UDC 519.85, 519.1
DOI: 10.15587/1729-4061.2025.322991

DEVISING A METHOD FOR SOLVING A MULTI-CRITERIA SHORTEST PATH PROBLEM WITH FUZZY INITIAL DATA

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The object of this study is the optimization of road freight transportation routes under conditions of martial law or emergencies. The paper addresses the task of building a model and devising a method for solving the multi-criteria shortest path problem, taking into account the uncertainty of input data and the multiplicity of optimization criteria. The input data consists of communication lengths, their safety level, and road surface quality, which are represented by elements of fuzzy sets with corresponding membership functions, as well as a road network graph. The introduction of a system of rules, according to which the communication optimal by three criteria is chosen, has made it possible to formulate a generalized fuzzy optimization criterion for the edges of the graph, represented by the membership function of the fuzzy goal. This criterion is used as the weight of the edges in the devised method for solving the problem and makes it possible to simultaneously take into account the uncertainty of the input data and several optimization criteria. The method for solving the problem is based on a modified Dijkstra's algorithm. For fuzzy data processing, fuzzy logical inference is used to form a generalized optimization criterion, and the Bellman-Zadeh approach is used for the optimization problem. The results of solving the problem are the optimal route, its length, safety level, and road surface quality. For the considered road network, the length of the optimal route (41 km) is not the shortest, compared to other methods (ranging from 19 km to 50 km), but the safety level of the route is high (0.75). This is due to the values of the weight coefficients of the optimization criteria. The application of this method for optimizing freight transportation routes under conditions of martial law could improve the efficiency and reliability of transport systems under conditions of uncertainty

Keywords: multi-factor optimization, optimal route, fuzzy data, membership function, fuzzy criterion

Received 23.12.2024
Received in revised form 21.01.2025
Accepted 10.02.2025
Published 21.02.2025

***How to Cite:** Matviienko, O., Miroshnichenko, O. (2025). Devising a method for solving a multi-criteria shortest path problem with fuzzy initial data. Eastern-European Journal of Enterprise Technologies, 1 (3 (133)), 48–56. https://doi.org/10.15587/1729-4061.2025.322991*

1. Introduction

Under martial law, there is a need to quickly rebuild the usual routes of road freight transportation. The reason for this may be the destruction of infrastructure, changes in customs regulations, road blockages, increased danger on certain sections of the road. A critically important task is to ensure timely delivery of goods and save material resources: vehicles, fuel, cargo.

The choice of route is influenced by various factors (road conditions, weather conditions, traffic jams, risks on the road), which are vague and difficult to accurately quantify. To take them into account, it is appropriate to use the apparatus of fuzzy logic and fuzzy sets. The situation on the roads can change very quickly, so it is necessary to have a flexible planning method that is able to adapt to new conditions.

The optimal route is chosen according to several criteria (length, speed, cost, safety), which may be contradictory, so it is advisable to use multifactor optimization methods for modeling.

To find the shortest path on the road network, well-known classical algorithms are used, such as the Dijkstra, Bellman-Ford, Floyd-Warshell algorithms, and others. But these algorithms are not designed to work with fuzzy data and many optimization criteria.

To solve the problem of the shortest route taking into account fuzzy initial data, combined methods are also used. They combine classical graph optimization algorithms and fuzzy logic methods: the Bellman-Zadeh approach, Mamdani, Sugeno, Tsukamoto algorithms, and others. But they do not take into account the multi-criteria nature of the problem.

Road freight transportation in high-risk areas is the most economical, flexible, and reliable way to deliver goods. Existing methods do not take into account all aspects of the problem of finding optimal routes. It is advisable to investigate the possibility of combining graph theory algorithms and fuzzy set theory to solve a multi-criteria optimization problem. Implementing the results of such research to optimize freight transportation routes will increase the efficiency and reliability of transport systems under conditions of uncertainty.

2. Literature review and problem statement

Study [1] is aimed at improving the system for calculating the cost of online motorcycle taxi trips. The shortest routes are calculated using the Dijkstra algorithm, and a fuzzy inference system based on the Mamdani algorithm is used to determine the cost of the trip. In this case, the distance and terrain features that complicate the route are taken into account, but only one criterion for finding the shortest route is considered – the length of the route.

In work [2], an approach combining simulation, metaheuristics, and fuzzy logic is proposed for solving optimization problems in logistics and transport. Both stochastic and fuzzy uncertainty are taken into account, but multi-criteria are not considered.

In [3], a particle swarm algorithm is proposed for solving a fuzzy transport problem. The high efficiency and accuracy of the method are shown, but the issues of multi-criteria optimization of routes under uncertainty remain unresolved. The reason for this may be the difficulty of simultaneously taking into account several criteria and fuzzy parameters. The solution to the problem may be the development of an algorithm that would combine the advantages of the modified particle swarm algorithm with multi-criteria optimization and fuzzy logic methods.

Improvement of the path finding algorithm based on particle swarms and graphs using fuzzy logic for solving multi-criteria optimization problems is presented in study [4]. However, the issues of optimizing the computational complexity of new algorithms remain unresolved. due to the need for fuzzification and defuzzification in fuzzy systems, which requires additional computational resources. Therefore, it is advisable to improve the proposed algorithms in order to achieve a better balance between the quality of routes and computational efficiency.

