Automatic balancing devices (ABDs) of the Leblanc type – passive ABDs of the liquid type – are used in rotary machines to reduce their vibration level when the distribution of masses around the geometric axis of rotation changes during machine operation or during its restart. To redistribute the masses during balancing, the movement of the working (correcting) liquid in the direction opposite to the imbalance is used. The object of this study is the motion modes (qualitative states) of the working liquid in the chamber of the balancing device for the vertical rotor system. The study is aimed at substantiating the existence of the auto-balancing mode at subcritical angular velocities of the rotor system and investigating its conditions and features. This paper reports the results of modeling the motion modes of the working liquid in the cylindrical chamber of the Leblanc ABP at a subcritical range of rotation speeds taking into account the vector relationships of the force factors depending on the design parameters of the auto-balancing device, the volume of the working fluid, and the shape of its free surface. Estimates of the angular velocities of switching on the working liquid under the rotational motion and under the auto-balancing mode have been analytically and experimentally substantiated, constituting, respectively, 1/3 and 1/2 of the critical angular velocities of the rotor system. For the practice of balancing an elastically deformable rotor and a rotor on elastic supports, the results of the study expand the range of rotation speeds where the balancing of the imbalance by the liquid and the reduction of the amplitudes of vibration processes are observed. This could help increase the service life, reliability, and accuracy of the technological process of machines with variable rotor imbalance by monitoring their vibration resistance through the use of liquid ABDs

Keywords: passive auto-balancing, Leblanc-type device, vertical rotor, auto-balancing mode

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DETERMINATION OF THE RANGE OF ANGULAR VELOCITIES OF THE AUTO-BALANCING MODE FOR A VERTICAL ROTOR SYSTEM WITH A LEBLANC-TYPE BALANCER

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1. Introduction

An unbalanced rotor is subjected to a general centrifugal force that causes it to deflect in the radial direction. This centrifugal force is transmitted to the bearings and the machine structure as a harmonically varying force [1]. Such forces can cause problems ranging from annoying noise levels or foundation vibrations to bearing failure or other structural components due to fatigue. Therefore, unbalance is considered an important potential cause of equipment failure. And vibrations caused by unbalance are a problem in the dynamics of rotary machines.

Regarding this issue, some balancing methods have been devised in the past [1–4]. However, the problem arises when the unbalance is variable or when balancing the rotor on the move (without stopping the machine) is not possible. In these cases, one effective way to solve such a problem is automatic balancing. Automatic balancing is a set of passive and automated methods by which unbalance (variable or constant) is eliminated by redistributing (moving) the masses of the rotor [4]. Potentially, this approach could solve the problems associated with the accumulation of unbalance caused by inaccuracy in rotor machining and assembly. For example, the aerodynamic efficiency of turbine engines is severely limited by the gaps between the blade tips [5]. It is effective for eliminating unbalance that depends on stochastic processes during the operation of a rotary machine, in particular, deformations, wear, and local destruction during rotor operation, as well as unbalance that varies due to the peculiarities of machine operation (for example, in washing machines or optical disc readers [6, 7]).

The Leblanc balancer is typically a rigid hollow ring (cylindrical or toroidal) partially filled with liquid [8]. Due to its simplicity and availability in manufacturing, reliability and noiselessness in practical implementation, as well as being unfavorable to wear of working surfaces, the use of the Leblanc balancer has become a promising method of automatic balancing. The advantage of the Leblanc autobalancer is also that its design parameters can provide full compensation for the most possible unbalance in the rotor system. However, according to [7], such a balancer cannot provide complete balancing of the rotor since it requires the presence of residual unbalance. Because of this, the scope of application of such ABDs is considered to be the class of serial machines (in particular, household ones). For them, the reduction in the cost of the balancer, the simplicity of its design, unpretentiousness in maintenance, and the possibility of repeated use without readjustment are of great importance, and the requirements for balancing accuracy are usually reduced [9, 10].

The general classical theory of passive ABDs contains a number of unsolved problems. In particular, there is no fundamental approach to studying the operation of a liquid-based ABD over the entire range of angular velocities. As a result, there are no methods for determining the conditions for the onset of auto-balancing, in particular, in the subcritical range of angular velocities of the rotor [11]. In general, the principle of operation of all passive ABDs is based on the phenomenon of rotor self-centering in the subcritical range of rotation speeds. This limits the practical use of devices that are unpretentious in operation and maintenance.

Therefore, it is a relevant task to carry out studies on justifying the Leblanc type passive balancing method for balancing and ensuring vibration resistance of rotors of machines with variable imbalance. And ways of comprehensively studying the dynamics of the rotor-balancer-liquid system and assessing the influence of the design parameters of ABDs, rotor characteristics, as well as dynamic processes in the flow of the corrective fluid, on the presence and effectiveness of auto-balancing are important.

2. Literature review and problem statement

It is known that at supercritical angular velocities the working fluid can act as a correcting body and thereby reduce the amplitudes of oscillations and therefore balance the unbalanced rotor. Most often, the property of a flexible or elastically supported rotor to rotate at supercritical speeds "with the light side outwards", i.e., the phenomenon of rotor self-centering, is used to explain the operation of the correcting fluid [12]. According to this approach, the kinetostatic method is used to study the process of rotor balancing with liquid balancers, which does not take into account the dynamics of fluid motion in the ABD chamber.

In general, there is a rather limited number of scientific papers that have investigated the motion of rotating bodies with cavities partially filled with fluid in connection with the problem of passive auto-balancing. Thus, the simplest models of concentrated masses are considered in [6, 13]. It was found that the liquid acts as an added mass, reducing the critical angular velocity to a value that is sometimes called the "reduced critical velocity" [13]. In synchronous precessional motion, the critical angular velocity of the rotor does not depend on the degree of filling of the chamber with liquid [6]. These models are oversimplified and do not take into account the gravitational force of the liquid under the assumption that the liquid in the cavity acquires a cylindrical shape and moves with the chamber as one solid body. The issues related to the hydrodynamic component of the motion of the working fluid remain unresolved.

