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MULTI-OBJECTIVE OPTIMIZATION OF A TWO-STAGE HELICAL GEARBOX WITH DOUBLE GEARS IN SECOND STAGE USING SAW TECHNIQUE TO REDUCE BOTTOM AREA AND ENHANCE EFFICIENCY

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Two-stage helical gearboxes featuring two gears in the second stage are widely used across various industries. This gearbox design helps enhanced load capacity, increases consistent force distribution, and minimizes operational noise. One significant challenge in the design of this type of gearbox is the simultaneous optimization of multiple design criteria, including transmission efficiency and overall size. Optimizing gearbox design involves more than selecting an appropriate gear configuration; it necessitates a complete approach that balances performance with size, ensures sustainable operation, and minimizes manufacturing costs. This study was conducted to develop a method for solving the multi-objective optimization problem (MOOP) related to the design of a two-stage helical gearbox (TSHG) containing double gears in the second stage (DGSS). The primary focus is on two single-objective functions: maximizing gearbox efficiency and minimizing the gearbox bottom area. This study investigates three primary design parameters: the gear ratio of the first stage (u_1), the gear width coefficient of the first stage (X_{ba1}), and second stage (X_{ba2}). The optimization process was carried out in two distinct stages. The initial phase focused on the single-objective optimization problem aimed at minimizing the gap among the variable levels. The second stage concentrated on dealing with the MOOP to identify the optimal design parameters. The Simple Additive Weighting (SAW) method was employed to solve the multi-criteria decision-making (MCDM) problem, while the MEREC technique was utilized to establish the weights of the criteria. The implementation of SAW in this context introduces a novel methodology that streamlines the identification of the optimal solution while enhancing the precision of the outcomes. Moreover, addressing the MOOP problem in a two-stage approach reduces the solution process and enhances the precision of the outcomes. The proposed optimized values of the primary design parameters aim to enhance gearbox efficiency and maximize installation space, thereby facilitating potential applications across diverse industries

Keywords: SAW method, MEREC method, helical gearbox, gear ratio, gearbox efficiency, gearbox bottom area, MCDM, MOOP

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1. Introduction

In mechanical engineering, the transmission system is essential to the operating performance and reliability of machinery used in industry. A conventional mechanical transmission system includes a motor, a gearbox, and interconnecting components such as couplings, V-belts, or chains. The gearbox serves a fundamental function by decreasing speed and transmitting torque from the motor shaft to the working shaft [1].

Gear transmissions have significantly superior efficiency compared to alternative mechanical transmission methods, as well as electric, hydraulic, and pneumatic systems. Due to their exceptional performance, gearboxes have emerged as the predominant solution for translating rotary motion from the primary motor to the transmission system, serving an essential purpose in numerous industrial applications. The ongoing advancement of gearbox technology enhances efficiency and dependability, addressing the rising needs in production and operation [2].

The primary advantage of two-stage helical gearboxes is the twin gear configuration in the second stage, which yields significant benefits including enhanced load distribution, less operational noise, and improved system longevity. In heavy sectors including machine manufacture, metallurgy, mining, and wind power generation, this sort of gearbox is crucial for ensuring steady operation, reducing wear, and optimizing energy usage. To attain an optimal gearbox design, engineers must examine various elements, including dimensions, transmission efficiency, durability, and manufacturing costs. Identifying a solution that reconciles these elements continues to be a significant problem for both design engineers and researchers. Therefore, studies devoted to multi-objective optimization of gearboxes are scientific relevance.

2. Literature review and problem statement

Numerous researches on the best design of two-stage helical gearboxes have been conducted too far. The authors in [3] introduced an innovative way to address the Multi-Objective Optimization Problem (MOOP) for the design of a THHG utilizing the Taguchi method and Grey Relation Analysis (GRA). The aim of this study is to determine the ideal values of u_1, X_{ba1}, and X_{ba2} to optimize gearbox efficiency and reduce gearbox mass. However, this study is only for TSHG, not research on TSHG with DGSS.

Recently, MCDM methods have been employed to address MOOP in order to identify the optimal primary design factors for TSHG with SSDG, utilizing the Multi-Attributive Ideal-Real Comparative Analysis (MAIRCA) approach [4] or the Evaluation by an Area-based Method of Ranking (EAMR) method [5], as well as for TSHG through the Measurement of Alternatives and Ranking according to COmpromise Solution (MARCOS) technique [6]. These studies propose optimal values for key design factors (u_1, X_{ba1}, and X_{ba2}) aimed at enhancing gearbox efficiency and minimizing gearbox volume (as discussed in [4] and [6]) or reducing gearbox mass (as noted in [5]). Currently, there is a lack of research utilizing the MCDM method to address the MOOP problem for a TSHG with DGSS focused on enhancing efficiency and minimizing gearbox bottom area.

