

The object of this study is a helical sweeping surface or a helicoid torso and the process of its design according to predefined structural parameters. Helical surfaces are widely used in engineering practice. They have become widespread in devices for transporting various materials, as well as in agricultural machinery. The problem is that when they are manufactured, the technique of their formation from the point of view of analytical description is not always taken into account. Helical surfaces can be linear and nonlinear. Linear surfaces, or helicoids, are formed by the helical motion of a straight-line generatrix around an axis, and the generatrix can intersect it or be coincident. If the straight-line generatrix intersects the axis at a right angle, then the helicoid will be a helical conoid, which is very common in technology under the name of a screw. Certain conditions are imposed on the helical motion of the straight-line generatrix of a helicoid torso. Its main advantage among other helicoids is the possibility of constructing an exact sweep. All other helicoids cannot be swept. For their manufacture, an approximate sweep is found, which is deformed into the desired shape. At the same time, the energy intensity of the process of deformation of this sweep into the finished product increases due to overcoming significant plastic deformations.

As a result of this research, dependences were established that make it possible to construct a set of helicoid torsos that pass through the predefined helical line. The results are based on the differential characteristics of the surface. These are their distinctive features from known results, according to which only one helicoid torso corresponds to the predefined helical line.

This paper shows the practical application of the helicoid torso as a supporting turn of the narrowing screw of a forage harvester with a radius of  $R=0.25$  m of the outer edge and  $r=0.125$  m of the inner one

**Keywords:** helicoid torso, return edge, parametric equations, tapering screw, support coil

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# MATHEMATICAL DESCRIPTION OF WINDING HELICOID SECTION CONSTRUCTION BASED ON THE PREDEFINED STRUCTURAL PARAMETERS

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## 1. Introduction

Helical surfaces are widely represented in various mechanisms and devices. In particular, the justification and calculation of the design and power parameters of screw loaders is proposed in [1]. In [2], a screw electrothermomechanical converter for additive manufacturing is considered. In [3], the helical surface is used as the surface of the cutter profile.

lation of the design and power parameters of screw loaders is proposed in [1]. In [2], a screw electrothermomechanical converter for additive manufacturing is considered. In [3], the helical surface is used as the surface of the cutter profile.

In [4], the helicoid torso is considered as the working body of a tillage tool for surface tillage of the soil. In [5], the study of helical working surfaces of auger mechanisms of agricultural machines was carried out. In [6], the design of helical knives of a forage harvester was proposed. In [7], the structure of a ramp in the form of a sweep helicoid was calculated. The practical significance of using such surfaces is important from two points of view. First, for the helicoid torso it is possible to find an exact sweep from sheet material. Secondly, the technology of manufacturing a surface coil is reduced to simple bending with minimal plastic deformations, that is, it is energy-saving. But in engineering practice, little attention is paid to the internal structure of helical surfaces from the point of view of differential geometry. It is clear that for a helicoid torso, the edge of the turn is a helix. Based on this, it is very easy to construct parametric equations of a helicoid torso. However, through the predefined helicoid, one can draw a set of helicoid torsos with different angles of inclination of the rectilinear generatrices, which makes it possible to choose the desired surface. Therefore, research in this area is relevant.

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## 2. Literature review and problem statement

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In [8], a screw electrothermomechanical converter is studied as a source of multiphysical effects on the technological environment. The authors consider the design of the converter and its ability to generate mechanical and thermal effects that contribute to increasing the efficiency of industrial processes. However, a limitation of the study is the lack of an analytical model due to its complexity in such a process. In [9], the structure of a screw electromechanical hydrolyzer for processing poultry by-products is investigated. The study focuses on optimizing the screw mechanism to increase the efficiency of organic waste processing but there is also no analytical description of this process for the same objective reason. This drawback is eliminated in [10], in which the oil extrusion process is analyzed, in particular, the effect of preliminary grinding of raw materials in a twin-screw extruder. The authors offer a detailed mathematical model and simulation results that confirm the optimization of this process. The main drawback is the limited shape of the proposed screw working body.

