

The object of this study is the deformation processes in a protective structure of the preparatory working, maintained in the excavated space of the excavation area. The task addressed was ensuring the stability of the preparatory working in a deep coal mine to improve the safety of miners and coal mining. The assessment of the strength of the cast strip for the protection of preparatory workings in excavation areas has been substantiated.

It was experimentally established that in the zone of active influence of mining pressure on the section $0 < l < 50$ m behind the treatment face, there is an exponential dependence between the length of the working l (m) and the relative change in the volume δV of the protective structure. It characterizes the safe deformation resource of the cast strip. Within its limits, the process of gaining strength of the protective structure occurs. At $\delta V < 0.18$ and the failure rate of the arched flexible support $\omega < 0.2$, the losses of the cross-sectional area of the working do not exceed 20 %. Under conditions when $\delta V \leq 0.22$, the resistance of the protective structure increases, which makes it possible to limit the movement of lateral rocks on the contour of the working, ensuring its overall dimensions. In cases when $\delta V > 0.22$, and $0.22 < \omega < 0.68$, the losses of the cross-sectional area are more than 40 %. At a distance of $l > 80$ m behind the breakage face, the level of threat of roof collapses approaches a critical state, which is associated with an uncontrolled increase in lateral rock displacements due to the loss of strength of the cast strip.

The cast strip performs the functions of a supporting structure only in a certain range of physical, mechanical, and deformation properties.

The operational condition of the preparatory working is ensured within the deformation resource of the cast strip. After reaching the rated strength of the cast strip, its strength makes it possible to limit the movement of lateral rocks on the contour of the working and ensure its stability at a distance of $l \leq 80$ m along the length of the excavation section

Keywords: preparatory working, cast strip, strength of the protective structure, deformation properties, occupational safety

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EVALUATING THE STIFFNESS OF A CAST STRIP FOR PROTECTING A PREPARATORY MINE WORKING

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1. Introduction

One of the main ways to improve the safety and efficiency of complex-mechanized breakage faces in deep coal mines

is ensuring the stability of preparatory mining operations. With an increase in the depth of mining operations, the mining and geological conditions for the development of coal seams deteriorate. In the current situation, the problem of

maintaining and protecting preparatory mining operations becomes particularly important. Intensive deformation of the fastening, losses of cross-sectional area of up to 50 % occur in preparatory mining operations exposed to cleaning operations. Deterioration of the stability of the workings negatively affects the conditions for ventilation of excavation areas, the operation of mine transport, and generally reduces the safety of mining operations.

The unsatisfactory condition of preparatory mining operations is explained by the inconsistency of the applied protection techniques with the nature of the development of geomechanical processes in the coal massif. Under such conditions, the use of special fastening, increasing the density of installation of arched flexible fastening is insufficient. The average labor intensity of maintenance of workings exceeds 90 man-shifts per 1000 tons of coal production. Over 15 % of underground workers are engaged in repair of workings.

In many cases, the stability of preparatory workings can be ensured by using protective structures that reduce the negative impact of mining pressure on the contour massif. But this requires an assessment of the strength of protective structures, which takes into account their deformation properties during unloading of the coal massif. Therefore, the problem of ensuring the stability of preparatory workings in deep mines is relevant. It could be solved by improving the techniques of protecting preparatory workings taking into account the operability of arched flexible fastening. When solving this problem, it is necessary to assess the strength of protective structures, compare their deformation characteristics with the strength characteristics of lateral rocks. Scientific research on this issue is important because using the results of such research could make it possible to devise measures to prevent lateral rock collapses, maintain the stability of preparatory excavations, and improve occupational safety.

2. Literature review and problem statement

In a coal mine, in the set of mining operations, a special place is occupied by the protection and maintenance of preparatory workings, which ensures the proper functioning of the excavation sections and generally affects the efficiency of production [1]. The most difficult conditions for maintaining excavations are formed at the border with the excavated space along the length of the excavation section, which is equivalent to maintaining the workings in unstable rocks with constant deformation creep [2]. The authors draw attention to the fact that in order to make decisions on the feasibility of using certain techniques of protection and maintenance of mining workings, it is necessary to observe the magnitude and dynamics of lateral rock displacements and changes in the residual cross-sectional area.

One of the most technological is the technique of protecting workings with cast strips, which involves constructing a rigid structure from a quick-hardening mixture on the side of the excavated space. The advantages of cast strips include full mechanization of the construction of structures that provide protection of excavations on layers with medium and heavy rockfall roof rocks [3]. At the same time, with an increase in the thickness of the layers, the increase in the cost of the cast strip becomes a separate issue [4]. The directions for solving this problem can be the use of empty rock as a filler [5], the selection of the optimal composition of hardening mixtures, and the determination of the optimal

width of the strip, which significantly affects the deformation characteristics of the protective structure.

It is believed [6–8] that protective cast strips are effective on shallow layers up to 2.5 m thick with medium and heavy collapsing rocks. In this case, the width of the strip should not be less than 1.0 m, and the optimal ratio of the strip width to the thickness of the layer should be in the range of 1.0–1.5 [7, 9]. In [7, 8], technological recommendations are given for substantiating the parameters of standard cast strips; in [9], combined, two-layer strips are also considered – with flexible and rigid elements. At the same time, not enough attention is paid to analyzing the state of the workings at different stages of deformation of protective structures. Cement, natural and artificial anhydride, phosphogypsum are used as a binder in the manufacture of cast strips [7]. Under the conditions of the Pokrovske mine, the cement-mineral mixture "BI-fastener-3" is mainly used for the construction of cast strips.

Studies [10, 11] show that the descending branch of the deformation diagram of the cast strip material has a sharp character. After the creation of a block structure, a further change in the strength of the cast strip material practically does not increase the strength of the protective structure [10]. This has a negative impact on the stability of the side rocks and the deformation of the preparatory excavation contour. It is important to note that in such a situation, resistance to roof subsidence is not ensured and the load on the preparatory excavation fasteners increases.

In [12], the authors draw attention to the fact that, along with the width of the cast strip, the strength of the support and its moderate shrinkage are important. Important factors are the rate of hardening of the hardening material and its strength [7, 13, 14]. Under real mine conditions of loading of protective structures, concrete operates under the mode of supra-limit deformation [14, 15]. It should be noted that in such highly loaded structures, a significant role is assigned to the energy indicators of destruction. It is such indicators that determine the nature of the deformation of the material in the supra-limit state, which affects the assessment of the strength of the protective structure.