The authors in [7] employed a combination of the min-max approach and univariate search method to optimize a two-stage spur gear train. The objective functions considered were gear volume, center distance, and five dynamic variables of the shaft and gear. In [6] examined four objective functions, namely the lowest size, weight, tooth deflection, and maximum life of a spur gear pair, using the Modified Iterative Weighted Tchebycheff (MIWT) method. The drawback of this strategy is in the challenge of achieving convergence, which is conditional upon the initial sample vector. Additionally, the time required for the solution to converge is frequently excessive. Besides, an optimization work using an enhanced Genetic Algorithm (GA) to optimize the weight of a spur gear pair was implemented [8, 9]. In this study, the bending strength of the tooth, interference, and torsional strength of shafts as active constraints were taken into account. The authors in [10] conducted a study on the multi-objective optimization of the three-stage spur gear train using interactive physical programming. The design objectives included minimizing volume, maximizing surface fatigue life, and maximizing load capacity. The basic procedures of the MATLAB Optimization Toolbox to reduce the volume of the gearbox have been applied in [11]. The graphs derived from the optimization findings were used to estimate various parameters such as the number of stages, face width of gears, and shaft diameters by selecting different inputs such as input power, gear ratio, and the hardness of the gear materials. Nevertheless, the capacity of the gearbox casing was estimated under the assumption that it has a consistent thickness throughout.

A multi-objective optimization study on a two-stage spur gearbox, using tribological limitations and power loss as objective functions, in addition to volume minimization was conducted [12]. The results of the study led to the development of a safer design with reduced power losses. In addition, a multi-objective optimization problem involving two-stage helical gear sets in a helical hydraulic rotary actuator was introduced in [13]. The authors employ the Non-Dominated Sorting Genetic Algorithm II approach (NSGA-II) to tackle this problem. This study focused on examining three specific objectives: volume, transmission efficiency, and maximum contact stress. The results indicate that there is a compromise between the highest level of stress on a surface and the amount of space available in the optimal area. Several scientists [14] devised a method to optimize the design of spur gear sets to minimize gear weight, reduce contact stress over the whole contact path, and achieve an appropriate film thickness at the contact site. This study utilized a combination of particle swarm optimization, particle swarm optimization-based teaching learning optimization, and Jaya approaches to achieve a substantial reduction in weight and contact stress of a profile-modified spur gear set. The aim was to provide an adequate film thickness at the contact site. The study's findings indicate that gear design is greatly enhanced by utilizing optimal addendum coefficient values inside the designated design space, as opposed to traditional designs. The authors in [15] undertook a study to optimize both an electric motor and a gearbox together to build a drive system specifically for electric vehicles. The primary goals of this effort are to minimize both the overall energy loss and the weight of the driving system. The optimization results are contrasted with earlier results to illustrate the additional possibilities of cooperative optimization. According to the paper, optimizing the entire drive system results in improved efficiency when the gear ratio is increased.

The Taguchi and Grey relation analysis (GRA) techniques were utilized to examine the multi-objective optimization issue of constructing a two-stage helical gearbox [3]. This study selected two objectives: minimizing the mass of the gearbox and maximizing its efficiency. The study's results were utilized to establish the optimal values for the five essential design components involved in the production of a two-stage helical gearbox. In [16], same methods were also utilized to reduce the cross-sectional area of the gearbox and enhance its efficiency. The techniques described in [17] were used to optimize a two-stage helical gearbox with second stage double gear-sets, with the goal of improving efficiency and reducing gearbox mass. In a recent study [18], the TOPSIS technique was employed to solve the multi-objective optimization issue of a two-stage helical gearbox. The aim of this project is to reduce the cross-sectional area of the gearbox and enhance its efficiency. The authors in [4] employed the Multi-Attributive Ideal-Real Comparative Analysis (MAIRCA) approach to do a multi-objective optimization of a two-stage helical gearbox with second-stage double gear sets. The objective of the study is to identify the optimal key design elements that can improve the efficiency of the gearbox while simultaneously reducing its size.

The studies [3, 4, 17–19] focus on the outcomes of solving the MOOP problem to determine the optimal main design parameters. Studies [3, 16, 17] employ the Taguchi and GRA methods, whereas study [4, 18] utilize the MCDM method, specifically the TOPSIS and MAIRCA techniques. The studies focus on enhancing gearbox efficiency while minimizing gearbox mass (as noted in [3, 17]), reducing gearbox cross-sectional area (as indicated in [16, 18]) or decreasing gearbox volume (as reported in [4]). Nonetheless, there has been no research using SAW method to solve MOOP for TSHG with DGSS aimed at reducing bottom area and enhancing gearbox efficiency.

