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DEVELOPMENT OF A METHOD FOR COMBINED CONTROL OF THICKNESS, TENSION AND FLATNESS OF STRIPS IN CONTINUOUS COLD ROLLING TO REDUCE LONGITUDINAL THICKNESS VARIATION AND NONFLATNESS

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The object of study is the process of continuous cold rolling of strips, in which the parameters of thickness, tension and flatness of the metal.

Obtaining high flatness of thin strips during cold rolling at industrial 4-stand mills, where the last stand realizes significant degrees of compression in the working rolls with a diameter of up to 500 mm, when implementing the mode with a constant rolling force causes an increase in the longitudinal thickness deviation of strips. And the accentuated obtaining of minimum longitudinal thickness variation of strips leads to a change in the rolling force and as a result, due to the change in the deflection of rolls worsens the flatness of strips. The results of control actions can worsen both main quality indicators at the same time. The known methods of thickness and flatness control during cold rolling are analyzed and an alternative method of combined influence on these quality indicators is proposed. An optimization criterion is proposed. The quantitative estimation of the achieved simultaneous improvement of accuracy in thickness and flatness of strips as a result of the combined impact due to the realization of the optimal combination of interrelated control actions is performed. The method and algorithm of combined influence on thickness, tension and flatness of strips taking into account speed capabilities of actuators and their current position are proposed to reduce longitudinal fluctuations of thickness and nonflatness.

Simultaneous decrease of strip nonflatness and longitudinal thickness fluctuations is explained by the optimal combination of regulation channels. This method can be used on modern mills based on local systems of automatic control of strip thickness and flatness.

Numerical estimations of probable practical results achieved when using the proposed method. A simultaneous reduction in strip flatness of 20–30 % and the provision of limited thickness limit deviations to EN 10131(S) in at least 80 % of the intergrade transitions are expected

Keywords: continuous cold rolling, flatness and strip thickness control system, combined control

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1. Introduction

Historically, previous generations of continuous cold rolling mills for thin strips usually had 4 quarto stands. This number of stands in mills with working roll diameters up to 500 mm was considered sufficient to produce strips with a minimum thickness of 0.4–0.5 mm. The development of continuous cold rolling mills followed the way of construction of high-speed and high-capacity 5-stand mills with increased diameter of working rolls (usually up to 615 mm). This made it possible to carry out the rolling process in the last stand with low reduction in the mode with constant rolling force. Such mode of operation, in its turn, provided improvement of flatness of thin strips, stable surface roughness of strips, about 0.8–1.2 microns (Ra), necessary for subsequent annealing of strips in coils. Very small thin strips reduction in the last stand (1–5 %) in the mode of maintaining a constant rolling force on one side simultaneously guaranteed the best flatness and stable surface roughness, which did not increase the longitudinal thickness variation too much due to the small compression, while the main fine control of strip thickness was carried out in the penultimate 4th stand. This approach does not allow to fully utilize the last stand for its main purpose – strip reduction.

There are many compact 4-stand mills in the world, where each stand works according to its main purpose, realizing in the last stand in rough working rolls a rather large reduction up to 15–20 %. And in such mills, where the last stand is loaded by reduction, it is very important to realize effective combined control of thickness and flatness. It is not possible to aware any effectively working systems of interconnected regulation of strip thickness and flatness at such mills from the world practice.

The flatness of strips is the most important characteristic of their quality and is interrelated with the cross-sectional profile. Accordingly, in automatic profile and shape control systems used in cold rolling mills, the cross-sectional profile is controlled in such a way that the flatness of strips is not impaired [1, 2]. Achieving high flatness in cold rolling is often associated with the aggravation of profile irregularity. This is a consequence of the fact that the algorithms embedded in the thickness and flatness control systems are separate and do not interact with each other. Sometimes the results of control actions can be mutually compensated. In this case, they either do not achieve the goal or worsen the regulated parameters.

Thus, the research devoted to the development of an effective method of combined regulation of thickness and flatness of strip rolled products is relevant.

2. Literature review and problem statement

The issues of providing invariance of the system of automatic control of strip thickness and tension in relation to the flatness of strips during rolling are considered in [3–9]. These issues have gained practical significance in connection with the need to create a strip thickness control system invariant to flatness for reversible cold rolling mills, as well as in connection with the need to work out the modes of strip threading into the mill, stands of which are equipped with hydraulic pressure devices with systems for maintaining their constant positions.