The solution to the problem of maintaining preparatory workings is also related to the requirements for the reliability of the arched fastening structures used [16, 17]. The authors of [16] draw attention to the fact that the current procedures for calculating the fastening parameters of reusable workings do not take into account the strength of protective structures but are limited to the influence coefficients of these methods. In paper [17], a statistical analysis of arched fastening failures was carried out with different protection techniques, but the influence of the deformation characteristics of protective structures was not shown. The presence of intensive lateral displacements of rocks in the zone of active influence of mining pressure leads to deformation of the working contour [2, 4]. In the cited studies, the authors do not pay attention to the operability of the installed fastening. It is precisely to ensure the operational condition of preparatory workings along the length of the excavation section that their stability should be considered taking into account the assessment of the strength of protective structures and the operability of the fastening.

Assessment of the influence of the deformation properties of the cast strip on the change in the cross-sectional area of the preparatory workings along the length of the excavation section could make it possible to devise measures to ensure their stability. To prevent the collapse of lateral rocks, improve the safety of miners and the efficiency of coal mining, it is also necessary to take into account the performance of the

main fastening. All this allows us to state that it is advisable to conduct a study on the deformation properties of the cast strip and assess its stability under mine conditions.

3. The aim and objectives of the study

The purpose of our study is to assess the strength of the cast strip along the length of the excavation section to ensure the stability of the preparatory workings, improve the safety of mining operations and coal mining in a deep coal mine. This will make it possible to predict the losses of the cross-sectional area of the preparatory workings when using cast strips under specific mining and geological conditions of the development of shallow coal seams.

To achieve the goal, the following tasks were set:

- to investigate the stability of the preparatory working when using the cast strip protection technique, taking into account the change in the cross-sectional area and the frequency of failures of the arch fastening along the length of the excavation section;
- to investigate the deformation properties of the cast strip, taking into account the increase in lateral rock displacements on the contour of the preparatory working;
- to assess the strength of the protective structure and its influence on the stability of the preparatory working along the length of the excavation section.

4. The study materials and methods

The object of our study is the deformation processes in the protective structure of the preparatory working, maintained in the excavated space of the excavation area.

Research hypothesis assumes that critical values of the specific potential energy of deformation of the cast strip determine its stress-strain state during unloading of the coal massif. Beyond these values, the strength of the protective structure decreases, which significantly affects the stability of the preparatory workings. In this study, the stress-strain state of the cast strip is considered through the relative change in its volume. This parameter is determined according to the data of the convergence of reference points along the contour of the preparatory working, through the relative deformation of the cast strip. This approach simplifies the calculations.

To assess the strength of the cast strip for the protection of the preparatory working, field studies were conducted at the Pokrovske mine company (Pokrovsk, Ukraine). Field studies were conducted at a horizon of 930 m in the preparatory working of the d_4 layer. Observations were carried out at two experimental sections along the preparatory working of the extraction section. The length of each experimental section is $l=120$ m. Mining and geological conditions of the d_4 layer are given in Table 1.

Mining and geological conditions of the excavation area

Layer index	Angle of incidence, deg.	Thickness, m	Lateral rocks			
			Roof		Outsole	
			Immediate	Basic	Immediate	Basic
d_4	4°	1.0 m	Siltstone of medium strength, up to 8.85 m thick	Fissured sandstone, up to 2.5 m thick	Siltstone of medium strength, up to 7.95 m thick	Hard sandstone, up to 2 m thick

At the experimental sites (Fig. 1), measuring stations were installed (Fig. 2). During the observations, the movements of the reference points, which were installed along the contour of the working, were recorded. Using a surveying tape measure, the amount of convergence of reference points 1, 4 in the direction of reference points 2, 3 was determined (Fig. 2). The measurement error was ± 2 mm.

The change in the cross-sectional area of the preparatory working was determined as the breakage face advanced by measuring the height H (m) and width B (m) of the working (Fig. 2).



Fig. 1. Scheme of experimental sites during the mining of coal seam d_4 : M.S. – measuring station

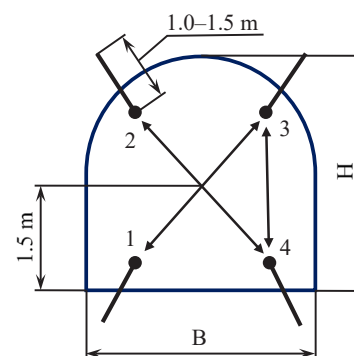


Fig. 2. Scheme of the measuring station in the preparatory working of the excavation section of the layer d_4 : 1–3; 4–2 and 4–3 – displacement of reference points 3 and 2 in directions 1 and 4; H – working height, m; B – working width, m

After determining the dimensions H (m) and B (m), the cross-sectional area of the excavation along the length of the excavation section was registered.

The design cross-sectional area of the preparatory excavation is $S=25$ m². Arched fastening KMP-A5R3 with a distance between the frames of 0.6 m. The protection technique is a cast strip. The width of the cast strip is 1.6 m. From the side of the excavated space, the cast strip is separated by a double organic row of wooden risers. The formation of the cast strip takes place in a space secured by two rows of temporary individual fastening (wooden risers under the top of the rock from a linear special profile SVP-33).

A special quick-hardening cement-mineral mixture "BI-kriplenya-3" and water were used to arrange the protective cast strip. The solution begins to harden 25–30 min after mixing. The uniaxial compressive strength of the hardening material reaches 7 MPa after one day, 15 MPa after seven days, and 30 MPa after 28 days.

The speed of the excavation is $V_e=240$ m/month, the technique of excavation is combine-based. The speed of the breakage face is 135 m/month. The roof control technique is a complete collapse.

Table 1

To determine the number of failures of the arched flexible fastening in the experimental sections, the fastening frames that were subjected to critical overload were fixed: there were breaks in the clamps in the lock joints and/or bending with twisting of the frame links. In total, the observation was carried out for $N=400$ pcs. of the same type of arched flexible fastening frames ($N=200$ pcs. fastenings in each section). The relative frequency ω of failures was determined from the following expression [17]:

$$\omega = \Delta N / N, \quad (1)$$

where ΔN is the number of arched reinforcement frames with critical deformations (number of failures, pcs.).