Numerous studies on the optimization of various gearbox types have been conducted to date. The primary design parameters for a TSHG with DGSS have not been established in any study employing the SAW approach to address the MOOP with two distinct objectives: maximizing efficiency and minimizing gearbox bottom area. The complexity of employing the MCDM method to address the MOOP with two distinct objectives might result from the programming challenges associated with calculating both single and multi-objective functions, particularly when there is a substantial gap between variable levels. Utilizing the approach of solving the problem through both single and multi-objective stages, as demonstrated in [4, 18], supports the idea that investigating the application of the SAW method to resolve the MOOP for enhancing efficiency and minimizing the gearbox bottom area of a TSHG with DGSS is both necessary and feasible.

3. The aim and objectives of the study

The aim of this study is to identify the optimal configuration of primary design parameters that can reduce the cross-sectional area and enhance the efficiency of the gearbox.

To achieve the aim, the subsequent objectives were established:

– determine the single objectives of the MOOP;

– analyze the impact of the primary design parameters on the response;

– solving the MOOP to identify the primary design parameters.

4. Materials and methods

4. 1. Object and hypothesis of the study

This study examines two critical variables in gearbox design: spatial efficiency and energy efficiency. A compact gearbox design conserves material and diminishes total weight while optimizing installation space, thereby enhancing applications that necessitate great flexibility and integration. Simultaneously, enhancing transmission performance contributes to increased energy efficiency, less power loss, and prolonged system longevity, aligning with the trend of sustainable and eco-friendly technological advancement.

Despite several studies on gearbox design optimization, the majority have not concentrated on multi-objective optimization techniques that simultaneously address the dual challenges of minimizing the gearbox base area and enhancing transmission efficiency in two-stage helical gearboxes. This work seeks to address that deficiency by employing the SAW approach, a MCDM technique, to systematically assess and enhance gearbox designs. This method facilitates the selection of best design solutions based on critical technical criteria, ensuring feasibility and efficiency in practical application.

The investigation and implementation of multi-objective optimization techniques enhance the gearbox design process and significantly contribute to the performance enhancement of mechanical systems in industrial production. This study presents a scientific analytical methodology that balances critical design aspects and demonstrates practical application in addressing current technical issues. The research findings enhance the domain of mechanical design and offer an optimal methodology for engineers and researchers, facilitating improved operational performance and promoting sustainability in contemporary transmission systems.

The main hypothesis of this study is that the gearbox utilizes oil bath lubrication, which facilitates effective lubrication. This allows the design of gears based on contact strength to avoid fatigue peeling. Furthermore, to simplify the calculation process, the shaft lengths within the gearbox are assumed to be identical. Also, the bearings utilized in the gearbox are only angular contact ball bearings.

4. 2. Objective functions and constraints

4. 2. 1. Objective functions

In this work, the MOOP consists of the following objectives:

– minimizing gearbox bottom area:

$$\min f_1(X) = A_b; \quad (1)$$

– maximizing gearbox efficiency:

$$\min f_2(X) = \eta_{gb}. \quad (2)$$

Let X denote the vector that the design variables are represented by. The five main design parameters of a two-stage helical gearbox are u_1, X_{ba1}, X_{ba2}, AS_1, and AS_2 [19]. Additionally, studies showed a relationship between the maximum values and the optimum values of AS_1 and AS_2 [19]. Therefore, u_1, X_{ba1}, and X_{ba2} were the three main design features that were used as variables in this study's optimization problem. As a result, let's currently distribute:

$$X = \{u_1, X_{ba1}, X_{ba2}\}. \quad (3)$$

The specific values of u_1, X_{ba1}, and X_{ba2} are easily determined according to [18].

4. 2. 2. Constrains

For i^{th} stage of this gearbox ($i=1÷2$), $u_i=1÷9$; $X_{bai}=$ $=0.25÷0.4$ [20]. Hence, the MOOP has the following constraints:

$$1 \le u_i \le 9, \quad (4)$$

$$0.25 \le X_{bai} \le 0.4. \quad (5)$$

The values of u_i and X_{bai} must be in the above range.

4. 3. Method for solving MOOP

The effort tries to accomplish two goals: decreasing the lower region of the gearbox and improving its efficiency. Table 1 presents the three essential design elements that serve as input for this research. Furthermore, the approach outlined in reference [20] was employed to address the MOOP. The procedure for performing this operation is illustrated in Fig. 1.