In [5], a fuzzy mathematical model is proposed for optimizing cargo transportation by various modes of transport on a route. The issue of simultaneously taking into account the fuzziness of input data and multi-criteria optimization remains unresolved. The reason for this may be the need to develop complex algorithms that require significant computational resources. The development of a hybrid approach that would combine fuzzy logic and multi-criteria optimization methods is promising.

The development of a decision support model for choosing the optimal route for transporting hazardous materials, which uses fuzzy logic to take into account various criteria and uncertainties, is presented in [6]. However, issues related to optimizing the computational complexity of the algorithm remain unresolved. The reason for this may be the need for fuzzification and defuzzification in fuzzy systems, which requires additional computational resources. An option to overcome these difficulties may be to develop a hybrid approach that would combine the advantages of the speed of classical optimization algorithms and the flexibility of fuzzy systems. This approach is partially used in this work but does not take into account the possibility of dynamic changes in the weights of the criteria.

In [7], a modification of the genetic algorithm is proposed to solve the shortest path problem in networks with fuzzy parameters. The efficiency of the algorithm is shown, but the issues of multi-criteria optimization and taking into account a larger number of fuzzy parameters remain unresolved. The reason may be the complexity of simultaneously taking into account several optimization criteria and different types of fuzzy data. A solution may be a hybrid approach that would combine the genetic algorithm with multi-criteria optimization methods and extended fuzzy logic models. This approach is partially used in the work, but it is limited to only one type of fuzzy numbers (trapezoidal) and does not take into account multi-criteria.

Research [8] considers the development of a multi-criteria decision support system using the method of hierarchy analysis and fuzzy logic to reduce traffic congestion in the city. However, the issues of route optimization remain unresolved. The reason for this may be the difficulties associated with the need to take into account the fuzziness of the initial data on traffic flows and the variability of optimization criteria over time. An option to overcome these difficulties may be the development of a comprehensive model that would combine the methods of fuzzy optimization and multi-criteria decision-making. This approach is partially used in the work, but it is limited only to the choice of the location of objects and does not consider the optimization of routes between them.

Having analyzed works [1–9], it can be stated that it is advisable to conduct a study on the development of a model and a method for solving the multi-criteria problem of the shortest route, taking into account the fuzziness of the initial data and the multiplicity of optimization criteria. Such research will expand the possibilities of applying optimization methods under real-world conditions, where it is often necessary to take into account several factors simultaneously, such as time, cost, safety, and route quality, while working with inaccurate or fuzzy input data.

3. The aim and objectives of the study

The aim of our research is to devise a method for solving the multi-criteria problem of the shortest route, which allows taking into account the fuzzy input data and several optimization criteria. This will make it possible to find optimal routes for road freight transportation in emergencies, when the main criterion for delivering goods is safety.

To achieve the goal, the following tasks were set:

– to build a mathematical model of the multi-criteria problem of finding the optimal route with fuzzy input data;

– to modify the Dijkstra algorithm for solving the multi-criteria problem of finding the optimal route with fuzzy input data;

– to test the model and method using examples.

4. The study materials and methods

The object of this study is the optimization of road freight transportation routes under conditions of martial law or emergencies.

The study hypothesizes the effectiveness and ease of use of the devised method for optimizing freight transportation routes. The method takes into account the fuzziness of the initial data and several optimization criteria (safety, road surface quality, route length). It is assumed that the use of this method will contribute to increasing the reliability of transport systems under conditions of martial law or emergencies.

It is assumed that the initial data can be represented in the form of fuzzy sets, all optimization criteria can be reduced to one generalized fuzzy optimization criterion. The weighting factors of the optimization criteria adequately reflect their significance.

To simplify the study, it is assumed that the optimization criteria (safety, road surface quality, route length) are independent of each other, their weighting factors are given by constant values.

Mathematical model.

The transport network is presented as a mixed graph, for each edge clear parameters are given: length, quality of road surface, safety. The parameters are fuzzified and represented by elements of discrete (for the level of safety and quality of road surface) and continuous (for the length of communication) fuzzy sets with the corresponding membership functions. The weight coefficients of each of the optimization criteria are given. To take into account all the fuzzy parameters and optimization criteria, a membership function of a fuzzy goal is introduced for each edge.

Optimization problem.

It is necessary to find the optimal path from the first vertex to the last one according to all the criteria. The result is the optimal path, its length in the form of a clear number, the level of safety and quality of road surface in the form of the values of the corresponding membership functions (the Bellman-Zadeh approach is used [10, 11]).

Justification of the selected methods.

Representation of the parameters of the edges of the graph (length, safety, quality of the road surface) as elements of fuzzy sets is the most convenient for solving the optimization problem by the proposed method. It was also possible to represent these parameters as fuzzy triangular or trapezoidal numbers, or as linguistic variables with the corresponding terms. However, since the criteria of safety and quality of the road surface take a small number of discrete values, such a representation would be excessive and would complicate the calculations.

Fuzzy inference is used to process fuzzy data when a system of rules is formed to create a formula for the membership function of a fuzzy goal. The Bellman-Zadeh approach is used to calculate the values of the safety and road surface quality membership functions of the optimal route.

Defuzzification algorithms are not used in the work, because the values of the safety (road surface quality) membership functions of the optimal route are represented by one fuzzy number. It can be interpreted as the degree of confidence that the route is safe (has good road surface quality).

