.18 m

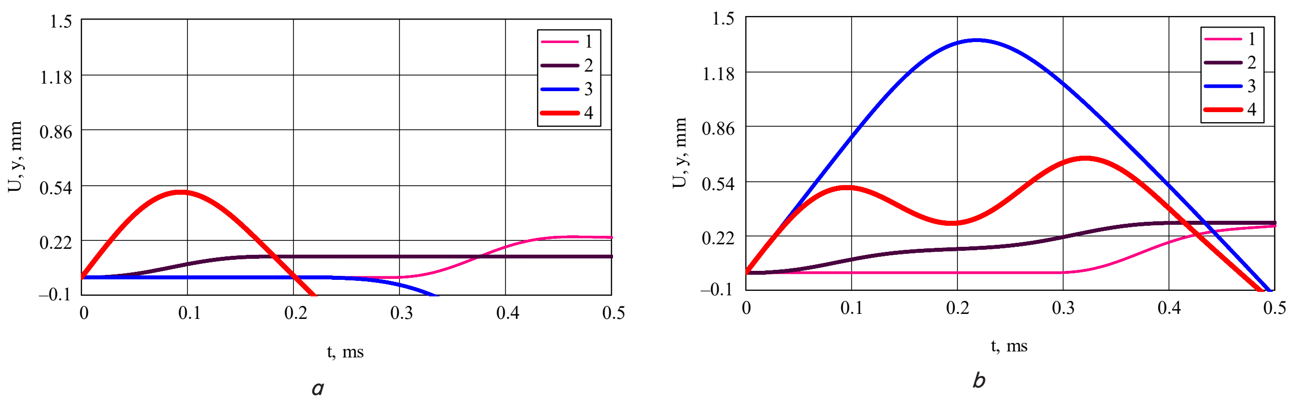

Fig. 7. Impact prolongation: 1 – U_N; 2 – U_0; 3 – Y_1; 4 – Y_2; *a* – W_1=0, W_2=8 m/s, m_1=7 kg, m_2=5 kg; duration of impact t_1=0.18 ms is determined under condition $Y_2 \geq U_0$; *b* – $W_1=W_2$=8 m/s, t_2=0.42 ms, $t_2>t_1$; 1 – U_0; 2 – U_N; 3 – Y_1; 4 – Y_2

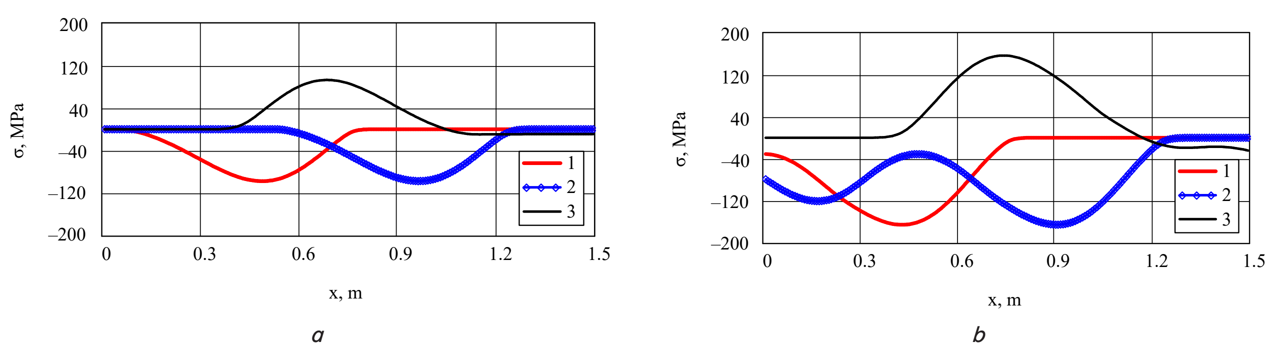

Fig. 8. The effect of a double impact on the stress distribution: *a* – no double impact; *b* – presence of a double impact; 1 – t_1=0.15 ms; 2 – t_2=0.24 ms; 3 – t_3=0.5 ms