Study [11] considers the optimal parameters of articulated working bodies of screw conveyors. It provides a detailed analysis of the influence of different configurations on the efficiency of conveyor systems but does not take into account possible variations in the geometry of screw bodies and the features of their manufacture, which may be due to the authors' focus on standard designs and their parameters, without taking into account non-standard solutions and technological deviations during production.

In [12], a method for calculating the maximum torque when the screw of a screw conveyor jams is proposed. The study provides practical recommendations for the design of stronger conveyors; however, the variability of geometric shapes was not taken into account, which is probably due to the authors' desire to simplify the calculation model.

In [13], six techniques for manufacturing helical surfaces are given, including stamping, rolling, and winding. Paper [14] is aimed at manufacturing such surfaces by milling. However, the studies do not take into account that helical surfaces can be unfolded. The sweep of non-folding helicoids can only be approximate, and their bending into the desired

shape increases the energy intensity of the process. In [15, 16], surface plastic deformations and their influence on the microgeometry, structure, wear resistance, and efficiency of coatings were investigated. Work [17] considers the synthesis of energy-saving transport and technological systems with helical working bodies, and [18] – power losses in the rotor of a helical electrothermomechanical converter. The cited studies provide information on the optimization of various systems to increase their energy efficiency but do not take into account the influence of surface geometry on such indicators at the stage of their design. This may be due to the complexity of their modeling in computational methods.

In [19] it is proposed to use a helicoid torso as a tillage tool for surface tillage. The equation of a helicoid torso with a horizontal axis of rotation is given, but the relationship between the angle of elevation of the helical line and the angle of inclination of the rectilinear generating surfaces passing through this line is not considered. This is probably due to the authors' focus on the basic geometric parameters of the design, which affect the main functionality, without a detailed analysis of secondary factors. In [20], the movement of soil particles along the surface of a deployable helicoid with a horizontal axis of rotation and the predefined angle of attack is investigated. This study provides important theoretical information for the development of agricultural machinery, but the issues of designing such a working body remain out of focus. This is probably due to the authors' focus on analyzing the dynamics of particle motion, rather than on the design aspects of helicoid fabrication.

Our review of the literature above has demonstrated that regarding the design of a helicoid torso, available body of research does not reveal the possibility of its construction along the predefined helical line, on condition that not one but a set of helicoid torsos pass through it. This indicates the feasibility of conducting research to fill the identified gap.

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## 3. The aim and objectives of the study

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The aim of our research is a mathematical description of the design of helicoid torsos that must pass through the predefined helical line. This will make it possible to select a surface according to the predefined design parameters.

To achieve this goal, the following tasks were set:

- to derive parametric equations for the set of helicoid torsos that pass through the predefined helical line;
- to calculate the helicoid torso for the support surface of the narrowing auger of a forage harvester with the possibility of choosing options.

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## 4. The study materials and methods

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The object of our study is the helicoid torso and the process of its construction according to the predefined design parameters. The hypothesis of the study is the assumption that through the predefined helical line it is possible to draw a one-parameter set of helicoid torsos with different angles of inclination of the rectilinear generatrices. In this case, a simplification is introduced, in which the thickness of the helicoid torso surface is zero.

The study is based on the classical theory of the formation of sweep surfaces, according to which such a surface is formed by a set of rectilinear generatrices tangent to the spatial curve.

The curve itself in relation to the sweep surface is called the edge of the turn. If the edge of the turn is a helical line of constant pitch, then the formed surface is a helicoid torso. The parametric equations of the helical line with a vertical axis located on a cylinder of radius  $a$  are written as follows:

$$\begin{aligned} x &= a \cos \alpha; \\ y &= a \sin \alpha; \\ z &= a \alpha \operatorname{tg} \beta, \end{aligned} \quad (1)$$

where  $\alpha$  is an independent variable, which is the angle of rotation of the point around the axis of the helical line when it moves along it;  $\beta$  is the angle of elevation of the helical line – a constant value.