It should be noted that in the event of critical failure of the arched reinforcement frames, they were reinforced.

The relative deformation ε of the protective structure was determined from the following expression [18, 19]:

$$\varepsilon = \Delta h / h, \quad (2)$$

where $\Delta h = U_{3-4}$ is the vertical convergence of lateral rocks, which corresponded to the distance between reference points 3 and 4 (m); h – initial height of the protective structure, which corresponded to the thickness of the coal seam d_4 , m.

The relative change in the volume δV of the cast strip during unloading of the coal massif along the length of the excavation section was determined from the following expression [20]:

$$\delta V = (1 - 2\nu)\varepsilon, \quad (3)$$

where ν is Poisson's ratio.

The increase in roof displacements ΔU (mm) with distance from the breakage face was determined from the following expression:

$$\Delta U = U_{(3-4)i} - U_{(3-4)i-1}, \quad (4)$$

where i is the measurement interval number ($i=1$ at a distance from the face to the measuring station of 10 m, $i=2$ at a distance of 20 m, etc.).

To smooth short-term fluctuations and identify characteristic trends in changes in the analyzed parameter, the simple moving average method (with an averaging period of 2) was used [21, 22]:

$$\Delta \bar{U} = \frac{\Delta U_i + \Delta U_{i-1}}{2}. \quad (5)$$

The assessment of the strength of the cast strip under field conditions was carried out in conjunction with monitoring the performance of the arched flexible support installed in the preparatory working along the length of the experimental section.

The specific change in the volume of the protective structure per unit of convergence of lateral rocks $\Delta \bar{V}$ (m^{-1}) during unloading of the coal massif was determined from the following expression:

$$\Delta \bar{V} = 1000(dV / \Delta \bar{U}), \quad (6)$$

where $\Delta \bar{U}$ is the average increase in lateral rock displacements, mm.

This approach has made it possible to assess the dynamics of strength of the cast strip along the length of the excavation section, taking into account the deformation characteristics of the protective structure.

5. Results of investigating the deformation properties of the cast strip for assessing the strength of the protective structure

5.1. Results of investigating the stability of the preparatory working along the length of the excavation section

Studies on the stability of the preparatory working were performed in the zone of active influence of mining pressure on the experimental sections of the layer d_4 (the length of each experimental section is $l=120$ m).

Fig. 3 shows plots of displacements U (mm) of lateral rocks along the contour of the preparatory working depending on the distance l (m) behind the breakage face.

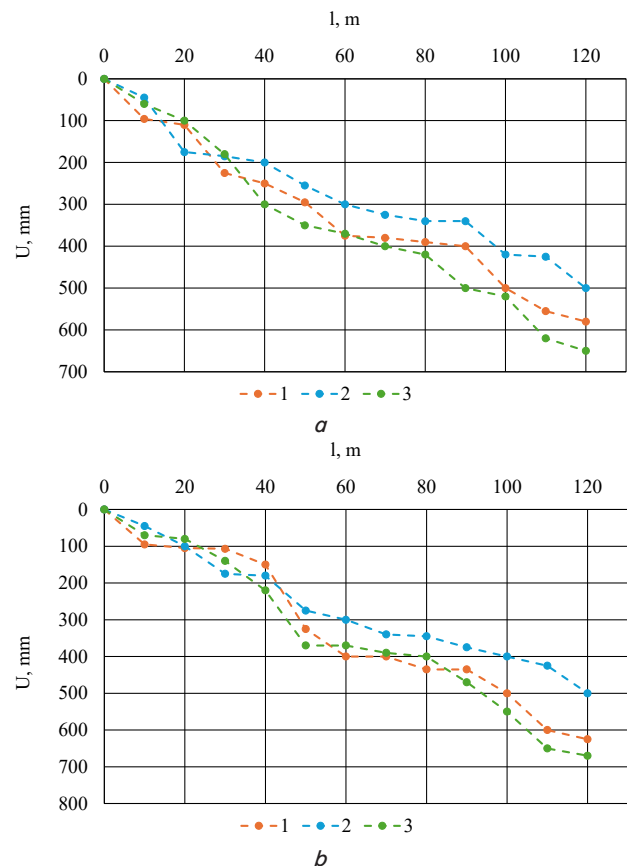


Fig. 3. Plots of lateral rock displacements along the contour of the preparatory working at a distance l (m) behind the breakage face: a – experimental area No. 1; b – experimental area No. 2: 1 – displacement of reference points 1–3; 2 – displacement of reference points 2–4; 3 – displacement of reference points 3–4

It was recorded that the roof displacements along the contour of the preparatory working (the displacement of reference points 2 and 3 in the direction of reference points 4 and 1) increase from the values $U_{2-4}=46$ mm and $U_{1-3}=95$ mm at $l=10$ m to the values $U_{2-4}=500$ mm and $U_{1-3}=595$ mm at $l=120$ m behind the breakage face (Fig. 3, a, b , curves 1, 2).

The convergence of the lateral rocks, which was recorded when moving reference points 3–4, changed from the values $U_{3-4}=60\text{--}70\text{ m}$ at $l=10\text{ m}$ to the values $U_{3-4}=650\text{--}670\text{ mm}$ at $l=120\text{ m}$ behind the breakage face (Fig. 3, curve 3).

Taking into account the deformation of the lateral rocks along the contour of the preparatory working, the change in its cross-sectional area $S\text{ (m}^2\text{)}$, the number of failures of the arch fastening $\Delta N\text{ (pcs.)}$, and the frequency of their failures ω along the length of the excavation section were recorded. The experimental data are summarized in Table 2.