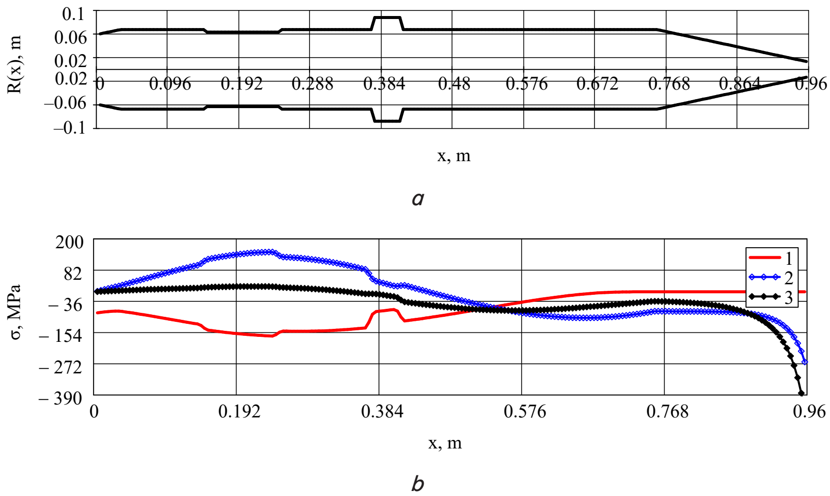

Fig. 9. Dynamics of changes in stresses of a tool of variable profile: *a* – variable profile of the tool, reduced to cylindrical; *b* – distribution of normal stresses at different times; 1 – t=0.135 ms; 2 – t=0.355 ms; 3 – t=0.5 ms

6. Discussion of results based on modeling the oscillation process of the elements of an impact device

The discrete-continuous model of the impact device, in which the striker consists of two elements, turned out to be an effective tool for studying oscillatory processes in the system "two consecutive elements of the striker – tool – machined medium" under external impulse loads.

The use of discrete and continuous elements in the model has made it possible to simplify the algorithm for solving the initial-boundary problem (1) to (7) while preserving the essential features of the interaction of the two elements of the striker with the tool in the presence of resistance of the working medium.

The application of a numerical method that combines the finite difference method and the implicit Euler scheme has made it possible to determine the dependence of internal force factors in the tool sections on external loads (velocities of discrete elements before impact, magnitudes and time of action of external forces).

In this model, the duration of the impact interaction of the two elements of the striker and the external end element of the striker with the tool was determined under the conditions $\Delta y \geq 0$, $\Delta u \geq 0$. The dependence of duration of the co-impact on the difference in velocities of discrete elements before the impact and on the stiffness of the connections between them and the end of the tool has been shown. An increase in the co-impact time (prolongation) was observed only at the values of c_1, $c_2 > 2\times10^{10}$ N/m$^{\alpha+1}$ and was 0.2–0.45 ms (Fig. 6, 7). Thus, this value can be changed by choosing the parameters of the striker elements (mass, velocity before the impact, stiffness of the connections).

The use of a mixed scheme of the finite difference method has made it possible to ensure a sufficiently wide range of stability of the numerical method with an acceptable error value.

The scheme parameters (γ, τ, h) were determined using the model problem (27) to (31). The model problem was a similar interaction of three discrete elements, the third element was replaced by a tool with an equivalent mass (Fig. 3, *a*), which was determined from formula (34). With a large mass of the third element, the results obtained practically coincide with the solution to the initial-boundary value problem (Fig. 3, 5, *b*).

Regarding the advantages of the model, it is possible to distinguish the possibility of determining the stresses in the tool cross-sections at different times (Fig. 6, 8, 9). The maximum stress was observed in the conical contact part of the tool and was about 200–380 MPa at a pulse load of 60 kN for 0.1 ms (Fig. 9). It should be noted that this is an average value; for a more accurate stress distribution, it is necessary to consider a more accurate geometric model of the tool.

It should be noted that, unlike the models given in [1–3], our model describes the process in the "striker – tool – working environment" system, which complicates the problem and the algorithm for its solution. Therefore, only the numerical method turned out to be effective, the rational parameters of which were found when comparing the solutions to the main and model problems in a general computer program.