The classical Dijkstra algorithm was chosen as the basis for the optimization method since it is well suited for solving the shortest route problem for mixed graphs with non-negative edges. It is simple and effective. The work considers small road networks in the territory of an emergency or military operations, therefore the number of vertices of the graph is small. The well-known drawback of Dijkstra's algorithm regarding the increase in computational complexity of processing graphs with a large number of vertices is not significant in this case.

To take into account multi-criteria, the convolution method is used, because it is simple, understandable, and does not require significant computational resources.

The research was conducted using mathematical and simulation modeling methods.

Thus, this study combines the methods of multi-criteria optimization, graph optimization, and fuzzy optimization.

5. Results of research on the multi-criteria optimal route problem with fuzzy input data

5. 1. Mathematical model of the multi-criteria optimal route problem with fuzzy output data

When organizing traffic under conditions of natural disaster or military operations, the problem of finding the shortest route connecting two given points is of great interest. A network containing n points connected by roads is represented as a directed (or mixed) graph $G(V, E)$, where V is a set of vertices, E is a set of arcs, or communications. All points (vertices) are ordered and numbered from 1 to n. Each communication (arc) (i, j) connecting the i-th and j-th vertices corresponds to the vector $(d_{ij}, r_{ij}, s_{ij})^T$, where the number d_{ij} is the length of the communication (i, j), r_{ij} is the quality of the road surface of the communication (i, j), s_{ij} is the safety of the communication (i, j). The quantities d_{ij}, r_{ij}, s_{ij} are independent of each other. It is necessary to find a mixed path from vertex 1 to vertex n that is safe, has a minimum length and high-quality coverage.

This is a multi-criteria optimization problem. Optimization criteria: the length of the path that must be minimized, the quality of the road surface must be maximized, and the safety must be maximized.

In military situations, the quality of the road surface can change unpredictably and quickly (deteriorate), so this parameter is considered fuzzy.

The same applies to security. The safety of a road section is affected by the operational military situation, the number of guards, weapons, the depth of the area's view, and many other factors that cannot be predicted or taken into account, so this parameter is considered fuzzy.

The values r_{ij}, which determine the quality of the road surface, are elements of a fuzzy set $\hat{R}$, given on the universal set $R=\{0,1,2,3\}$, where the value $r=0$ means the quality of the road surface, impossible to drive; $r=1$ is poor coverage, but travel is possible; $r=2$ is satisfactory coverage; $r=0$ is good coverage. The fuzzy set "High-quality road surface" has the form:

$$\hat{R}=\{(0;0),(1;0.33),(2;0.66),(3;1)\}. \quad (1)$$

The membership function of the fuzzy set "Quality road surface" is as follows:

$$\mu_i(r)=\begin{cases}0, r=0,\\ \frac{r}{3}, r=1,2,3,\end{cases} \quad (2)$$

where i is the communication number, $i=\overline{1,card(E)}$.

The values s_{ij}, which determine the safety of communications, are elements of a fuzzy set $\hat{S}$, given on the universal set $S=\{0,1,2,3,4\}$, where the value $s=0$ means extreme danger, which makes it impossible to travel; $s=1$ is very dangerous, but travel is possible; $s=2$ is dangerous; $s=3$ is a satisfactory level of safety; $s=4$ is safe.

The fuzzy set "Safe section of the road" has the form:

$$\hat{S}=\{(0;0),(1;0.25),(2;0.5),(3;0.75),(4;1)\}. \quad (3)$$

The membership function of the fuzzy set "Safe road section" has the form:

$$\mu_i(s)=\begin{cases}0, s=0,\\ \frac{s}{4}, s=1,2,3,4,\end{cases} \quad (4)$$

where i is the communication number, $i=\overline{1,card(E)}$.

The lengths of the communications d_{ij} are also elements of the fuzzy set $\hat{D}$ "The road section is short", given on the universal set $D=(0, d_{\max}]$ with the membership function $\mu_{ij}(d)$, which is shown in Fig. 1. $d_{\max}$ is the length of the longest section in the network.

The membership function of the fuzzy set "The road section is short" has the form:

$$\mu_i(d)=\frac{d_i-d_{\min}}{d_{\max}-d_{\min}}, \quad (5)$$

where $d_{\max}$, $d_{\min}$ are the lengths of the longest and shortest communication in the network.

The most efficient algorithm for solving the classical graph theory problem of the shortest route is Dijkstra's algorithm. This algorithm is modified for a multi-criteria problem with fuzzy input data.

When choosing the best section from several sections of the path, the following rules are used:

1. If for any of the sections (s=0 or r=0), then travel along this section is not allowed.

2. If for two sections (k and m): (r_k≥1 and r_m≥1 and s_k≥1 and s_m≥1 та s_k≠s_m), then the one for which s is greater is chosen.

3. If for two sections (k and m): (s_k=s_m and r_k=1 and r_m>1), then section m is chosen.

4. If for two sections (k and m): (s_k=s_m and ((r_k>1 and r_m>1) or (r_k=r_m))), then the section with the smallest length is chosen.

To formalize these rules, the concept of a fuzzy goal membership function is introduced. Since the criteria "security" and "coverage quality" must be maximized, and the path length must be minimized, the fuzzy goal membership function for a certain communication is determined from the formula:

$$\mu_c = \lambda_1\mu_d + \lambda_2\left(1-\mu_r\right) + \lambda_3\left(1-\mu_s\right), \tag{6}$$

where λ_1–λ_3 are weighting factors that determine the importance of the criteria: length, road surface quality, safety, respectively:

$$\sum_{i=1}^{3}\lambda_i = 1. \tag{7}$$

Formula (6) has been tested with examples. Let:

$$\lambda_1 = 0.1;\ \lambda_2 = 0.2;\ \lambda_3 = 0.7;\ d_{\min} = 4;\ d_{\max} = 25. \tag{8}$$

Table 1 gives the initial data (lines 2–4) and the results of the calculations (lines 5–8).