In widely known in the world fundamental publications, for example, in the works [1, 2], devoted to the theory and technology of rolling and processing of flat metal products, including improving the efficiency of cold strip rolling mills, optimization of the rolling process and achieving the required strip thickness with minimum deviations [2], there is no information about solutions to the problem of combined regulation of strip thickness and flatness [2]. This is due to the fact that in the years of publication of these works have not yet received sufficient development means of digital control of parameters of rolling processes, although the basic ideas of simultaneous automatic control of thickness and flatness of strips were considered.

In [3], control methods based on quadratic optimization and delay compensation are proposed to solve the problem of automatic flatness control (AFC) in cold tandem mills. This work focuses on multivariate optimization in controlling the flatness of strips in the six-roll last stand of a 5-stand mill. Importantly, multivariate optimization of the control process and delay compensation is used, which allows the speed characteristics of the actuators to be taken into account. In [4], a new flatness prediction and optimization paradigm for a 5-stand cold rolling mill with six-roll stands based on cloud-edge interaction platform is proposed. These works [3, 4] do not address the problems of simultaneous fine control of strip thickness and flatness in the same stand. The authors concentrated only on strip flatness control.

In [5], multi-objective optimization of continuous cold rolling compression and inter-stand tension using NSGA-II and Pareto-optimal front is investigated. The target functions and optimization criteria for strip flatness in rolling process are not considered. In multi-objective optimization, the total energy consumption and uniform power distribution are proposed as target functions, and the reduction thickness in each stand and inter-stand tensions are selected as decision variables of the problem.

In [6], an interpretable mechanism driven by multi-channel distributed meta-learning is proposed to predict the flatness of strips. Comparative results show that the proposed system outperforms existing flatness prediction methods and other state-of-the-art machine learning techniques. The control system is considered on the example of a 5-stand mill, where the compression in the last stand is not large and therefore independent control of flatness in this stand does not cause large deviations in thickness.

In [7] an original fast mathematical model for the calculation of flatness of rolled products taking into account the lateral flow of metal in the rolling process is proposed, which allows its use in real-time control systems. However, the issues of developing methods and algorithms for automatic control of strip flatness, as well as thickness, were not considered in this paper. The authors focused on the issues of the most accurate prediction of strip flatness.

The paper [8] presents a solution to the problem of optimizing the mill stands reduction from a probabilistic point of view, which provides high stability of the rolling process, minimum dispersion of the rolling force in the last 4th stand of the mill, which allows, along with providing a given strip thickness, to improve its flatness. It is shown that the mode of crimping with maintenance of rolling force uniformly distributed over the stands corresponds to such conditions. However, this study deals with the creation of such conditions from a probabilistic point of view, it does not solve the problem of finding the optimal combination of different control actions in last stand at each control cycle. The approach proposed by the authors only indirectly, in probabilistic aspects, improves the conditions for increasing flatness and reducing strip thickness deviations.

The book [9] considers methods of optimization of continuous cold rolling of strips according to various criteria that ensure the achievement of various targets, including minimum longitudinal strip thickness variation and minimum non- flatness. These tasks were solved by searching for optimal distribution of pressings on stands and in one case the maximum natural alignment of longitudinal variation of strip thickness in the case when the mill is not equipped with automatic thickness control systems at all, or minimum strip flatness was achieved when a given ratio of rolling forces on stands was achieved, providing the distribution of roll deflections with known roll profiles, so that the ratio of absolute reductions to strip thickness was not violated on stands of the mill. Such approaches to optimization of the process of continuous cold rolling of strips can be considered outdated, as today almost all mills are equipped with systems of automatic control of thickness and flatness of strips.

At present, the following scheme of influence of strip thickness control means on flatness is applied in practice. As a result of changes in rolling conditions, the rolling force and roll bending change, hence the transverse strip thickness varies. The unevenness of tension at the stand entrance and exit plays a significant role. This gives rise to the redistribution of linear pressure in the deformation center and to the non-uniformity of elastic flattening of rolls across the strip width. Following the chain of interactions: non-uniformity of pressing $\rightarrow$ tension $\rightarrow$ line pressure $\rightarrow$ flattening.

Thus, due to the fact that the algorithms embedded in the known local control systems of strip thickness and flatness are separated and do not interact with each other, the results of control actions can deteriorate both main quality indicators simultaneously.

3. The aim and objectives of the study

The aim of the study is to develop a method and algorithm of combined effect on thickness, tension and flatness of strips, taking into account the speed capabilities of actuators and their current condition for the last stand, which equipped with the systems of axial shifting and bending of working rolls. This approach will simultaneously reduce both strip thickness deviation from the target value and strip flatness, which is not normally achieved with non-interdependent localized control systems for strip thickness and flatness.