In [14], the authors considered a flat model of a rotor with an ABD and analyzed the centrifugal forces acting on the system in a stationary state. A mathematical description of the distribution of liquid in the balancer within the equations of kinetostatics is presented, and the influence of the balancer on the dynamics of the centrifugation process of a washing machine with a vertical axis is also substantiated. Since the results of the study are valid only for a stationary state, they could hardly be used to analyze transient regimes.

Further studies are mainly focused on analyzing the stability of rotor motion in the presence of a liquid in the cavity. In [15], the motion of a viscous liquid was considered within the linearized Navier-Stokes model, and it was found that the viscosity of the liquid reduces the critical angular velocity of the rotor system. The important role of internal resonances in the occurrence of instability of stationary rotation of rotor systems containing a liquid was established. It was also established that without taking into account the viscosity of the liquid and external damping, it is impossible to correctly construct the boundaries of the stability regions in the parameter space. In [16], the theory of waves on the liquid surface was used within the nonlinear Korteweg-de Vries model to explain the dynamics of a liquid balancer at supercritical rotation speeds of the rotor system. In these studies, the operation of the liquid passive balancer is based on the phenomenon of rotor self-centering, which occurs for flexible rotors in the subcritical range of rotation speeds, and in general, does not depend on the presence of a working fluid in the autobalancer chamber. Because of this, the authors do not consider the subcritical range of motion of the rotor system. A significant drawback of these studies is the lack of experimental verification of the considered mathematical models.

The results of the construction and analysis of a numerical model of the Leblanc type ABD operation in the transient mode are reported in [17]. In [18], an analysis of experimental studies during the transition of the rotor with a liquid balancer through resonance is presented. It should be noted that the results of these studies expand the range of rotor rotation speeds, which correspond to the balancing of the imbalance by the liquid and the reduction of the amplitudes of vibration processes, in particular, the critical mode of motion of the rotor system. Also, in [18], to increase the efficiency of the operation of the liquid ABD in the pre-resonance range, it is proposed introducing radial partitions into the chamber design. However, there is no theoretical justification for such a solution.

Liquid balancing devices have existed for over a century. However, there are still gaps in knowledge about their effective operation. However, these devices have been widely used by manufacturers of household appliances since the 1940s. We believe that the main reasons for the incompleteness of knowledge are the high competition among manufacturers of household appliances, which adhere to a restrictive policy regarding publications on available technologies for designing this type of component; insufficient attention from scientists and researchers to non-classical approaches in studying the operation of such a device, which leads to the absence of new theoretical developments and in-depth knowledge about such an ABD.

In previous studies [11], a theoretical justification for the effective operation of a liquid balancer in the subcritical range of rotation speeds of a flexible rotor system for a rotor tilted at an angle to the horizon line was given. It was shown that the stability of the balancer operation over the entire range of angular speeds depends on the ratio between the geometric parameters of the balancer chamber (the radius of the chamber must significantly exceed its height), as well as on the volume of the working fluid. It was also shown that the force of gravity, and therefore the angle of inclination of the rotor axis, have a significant impact on the effective operation of the Leblanc-type

ABDs. Thus, it can be assumed that the processes of balancing the vertical rotor with a corrective fluid have their own specific features, which should be considered as a separate case.

All this gives grounds to argue that it is advisable to conduct a study on the existence of an auto-balancing mode at subcritical speeds (relative to the reduced critical speed) of the vertical rotor movement, which aims to investigate its conditions and features.

3. The aim and objectives of the study

The purpose of our work is to substantiate the modes of motion of the working (active) fluid in the chamber of the Leblanc-type auto-balancing device in the subcritical range of rotation speeds of the rotor system. This will make it possible, taking into account the vector relations of force factors depending on the design parameters of the ABD, the volume of the fluid, and the shape of its free surface, to estimate the limits of range of the angular velocities of an auto-balancing mode for a vertical rotor system.

To achieve the goal, the following tasks are set:

– within the framework of the Euler equilibrium equation for an ideal fluid, to construct and analyze models of the modes of motion of the working fluid in the ABD chamber;

– to analytically determine estimates of the angular velocities of rotation of the rotor system that correspond to each mode of motion;

– to perform experimental verification of the results of theoretical research on a bench that models a rotor on elastic supports with a vertical axis based on measuring the amplitudes of oscillations and observing the position of the working fluid in the ABD chamber;

– to substantiate estimates of angular velocities at which the working fluid is included in auto-balancing in the subcritical range of angular velocities.

4. The study materials and methods

4. 1. The object and hypothesis of the study

The object of our study is the motion modes (qualitative states of motion) of the working fluid in the chamber of the Leblanc-type balancing device for a vertical rotor system.

Research hypothesis assumes that the Leblanc-type ABD effectively operates both in the subcritical and supercritical range of angular velocities of the rotor and also ensures the safe transition of the machine through critical modes.

The basis of theoretical studies of the motion modes of the working fluid is a simplified model – the Euler equilibrium equation for an ideal fluid. This makes it possible to analytically estimate the ranges of angular velocities of the rotor rotation, at which a particular mode of fluid motion occurs. It can be assumed that the presence of fluid viscosity, i.e., the friction force between the layers of the fluid and between the fluid and the walls of the cavity, insignificantly reduces the calculated values of the estimates of the rotation speeds of the working fluid in the cylindrical chamber since for the auto-balancing process, the viscosity of the working fluid is a less significant characteristic compared to its density [19].

For experimental studies, a research setup was used that simulates a rotor on elastic supports with a vertical axis. For clarity, the results of experimental studies for cases of optimal filling of the ABD chamber with a working fluid are reported in our work.

The empirical balancing efficiency was calculated using a simplified approach – as the ratio of the amplitudes of rotor oscillations without fluid and with fluid.

4. 2. Description of the experimental bench

The experimental setup was designed on the basis of a drum-type washing and spinning machine (Fig. 1).