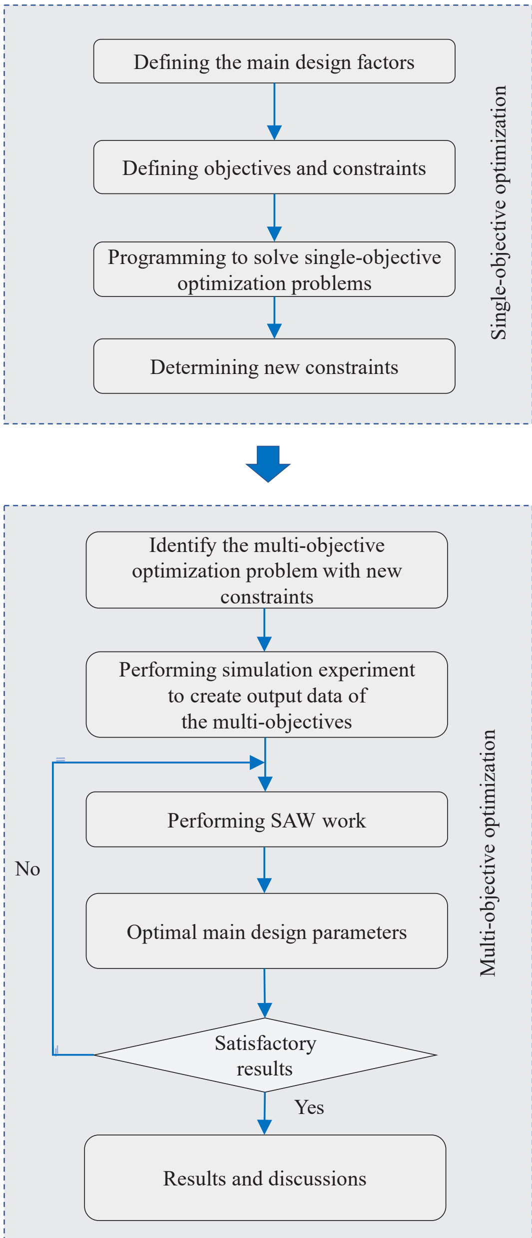

Fig. 1. The process to solve MOOP

The approach to solve the MOOP comprises two distinct stages: To solve the single-objective optimization issue, the first step is to minimize the discrepancies between the input variables, as specified in Table 1. Also, the single-objective optimization problem through the application of a direct-search methodology. This method is employed due to its straightforward programming and its ability to converge rapidly to an optimal solution. The next phase, however, seeks to tackle the multi-objective optimization problem by selecting the most optimal core design variables. If the levels of the variables in the second stage change by less than 0.02, the SAW issue will be reassessed by taking into account the smaller difference between the two levels of the input components.

Table 1

Input parameters

Parameter	Minimal value	Maximal value
u_1	1	9
X_{ba1}	0.25	0.4
X_{ba2}	0.25	0.4

4. 4. Method to solve MCDM problem

The SAW approach was employed in this study to address the MCDM task. In order to effectively implement this strategy, it is crucial to follow strictly to the subsequent rules [21]:

– build the initial decision-making matrix:

$$X = \begin{array}{c} \\ A_1 \\ A_2 \\ \dots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \dots & C_n \end{array} \\ \begin{bmatrix} y_{11} & y_{12} & \dots & y_{1n} \\ y_{21} & xy_{22} & \dots & y_{2n} \\ \dots & \dots & \dots & \dots \\ y_{m1} & y_{m2} & \dots & y_{mn} \end{bmatrix} \end{array}. \tag{6}$$

where m and n are option and criterion numbers;

– find the normalized matrix by:

$$n_{ij} = \frac{r_{ij}}{\max r_{ij}}, \tag{7}$$

$$n_{ij} = \frac{\min r_{ij}}{r_{ij}}. \tag{8}$$

Let's note that (7) is used for the gearbox efficiency objective, and (8) for the gearbox bottom area;

– determine the preference value for each alternative:

$$V_i = \sum_{j=1}^{n} w_j \cdot n_{ij}; \tag{9}$$

– rank the option's order by maximizing V_i.