Table 1

Comparison of communication characteristics

Indicator	Communication number									
	1	2	3	4	5	6	7	8	9	10
d	10	10	10	10	20	10	10	18	25	25
r	2	2	2	2	2	3	1	3	1	1
s	4	3	2	1	2	2	2	2	1	2
μ_d	0.286	0.286	0.286	0.286	0.762	0.286	0.286	0.667	1.000	1.000
μ_r	0.667	0.667	0.667	0.667	0.667	1.000	0.333	1.000	0.333	0.333
μ_s	1.000	0.750	0.500	0.250	0.500	0.500	0.500	0.500	0.250	0.500
μ_c	0.095	0.270	0.445	0.620	0.493	0.379	0.512	0.417	0.758	0.583

Of the two communications, the one for which the value of μ_c is smaller is selected. Analysis of the calculation results:

1. Of the two communications 1 and 2, which differ in the values of the parameter s, the one for which s is larger is selected, that is, the first, because the safety criterion has the highest priority. From the last line of Table 1 it is seen that μ_c for the first communication is smaller than for the second (0.095<0.27).

2. Of the two communications 6 and 7, which differ in the values of the parameter r, the one for which r is larger is selected, that is, the sixth. From the last line of Table 1 it is seen that μ_c for the sixth communication is smaller than for the seventh (0.379<0.512).

3. Of the two communications 7 and 10, which differ in the values of the parameters d, the one for which d is smaller is selected. From the last line of Table 1 it is seen that μ_c for the seventh communication is less than for the tenth (0.512<0.583).

And so on. Summing up, it is obvious that the calculations confirm the introduced rules and formulas.

5. 2. Modification of Dijkstra's algorithm for solving the multi-criteria problem of finding the optimal route with fuzzy initial data

The devised method is based on the classical Dijkstra's algorithm for finding the shortest path on a directed graph. The modification consists of the following:

1. In the classical Dijkstra's algorithm, the parameters of the vertices and edges are clear values (for example, length). In this work, the values of the membership function of the fuzzy goal are used, i.e., fuzzy values.

2. The form of representation of the vertex labels has been changed. In the Dijkstra's algorithm, the vertex labels are the distance from the first vertex to the given vertex and the number of the vertex from which the given one came. In the devised method, the vertex label consists of three components: the previous vertex of the path, the value of the membership function of the fuzzy goal, and the number of communications from the first vertex to the given one.

3. The formula for calculating time stamps has been changed due to the use of fuzzy values (membership functions of a fuzzy goal).

The essence of the proposed method.

If vertices i and j are not connected by any communication, then μ_{cij}=1, if there is only an arc (i, j) between vertices i and j, then μ_{cji}=1. For undirected arcs (edges), μ_{cij}=μ_{cji}. If there are arcs (i, j) and (j, i) on the network, then it is not necessary that μ_{cij}=μ_{cji}. The set of vertices to which the desired routes have already been found is successively expanded. Such vertices are given permanent labels, the rest are given temporary labels.

The permanent label consists of three parts and for the vertex with number i has the form: $\overline{(p;M_{ci};t)}$, where M_{ci} is the value of the membership function of the fuzzy goal of the best (according to all criteria) route from the first vertex to the vertex with number i; p is a vertex such that the communication (p, i) lies on this route; t is the number of communications in the route from the first vertex to the vertex i.

The temporary label consists of three parts and for the vertex with number i has the form: (p; μ_{ci}; t), where p is the vertex adjacent to vertex i (i.e. vertices p and i are connected by communication (p, i)); μ_{ci} is the value of the membership function of the fuzzy goal of the route passing through vertex p and connecting the first vertex with vertex i; t is the number of communications in the route from the first vertex to vertex i.

Preparatory stage.

Each communication (i, j) is assigned a value of μ_{cij}. The vertices of the graph are labeled. The first vertex receives a permanent label $\overline{(1;1)}$, the other vertices receive temporary labels (1; μ_{c1i}; 1) (i is the number of the vertex under consideration). It is assumed that each vertex of the network is connected to each. If in fact there is no communication between vertices i and j, we assume that it exists but has a fuzzy goal membership function value equal to 1. Therefore, if there is no communication (1, i) in the network, then μ_{c1i}=1.

Further, the algorithm consists of a sequence of iterations, each of which consists of the following steps:

1. A vertex i is found such that $\mu_{ci}=\min\mu_{cu}$. Here, the minimum is taken over all vertices u with temporary labels.

It turns out that μ_{ci} is the smallest value of the fuzzy goal membership function from the first vertex to vertex i.

2. The temporary label $(p; \mu_{ci}; t)$ of vertex i is replaced by a permanent label $\overline{(p;M_{ci};t)}$, where $M_{ci}=\mu_{ci}$.