In order to draw tangents to the helical line (1), it is necessary to have projections of the unit direction vector, which specifies the direction of the tangent at the current point. The projections are determined by differentiating equations (1). After differentiating equations (1) and reducing the vector to the unit, its projections will be written as follows:

$$\{-\cos \beta \sin \alpha; \cos \beta \cos \alpha; \sin \beta\}. \quad (2)$$

Taking into account the projections of the directional unit vector (2), the surface of the helicoid torso will be written as follows:

$$\begin{aligned} X &= a \cos \alpha - u \cos \beta \sin \alpha; \\ Y &= a \sin \alpha + u \cos \beta \cos \alpha; \\ Z &= a \alpha \operatorname{tg} \beta + u \sin \beta, \end{aligned} \quad (3)$$

where  $u$  is the second independent variable of the surface of the helicoid torso, which is the length of the straight-line generatrix, the count of which starts from a point on the edge of the return.

## 5. Results of constructing a set of helicoid torso surfaces under the predefined initial conditions

### 5.1. Finding parametric equations of a helicoid torso for the predefined angle of elevation of rectilinear generatrices

If we cross the helicoid torso surface (3) with a horizontal plane at  $Z=0$  and equate the last equation in (3) to zero, we shall obtain the dependence  $u=u(\alpha)$ :

$$u = -\frac{a\alpha}{\cos \beta}. \quad (4)$$

By substituting (4) into (3), we can obtain the parametric equations of the known cross-section curve of the helicoid torso (involute of a circle of radius  $a$ ) lying in the horizontal plane:

$$\begin{aligned} x &= a \cos \alpha + a\alpha \sin \alpha; \\ y &= a \sin \alpha - a\alpha \cos \alpha; \\ z &= 0. \end{aligned} \quad (5)$$

In Fig. 1, a helical line (1) and an involute of a circle (5) are constructed with straight-line generatrices connecting the corresponding points on the curves.

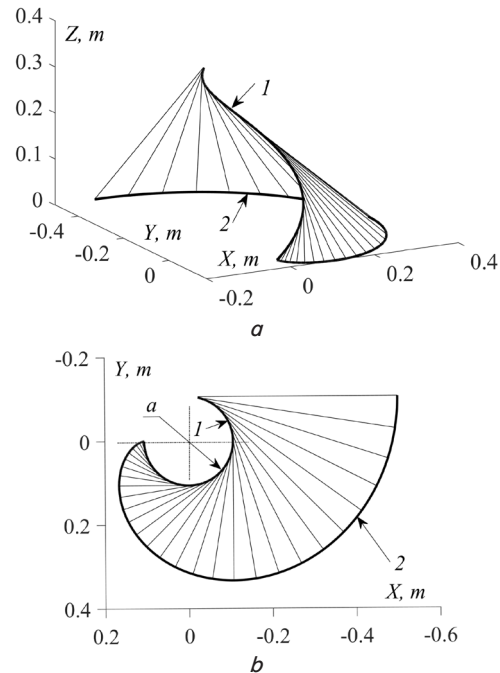


Fig. 1. Projections of the helicoid torso: 1 – helical line (return edge); 2 – involute of a circle (curve of the cross-section of the helicoid torso by a horizontal plane); a – axonometric image; b – horizontal projection

From Fig. 1 it is seen that the surface does not exist in the middle of the region bounded by the cylinder of radius  $a$ . If we intersect the surface (3) with a coaxial cylinder of radius  $R > a$ , then the radius  $R$  can be defined as the distance from the axis of rotation  $Z$  of the points of the surface (3) as follows:

$$R = \sqrt{X^2 + Y^2} = \sqrt{a^2 + u^2 \cos^2 \beta}. \quad (6)$$

After solving (6) with respect to  $u$ :

$$u = \frac{\sqrt{R^2 - a^2}}{\cos \beta}. \quad (7)$$

Substituting (7) into the equation of the helicoid torso (3) gives the line of intersection of the cylinder of radius  $R$  with its surface:

$$\begin{aligned} x &= a \cos \alpha - \sqrt{R^2 - a^2} \sin \alpha; \\ y &= a \sin \alpha + \sqrt{R^2 - a^2} \cos \alpha; \\ z &= \left( a\alpha + \sqrt{R^2 - a^2} \right) \operatorname{tg} \beta. \end{aligned} \quad (8)$$

Parametric equations (8) describe a helical line with a pitch angle  $\gamma$ , which is determined from the following expression:

$$\operatorname{tg} \gamma = \frac{a}{R} \operatorname{tg} \beta. \quad (9)$$

Fig. 2 shows that the generatrices of the helicoid torso (3) with an inclination angle  $\beta$  pass through a helical line with an angle of rise  $\gamma$  on a cylinder of radius  $R$ . But the helical line on a cylinder of radius  $R$  can be taken as an edge of the return and another helicoid torso with an inclination angle of the generatrices  $\gamma$  can be constructed.