4. 5. Method to calculate the criteria weights

The criteria weights for the current research were derived using the MEREC technique. The implementation of this strategy requires the performance of the following procedures [22]:

1. Create the initial matrix using the identical procedure employed at the start of the SAW method.

2. Determine the normalized values of the elements in the matrix by adhering to the process outlined below:

– for gearbox efficiency target:

$$h_{ij} = \frac{\min x_{ij}}{x_{ij}}; \tag{10}$$

– for gearbox bottom area target:

$$h_{ij} = \frac{x_{ij}}{\max x_{ij}}. \tag{11}$$

3. Calculate the effectiveness of the options S_i by:

$$S_i = \ln\left[1 + \left(\frac{1}{n}\sum_j \left|\ln\left(h_{ij}\right)\right|\right)\right]. \tag{12}$$

4. Determine the efficiency of the i^{th} option S'_{ij} by:

$$S'_{ij} = \ln\left[1+\left(\frac{1}{n}\sum_{k,k\neq j}\left|\ln\left(h_{ij}\right)\right|\right)\right]. \quad (13)$$

5. Compute the removal effect of the j^{th} criterion E_j:

$$E_j = \sum_j \left|S'_{ij} - S_i\right|. \quad (14)$$

6. Calculate the criteria's weight by:

$$w_j = \frac{E_j}{\sum_k E_k}. \quad (15)$$

The creatia weight calculated by (15) is used when using the SAW technique to solve the MOOP.

5. Results of solving optimization problem

5. 1. Results of determining single objectives

5. 1. 1. Determining gearbox bottom area

For this gearbox, the bottom area A_b is calculated by (Fig. 2):

$$A_b = \left(L \cdot B\right). \quad (16)$$

In the above equation, L and B are found in Fig. 1:

$$L = d_{w11} + \frac{d_{w21}}{2} + \frac{d_{w12}}{2} + d_{w22} + 2\cdot\delta, \quad (17)$$

$$B = b_{w1} + b_{w2} + 2\cdot\delta, \quad (18)$$

in which, δ=7÷10 mm [15]; b_{wi}, d_{w1i}, d_{w2i} are with of the gear, the pitch diameter of the pinion and the gear of i-th stage (i=1÷2) which are determined by:

$$b_{wi} = X_{bai}\cdot a_{wi}, \quad (19)$$

$$d_{w1i} = 2\frac{a_{wi}}{\left(u_i+1\right)}, \quad (20)$$

$$d_{w2i} = 2\cdot\frac{a_{wi}\cdot u_i}{\left(u_i+1\right)}. \quad (21)$$

In which, X_{bai} and a_{wi} (i=1÷2) denote the coefficient of wheel face width and the gearbox center distance of stage i; a_{wi} is found by [19]:

$$a_{wi} = k_a\cdot\left(u_i+1\right)\cdot\sqrt[3]{\frac{T_{1i}\cdot k_{H\beta}}{\left[AS_i\right]^2\cdot u_i\cdot X_{bai}}}, \quad (22)$$

where AS_i is the allowable contact stress of stage i; T_{1i} (i=1÷2) is the torque on the pinion of i^{th} stage which is found by:

$$T_{11} = \frac{T_{out}}{\left(u_{gb}\cdot\eta_{hg}^2\cdot\eta_b^3\right)}, \quad (23)$$

$$T_{12} = \frac{T_{out}}{\left(2\cdot u_2\cdot\eta_{hg}\cdot\eta_{be}^3\right)}, \quad (24)$$

where T_{out} is the output torque (Nmm).

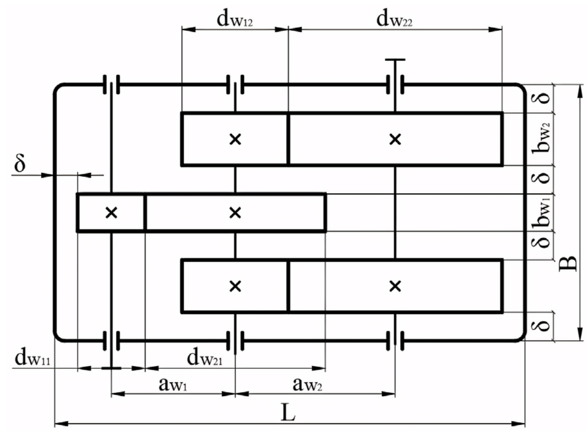

Fig. 2. For finding the gearbox bottom area

5. 1. 2. Calculating the gearbox efficiency

The gearbox efficiency (%) can be calculated by:

$$\eta_{gb} = 100 - \frac{100\cdot P_l}{P_{in}}. \quad (25)$$

In (25), P_l is the total power loss in the gearbox; P_l is computed by [2]:

$$P_l = P_{lg} + P_{lb} + P_{ls} + P_{z0}, \quad (26)$$

in which, P_{lg}, P_{lb}, P_{ls}, and P_{z0} represent the power loss in the gearings, in bearings, in seals, and in the idle motion; These elements can be determined as in [19].