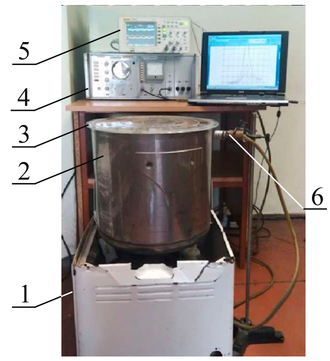


Fig. 1. General view of the experimental bench with an experimental setup for studying a rotor system with a vertical axis of rotation: 1 — housing; 2 — drum (rotor); 3 — Leblanc-type autobalancer; 4 — BB-10N device; 5 — RIGOL DS1052E digital oscilloscope; 6 — inductive sensor

An experimental sample of an autobalancer (three concentric cylindrical chambers made of optically transparent material) is installed on the upper edge of the drum (rotor) (Fig. 2).

Fig. 2. Experimental model of a Leblanc-type auto-balancing device

The rotor axis is connected to a rigid platform through rolling bearings. The platform is connected to the installation body by four elastically damped suspensions. The natural frequency of oscillations of the suspended system is ~9–10 Hz. To measure the range of movements of the upper edge of the rotor, a non-contact inductive sensor connected to the VV-10N device was used (Fig. 1).

To measure the drum rotation speed, a specific speed marker was used in the form of a small ferromagnetic plate installed on the end of the drum pulley. For each drum rotation, the plate once crossed the magnetic lines of force of the sensor, which was designed on the basis of a Hall sensor.

The signals from the sensors were amplified, their initial processing was carried out, they were digitized using a specially designed analog-to-digital converter and fed to a PC. For observation and measurement of waveform parameters, a digital oscilloscope RIGOL DS1052E (PRC) was used, which can save data images via a USB port as a file with the BMP extension or event table data as a CSV (tabulated) file, which can be edited in MS Excel (USA).

The study was carried out with a stepwise increase in the mass of the simulated imbalance (the rotor imbalance D was 0 g·cm, 1,000 g·cm, 2,000 g·cm, 3,000 g·cm.); with a stepwise increase in the volume of the liquid (from 0 ml to 350 ml) in chambers of different radii R=0.2 m, R=0.15 m, and R=0.1 m. Fresh water (density 1000 kg/m^3, kinematic viscosity $1.01 \cdot 10^{-6}$ m^2/s at a temperature of 20 °C) was used as the working fluid.

Before the series of experiments, the rotor of the experimental setup was dynamically rebalanced in place (in its own supports).

For detailed studies into the operation of the autobalancer, we stopped it temporarily for 5–10 s in the pre-resonant, resonant, and post-resonant zones of rotor rotation. For each case, the amplitude-frequency characteristic of the vibrations of the upper edge of the drum was constructed. After that, the empirical balancing efficiency was calculated as the ratio of the amplitude of vibrations without liquid and with liquid.

4. 3. High-speed video recording method for studying the movement of the working fluid in the autobalancer chamber

It is not possible to observe the fast-moving behavior of the fluid in the Leblanc autobalancer chamber and record its location relative to the imbalance even at low angular speeds of rotation of the rotor. Such observation is advisable to carry out by the high-speed video recording method with subsequent slow-motion viewing and storyboarding of digital recordings.

This method is implemented on a designed bench with an experimental setup that simulates a vertical rotor system (Fig. 1). The equipment connection diagram is shown in Fig. 3.

The video camera was placed along the axis of rotation of the rotor vertically above the surface of the balancer.

For operational observation of the location of the corrective fluid in the autobalancer chamber, a stroboscope SSh-1 was used, synchronized with the signal of the non-contact sensor-marker of the rotor revolutions.

The rotor launch was synchronized with the launches of the digital oscilloscope and video camera.

By comparing the video recordings with the oscilloscope recordings, the correspondence of the video material to certain ranges of rotation speeds of the rotor system and the value of the rotor rotation speed for certain processed video frames were determined. The results of the video recording were recorded on a PC.

The method of high-speed video recording of the movement of the corrective fluid in the chambers of the autobalancer was implemented in the following steps:

– balancing the rotor with a sample of the autobalancer in the absence of liquid in it;

– selection of the location and the magnitude of the simulated rotor imbalance;

– determining the volume of the corrective fluid in the autobalancer chamber;

– determining the optimal lighting of the shooting field;

– determining the optimal location of the video camera (viewing angle and distance to the balancer surface);

– determining the required video recording speed;

– trial runs of the entire system to coordinate the interaction of its individual elements;

– synchronous recording of fluid movements in the autobalancer and oscillographic data on the vibration sensor signals and the speed marker;

– signals transmission and recording them in files on a PC.

The range of resonant speeds of rotation of the vertical rotor system ω_r was from 49 s^{-1} to 57 s^{-1} depending on the mass of the simulated imbalance and the mass of the working fluid. The study of the motion of the corrective fluid in the autobalancer at pre-resonant, resonant, and post-resonant rotor rotation speeds was carried out at angular velocities of 0.3 ω_r, 0.5 ω_r, and 0.75 ω_r (pre-resonant zone), ω_r (resonant zone), 1.5 ω_r (post-resonant zone).

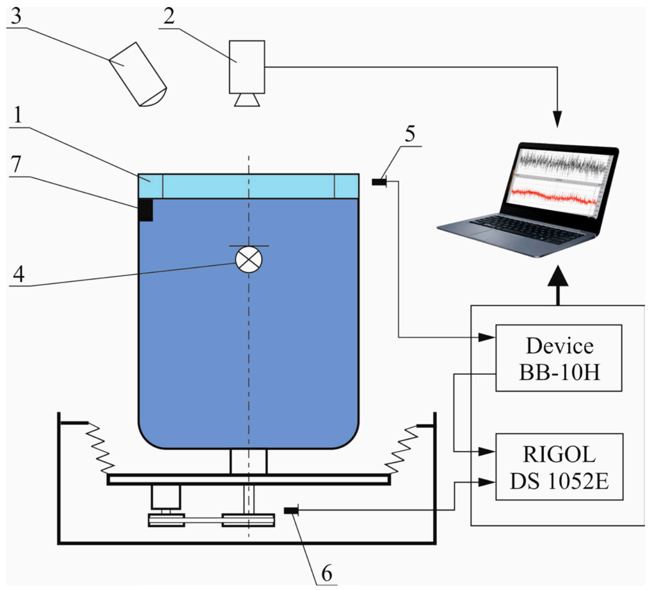


Fig. 3. Schematic showing the bench with an experimental setup for studying the behavior of the working fluid in the autobalancer chamber using the high-speed video recording method: 1 — Leblanc-type autobalancer; 2 — Panasonic AG-DVX100BE video camera; 3 — stroboscope; 4 — light source; 5 — inductive sensor; 6 — inductive sensor (speed marker); 7 — simulated unbalance

5. Results of research on the motion of the working fluid in the autobalancer at the subcritical range of rotation speeds

5. 1. Results of modeling the modes of motion of the working fluid in the autobalancer chamber

A fluid of density ρ is contained in a closed cylindrical cavity of radius R and height h, closed from above and below, which rotates around a vertical axis with a constant angular velocity ω ($[\omega]=s^1$) (Fig. 4).