This work is a generalization of the study reported in [10]. An additional discrete mass models the internal element of the striker, and the three-mass model has made it possible to estimate the parameters of the finite difference method. This model has made it possible to estimate the influence of the additional element of the striker on the parameters of the impulse interaction since at zero velocity of the internal element of the striker, a model of a striker with a single-element monolithic striker was built (Fig. 8).

Possible limitations of the model include the following aspects:

1. The model takes into account only longitudinal vibrations of the tool; in the real system, there are also transverse vibrations that affect the reliability of the structural elements.
2. A linear model of the resistance of the machined medium is considered using stiffness and dissipation elements. In the real process, nonlinear processes, for example, processes of a hysteretic nature, must also be taken into account.
3. The numerical method has not been studied by qualitative methods regarding the range of stability and approximation error. The model problem only approximately makes it possible to establish rational parameters of the numerical method, and they are corrected in the process of conducting numerical experiments.
4. The problem of identifying the model relative to experimental data, or models that have been studied using effective computer programs, for example, systems such as Ansys (USA), remains unsolved.

Possible areas for further research:

– taking into account the nonlinear resistance of the machined medium;

– carrying out model identification with respect to experimental data;

– determining the efficiency of energy transfer to the machined medium;

– taking into account transverse loads on the tool and transverse vibrations;

– finding the error of the numerical finite difference method.

When applying the model, the following limitations should be taken into account:

1. The model can be used for a tool that has a cylindrical shape or is reduced to such a shape.

2. The elements of the striker are replaced by discrete elements, therefore, the use of the model to assess the influence of the geometric shape of the striker on the pulse parameters is not provided.

3. The time interval over which the initial-boundary problem is solved is limited, which is associated with high and low frequency tool oscillations. To obtain high frequency parameters, such an interval is of the order of the impact time, i.e., 0.1…0.5 ms. An increase in the time interval leads to the loss of information about high frequency oscillations.

4. The characteristics of the machined medium are modeled by elastic and dissipative elements. The parameters of these elements must be additionally determined using statistical data.

7. Conclusions

1. A computational scheme of a discrete-continuous type of adaptive impactor with a two-element striker has been constructed, which describes the impulse interaction in the system "two consecutive striker elements – a tool of variable cross-section – a working medium". The striker elements are represented by discrete elements, and the tool is represented by a rod of variable cross-section. The interaction of the striker elements with the tool and the tool with the working medium is modeled by nonlinear and linear rigid and dissipative links.

2. A mixed initial-boundary-value problem with two ordinary differential equations and a wave equation in partial derivatives has been stated. The interaction force of discrete elements and the contact end of the rod is represented as a power dependence on the difference in displacements of the corresponding elements. The loads on the tool are determined by the pre-impact velocities of the two striker elements and external short-term active forces. The boundary conditions for the rod describe the interaction of the rod ends with discrete elements and the working medium. The mathematical model has made it possible to estimate the increase in the co-impact time relative to the device with a monolithic striker by 1.5…2 times, with a range of 0.35…0.5 ms.

3. The differential problem is approximated by a difference problem, the solution search algorithm of which is built on the basis of the sweep method on each time layer with partial linearization of nonlinear rigid connections. The parameters of the mixed difference scheme provide the necessary stability and accuracy, which is controlled using the model problem. The rational values of the parameters were in the following ranges: weight coefficient 0.3…0.9, time step $(1…5)\times10^{-5}$ *s*, length step (0.01…0.04) of the tool length. To estimate the parameters of the mixed difference scheme, a discrete model with three masses was used. The solution to the model problem was compared with the solution to the initial-boundary value problem with increased geometric parameters of the rod.

4. The model of the interaction between the impactor and the tool makes it possible to determine the distribution of normal stresses along the length of the tool of variable cross-section; their dependence on external impulse loads and pre-impact velocities of the external and internal elements of the striker. The maximum values of normal stresses in the conical part of the tool were 200…380 MPa. Our calculation scheme can be used in the design of impact mechanisms and the selection of optimal modes for their operation and in adaptive control over the length and shape of the pulse in accordance with the characteristics of the medium machined by the tool.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study, as well as the results reported in this paper.

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Data availability.

The data will be provided upon reasonable request.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

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