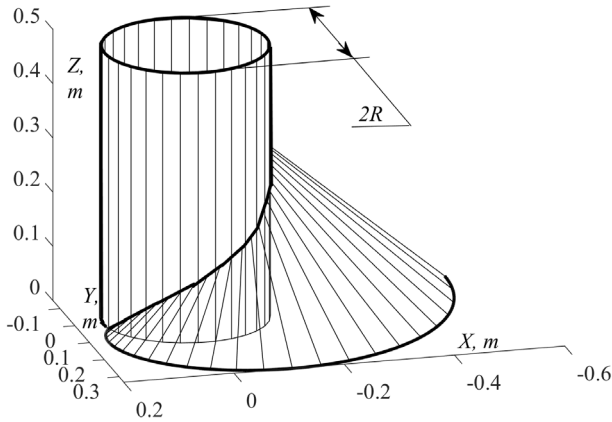


Fig. 2. Intersection of the surface of a helicoid torso with a coaxial cylinder

Thus, two helicoid torsos can pass through the helical line with the angles of inclination of the generatrices  $\beta$  and  $\gamma$ . It is evident that between these two angles there can be other values of angles. This means that through the predefined helical line it is possible to draw a set of helicoid torsos, which would differ from each other in the angle of inclination of the rectilinear generatrices to the horizontal plane.

From (9) it is necessary to determine the expression  $aa = R \tan \gamma \tan \beta$  and substitute it into equation (3). After that they take the following form:

$$\begin{aligned} X &= R \tan \gamma \tan \beta \cos \alpha - u \cos \beta \sin \alpha; \\ Y &= R \tan \gamma \tan \beta \sin \alpha + u \cos \beta \cos \alpha; \\ Z &= R \alpha \tan \gamma + u \sin \beta. \end{aligned} \quad (10)$$

Parametric equations (10) describe a helicoid torso that passes through the predefined helical line with a rise angle  $\gamma$ , which is located on a cylinder of radius  $R$ . The angle  $\beta$  specifies the inclination of the rectilinear generatrices to the horizontal plane. The count of their length  $u$  starts from the edge of the radius  $a = R \tan \gamma \tan \beta$ . However, it is more convenient to pass from the length of the generatrix  $u$  to a new variable  $\rho$  – the distance of the surface points from its axis. For equations (10), we can write:

$$\rho = \sqrt{X^2 + Y^2} = \sqrt{u^2 \cos^2 \beta + R^2 \tan^2 \gamma \tan^2 \beta}. \quad (11)$$

The solution of equation (11) with respect to  $u$  gives the following result:

$$u = \frac{1}{\cos \beta} \sqrt{\rho^2 - R^2 \tan^2 \gamma \tan^2 \beta}. \quad (12)$$

After substituting (12) into (10), the final result is as follows:

$$\begin{aligned} X &= R \tan \gamma \tan \beta \cos \alpha - \sqrt{\rho^2 - R^2 \tan^2 \gamma \tan^2 \beta} \sin \alpha; \\ Y &= R \tan \gamma \tan \beta \sin \alpha + \sqrt{\rho^2 - R^2 \tan^2 \gamma \tan^2 \beta} \cos \alpha; \\ Z &= R \alpha \tan \gamma + \sqrt{\rho^2 - R^2 \tan^2 \gamma \tan^2 \beta} \tan \beta. \end{aligned} \quad (13)$$

In parametric equations (13), one can set the bounds of the variable  $\rho$  within the limits  $\rho = \rho_1 \dots \rho_2$ . In this case, they

will describe the section of the helicoid torso between two coaxial cylinders of the specified radii.