5. 2. Results of analyzing the impact of the primary design parameters on the response

The direct search approach was used in this study to optimize a single objective. Furthermore, a computational program implemented in Matlab was utilized to evaluate two distinct single-objective issues: enhancing η_{gb} and minimizing Ab. The program's findings can be succinctly summarized in the following observations: Fig. 7 illustrates the association between variable u_1 and variable A_b. The value of u_1 that yields the best results (as shown in Fig. 3) is the one that corresponds to the lowest value of A_b. The findings align with the results presented in reference [2] regarding the gearbox volume function, as well as with reference [3] concerning the gearbox mass function. Fig. 4 depicts the correlation between η_{gb} and u_1. From the Fig. 8, it is clear that there is a specific value of u_1 where η_{gb} reaches its maximum value, indicating an optimal value. Furthermore, Fig. 5 depicts the relationship between the ideal values of u_1 and the u_{gb}. The restrictions for the variable u_1 were determined based on the optimal values of u_1, as specified in Table 2.

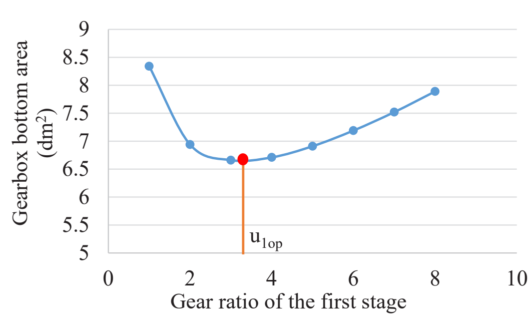

Fig. 3. Relationship between u_1 and A_b

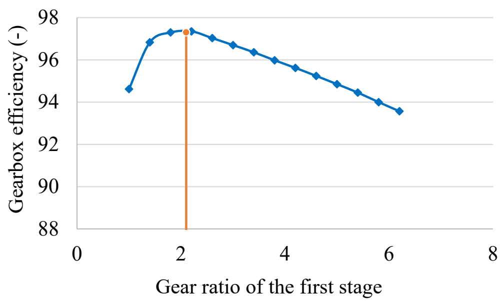

Fig. 4. Relation between u_1 and η_{gb}

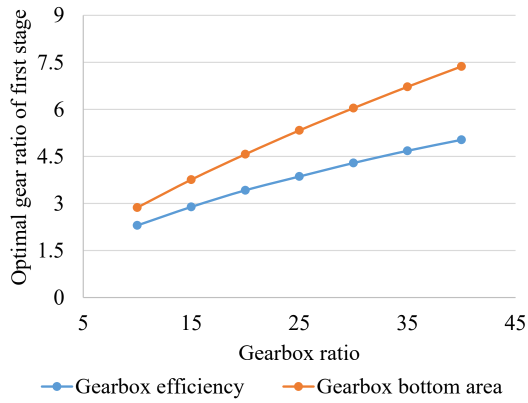

Fig. 5. Relation between u_{gb} and optimum u_1

Table 2

New restrictions of u_1

u_{gb}	u_1	
	Lower limit	Upper limit
10	2.20	2.97
15	2.79	3.86
20	3.32	4.67
25	3.76	5.43
30	4.19	6.14
35	4.58	6.82
40	4.93	7.47

Table 2 provides the input data for u_1 to address the MOOP. The use of this method can significantly reduce the time required to execute computer programs.

5. 3. Results of solving multi-objective optimization problem to identify the primary design parameters

A computer program has been created to do simulation experiments. The investigation examined a range of values for u_{gb}, which varied from 10 to 40 with steps of 5. Below are the solutions for the issue with u_{gb}=40. The gearbox ratio that had been previously set was utilized for 125 initial testing cycles, as specified in Section 3. The experiment will provide output data that will be used as input parameters for the SAW algorithm to solve the MOOP, notably focusing on the efficiency of the gearbox bottom section. This method will continue until the discrepancy between the two levels of each variable is below 0.02. Table 3 presents the primary design parameters and output responses for the fifth and final iteration of the SAW experiment, with an u_{gb} value of 40. The weights for the criterion were established utilizing the MEREC technique, as elucidated in Section 3. 3, subsequently: the values h_j were standardized using (10), (11). The values S_i and S'_{ij} were derived from (12) and eq. (13). The elimination impact of the criterion was calculated using (14). The criteria weights, w_j, were obtained using the utilization of (15). Section 3. 2 provides guidance on the optimal utilization of the SAW technique to tackle the challenges associated with MCDM. Commence the process by computing the decision-making matrices utilizing (6). The original matrix should be normalized using equations (7), and (8). Afterwards, the calculation of V_i is carried out using the eq. (9). Ultimately, organize the options in a way that ensures the most advantageous solution has the highest V_i.