3. The time labels are recalculated. For this purpose, vertices with time labels adjacent to vertex i (which has just received a permanent label) are considered, and it is checked whether the value of the fuzzy goal membership function from the first vertex to the analyzed one will not be smaller if we move through vertex i. If so, then the value of its fuzzy goal membership function is found and a new time label t is assigned to vertex j (j is the number of the vertex under consideration) $\left(i; \dfrac{M_{ci}t+\mu_{cij}}{t+1}\right)$, t – the number of communications in the route to vertex i; otherwise, the time label of the vertex does not change. That is, for each vertex j adjacent to vertex i and having a time label, the following is calculated: $\dfrac{M_{ci}t+\mu_{cij}}{t+1}$. If $\dfrac{M_{ci}t+\mu_{cij}}{t+1}<\mu_{cj}$, then the vertex receives a new time label $\left(k; \dfrac{M_{ci}t+\mu_{cij}}{t+1}\right)$. Otherwise, the time label of vertex j does not change.

4. If not all vertices have permanent labels yet, then the transition to step 2 occurs, otherwise all iterations are completed, and it remains to build the shortest routes.

The algorithm ends with a stage in which the best routes from the first vertex to each are restored. Let the vertex with the number k have a constant label $\overline{(r;M_{ck};t)}$, which means that the best route to it from the first vertex passes through the vertex r. If the left part of the constant label of the vertex r is equal to w, then the best route to it passes through the vertex w, and so on, to the first vertex. At this point, the algorithm ends its work.

The length, safety, and quality of this route are calculated.

Let the optimal route consist of n communications: $(1, i_1)$, (i_1, i_2), .., (i_{n-1}, i_n).

The length of the route is calculated as the sum of the lengths of each communication:

$$d_{opt}=\sum_{i=1}^{n} d_i. \qquad (9)$$

The safety of a route, or its reliability from a safety point of view, is determined by the membership function $\mu_s=$ $=\min\{\mu_{s1}, \mu_{s2},..., \mu_{sn}\}$.

The quality of the road surface of a route, or its reliability from a road surface quality point of view, is determined by the membership function $\mu_r=\min\{\mu_{r1}, \mu_{r2},..., \mu_{rn}\}$.

5. 3. An example of solving a multi-criteria problem of finding the optimal route with fuzzy initial data

We consider a network of roads represented by the graph in Fig. 1; here, the triplets of numbers on communications mean the distance, the quality of the road surface, and the level of safety of the corresponding communication.

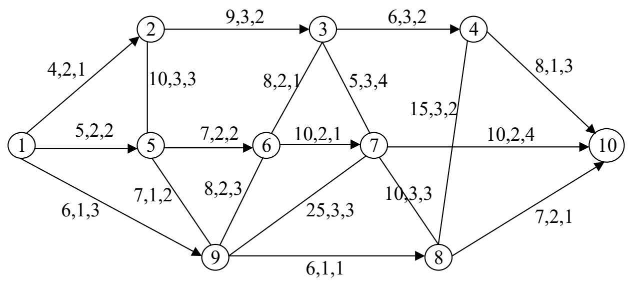

Fig. 1. Graph of the road network

The initial data for the problem are given in Table 2.

Step 0. Each communication is assigned the value μ_c from Table 2. The vertices of the graph are marked. The first vertex receives a permanent mark $\overline{(1;1)}$, the other vertices receive temporary marks $(1; \mu_{c1i})$ (i is the number of the considered vertex). If there is no communication $(1, i)$ in the network, then $\mu_{c1i}=1$ (Fig. 2).

Step 1. Among the vertices with temporary labels, vertex 9 has the smallest μ_c value. Min(0.318;0.421;0.592)=0.318. The temporary label of vertex 9 is replaced by a permanent label $\overline{(1;0.318;1)}$.

The temporary labels are recalculated. To do this, the vertices with temporary labels adjacent to vertex 9 (vertices 5, 6, 7, 8) are considered and it is checked whether the membership function of the fuzzy goal from the first vertex to the analyzed one will be smaller if we move through vertex 9.

For vertex 5: (0.318+0.498)/2=0.408. 0.408<0.421, so the temporary label of vertex 5 is changed to (9; 0.408; 2).

For vertex 6: (0.318+0.261)/2=0.290. 0.290<1, so the temporary label of vertex 6 changes to (9;0.290;2). The first number indicates which vertex they came from, the second is the value of the fuzzy goal membership function, and the last is the number of communications from vertex 1 to vertex 6.

For vertex 7: (0.318+0.275)/2=0.297. 0.297<1, so the temporary label of vertex 7 changes to (9;0.297; 2).

For vertex 8: (0.318+0.668)/2=0.493. 0.493<1, so the temporary label of vertex 8 changes to (9;0.493;2) (Fig. 3).

Table 2

Initial data for the problem

Parameter	Communication																		
	1–2	2–5	1–9	5–9	5–6	2–3	9–6	9–7	9–8	3–4	3–6	3–7	6–7	7–8	7–10	4–10	8–10	4–8	1–5
d, km	4	10	6	7	7	9	8	25	6	6	8	5	10	10	10	8	7	15	5
r	2	3	1	1	2	3	2	3	1	3	2	3	2	3	2	1	2	3	2
s	1	3	3	2	2	2	3	3	1	2	1	4	1	3	4	3	1	2	2
μ_c	0.592	0.204	0.318	0.498	0.431	0.374	0.261	0.275	0.668	0.360	0.611	0.005	0.620	0.204	0.095	0.327	0.606	0.402	0.421