In Fig. 4, the coordinate axes x, y, z are plotted with the origin at point O, located in the geometric center of the cavity bottom. We assume that the fluid has a volume V less than the volume of the cavity.

At each point in the fluid, the movement occurs in circles, the center of which lies on the Oz axis. The circular velocity of a fluid particle is directed tangentially to the corresponding circle and is equal to:

$$v = \omega \times r.$$

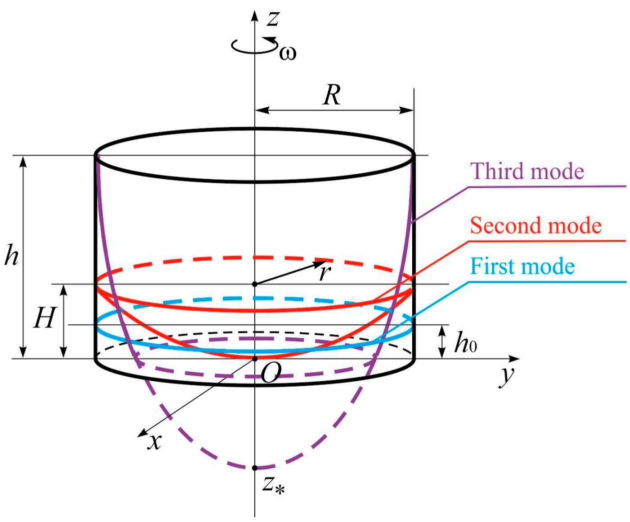


Fig. 4. Schematic representation of the shape of the free surface of the liquid in the chamber of a Leblanc-type auto-balancing device for a vertical rotor depending on the mode of movement of the working fluid

The mass forces F acting on each particle during rotation are the gravitational force $m\cdot g$ and the centrifugal force of inertia $m\omega^2\cdot r$, their resultant is normal to the surface of the liquid. Let us denote by XX$=\omega^2 x$, $Y=\omega^2 y$, $Z=-g$ the projections of the mass forces referred to the unit mass.

From the Euler equation of the stationary motion of an ideal liquid:

$$F-\frac{1}{\rho}\text{grad}\left(p\right)=0,$$

the law of pressure distribution in the volume of liquid is obtained:

$$p=\frac{1}{2}\omega^2\rho\left(x^2+y^2\right)-g\rho z-C_0. \tag{1}$$

By substituting the boundary condition for the hydrodynamic pressure on the free surface of the liquid $p=p_0$=const into expression (1), the equation of the free surface during stationary motion of a vertical rotor system with an ABD is obtained:

$$z=\frac{1}{2}\cdot\frac{\omega^2}{g}\cdot\left(x^2+y^2\right)+\frac{C-p_0}{\rho\cdot g}, \tag{2}$$

where the constant C characterizes the filling of the chamber and depends on the relative volume of liquid in the cavity ($V/\pi\cdot R^2\cdot h$).

From the analysis of equation (2) the location of the free surface of the liquid in the cylindrical chamber of the ABD is determined depending on the values of the angular velocity ω. The initial level of the liquid in the cavity is h_0 (Fig. 4). In the cylindrical coordinates r, φ, z the equation of the free surface (2) takes the form:

$$z_0=C_0+\frac{\omega^2}{2g}r_0{}^2.$$

If the coordinate of the vertex of the parabolic free surface is denoted by z_*, then at $r_0=0$, $z_0=z_*$. From the previous equation, we obtain the identical substitution $C_0=z_*$. Therefore, the equation of the free surface (2) takes the form (3):

$$z_0=z_*+\frac{\omega^2}{2g}r_0{}^2. \tag{3}$$

In the case of direct-acting liquid auto-balancing devices for rotors with a vertical axis of rotation, the working fluid is simultaneously exposed to the fields of gravity, friction forces, and centrifugal forces. For rotors with a vertical axis of rotation, with slow acceleration of the rotor, the liquid is gradually involved in the movement together with the body of the auto-balancing chamber and is gradually thrown back to the periphery of the ABD and pressed against the wall of the auto-balancing chamber. With a gradual increase in speed, the value of the centrifugal force increases, and with it the friction between the liquid and the wall of the auto-balancing chamber, as well as between the layers of the liquid. All this contributes to the fact that after some time the liquid and the auto-balancing chamber rotate as one solid body.

In this case, the working fluid passes through four modes of motion (qualitative states of motion): mode of full priority of gravity forces; mode of partial priority of gravity forces; mode of partial priority of centrifugal forces; mode of full priority of centrifugal forces. Only after the fourth motion mode does the fluid begin to function as the working fluid of the auto-balancing process. Video observations of the fluid behavior in the auto-balancing chamber during acceleration of the vertical rotor made it possible to identify and describe these four qualitative states of motion of the working fluid.

The first mode, the mode of full priority of gravitational forces, takes place at the beginning of the rotor movement at low speeds under the active influence of the fluid's gravity. The mass of the fluid under the action of gravity is pressed to the bottom of the chamber and does not participate in the rotation except for the liquid layer that touches the bottom of the cavity. Visually, the fluid slips relative to the bottom of the autobalancer chamber.

The second mode is characterized by the formation of a parabolic shape of the free surface of the fluid with increasing rotation speed. This mode ends when the paraboloid of the free surface of the fluid touches the bottom of the cavity or $z_*=0$ (Fig. 4).