Table 2

Experimental data on the change in the cross-sectional area $S\text{ (m}^2\text{)}$ of the preparatory working, the number of failures $\Delta N\text{ (pcs.)}$, and the failure frequency ω of the arched flexible anchorage over the length $l\text{ (m)}$ of experimental sections No. 1 and No. 2

$l, \text{ m}$	Section No. 1			Section No. 2		
	$S, \text{ m}^2$	$\Delta N, \text{ pcs.}$	ω	$S, \text{ m}^2$	$\Delta N, \text{ pcs.}$	ω
10	24.5	2	0.01	24.0	3	0.015
20	24.2	6	0.04	24.0	5	0.04
30	23.5	11	0.09	23.8	9	0.08
40	21.0	12	0.15	22.5	11	0.14
50	19.2	14	0.22	20.1	11	0.19
60	19.0	16	0.30	20.1	12	0.25
70	16.0	16	0.38	18.2	12	0.31
80	15.3	15	0.46	17.4	12	0.37
90	15.1	14	0.53	16.0	10	0.42
100	15.0	12	0.59	16.0	9	0.47
110	14.9	11	0.64	15.5	9	0.51
120	14.8	8	0.68	15.0	8	0.57

Fig. 4 shows plots of changes in the cross-sectional area $S\text{ (m}^2\text{)}$ of the preparatory working along the length of experimental sections.

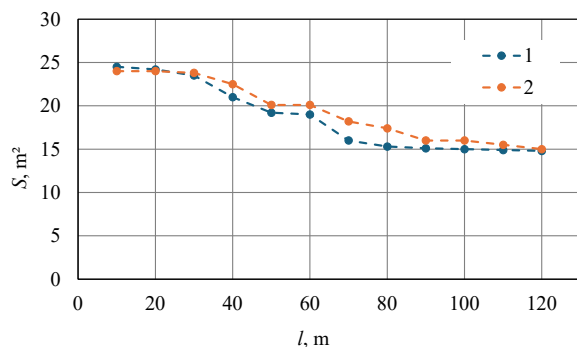


Fig. 4. Plot of change in the cross-sectional area $S\text{ (m}^2\text{)}$ of the preparatory working over the length $l\text{ (m)}$ of experimental section: 1 – No. 1; 2 – No. 2

A change in the cross-sectional area $S\text{ (m}^2\text{)}$ of the preparatory working depending on the value $S \approx 24.5\text{ m}^2$ at $l=10\text{ m}$ to the value $S \approx 14.8\text{ m}^2$ at $l=120\text{ m}$ behind the breakage face was recorded (Fig. 4, curves 1, 2); the losses in the cross-sectional area are 40–42 %.

Fig. 5 shows a histogram of the number of failures $\Delta N\text{ (pcs.)}$ of the arched flexible fastening in the preparatory working along the length $l\text{ (m)}$ of experimental sections and the failure frequency.

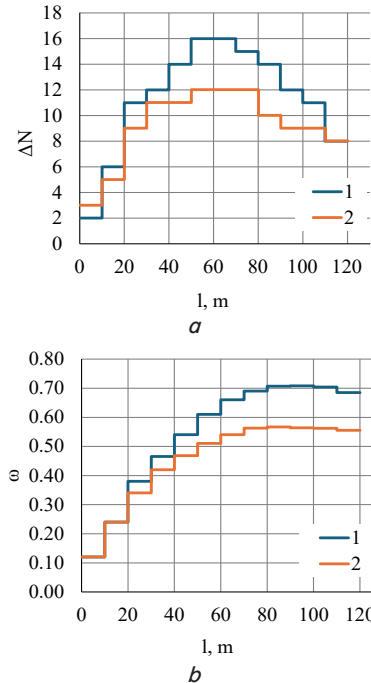


Fig. 5. Histograms: *a* – changes in the number of failures $\Delta N\text{ (pcs.)}$ of arched flexible fastening in the preparatory working along the length $l\text{ (m)}$ of the experimental section; *b* – failure frequency ω along the length $l\text{ (m)}$ of the experimental section; 1 – section No. 1; 2 – section No. 2

At the experimental site No. 1, the number of arched flexible fastening frames that failed was $\Delta N=137\text{ pcs.}$, and the failure rate was $\omega=0.68$. At the same time, along the length of the experimental site, the failure rate of the fastening changed from $\omega=0.01$ at a distance of $l=10\text{ m}$ to $\omega=0.68$ at a distance of $l=120\text{ m}$ behind the breakage face (Fig. 5, curve 1). The maximum increase in failures was observed at the distance from the excavation $l=50\text{--}70\text{ m}$.

At the experimental site No. 2, the number of arched flexible fastening frames that failed was $\Delta N=111\text{ pcs.}$, and the failure rate was $\omega=0.55$. Along the length of the experimental section, the failure rate of the fastening changed from $\omega=0.015$ at $l=10\text{ m}$ to $\omega=0.55$ at a distance of $l=120\text{ m}$ behind the breakage face (Fig. 5, curve 2). The maximum increase in failures was observed in the distance interval from the excavation $l=50\text{--}80\text{ m}$.

The use of the Pearson criterion [23] confirms the statistical hypothesis of a normal distribution of experimental data on the number of failures of the arched flexible fastening in the preparatory working.

5. 2. Results of investigating the deformation properties of a protective structure in the preparatory working

As a result of unloading a coal massif, the cast strip is loaded. With the main roof of average collapse, the load on the cast strip when moving the breakage face, $V_{b,f}=135\text{ m/month}$, increases long before the complete solidification of the protective structure material.

Table 3 gives deformation parameters of the cast strip on the experimental section with a length of $l=120\text{ m}$. The data demonstrate the deformation of the cast strip in the zone of active influence of mining pressure behind the breakage face.

Table 3

Deformation indices of the cast strip along the length l (m) of experimental sections No. 1 and No. 2

l, m	Section No. 1		Section No. 2	
	ε	δV	ε	δV
10	0.06	0.03	0.07	0.035
20	0.10	0.05	0.08	0.04
30	0.18	0.09	0.14	0.07
40	0.30	0.15	0.22	0.11
50	0.35	0.18	0.37	0.18
60	0.37	0.19	0.37	0.18
70	0.40	0.20	0.39	0.19
80	0.42	0.21	0.40	0.20
90	0.50	0.26	0.47	0.23
100	0.52	0.27	0.55	0.275
110	0.62	0.32	0.65	0.325
120	0.65	0.33	0.67	0.335

Fig. 6 shows the plots of change in the cross-sectional area S (m^2) of the preparatory working depending on relative deformation ε of the cast strip.