The root expression in equations (13) indicates that there are restrictions between the ratio of radii and angles included in it. With the condition that the root expression cannot be negative:  $\rho \geq R \tan \gamma \tan \beta$ . The initial value  $\rho = R$ , i.e., in this case the limiting cylinder is the cylinder of radius  $R$ . The second limiting cylinder could be external ( $\rho > R$ ) or internal ( $\rho < R$ ). At  $\rho = R$ , we have  $R \geq R \tan \gamma \tan \beta$  or  $\gamma \geq \beta$ . If  $\gamma = \beta$ , then at  $\rho = R$ , parametric equations (13) will describe the predefined helical line, which will be an edge of the helicoid torso. Thus, the minimum possible angle of inclination of the generatrices  $\gamma$  is equal to the angle  $\beta$  of the rise of the given helical line ( $\gamma = \beta$ ), and in general  $\gamma \geq \beta$ . This applies to the case when one limiting cylinder is a cylinder of radius  $R$ , and the second is a cylinder of radius  $\rho > R$ . If the second bounding cylinder is a cylinder of radius  $\rho < R$ , then in the root expression it is necessary to swap  $\rho$  and  $R$ . After that, we can obtain:  $\gamma \geq \arctan(R \tan \beta / \rho)$ . If  $\rho = R$ , then  $\gamma \geq \beta$ , that is, the result is similar to the previous case. Fig. 3 shows an axonometric example of constructing two helicoid torsos passing through a common helical line of radius  $R = 5$  and elevation angle  $\beta = 10^\circ$ . The torsos are bounded by external cylinders (i.e., at  $\rho > R$ ). Therefore, the minimum possible value is  $\gamma = \beta = 10^\circ$ . One helicoid torso is constructed at  $\gamma = 60^\circ$ , the other at  $\gamma = 11^\circ$ . Each torso corresponds to its own turning edge. It can be seen from Fig. 3 that as the angle  $\gamma$  decreases, the turning edge approaches the given helical line and coincides with it at the minimum value  $\gamma = 10^\circ$ .

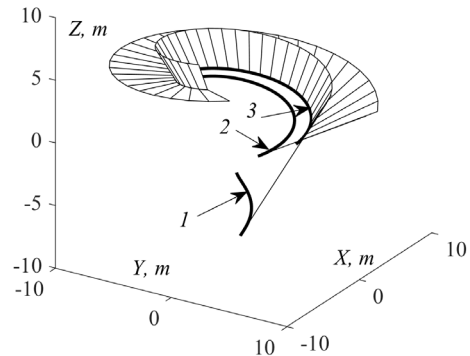


Fig. 3. Two helicoid torsos that pass through the predefined helical line 3 and have different angles of inclination of the rectilinear generatrices: 1 – the turning edge for the first torso; 2 – the turning edge for the second torso

The turning edges are located inside the bounding cylinder of radius  $R$  and lie on its surface at the minimum value of the angle  $\gamma$ . The same happens if the radius of the bounding cylinder  $\rho = r < R$ . Then the turning edges are located inside the cylinder of radius  $r$ .

## 5. 2. Calculation of the helicoid torso for the supporting surface of the narrowing screw of a forage harvester

Fig. 4, a shows a fragment of the narrowing screw, and Fig. 4, b – an axonometric image.

The given helical line is taken to be the line of intersection of the working and supporting surfaces, which is shown in Fig. 4, b by a thick line. The approximate dimensions of the auger of forage harvesters are as follows: outer edge (intersection line)  $R = 0.25$  m, inner (shaft radius)  $r = 0.125$  m. Usually, the helical line is given not by the pitch angle  $\beta$  but by the



pitch  $H$ . Let the approximate pitch  $H=0.5$  m. There is a relationship between the pitch of the helical line  $H$ , the radius  $R$ , and the elevation angle  $\beta$ :  $\operatorname{tg}\beta=H/(2\pi R)$ . Hence, the elevation angle is determined:  $\beta=18^\circ$ .