The results of the option ranking and parameter computation using the SAW approach are shown in Table 4 (for the final iteration of SAW analysis). According to the table, option 51 is the most optimal decision out of all the available possibilities. The optimal values for the crucial design components are u_1=5.67, X_{ba1}=0.25, and X_{ba2}=0.25, as specified in Table 5. For the first time, the values of the primary design factors for a two-stage helical gearbox with double gears in the second stage have been satisfactorily ascertained using the SAW approach. Fig. 6 depicts the relationship between the ideal values of u_1 and u_{gb}:

$$u_1 - 0.1093 \cdot u_{gb} + 1.4405. \quad (27)$$

The given regression equation, which has an R^2 coefficient of determination of 0.9918, can be used to calculate the optimal values of u_1.

Equation (12) determines the optimal gear ratio for the first stage in the design of a two-stage helical gearbox, which incorporates two gears in the second stage to achieve minimal bottom area and maximum gearbox efficiency simultaneously. These results can be used to design a two-stage helical gearbox having two gears in the second stage in practice or for mechanical engineering students when doing their projects.

Table 3

Input factors and output results for u_{gb}=40

Trial.	u_1	X_{ba1}	X_{ba2}	A_b (dm^2)	η_{gb} (%)
1	5.64	0.25	0.25	7.15	94.95
2	5.64	0.25	0.29	7.32	94.87
3	5.64	0.25	0.33	7.49	94.82
4	5.64	0.25	0.36	7.66	94.75
5	5.64	0.25	0.40	7.82	94.68
6	5.64	0.29	0.25	7.19	94.11
...	...	...	...	...	...
50	5.66	0.40	0.40	7.90	90.66
51	5.67	0.25	0.25	7.14	94.92
52	5.67	0.25	0.29	7.32	94.84
...	...	...	...	...	...
80	5.69	0.25	0.40	7.81	94.63
81	5.69	0.29	0.25	7.18	94.05
82	5.69	0.29	0.29	7.35	93.98
...	...	...	...	...	...
123	5.70	0.40	0.33	7.62	90.71
124	5.70	0.40	0.36	7.76	90.64
125	5.70	0.40	0.40	7.90	90.57

Table 4

Calculated results and rankings of options by SAW for u_{gb}=40

Trial.	n_{ij}		V_i	Rank
	A_b	η_{gb}		
1	0.9986	1.0000	0.9995	4.0000
2	0.9754	0.9992	0.9910	13.0000
3	0.9533	0.9986	0.9830	23.0000
4	0.9321	0.9979	0.9753	41.0000
5	0.9130	0.9972	0.9683	58.0000
6	0.9930	0.9912	0.9918	7.0000
...	...	...	...	...
50	0.9038	0.9548	0.9373	121.0000
51	1.0000	0.9997	0.9998	1.0000
52	0.9754	0.9988	0.9908	15.0000
...	...	...	...	...
80	0.9142	0.9966	0.9683	57.0000
81	0.9944	0.9905	0.9919	6.0000
82	0.9714	0.9898	0.9835	20.0000
...	...	...	...	...
123	0.9370	0.9553	0.9490	110.0000
124	0.9201	0.9546	0.9428	120.0000
125	0.9038	0.9539	0.9367	125.0000

Table 5

Optimum main design factors

No.	u_{gb}						
	10	15	20	25	30	35	40
u_1	2.38	3.10	3.74	4.25	4.80	5.28	5.67
X_{ba1}	0.25	0.25	0.25	0.25	0.25	0.25	0.25
X_{ba2}	0.25	0.25	0.25	0.25	0.25	0.25	0.25

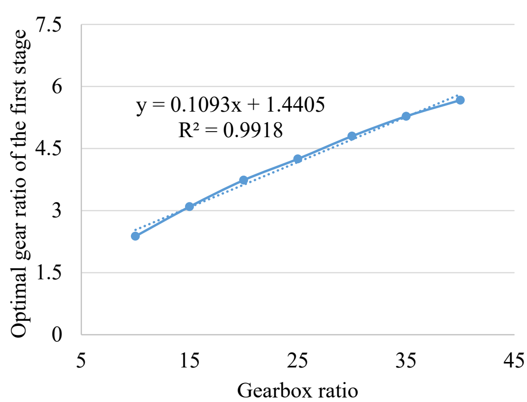

Fig. 6. Relation between optimum u_1 and u_{gb}

Table 5 shows that X_{ba1} and X_{ba2} consistently choose the lowest values (=0.25), even as u_{gb} increases. This is because the gearbox bottom area is the smallest with the smallest X_{ba1} and X_{ba2}. Additionally, a first-order link between the ideal values of u_1 and u_{gb} is noted (equation (12)). Additionally, this first-order functional form agrees with the findings of earlier research, including [16, 17]. When the value of u_{gb} is known, this functional form is particularly useful for figuring out the ideal u_1. Furthermore, equation (12) fits the data derived from the simulation experiment quite well, as evidenced by the R^2 coefficient of determination of 0.9918.