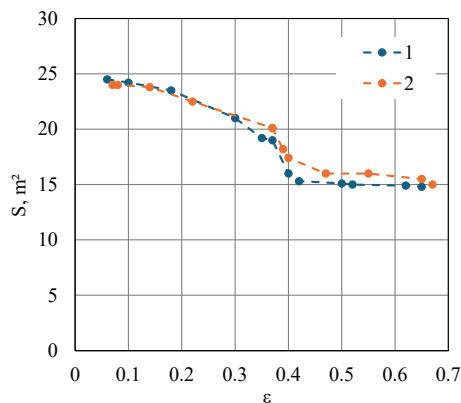


Fig. 6. Plot of change in the cross-sectional area S (m^2) of the preparatory working depending on the relative deformation ε of the cast strip during unloading of the coal massif: 1 – experimental site No. 1; 2 – experimental site No. 2

At experimental site No. 1, a decrease in the cross-sectional area from $S=24.5 \text{ m}^2$ to $S=14.8 \text{ m}^2$ was recorded with an increase in the relative deformation ε of the cast strip from $\varepsilon=0.06$ to $\varepsilon=0.65$ (Fig. 6, curve 1).

At experimental site No. 2, when the relative deformation of the cast strip changes from $\varepsilon=0.07$ to $\varepsilon=0.67$, the cross-sectional area of the preparatory working decreases from $S=24.0 \text{ m}^2$ to $S=15.0 \text{ m}^2$ (Fig. 6, curve 2).

Fig. 7 shows the plots of change in the cross-sectional area S (m^2) of the preparatory working and the failure frequency ω of the arched flexible fastening depending on relative change in the volume δV of the cast strip during unloading of the coal massif.

At experimental site No. 1 with a relative change in the volume of the cast strip from $\delta V=0.03$ to $\delta V=0.33$, the cross-sectional area of the preparatory excavation decreases from $S=24.5 \text{ m}^2$ to $S=14.8 \text{ m}^2$. At the same time, the failure frequency of the arched fastening increases from $\omega=0.01$ to $\omega=0.68$ (Fig. 7, a; curves 1, 2).

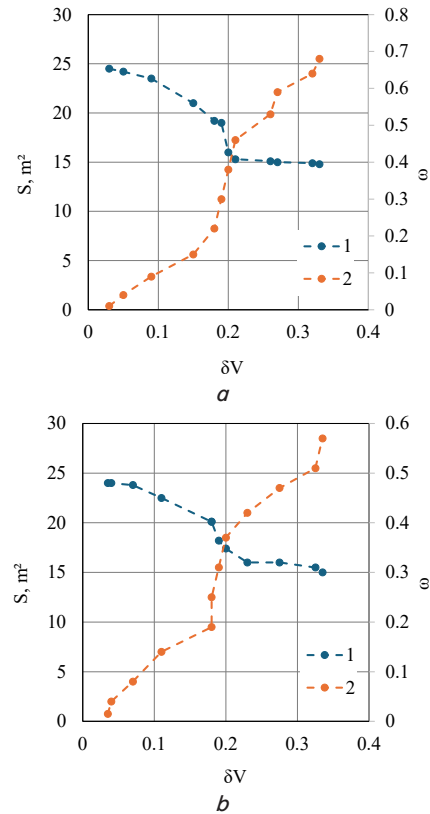


Fig. 7. Plots of changes in the cross-sectional area S (m^2) of the preparatory working and the failure frequency ω of the arched flexible support depending on relative change in the volume δV of the cast strip during unloading of the coal massif: a – at experimental site No. 1; b – at experimental site No. 2; 1 – S (m^2); 2 – ω

At experimental site No. 2, under conditions when the relative change in the volume of the cast strip is $0.035 < \delta V < 0.335$, the cross-sectional area of the excavation decreases from $S=24.0 \text{ m}^2$ to $S=15.0 \text{ m}^2$. The failure frequency of the arched flexible fastening increases from $\omega=0.015$ to $\omega=0.55$ (Fig. 7, b; curves 1, 2).

Intensification of failures of the arched flexible fastening occurs at $\delta V > 0.18$ in both experimental sections (Fig. 7, a, b).

Fig. 8 shows the plots of change in the failure frequency ω of the arched flexible fastening in the preparatory work depending on the relative deformation ε of the cast strip.

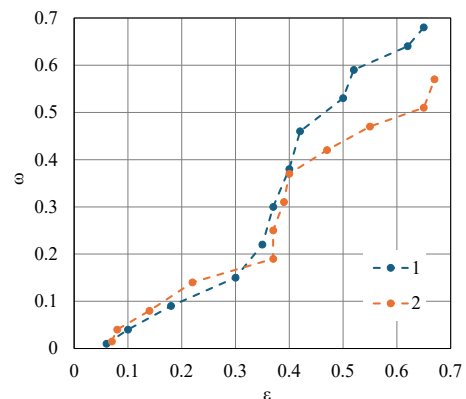


Fig. 8. Plots of change in the failure frequency ω of the arched flexible fastener in the preparatory work depending on relative deformation ε of the cast strip: 1 – at experimental site No. 1; 2 – at experimental site No. 2

It was recorded that with an increase in the relative deformation of the cast strip to the maximum values $\varepsilon=0.65$ at experimental site No. 1 and $\varepsilon=0.67$ at experimental site No. 2, the failure frequency is, respectively, $\omega=0.68$ and $\omega=0.55$ (Fig. 8; curves 1, 2). In the range of relative deformation values $\varepsilon=0.35-0.4$, a significant increase in the failure frequency is observed.

5. 3. Assessing the strength of a cast strip to substantiate the stability of the preparatory working

The assessment of strength of a protective structure was carried out based on the analysis of experimental data on the increase in displacements $\Delta\bar{U}$ (mm) of the side rocks along the contour of the preparatory working. The relative deformation ε of the cast strip and the relative change in its volume δV along the length l (m) of the experimental sections during the unloading of the coal massif were also taken into account.

Fig. 9 shows the plots of change in the increase in displacements $\Delta\bar{U}$ (mm) of the side rocks along the contour of the preparatory working along the length l (m) of the experimental section.