The given helical line of radius  $R=0.25$  m is the outer common edge of both surfaces. The inner edge  $r=0.125$  m corresponds to the case  $r=\rho<R$ . For this case, the minimum value of the angle  $\gamma$  is determined from formula  $\gamma\geq\operatorname{Arctg}(R\operatorname{tg}\beta/\rho)$ , where  $\rho=r=0.125$  m. The result of the calculations is  $\gamma\geq 33^\circ$ . In Fig. 5, the working surface (screw conoid) and the supporting surface (helicoid torso) are constructed in two versions.

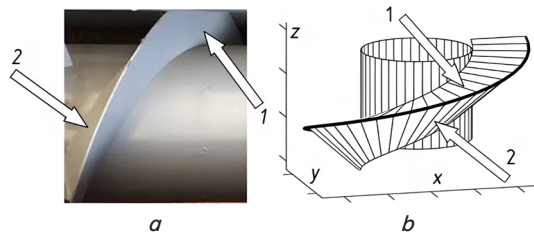


Fig. 4. Graphic illustrations of the structure of the narrowing screw [4] with working surface 1 and supporting surface 2: *a* – fragment of the screw; *b* – axonometric image

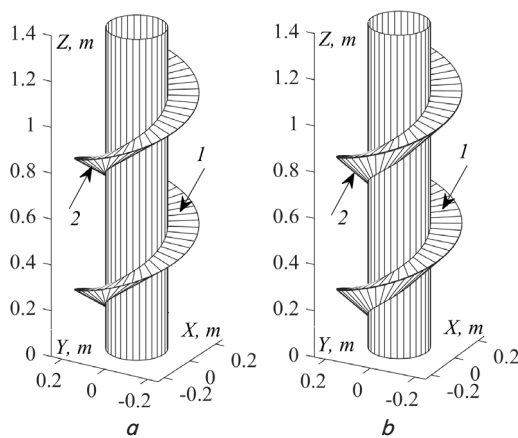


Fig. 5. Narrowing screws with the same working surface 1 and different supporting surfaces 2: *a* – supporting surface constructed at  $\gamma=33^\circ$  (minimum value); *b* – supporting surface constructed at  $\gamma=45^\circ$

The construction was carried out according to equations (13) with the predefined parameters of the helical line:  $R=0.25$  m and the angle of elevation  $\beta=18^\circ$ . In one case (Fig. 5, *a*), the value of the angle  $\gamma$  is minimal ( $\gamma=33^\circ$ ), in the other (Fig. 5, *b*) – one of the possible ( $\gamma=45^\circ$ ). When constructing the surface, the variable  $\rho$  was given values within  $\rho=0.125\dots 0.25$ ). The distance between the welds along which the working and supporting surfaces are welded to the shaft depends on the value of the angle  $\gamma$ . With increasing distance, that is, with increasing angle  $\gamma$ , the rigidity of the turn increases, but the usable volume of the inter-turn space decreases. The value of the angle  $\gamma$  can be selected depending on the specific operating conditions of the narrowing screw.

## 6. Discussion of results based on constructing the surfaces of helicoid torsos along the predefined helical line

The resulting parametric equations (13) are a consequence of analysis of the structure of helical surfaces from

the point of view of differential geometry. The decisive thing is that with their help it is possible to write down a section of the surface bounded by two coaxial cylinders. In this case, on the initial cylinder of radius  $R$ , a helical line is given by its elevation angle  $\gamma$ , from which, if necessary, it is possible to proceed to a step. The angle  $\beta$  gives the inclination of the rectilinear generatrices of the helicoid torso to the horizontal plane. The independent variable  $\rho$  varies within certain limits. These limits are the values of the radii of the bounding cylinders. One of these cylinders is the initial cylinder of radius  $R$ , and the second can be both external to the initial and internal. If the radius of the external bounding cylinder is denoted by  $R_o$ , then the variable  $\rho$  will have the following limits:  $\rho=R\dots R_o$ . If the bounding cylinder is the inner radius  $r_o$ , then the variable  $\rho$  will have the limits:  $\rho=r_o\dots R$ . The limits of the second independent variable  $\alpha$  determine the number of turns. For example, at  $\alpha=0\dots 2\pi$ , one turn of the surface will be constructed. Among the helicoids formed by the helical motion of a straight generatrix, a direct closed helicoid, a direct open helicoid, an oblique closed helicoid, an oblique open helicoid, and a revolute helicoid or torso helicoid are distinguished. Among them, only the latter can be manufactured by bending a revolute, which is determined exactly. To manufacture the turns of the remaining helicoids, for example, as in work [3], special equipment is required to form an approximate revolute into a finished product. In this case, significant resistance must be overcome to overcome plastic deformations. It is these two factors, namely the ability to find an exact sweep and the simplification of the coil manufacturing technology by simply bending it, that provide the advantage of the sweep helicoid over other helicoids. This advantage is especially noticeable when one considers that in engineering practice the subtleties of the internal geometry of helicoids are not necessarily accounted for.