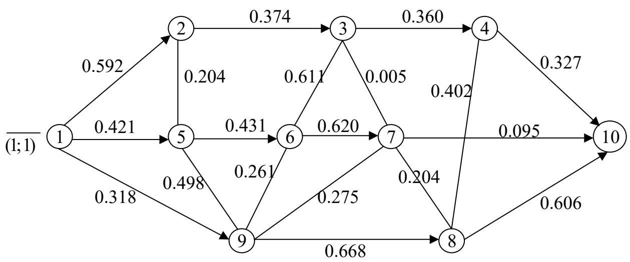

Fig. 2. Graph after step 0

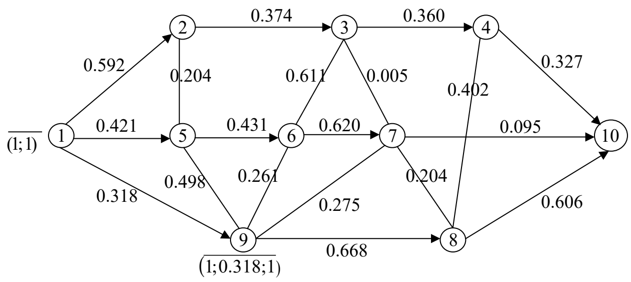

Fig. 3. Graph after step 1

Step 2 (the algorithm is repeated from step 1). Among the vertices with temporary labels, vertex 6 has the smallest μ_c value. The temporary label of vertex 6 is replaced by a permanent label $\overline{(9;0.290;2)}$.

The temporary labels of neighboring vertices are recalculated.

For vertex 3: (0.290·2+0.611)/3=0.402. 0.402<1, so the temporary label of vertex 3 is changed to (6;0.402; 3).

For vertex 7: (0.290·2+0.620)/3=0.4. 0.4>0.297, so the temporary label of vertex 7 does not change (Fig. 4).

Step 3. Among the vertices with temporary labels, vertex 7 has the smallest μ_c value. The temporary label of vertex 7 is replaced by a permanent label $\overline{(9;0.297;2)}$.

The temporary labels of neighboring vertices are recalculated.

For vertex 8: (0.297·2+0.204)/3=0.266. 0.266<0.493, so the temporary label of vertex 8 is changed to (7; 0.266; 3).

For vertex 10: (0.297·2+0.095)/3=0.230. 0.230<1, so the temporary label of vertex 10 is changed to (7;0.230;3).

For vertex 3: (0.297·2+0.005)/3=0.20. 0.20<1, so the temporary label of vertex 3 changes to (7; 0.20; 3) (Fig. 5).

Step 4. Among the vertices with temporary labels, vertex 3 has the smallest μ_c value. The temporary label of vertex 3 is replaced by a permanent label $\overline{(7;0.20;3)}$. The temporary labels of neighboring vertices are recalculated.

For vertex 4: (0.20·3+0.360)/4=0.24. 0.24<1, so the temporary label of vertex 4 is changed to (3; 0.24; 4) (Fig. 6).

Step 5. Among the vertices with temporary labels, vertex 10 has the smallest μ_c value. The temporary label of vertex 10 is replaced with a permanent label of $\overline{(7;0.230;3)}$, there are no neighboring vertices.

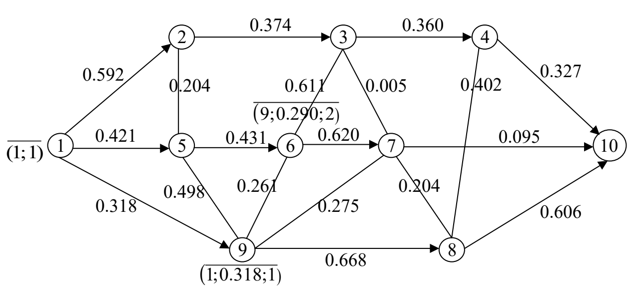

Fig. 4. Graph after step 2

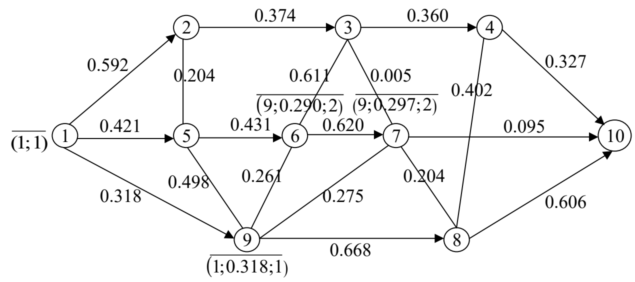

Fig. 5. Graph after step 3

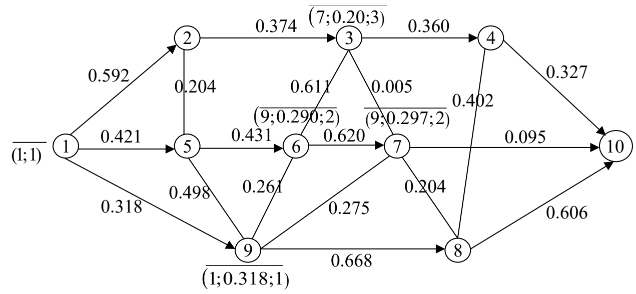

Fig. 6. Graph after step 4

Step 6. Among the vertices with temporary labels, vertex 4 has the smallest μ_c value. The temporary label of vertex 4 is replaced with a permanent label of $\overline{(3;0.24;4)}$. No neighboring vertices with temporary labels (Fig. 7).