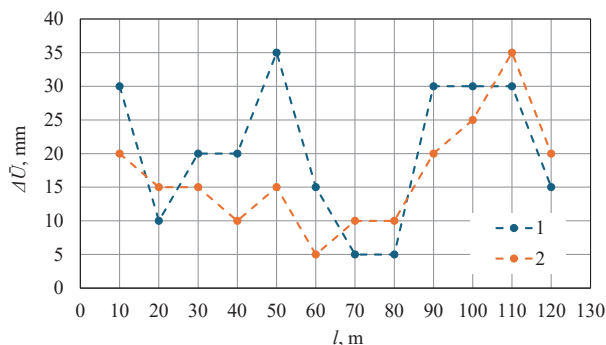


Fig. 9. Plots of change in the increase in displacements $\Delta\bar{U}$ (mm) of lateral rocks along the contour of the preparatory working along the length l (m) of the experimental section: 1 – No. 1; 2 – No. 2

Qualitative analysis of the diagrams shown in Fig. 9 (curves 1, 2) reveals that at the initial stage of deformation (section 0–50 m behind the breakage face) the cast strip gains strength. At the same time, the increase in displacements gradually decreases or increases, which largely depends on the resistance efficiency of individual and organ fastening and the intensity of subsidence of roof rocks in the excavated space. After gaining rated strength, the cast strip operates under the resistance mode (section 50–80 m). The increase in displacements decreases to values $\Delta\bar{U}=5-10$ mm. A sharp drop in the resistance of the cast strip with an increase in the growth of lateral rock displacements to values $\Delta\bar{U}=30-35$ mm is recorded at a distance $l>80$ m behind the breakage face. Further deformation of the cast strip leads to complete destruction and loss of strength of the protective structure (Fig. 9; curves 1, 2).

After processing the experimental data on the increase in displacements $\Delta\bar{U}$ (mm) of lateral rocks in the experimental areas, the average value of the parameter $\Delta\bar{U}$ (mm) and the specific change in the volume of the protective structure per unit of convergence of lateral rocks $\Delta\bar{V}$ (m^{-1}) were established. The values of the established parameters are given in Table 4, taking into account the length l (m).

Fig. 10 shows the plots of relative change in the volume of the protective structure per unit of convergence of lateral rocks $\Delta\bar{V}$ (m^{-1}) along the length l (m) of the excavation area.

Table 4

Experimental data on the increase in displacements $\Delta\bar{U}$ (mm) of lateral rocks and the specific change in the volume of protective structures per unit of lateral rock convergence $\Delta\bar{V}$ (m^{-1})

l, m	$\Delta\bar{U}, \text{mm}$	$\Delta\bar{V}, \text{mm}$
10	25	1.4
20	12.5	3.8
30	17.5	4.6
40	15	9.3
50	25	8.6
60	10	24.3
70	7.5	29.5
80	7.5	31.0
90	25	10.1
100	27.5	10.0
110	32.5	10.0
120	17.5	19.4

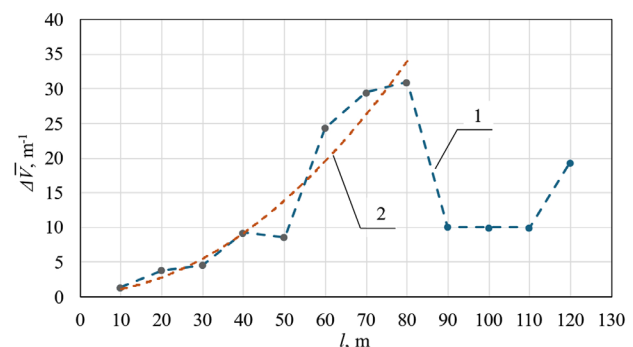


Fig. 10. Plots of relative change in the volume of the protective structure per unit of convergence of lateral rocks $\Delta\bar{V}$ (m^{-1}) by the length l (m) of the extraction section: 1 – according to experimental data; 2 – according to the results of approximation

An increase in the parameter under study from $\Delta\bar{V}=1.2 \text{ m}^{-1}$ to $\Delta\bar{V}=30.0 \text{ m}^{-1}$ on the section from $l=10$ m to $l=80$ m behind the breakage face is noted. With an increase in the length of the excavation section $l>80$ m, a decrease in this parameter to the value of $\Delta\bar{V}=9.4 \text{ m}^{-1}$ at a distance of $l=110$ m is recorded. This is due to the loss of strength of the cast strip (Fig. 10; curve 1).

Between the relative change in the volume of the protective structure per unit of convergence of lateral rocks $\Delta\bar{V}$ (m^{-1}) and the length l (m) of the excavation section at $10<l<80$ m, there is a quadratic dependence (Fig. 10; curve 2) of the form:

$$\Delta\bar{V} = 0.005l^2 + 0.0188l + 0.4427, \quad (7)$$

with coefficient $R^2=0.93$ and relative error 20 % [24, 25].

Fig. 11 shows the plots of relative change in the volume δV of the cast strip as a result of unloading of the coal massif and increase in displacements $\Delta\bar{U}$ (mm) of lateral rocks along the length l (m) of the experimental section.

It was established that at a distance from $l=10$ m to $l=50$ m behind the breakage face, the relative volume change rapidly increases from $\delta V=0.03$ to $\delta V=0.18$ since during this period the strength of the cast strip increases. Between the

parameters δV and l (m) (Fig. 11; curve 1) there is an exponential dependence of the form:

$$\delta V = 0.0201e^{0.0448l}, \quad (8)$$

with a coefficient of determination $R^2=0.98$ and a relative approximation error $|\Delta|=4.7\%$ [24, 25].

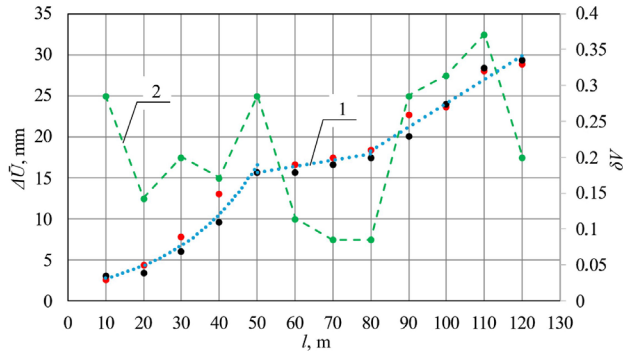


Fig. 11. Plots of relative change in the volume δV of the cast strip as a result of unloading the coal massif and the increase in displacements $\Delta\bar{U}$ (mm) of lateral rocks along the length l (m) of experimental section:
• No. 1; • No. 2; 1 – δV ; 2 – $\Delta\bar{U}$ (mm)

On the section from $l=51$ m to $l=80$ m, a relative change in the volume of the cast strip is recorded in the range of $0.18 < \delta V < 0.21$. In this range, there is a linear dependence of the following form between the parameters under study (Fig. 11; curve 1):

$$\delta V = 0.0008l + 0.136, \quad (9)$$

with a coefficient of determination $R^2=0.98$ and a relative error $|\Delta|=0.7\%$.