The third mode, the mode of partial priority of centrifugal forces, is characterized by the transformation of the parabolic shape of the free surface of the fluid with increasing rotation speed. This mode ends when the paraboloid of the free surface of the fluid touches the upper base of the cylindrical cavity or $z_0=h$ at $r_0=R$ (Fig. 4).

In the general case, with increasing rotation speed, the parabolic shape degenerates into a cylindrical one. However, under the condition $R>>h$ [19], further increase in $\omega\left(\omega>\omega_0^{VR}\right)$ almost does not lead to a change in the shape of the free surface of the liquid. The fourth mode of motion (qualitative state of liquid motion) is reached – the mode of full priority of centrifugal forces. After reaching the angular velocity at which the liquid is included in the rotational motion of the rotor system, the liquid gradually begins to be involved in the process of automatic balancing – the auto-balancing mode becomes apparent. Its limits depend on various parameters of the rotor system, such as rotor stiffness, damping, physical properties of the liquid, liquid volume, geometric parameters of ABD, etc. [20, 21].

5. 2. Determining the estimates of the angular velocities of rotation of the rotor system, which correspond to each mode of motion

Worth noting is at what speed of rotation of the rotor system the mode of partial priority of gravitational forces ends. To clarify, we shall determine z_* and write down the material balance by the volume of the liquid: how much of it was poured into the cavity (volume V – to the level h_0), the same amount is in the rotating chamber (it is taken into account that the volume of the paraboloid of rotation is equal to half the volume of the cylinder of the same height):

$$V=\pi R^2 \cdot h_0=\pi R^2 \cdot z_* + 1/2\pi R^2 \cdot (H-z_*),$$

where H is the height of the liquid rise near the wall of the rotating cylindrical chamber ($H<h$) (Fig. 4). From the last equality $H=2h_0-z_*$. We find H from the ratio of forces, that is, from the condition $z_0=H$ at $z_0=H$ (chamber wall):

$$H = z_* + \frac{\omega^2}{2g}R^2.$$

Therefore:

$$z_* = h_0 - \frac{\omega^2}{4g}R^2, \quad (4)$$

and the equation of the free surface takes the form:

$$z_0 = h_0 - \frac{\omega^2}{4g}R^2 + \frac{\omega^2}{2g}r_0{}^2.$$

The value of ω, at which the bottom of the cavity is touched, is determined from (4) at $z_*=0$: $\omega = \frac{2}{R}\cdot\sqrt{g\cdot h_0} = \frac{2}{R^2}\cdot\sqrt{g\cdot\frac{V}{\pi}}$.

For the parameters of the experimental sample of the ABD for a vertical rotor system with a working fluid volume V=100 ml, the value of $\omega=0.89$ s^1.

It is important that the ratio of forces (centrifugal and gravitational), regardless of the rotation parameters and geometric characteristics, is always expressed by the ratio (3), which is why the values of z_* and H do not depend on the density of the liquid.

Next, it is determined at what speed of rotation of the rotor system the third qualitative state of liquid motion is completed.

At larger ω, i.e., at $z_*<0$ and $H>2h_0$, the material balance for the volume of the liquid is compiled using a definite integral to calculate the volume of the liquid as the volume of the body formed by the rotation of a curvilinear trapezoid around the axis Oz:

$$V = \pi R^2 \cdot h - \pi\int_0^h r^2(z)\,\mathrm{d}z = \pi R^2 \cdot h - \pi\int_0^h \frac{2g}{\omega^2}(z - z_*)\,\mathrm{d}z = $$
$$= \pi R^2 \cdot h - \frac{\pi g h}{\omega^2}\cdot(h - 2z_*),$$

hence:

$$z_* = \frac{\pi g h^2 - (\pi R^2 h - V)\omega^2}{2\pi g h},$$

and the equation of the free surface (2) takes the form:

$$z_0 = \frac{\pi g h^2 - (\pi R^2 h - V)\omega^2}{2\pi g h} + \frac{\omega^2}{2g}r_0^2. \quad (5)$$

The value of the angular velocity of inclusion of the liquid volume in rotational motion ω_o, i.e., at which the paraboloid of the free surface of the liquid touches the upper base of the cavity, is determined from (5) at $z_0=h$, $r_0=R$:

$$\omega_o^{VR} = \sqrt{\frac{\pi g h^2}{V}}. \quad (6)$$

For parameters of the prototype of the ABD for the vertical rotor system R=0.2 m, h=0.05 m according to (6) with a working fluid volume V=100 ml, we obtained $\omega_o^{VR} = 28\ \mathrm{s}^{-1}$; at V=200 ml – $\omega_o^{VR} = 19.8\ \mathrm{s}^{-1}$; at V=300 ml – $\omega_o^{VR} = 16.2\ \mathrm{s}^{-1}$.

5. 3. Verifying the results of theoretical studies by the method of measuring the amplitudes of oscillations of the rotor of the experimental installation

The range of angular speeds of rotation of the rotor system with the Leblanc type ABD, which correspond to the auto-balancing mode, was experimentally determined based on the processing of oscillographic data.

The oscillation amplitude (2A) of the system for certain rotational speeds was determined from the oscillographic records. When conducting multiple repetitions of the tests, empirical data were collected on the oscillation amplitude of the upper edge (drum) of the rotor for certain parameters of the ABD (chamber radius, working fluid volume, simulated imbalance value) and the conditions of movement of the rotor system. After processing the data array using statistical methods, graphical dependences of the oscillation amplitude of the rotor on the change in a certain parameter or set of parameters of the rotor system with the auto-balancing system were obtained.

Thus, Fig. 5 shows the graphical dependences of the rotor oscillation amplitude on the filling of the autobalancer chamber with a radius of R=0.1 m (Fig. 5, *a*); R=0.15 m (Fig. 5, *b*); R=0.2 m (Fig. 5, *c*). Empirical data were obtained as a result of conducting a series of experiments with simulated rotor imbalance D=3000 g·cm, filling each ABD chamber separately with fresh water in dosed portions of 50 ml. Recording of vibrations of the upper edge of the drum was carried out after each change in the volume of liquid during its movement at relative angular speeds of rotation $0.6\omega_r$, $0.8\omega_r$, ω_r, $1.2\omega_r$.