Step 7. Among the vertices with temporary labels, vertex 8 has the smallest μ_c value. The temporary label of vertex 8 is replaced with a permanent label $\overline{(7;0.266;3)}$. There are no neighboring vertices with temporary labels.

Step 8. Among the vertices with temporary labels, vertex 5 has the smallest μ_c value. The temporary label of vertex 5 is replaced with a permanent label $\overline{(1;0.421;1)}$.

One unlabeled vertex 2 remains. The temporary label of vertex 2 is replaced with a permanent label. All vertices have permanent labels, the algorithm is completed (Fig. 8).

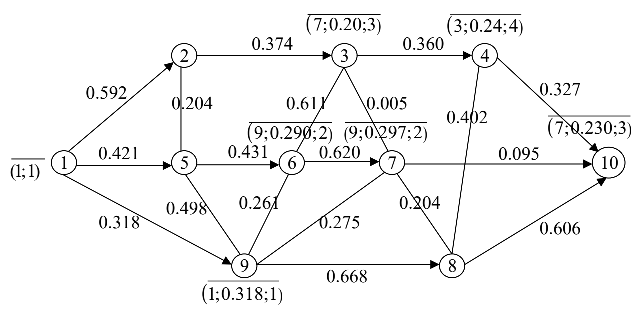

Fig. 7. Graph after step 6

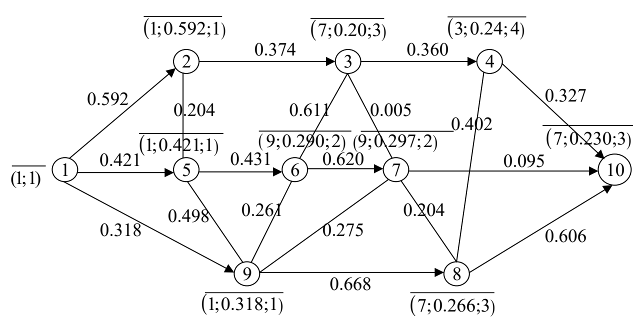

Fig. 8. Graph after step 8

The best route is restored from the first vertex to the last. Vertex 10 has the first value in the constant value 7, vertex 7 has the first value in the constant value 9, and vertex 9 has the first value in the constant value 1. Therefore, the best path according to the three criteria is: 1-9-7-10. Its length is 6+25+10=41 km; safety min(0.75; 0.75; 1)=0.75; road surface quality min(0.33; 0.66; 1)=0.33.

6. Discussion of results of investigating the multi-criteria optimal route problem with fuzzy input data

Comparative analysis of different route optimization methods, including the proposed method. Five scenarios with different optimization criteria are considered:

1. Classic Dijkstra algorithm – clear input data, one optimization criterion – route length.
2. Combination of Dijkstra algorithm with Bellman-Zadeh approach – fuzzy input data (safety), one optimization criterion – route safety.
3. Combination of Dijkstra algorithm with Bellman-Zadeh approach – fuzzy input data (road surface quality), one optimization criterion – route road surface quality.
4. Bellman-Zadeh approach – fuzzy input data (safety, road surface quality), two optimization criteria – route road surface quality.
5. The algorithm proposed in our work involves fuzzy input data (safety, road surface quality, length of communications), three optimization criteria – safety, road surface quality of the route, and route length.

The results of comparing all methods are given in Table 3.

The length of the optimal route, obtained by different methods, ranges from 19 km to 50 km. The shortest route length was obtained using the classical Dijkstra algorithm, but the safety and quality of the surface on the route are not taken into account. If we search for the optimal route by one criterion "safety" or "quality of surface", the route length is greater than 19 km, but in this case "length" was not taken into account as an optimization criterion, but the corresponding criterion is improved. If we take two criteria at the same time: both "safety" and "quality of surface", the length of the optimal route is the largest. The obvious conclusion is that the more optimization criteria (not by length), the greater the length of the optimal route. The method proposed in the work, which takes into account all three criteria, gives an optimal route length of 41 km.

When studying the model and the method for sensitivity to input data, the weight coefficients of the optimization criteria change. The results obtained are given in Table 4.

From Table 4, it can be seen that the weights of the direction criteria directly affect the parameters of the optimal route: the larger the weight of the corresponding criterion, the better this indicator in the optimal route.

Our study solves some problems that were not considered in works [1–8]. Unlike the classical Dijkstra algorithm [1] and the particle swarm algorithm [3], the method proposed in the work takes into account several optimization criteria simultaneously. In addition, several optimization criteria and the fuzziness of the output data are combined by using the membership function of a fuzzy goal for each communication of the graph. A similar problem was considered in work [4], but a different approach (particle swarm algorithm) was used there, which requires large computational resources. In the problem under study, the weight coefficients of the criteria (8) are chosen in such a way that the most important criterion is the "safety" criterion, and the least important is the "length" criterion ($\lambda_1 < \lambda_2 < \lambda_3$). This is much simpler than in works [6, 8], in which the emphasis is on determining the weight coefficients of the optimization criteria by experts. Instead, the sensitivity of the model and the method to changes in the weight coefficients of the optimization criteria was investigated (Table 4). From Table 4 it can be seen that the value of the weight coefficient of the criterion directly affects the corresponding indicator of the optimal route. Further research can tackle a more substantiated choice of the weight coefficients of the optimization criteria. Unlike work [5], in which the alpha-slice method was used, which involves solving a series of optimization problems, the proposed method takes into account data fuzziness and multi-criteria in one generalized fuzzy criterion. This makes it possible to solve one optimization problem, which simplifies the solution process. For the problem under study (with a small number of vertices), the proposed method is more efficient than the genetic algorithm [7], which is too complex to implement.