The increase in lateral rock displacements along the contour of the working decreases to a value of $\Delta\bar{U}=7.5$ mm (Fig. 11; curve 2), which indicates an increase in the resistance of the protective structure.

At a distance of $l > 80$ m behind the breakage face, a linear increase in the relative change in the volume of the cast strip $\delta V > 0.21$ is observed. In this range, a linear dependence of the following form occurs between the parameters δV and l (m) (Fig. 11; curve 1):

$$\delta V = 0.0033l - 0.057, \quad (10)$$

with a coefficient of determination $R^2=0.97$ and a relative error $|\Delta|=2.2\%$ [24, 25].

In this section, the increase in lateral rock displacements increases ($\Delta\bar{U} > 25$ mm), which indicates a decrease in the strength of the protective structure (Fig. 11; curve 2).

Fig. 12 shows a diagram of the regions of values of the relative change in the volume δV of the cast strip and the relative deformation ε during different periods of operation.

The plot characterizes the state of the cast strip. In zone I, when the relative deformation $\varepsilon < 0.37$ and the relative volume change $\delta V < 0.18$, the protective structure is in an unloaded state. The strength of the cast strip increases. Zone I corresponds to the limits of its safe deformation resource. Within these limits, the load falls on the organ and individual fastening due to the increase in the displacements of the lateral rocks.

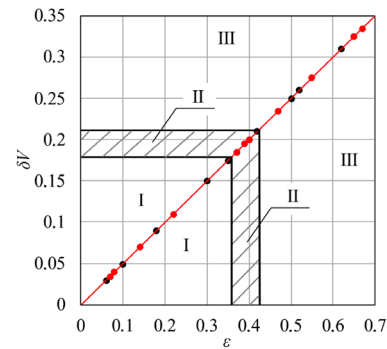


Fig. 12. Diagram of the regions of values of the relative change in volume δV and relative deformation ε of the cast strip in different periods of operation: I – region of the unloaded state; II – region of the normal state; III – region of the unstable state; • experimental data for section No. 1; • the same, for section No. 2

In zone II, a rigid mode of operation of the cast strip is recorded. The protective structure perceives a greater load from the roof side. The resistance of the structure increases. The strength of the protective structure allows it to resist the subsidence of the roof. In this zone, the relative deformation of the cast strip is in the range of values $0.37 < \varepsilon < 0.42$, and the relative volume change is $0.18 < \delta V < 0.21$.

The established deformation properties of the cast strip characterize the deformation resource of the protective structure, within which the displacements of the lateral rocks along the contour of the preparatory working are limited.

In zone III (at $\varepsilon > 0.42$ and $\delta V > 0.21$) there is a decrease in the strength of the cast strip, which characterizes the unstable state of the protective structure. After the loss of stability under conditions when the yield resistance of the cast strip decreases, the displacement of the lateral rocks occurs in the working. In this case, the working loses its cross-sectional area, the arched support is deformed.

The established deformation characteristics make it possible to assess the strength of the cast strip, which is intended for the protection of a preparatory working in the excavation section of a coal mine.

6. Discussion of results based on investigating the stability of a preparatory working for assessing the strength of a protective structure

From the results of mine studies on the stability of a preparatory working when using the cast strip protection technique, an assessment of its strength along the length of the excavation section has been established. To assess the strength of the cast strip, experimental data on the displacement of lateral rocks and their growth along the contour of the preparatory workings were used. On experimental sites in the zone of influence of cleaning works, the relative deformation of the cast strip (2) and the relative change in its volume (3) were established. At the same time, the change in the cross-sectional area of the workings and the number of damaged arch supports were determined (Table 2). Experimental data on arch support failures in the preparatory work follow a normal distribution (Fig. 5).

Our analysis of plots of lateral rock displacements along the contour of the preparatory working (Fig. 3) allows us to state that the deformation of the arched flexible support

occurs more intensively from the roof side. The maximum values of $U_{3-4}=650$ mm and $U_{3-4}=670$ mm were recorded when moving reference points 3–4 (Fig. 3, *a, b*; curve 3). Losses of the cross-sectional area of the preparatory working are more than 40 % at a distance of $l=120$ m behind the breakage face (Fig. 4).

According to the results of the observations (Table 2; Fig. 5), the state of the arched flexible support installed in the preparatory working along the length of the excavation section was recorded. It was recorded that the most loaded areas of the fastening are the elements located on the side of the largest displacements of the lateral rocks (references 3, 4; Fig. 3). Characteristic features of the deformation of the fastening are the ruptures of clamps in the lock joints and the bending with twisting of the frame links. Under such conditions, the loss of stability of the special profile and the decrease in the strength of the fastening are noted. The reason for the failure of the fastening is the formation of a one-sided directed load.

In the excavation area in the zone of active influence of the mining pressure at a distance of $l \leq 50$ m behind the breakage face, there is an exponential dependence between the length of the preparatory working and the relative change in the volume δV of the cast strip (Fig. 11, curve 1; equation (8)). The established dependence characterizes the safe deformation resource of the protective structure. It is recorded that at $\delta V \leq 0.18$ the failure frequency of the arched flexible fastening is $\omega < 0.2$. The losses of the cross-sectional area of the preparatory excavation do not exceed 20 %. Under such conditions, safe overall dimensions of the excavation are ensured.

Within the safe deformation resource of the protective structure, the process of gaining strength occurs. During hardening, the cast strip undergoes shrinkage. Shrinkage deformations occur in the protective structure, which are summed up with their values from the load that appears during the unloading of the coal massif.