Analysis of Fig. 5, *a–c* reveals the following:

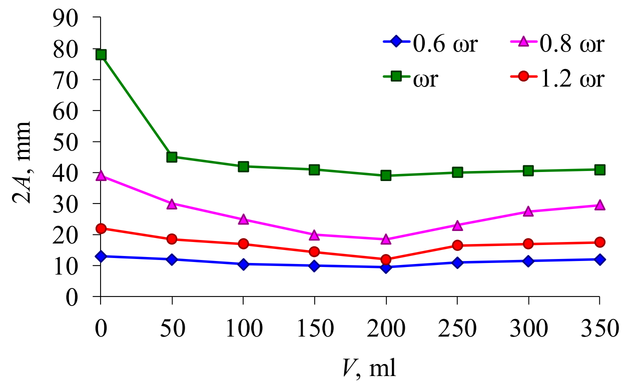
1) for a vertical rotor installation in the considered range of angular velocities of the system, the efficiency of liquid balancing is obvious (the values of the amplitude of oscillations of the upper edge of the drum, corresponding to V=0 ml, are significantly smaller when adding liquid);

2) there is a sufficient volume of working liquid to balance the system, which approximately creates an imbalance equal to the rotor imbalance: for a chamber with a radius of 0.2 m, it is 150 ml, for a chamber with a radius of 0.15 m – 150–200 ml and for a chamber with a radius of 0.1 m – 150–200 ml with an imbalance mass of 150 g;

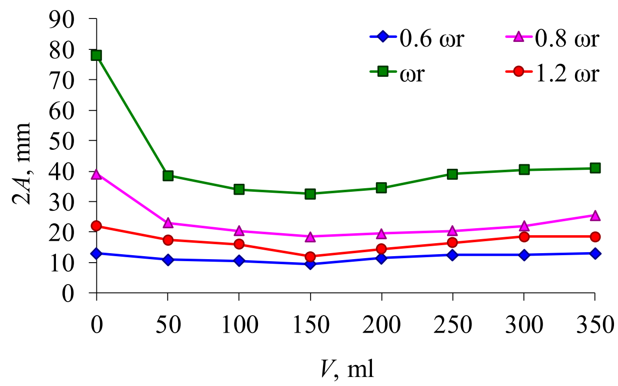
3) the amplitudes of rotor oscillations slightly increase if the condition of liquid sufficiency is not met.

The values of the efficiency coefficient of the auto-balancing mode were determined experimentally as the ratio of the amplitudes of oscillations of the upper edge of the drum of the experimental setup without liquid in the ABD and with liquid based on the processing of oscillographic data. Fig. 6 shows the distribution of empirical values of the efficiency coefficient of the auto-balancing mode with optimal filling of the ABD chambers for various simulated imbalances: D=1,000 g·cm, D=2,000 g·cm, D=3,000 g·cm.

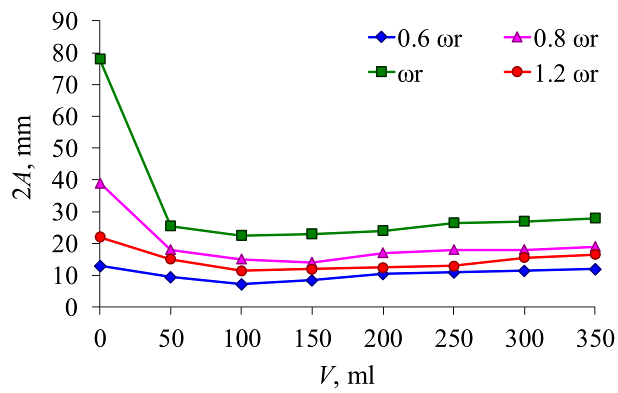
Analysis of the research results illustrated in Fig. 6 makes it possible to conclude: when the volume of the correction fluid is sufficient, effective balancing is observed over the entire range of the studied angular velocities of rotation of the vertical rotor system. Such a reduction in the amplitudes of oscillations of the upper edge of the rotor is present regardless of the size of the chamber radius. However, an autobalancer with a chamber of a larger radius (R=0.2 m) is significantly more effective in comparison with balancing chambers of smaller radii (R=0.1 m, R=0.15 m). The tendency of increasing balancing efficiency in the vicinity of resonance is preserved for studies with different Leblanc-type ABD chambers and for different values of simulated imbalances.