6. Discussion the results of multi-objective optimization problem

The findings of this study illustrate the effectiveness of the direct search technique in optimizing a single objective related to transmission efficiency. The MATLAB-based computational program effectively assessed two separate optimization issues: maximizing of gearbox efficiency (η_{gb}) and minimizing of gearbox volume (Ab). The findings underscore significant correlations between the decision variable u1 and the performance metrics of the system. Fig. 3 illustrates that the optimal value of u_1 is associated with the minimum Ab, thereby verifying prior studies concerning the volume and mass functions of gearboxes. This demonstrates the importance of selecting an appropriate u_1 in order to minimize the size of the transmission while preserving its structural integrity. In the same way, Fig. 4 depicts the relationship between η_{gb} and u_1, indicating that efficiency reaches its maximum at a particular value of u_1, after which additional increases may adversely affect performance. This implies the presence of an optimal design configuration that harmonizes efficiency with mechanical constraints. Moreover, Fig. 5 illustrates the correlation between the optimal values of u_1 and u_{gb}, thereby serving as a reference for establishing viable ranges for u_1. The constraints delineated for u_1, as presented in Table 2, facilitate a systematic methodology in multi-objective optimization (MOOP), thereby empowering designers to attain optimal trade-offs between efficiency and volume reduction.

The study's results, utilizing the SAW method, indicated that option 51 appeared as the most optimal option among those evaluated, as shown in Table 4. The optimal values for the key design parameters of the TSHG with DGSS were identified as u_1=5.67, X_{ba1}=0.25, and X_{ba2}=0.25, as presented in Table 5. This study successfully identifies the primary design factors of this type of gearbox using the SAW method, thereby validating the method's effectiveness in optimizing design selection.

Fig. 6 illustrates the correlation between the ideal values of u_1 and u_{gb}, as defined by the regression equation (12). The coefficient of determination for this equation is R^2=0.9918, indicating a strong correlation between the regression model and the experimental data. This factor is significant as it enables the prediction of the optimal value of u_1 based on the known value of u_{gb}. Equation (12) defines the optimal gear ratio for the first stage of TSHG with DGSS, aiming to minimize base area while maximizing gearbox efficiency. This result is applicable in the practical design of this type of gearbox and serves as a reference for mechanical engineering students undertaking course projects.

By cutting down on processing time, the direct search approach significantly increases computing efficiency. Furthermore, it is easier to simplify the computational process and increase the accuracy of the results when the optimization process is implemented in two distinct stages. The first stage concentrates on single-objective optimization to reduce the gap between variable levels, while the second stage solves a multi-objective optimization problem (MOOP) to identify the optimal design parameters.

In conclusion, this study shows the practical importance of the used method in gearbox design in addition to proving its effectiveness. In order to achieve a more thorough and useful optimization technique, this approach can be expanded in the future by incorporating more significant design considerations such manufacturing cost and durability of gearboxes.

6. Conclusions

1. Two single objectives of the MOOP including maximal gearbox efficiency and minimal gearbox bottom area were identified. These two single-objective functions and restrictions were then used to establish and solve the MOOP.

2. The influence of three main design factors (u_1, X_{ba1}, and X_{ba2}) were analyzed. It was found that there is an optimal value of u_1 at which the gearbox efficiency and the bottom area have the optimal values. When designing TSHG with DGSS, the values of X_{ba1} and X_{ba2} are crucial, particularly when it comes to minimizing the gearbox bottom area. The findings demonstrate that even when u_{gb} grows, both parameters consistently attain the lowest value (0.25), optimizing the design space. These values' stability attests to their contribution to increase efficiency and decrease overall size of the gearbox.

3. The primary design factors for a two-stage helical gearbox involving two gears in the second stage are presented in the table and regression formula. The proposed model for determining u_1 demonstrates a high level of consistency with validated data, as evidenced by a coefficient of determination (R^2) of 99.18 %. This outcome can be used in the actual design of this kind of gearbox or as a guide for students studying mechanical engineering who are working on course projects.

Conflict of interest

The authors declare that they have no conflict of interest in relation to this research, whether financial, personal, authorship or otherwise, that could affect the research and its results presented in this paper.

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Data availability

The manuscript has no associated data.

Use of artificial intelligence

We confirm that we did not use artificial intelligence technologies when creating the current work.

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