At a distance of $l > 50$ m behind the breakage face, there is a linear dependence between the length of the preparatory working and the relative change in its volume (Fig. 11, curve 1, (9), (10)). This dependence makes it possible to assess the strength of the protective structure beyond the safe deformation resource. With a relative deformation of the cast strip $\varepsilon \leq 0.42$ and a relative change in volume $\delta V \leq 0.22$, the resistance of the cast strip increases. Such a geomechanical situation is recorded in the area $50 \leq l \leq 80$ m behind the breakage face, which makes it possible to limit the movement of lateral rocks along the contour of the preparatory working and avoid collapses. It should be noted that the area $l \leq 80$ m behind the breakage face characterizes the deformation resource of the protective structure.

Under conditions when $\varepsilon > 0.42$ and $\delta V > 0.22$, there is an increase in the increment of lateral rock displacements along the contour of the preparatory working to values of $\Delta \bar{U} = 30-35$ mm (Fig. 11, curve 2). With an increase in the frequency of failures of the arch fastening in the working $0.22 < \omega < 0.68$, the losses of the cross-sectional area of the working increase, and the level of threat of roof collapses approaches a critical state. At a distance of $l > 80$ m behind the breakage face at values of $\omega = 0.68$, the losses of the cross-sectional area of the preparatory working are more than 40 % (Fig. 6).

Violation of the integrity of the cast strip is the cause of the loss of stability of the protective structure. This process introduces additional changes to the geometry of the protec-

tive structure, which is accompanied by a change in volume. At a certain stage of loading due to the violation of the integrity of the protective structure, a local loss of stability of the cast strip occurs. After the possibility of realizing the stability of local forms is exhausted, the process of final destruction of the cast strip begins. An increase in the increment in lateral rock displacements and a significant change in the volume of the protective structure are noted.

When unloading a coal massif under conditions where the relative change in the volume of the cast strip is in the range of values $0.06 < \delta V < 0.22$, and its relative deformation is $0.03 < \varepsilon < 0.42$, the operational condition of the preparatory working is ensured in the excavation section of the coal mine. Under such conditions, the convergence of lateral rocks along the contour of the excavation is limited. This result makes it possible to ensure the overall dimensions of the preparatory excavations and safety of work at the workplaces of the excavation section.

The cast strip belongs to rigid protective structures and is characterized by the following states (Fig. 12):

- underloaded, when forces and deformations are far from the limit values;
- normal, when forces and deformations correspond to the operating conditions of the cast strip;
- unstable, when the protective structure loses its strength and collapses.

Thus, the cast strip performs the functions of a supporting structure only in a certain range of physical and mechanical and deformation properties. After gaining strength, such structures acquire strength, which makes it possible, within the deformation resource of the protective structure, to limit the movement of lateral rocks along the contour. Under such conditions, the collapse of lateral rocks becomes unlikely, and losses in the cross-sectional area of the excavation make it possible to ensure its safe overall dimensions. However, at a distance of $l > 80$ m behind the breakage face along the length of the excavation section, the strength of the cast strip decreases. The performance of the flexible arch support deteriorates. Therefore, there is a need to apply special measures to increase the stability of the preparatory working.

Unlike [8, 10], our result makes it possible to determine the losses in the cross-sectional area of the preparatory workings along the length of the excavation section. This becomes possible by establishing the features of the deformation properties of the cast strip. Under such conditions, the assessment of the strength of the cast strip will ensure the stability of the lateral rocks around the working and its overall dimensions along the length of the excavation section.

The results of the research could be used when designing tools for protecting preparatory workings maintained in the excavated space of extraction sections. Application limitations are as follows: gentle coal seams with an angle of incidence of no more than 10° , up to 1.5 m thick, with medium and heavy collapsing roof rocks.

The disadvantages of this study include the indirect determination of the relative deformation of the cast strip. In further studies, it is advisable to use specialized equipment. A more qualitative assessment of the strength of a protective structure should take into account the change in its stiffness and the impact of this parameter on the stability of the working. This will make it possible to devise measures aimed at preventing roof collapses, maintaining the operational condition of the preparatory workings, and creating safe working conditions for miners.

7. Conclusions

1. It has been experimentally established that in the zone of active influence of mining pressure along the length of the excavation section $0 \leq l \leq 50$ m behind the breakage face there is an exponential dependence between the length of the preparatory working and the relative change in the volume of the cast strip. The established dependence characterizes the safe deformation resource of the protective structure, within which the formation of its strength and durability characteristics occurs. At $\delta V \leq 0.18$ and the failure frequency of the arched flexible fastening $\omega < 0.2$ (with a distance between the frames of 0.6 m), the losses of the cross-sectional area do not exceed 20 %, which makes it possible to ensure safe overall dimensions of the preparatory working.

2. Statistical relationships have been established that determine the relationship between the length of the preparatory working and the relative change in the volume of the cast strip beyond the safe deformation resource of the protective structure. The presence of such relationships makes it possible to estimate the strength of the cast strip at a distance of $l > 50$ m behind the breakage face along the length of the excavation section. Under conditions when the relative deformation of the protective structure $\varepsilon \leq 0.42$ and the relative change in its volume $\delta V \leq 0.21$, the resistance of the cast strip increases, which makes it possible to limit the movement of lateral rocks along the contour of the preparatory working. At $\varepsilon > 0.42$ and $\delta V > 0.22$, the increase in the displacement of lateral rocks along the contour increases to values of $\Delta U = 30\text{--}35$ mm. In the range of changes in the failure frequency of the arched fastening $0.22 < \omega < 0.68$, the loss of the cross-sectional area of the working is more than 40 %, and the level of threat of roof collapses approaches a critical state.

3. Based on our studies on the deformation properties of the cast strip, an assessment of its strength was established, the determining factor of which is the relative change in the volume δV of the protective structure. When unloading the coal massif, the operational condition of the preparatory working is ensured in the range of changes in the relative deformation of the cast strip within $0.03 < \varepsilon < 0.42$, and the

relative change in its volume within $0.06 < \delta V < 0.22$. Beyond these limit values, the cast strip is destroyed, and conditions are created for collapses of lateral rocks, which leads to loss of the arch support and deterioration of the stability of the preparatory working.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study, as well as the results reported in this paper.

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Data availability

All data are available, either in numerical or graphical form, in the main text of the manuscript.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

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