Table 3

Results of comparing different methods for solving the problem

Parameter	Optimization criterion				
	1 safety	1 surface quality	1 length	1 safety 2 surface quality	1 safety 2 surface quality 3 length
Optimal route	1-9-7-10	1-5-6-7-10	1-9-8-10	1-9-7-3-4-10	1-9-7-10
Length	41	32	19	50	41
Safety	0.75	–	–	0.5	0.75
Quality of coverage	–	0.66	–	0.33	0.33

Table 4

Optimal routes for different values of weight coefficients of optimization criteria

No.	Weight coefficients	Parameters of the optimal route			
		Route	Length, km	μ_s	μ_r
1	$\lambda_1=0.1$; $\lambda_2=0.2$; $\lambda_3=0.7$	1-9-7-10	41	0.75	0.33
2	$\lambda_1=0.2$; $\lambda_2=0.3$; $\lambda_3=0.5$	1-9-7-3-4-10	50	0.5	0.33
3	$\lambda_1=0.3(3)$; $\lambda_2=0.3(3)$; $\lambda_3=0.3(3)$	1-5-2-3-7-10	39	0.5	0.67
4	$\lambda_1=0.7$; $\lambda_2=0.1$; $\lambda_3=0.2$	1-2-3-4-10	27	0.25	0.33
5	$\lambda_1=0.5$; $\lambda_2=0.2$; $\lambda_3=0.3$	1-5-2-3-7-10	39	0.25	0.33

A feature of our study is that the optimal route is sought under conditions of increased danger, therefore it is advisable to give the highest priority to the criterion "safety", the lowest – "length" of the section. This additional restriction makes the subjectivity of the choice of weight coefficients of the optimization criteria not a critical drawback for the problem under study. It can also be noted that the road network graph contains two-sided edges, which allows the route to turn back or bypass dangerous sections if necessary.

A special feature of the proposed method is that the convolution of three criteria into one is applied for each edge of the graph and takes into account the fuzziness of the parameters, and there is no need to defuzzify the results.

The work considers a problem with three optimization criteria, but their number can be increased or decreased.

The limitations of the study are that the proposed model and method for finding the optimal route can be applied to directed graphs with non-negative edge weights that are static. Like Dijkstra's algorithm, the devised method is designed to find the shortest paths from one given initial node to all others.

The results fully correspond to the tasks set: a mathematical model and a method for solving the problem of finding the optimal route have been developed, which can be used to optimize road transportation in emergencies.

It should be noted that the use of fuzzy logic methods leads to obtaining an approximate solution, which is a drawback of this study.

Further research may focus on devising methods for determining the weight coefficients of criteria and exploring other ways of representing fuzzy input data.

7. Conclusions

1. We have proposed a mathematical model of a multi-criteria problem of finding an optimal route with fuzzy initial data. A system of rules has been devised by which the optimal communication according to three criteria is selected. Based on these rules, the concept of the fuzzy goal membership function μ_c is introduced as a criterion that generalizes several optimization criteria into one and takes into account the fuzziness of the initial data.

2. A method for solving the problem set has been devised, which is based on Dijkstra's algorithm. Unlike the classical Dijkstra's algorithm, the communication parameters are the values of the fuzzy goal membership functions μ_c; the form of representation of the graph vertex labels and the formula for their calculation have been changed.

3. The method performance was tested on examples. For a given network, the optimal route (1-9-7-10) was found for all criteria, its length (41 km), safety level (0.75), and road surface quality level (0.33) were calculated. When comparing the devised method with other methods for solving the optimal route problem, it was found that the length of the optimal route is from 19 to 50 km.

The smallest value of 19 km was obtained by the Dijkstra algorithm, which is 46 % less than 41 km, but safety and quality of the surface are not taken into account. The optimal route was calculated using the proposed method with the most important criterion "length". The obtained value of the optimal route length is 27 km, which is 42 % more than according to the Dijkstra algorithm. In this case, both the "safety" criterion and the "surface quality" criterion were taken into account. This result depends on the weight coefficients of the criteria.

The length of the optimal route according to one criterion "safety" is also 41 km, the safety level is high: 0.75, while the criteria "quality of surface" and "length" are not taken into account. This value of the length of the optimal route can be explained by the structure of a specific network and the initial data.

The length of the optimal route according to one criterion "quality of surface" is 32 km, which is 22 % less than 41 km, the quality of surface is high – 0.66.

The largest value of the length of the optimal route, obtained when taking into account two criteria simultaneously: both "safety" and "quality of surface", is 50 km – this is 22 % more than 41 km. At the same time, the safety level is 0.5, the quality of surface is 0.33. This is due to the values of the weight coefficients of the optimization criteria: the larger the weight coefficient of the corresponding criterion, the better this indicator is in the optimal route.

Thus, with an increase in optimization criteria (except length), the length of the optimal route increases. At the same time, with an increase in the weight coefficient of a certain criterion, its significance in the process of determining the optimal route increases.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study, as well as the results reported in this paper.

Funding

The study was conducted without financial support.

Data availability

All data are available, either in numerical or graphical form, in the main text of the manuscript.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

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