a

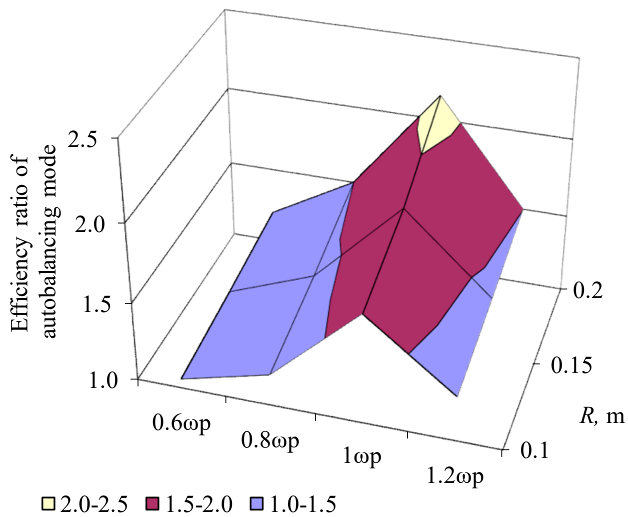


b

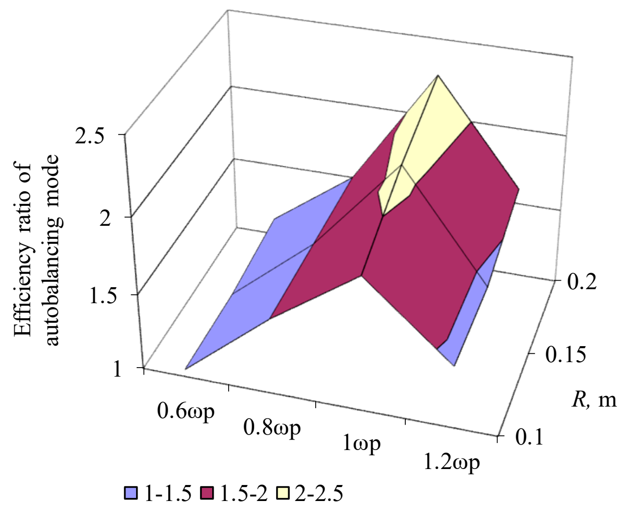


c

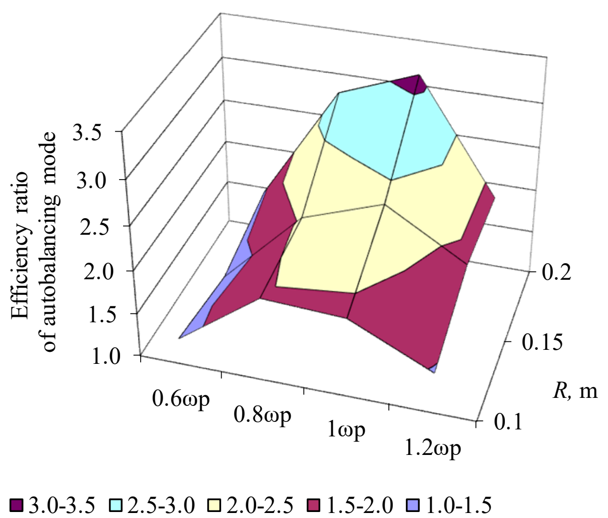
Fig. 5. The amplitude of oscillations of the upper edge of the drum for different volumes of working fluid in a Leblanc-type autobalancer when filling the chamber: *a* – radius R=0.1 m; *b* – radius R=0.15 m; *c* – radius R=0.2 m



a



b



c

Fig. 6. Distribution of values of averaged empirical efficiency coefficients of the auto-balancing mode with optimal filling of chambers of different radius of the Leblanc type auto-balancing machine under simulated imbalance: *a* – D=1,000 g·cm; *b* – D=2,000 g·cm; *c* – D=3,000 g·cm

5. 4. Substantiating the estimates of angular velocities of certain fluid motion modes using the high-speed video recording method

Let us consider a series of video frames (Fig. 7), which illustrates the movement of fluid in the chamber of an auto-balancer.

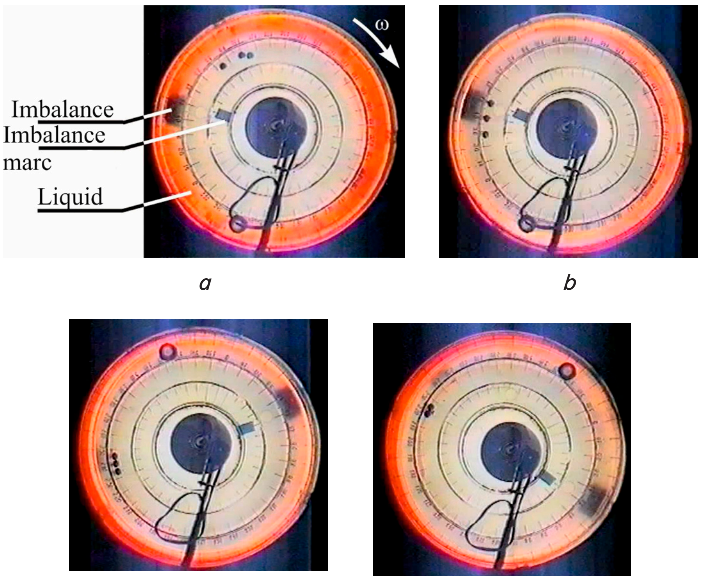


Fig. 7. Photo illustration of the qualitative states of the working fluid motion in the chamber of a Leblanc-type auto-balancing device: *a* – at ω=18 s^{-1}, *b* – at ω=30^{-1}, *c* – at ω=54^{-1}, *d* – at ω=80 s^{-1}

Our study was conducted for a vertical rotor system (Fig. 3) with the following parameters: mass of simulated unbalance – 150 g, rotor unbalance 3000 g·cm, resonant rotation speed of the rotor system ω_r – 54 s^{-1}. Parameters of the Leblanc type balancer: chamber radius R=0.2 m, chamber height h=0.05 m, volume of correction fluid – 300 ml.

The presented series of video frames visually make it possible to observe the change in the qualitative states of the working fluid in the autobalancer chamber with increasing values of the rotation speed of the rotor system. Fig. 7, *a*, which corresponds to ω=18 s^{-1}, illustrates the inclusion of the correction fluid in the rotational motion. The calculated value of the estimate of the angular velocity of rotation of the rotor, at which the working fluid is included in the rotational motion for a vertical rotor system, according to (6) is $\omega_o^{VR}=16.2\ \text{s}^{-1}$. Therefore, the empirical value is fully consistent with the theoretical one.

Fig. 7, *b*, which corresponds to ω=30 s^{-1}, shows the inclusion of the liquid in the process of effective auto-balancing in the pre-resonant zone of rotor rotation.

Fig. 7, *c* illustrates the position of the liquid in the balancer chamber at the resonant speed of the rotor rotation – 54 s^{-1}. The conclusion that the working fluid is located against the imbalance is visually confirmed.

In addition, a change in the shape of the free surface of the liquid is observed depending on the speed of rotation of the system: the paraboloid of rotation acquires a cylindrical shape, the horizontal section of the liquid placement area acquires the shape of a ring with a shifted center.

The placement of the working fluid in the resonant range of rotor rotation speeds is illustrated in Fig. 7, *d*. The video frame corresponds to the rotation of the rotor at a speed of 80 s^{-1}. Effective balancing is visually observed. This same result is observed when the rotor fully reaches the operating speeds of rotation.

The inclusion of the working fluid in the auto-balancing mode in the pre-resonant range of rotation speeds was visually observed during a series of experiments with different liquid filling, with different radii of the ABD chamber and different values of simulated imbalance.