To achieve this aim, the following objectives are accomplished:

– to determine quantitative dependences of the main characteristics of strip quality on the type and size of regulating influences on the available channels of influence;

– to determine the transfer functions of each channel of influence on the thickness, tension and flatness of the strip;

– to formulate the criterion, select the method and solve the optimization problem on a specific example;

– to justify and establish boundary conditions, taking into account the speed capabilities of the actuators;

– using a concrete examples to verify the solution of the optimization task;

– estimation of the probable practical result from the implementation of the proposed method.

4. Materials and methods

The object of study is the process of continuous cold rolling of strips, in which the parameters of thickness, tension and flatness of the metal.

Main hypothesis of the research. In the process of continuous cold rolling of strip as a result of various perturbations, changes in the input longitudinal and transverse thickness of the initial hot-rolled strip, variability of inter-roll gaps in stands due to the eccentricity of rolling rolls, changes in the resist of deformation of the strip due to the variability of the chemical composition of steel, changes in temperature and degree of deformation, changes in rolling speed and friction conditions in the contact of rolls with the strip, deviations of the final thickness and flatness of the finished strip occur.

The described perturbations are largely stochastic in nature and can be described from a probabilistic point of view. It is not possible to consider probabilistic descriptions in this study as an assumption.

As a simplification, let's consider deterministic models suitable for use in real-time control systems.

For solve the 1-st task as controlling influences mainly on the strip thickness let's take the change of the rolls rotation speed of the 4th stand Δv_4 (within ± 5 %), on the tension in the last interstand gap – the change of the position of the 4th stand pressure devices ΔZ_4 (within ± 0.5 mm). The force F_4 of rolls bending (additional bending '–' and counter-bending '+' within $\pm 1{,}000$ kN) and the axial roll shift movement S_4 (within ± 150 mm) were taken as control actions on strip flatness. Changes in thickness h_4, tension in the last intercellular gap T_{3-4}, rolling force P_4 and strip flatness index (nonflatness value P_4) were considered as response functions to the control actions.

The own methods of mathematical modeling of the process of continuous cold rolling of strips were used in the research. The mathematical model and computer modeling software [10] were used to estimate the impact on the transverse profile and flatness of strips by axial shear and roll bending. The computer system WinColdRolling© [11, 12] developed at the Iron and Steel Institute of the National Academy of Sciences of Ukraine was used. It was used to calculate the transfer functions of the response for different channels of influence. When solving optimization problems, it is possible to use the Excel computer program add-on 'Analysis Package' to perform analysis of complex data, where it is possible to implement the solution of the system of nonlinear equations by Newton's method. Also, for search of optimal solutions let's use the computer programme MathCad, where it is possible to implement the solution of the system of nonlinear equations by the Levenberg-Marquardt method. These methods allowed to perform a numerical evaluation of the results achieved when using the technology of combined control of thickness, tension and flatness of strips in continuous cold rolling.

5. Results of the study of combined thickness, tension and flatness control

5. 1. Determining the quantitative dependences of the main characteristics of strip quality on the type and size of regulating

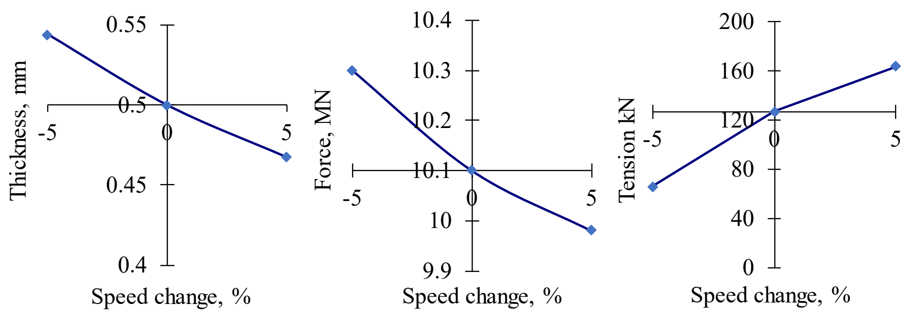
The process of rolling a strip of 08F steel with a width of 1,000 mm from the initial thickness of 2.0 mm to the final thickness of 0.5 mm was considered. The deformation rolling mode, providing uniform distribution of forces over the mill stands, which was determined using the developed optimization method [8], was taken as the basic one. This mode of distribution of compression with corresponding rolling speeds, roll rotation speeds and positions of pressure devices is the most rational from the point of view of minimum energy consumption and process stability. The initial (zero) values of bending force and axial shift movement of the rolls were also set, assuming that the roll profiling was chosen correctly and there was no flatness of the strip after rolling, the mill setting was set accurately, and all adjustable parameters corresponded to the nominal ones. During the rolling process, the nominal values of the controlled parameters change due to perturbations of the rolling characteristics. To compensate for the influence of this factor it is necessary to know the nature and magnitude of deviation of controlled parameters from the basic values under the influence of regulating influences.