Usually, the construction of a helicoid torso is associated with a helical line, which is its turning edge, as shown in Fig. 1. Our study has made it possible to solve the question of how to draw a helicoid torso through another helical line of the same pitch. To this end, it is necessary to cross the surface of the helicoid torso, given by the turning edge, with a coaxial cylinder whose radius is greater than the radius of the turning edge. The intersection line is the helical line through which the original torso passes (Fig. 2). However, the resulting helical line can be taken as the turning edge and thus another helicoid torso can be constructed. These two torsos would differ in the angle of inclination of the straight-line generatrices. Fig. 3 shows that two helicoid torsos pass through the given helicoid torso, in which the angle of inclination of the generatrices is quite significant. Obviously, a set of helicoid torsos can be drawn between them. From this set, one can choose a helicoid torso with a certain angle of inclination of the generatrices, which is demonstrated in Fig. 5 when constructing the support surface of the narrowing screw.

As a rule, torsos are described by parametric equations, one of the independent variables of which is the length of the straight-line generatrix (for example, equation (3)). This approach was proposed in [19]. The essential difference of our equations (13) is that they include two angles, not one, as in equations (3), and that instead of the independent variable – the length of the straight-line generatrix – the distance from the surface axis is introduced. By setting the limits of change of this distance from  $r$  to  $R$ , one can obtain a section of the surface bounded by coaxial cylinders of the specified radii.

It should be noted that when choosing the angle of inclination of rectilinear generating surfaces passing through the predefined helical line, there is a restriction on the minimum value of this angle. This paper describes how it is determined. The disadvantage of our study is that all calculations are carried out with the assumption that the surface of the helicoid torso has zero thickness.

This study shall be advanced by constructing the sweeps of the turns of helicoid torsos.

## 7. Conclusions

1. In the conventional description of helicoid torsos, only one angle of inclination of the straight-line generatrices appeared in the equations, which is equal to the angle of elevation of the return edge. Our equations introduce two angles since in the adopted technique for constructing a helicoid torso, the angle of elevation of the return edge is not equal to the angle of inclination of the straight-line generatrices. This has made it possible to describe a set of helicoid torsos that pass through the predefined helical line. The mutual dependence between these angles and the restriction on the value of one of them has been established. In addition, the resulting equations make it possible to describe the sections of helicoid torsos located between uniaxial cylinders, according to their given radii.

2. The devised technique for constructing a helicoid torso has found its application in calculating the supporting surface of a narrowing auger of a forage harvester or grain harvester. The application of our equations has been demonstrated by constructing two variants of the supporting surface, which differ in the angle of inclination of the rectilinear generatrices of helicoid torsos. The common edge of the main surface (screw conoid) and the supporting surface (helicoid torso) with the following parameters was taken as the helical line through which the helicoid torso passes –  $R=0.25$  m,  $\beta=18^\circ$ . The limiting cylinder with this helicoid is external. The internal limiting cylinder has a radius of  $r_o=0.124$  m. Two supporting surfaces were constructed using the equations derived: in one case, the angle of inclination of the rectilinear generatrices

is  $\gamma=33^\circ$ , in the other –  $\gamma=45^\circ$ . For both cases, an axonometric representation of the surfaces has been constructed.

## Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study, as well as the results reported in this paper.

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## Data availability

All data are available, either in numerical or graphical form, in the main text of the manuscript.

## Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

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