The results of joint processing of oscillographic data and video recordings indicate that the lower limit of the empirical estimate of the rate of inclusion of the liquid in the process of rotational motion is in the range from 15 s^{-1} to 21 s^{-1}. Its value depends on the filling of the chamber with the corrective liquid (its volume) and the magnitude of the simulated imbalance. This dependence has the nature of a negative feedback. It has been experimentally confirmed that on average over a series of experiments the lower limit of the empirical estimate of the rate of inclusion of the liquid in the process of rotational motion for a vertical rotor is ω_o=1/3 ω_r. And the empirical estimate of the rate of inclusion of the liquid in the process of effective balancing is about ω_b=1/2 ω_r.

In the presence of fluid viscosity, i.e., frictional forces between fluid layers and between fluid and cavity walls, the rotation of the working fluid in the cylindrical chamber will be correspondingly slower compared to the calculated values.

Analysis of the constructed models and the results of experimental studies have made it possible to substantiate the existence of an auto-balancing mode in the subcritical rotation zone of the vertical rotor system and thus to refute the existing limitation of the operating range of passive auto-balancing devices of the liquid type by the supercritical (or super resonant) rotor rotation zone.

6. Discussion of results based on investigating the operation of the autobalancer in the subcritical range of rotation speeds of the rotor system

Our research is based on an innovative comprehensive approach, which includes the problem of rotor dynamics with a fluid, which makes it possible to take into account the joint action of such factors as rotation, the presence of a free surface of the fluid, and experimental verification of theoretical models. The results reported in this paper make it possible to perform a substantiated analysis of the behavior of the working fluid in the chamber of the Leblanc-type ABD for vertical rotor systems in the subcritical range of rotation speeds.

The results of modeling according to the Euler equation of the stationary motion of an ideal fluid make it possible to determine an analytical estimate of the speed of inclusion of the working fluid in the rotational motion of the rotor system (6), and therefore to experimentally estimate the speed range that corresponds to the auto-balancing mode of motion and is achieved even in the subcritical zone of rotation of the rotor (Fig. 5, 7).

It was experimentally determined that after reaching the angular velocity at which the fluid is included in the rotational motion of the rotor system (Fig. 7, *a*), the fluid gradually begins to be involved in the automatic balancing process (Fig. 7, *b*). For the considered experimental setup under the specified experimental conditions at a speed of ω=30 s^{-1} (about 1/2 ω_r), which corresponds to the pre-resonant rotation zone, at constant speed the fluid is installed almost opposite the imbalance. Such results are qualitatively consistent with the results of modeling [11, 17] and experimental studies [18] for a vertical rotor. Unlike [6, 12–16], the results of our work were obtained taking into account the influence of gravity, and therefore the shape of the free surface of the working fluid, on its behavior in the pre-resonant range of motion of the rotor system.

The difference between theoretical and experimental estimates can be explained by not taking into account the viscosity of the working fluid in the model. This helps put forward a hypothesis for further research on the significant influence of the internal friction of the working fluid on the processes of its inclusion in the rotational motion, response to changes in the rotational speed of the rotor system and the efficiency of automatic balancing.

The method of passive balancing of the liquid type considered in the work is applicable only for rotors that are elastically deformable or mounted on elastic supports, where there is a phase difference between the direction of the force from the imbalance and the deflection or movement of the rotor. This limits the scope of application of the results of the study of rotor systems that have the specified mechanical characteristics.

For practical use of the research results, it is necessary to take into account the conditions substantiated in it: an increase in the mass of the correction fluid reduces the critical speed of rotation of the rotor system, and an increase in the ratio of the radius to the height of the device chamber contributes to an increase in the efficiency of the liquid autobalancer.

To study the stability of the autobalancer mode, it is necessary to analyze the parameters characterizing the mechanical properties of the rotor system (in particular, the friction damping coefficient, shaft deflection, etc.) [22, 23], which are not taken into account in the proposed model. Therefore, the next stage in the study of the passive balancer of the Leblanc type is to determine the analytical conditions for effective balancing with a liquid, taking into account these properties and the physical characteristics of the working fluid.

A thorough investigation of the operation of liquid balancers could ensure an increase in the operational life, reliability, and accuracy of the technological process of machines with variable rotor imbalance, in particular, for household purposes, by controlling their vibration resistance through the use of ABD. Therefore, it will contribute to reducing costs within production and individual use for repairs and maintenance of machines by increasing their usable power due to the reduction of energy spent on excess vibration, which will be eliminated by ABD.

7. Conclusions

1. As a result of modeling the fluid motion in a cylindrical chamber based on the vector relations of force factors depending on the shape of the free surface of the fluid, four modes (qualitative states) of the motion of the correction fluid have been established and characterized. Namely: the mode of full priority of gravity forces; partial priority of gravity forces; partial priority of centrifugal forces; full priority of centrifugal forces. The latter mode characterizes the inclusion of the fluid in the process of rotational motion of the rotor system and subsequently determines the onset of the auto-balancing mode of motion.

2. Based on the model of equilibrium of an ideal fluid in the Euler form, depending on the design parameters of the auto-balancing device and the volume of the working fluid, estimates of the angular velocities of rotation of the rotor system have been analytically determined, which correspond to the established modes of motion of the correction fluid.

3. Experimental analysis of theoretical results based on measurements of the amplitudes of oscillations of the rotor of the experimental installation has made it possible to establish that the estimate of the minimum speed in the fluid motion mode of full priority of centrifugal forces can be considered as the lower limit of the range of angular velocities of the rotor system at which automatic balancing is observed.

4. Direct visual observation of the location of the working fluid in the ABD chamber relative to the imbalance during system rotation indicates that the speed of inclusion of the fluid into the rotational motion in the experimental ABD sample is approximately 1/3 of the resonant speed. The lower limit of the range of angular velocities of the rotor system at which automatic balancing is observed is 1/2 of the resonant speed.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study, as well as the results reported in this paper.

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Data availability

All data are available, either in numerical or graphical form, in the main text of the manuscript.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

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