The last two stands of the mill were considered. Using the previously developed mathematical model of a continuous mill [8, 9]. The method of calculation of elastic deformations of the quarto roll unit, taking into account the change in the joint orientation of the rolls and the action of the system of rolls bending [10], as well as the method of calculation of the strip flatness value [8], the values of the control parameters were varied within the above mentioned limits. As a result, let's obtain quantitative dependences of changes in macro-geometric parameters of rolled strip (thickness h_4 and flatness P_4), as well as force parameters of the process (rolling force P_4 and back tension T_4) in the last 4th stand of the mill.

The analysis of the obtained data shows (Fig. 1, 2) that changing the rolls rotation speed is the means of preferential influence on the output strip thickness, and changing the position of the pressure devices is the means of preferential influence on the back tension. These data are logically linked with the available experimental data of strip thickness and tension control with the use of existing automated systems (automatic thickness control system; automatic tension control system; automatic strip thickness and tension control system) on continuous cold rolling mills.

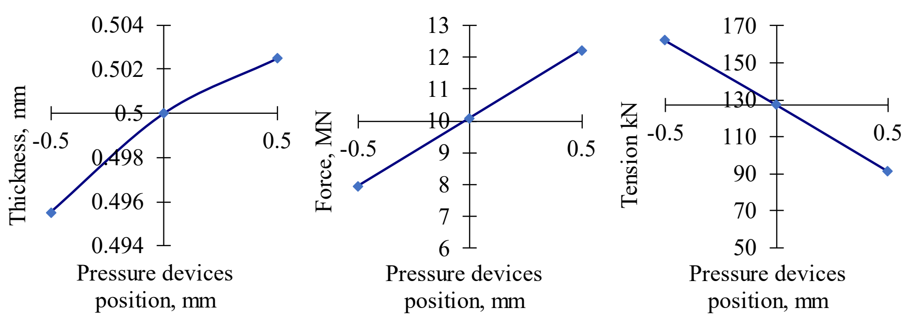
Fig. 3, 4 show similar dependences characterizing the influence of the variation of the bending force and the axial shift stroke of the 4th stand work rolls, and Fig. 5 – the dependence of rolling force influence on the final strip thickness obtained by force variation in the range of ±20 % from the working point (nominal value of 10.1 MN) at a constant level of strip back tension. The desired final strip thickness was determined using a mathematical model of the multi-pass rolling process [8].

It should be emphasized that the dependence in Fig. 5 reflects the resulting effect of different control channels, each of which was given specific values of control actions. 5 reflects the resulting effect of different control channels, each of which was given specific values of control actions, therefore, given as an example, it is not universal. Any other combination of control parameters will change the nature and magnitude of this dependence. At the same time, it follows from Fig. 1–4 that changing the control parameters for different channels under consideration affects the strip thickness differently. Therefore, when solving the problem of combined effect on thickness and flatness, it is very important to determine the influence of each control channel on strip thickness.



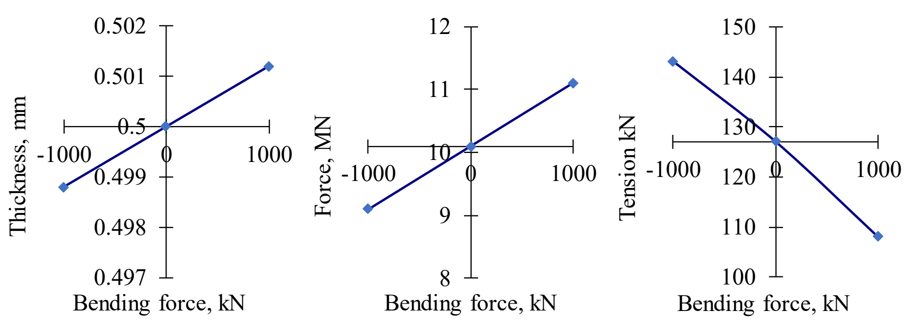
a *b* *c*

Fig. 1. Dependency graphs: *a* — influence of changing the roll speed of stand 4 on the final strip thickness; *b* — on the rolling force; *c* — on the tension in the last interstand gap



a *b* *c*

Fig. 2. Dependency graphs: *a* — influence of the position of the 4th stand pressure devices on the final strip thickness; *b* — on the rolling force; *c* — on the tension in the last interstand gap



a *b* *c*

Fig. 3. Dependency graphs: *a* — effect of changing the bending force of the 4th stand work rolls on the final strip thickness; *b* — on the rolling force; *c* — on the tension in the last interstand gap

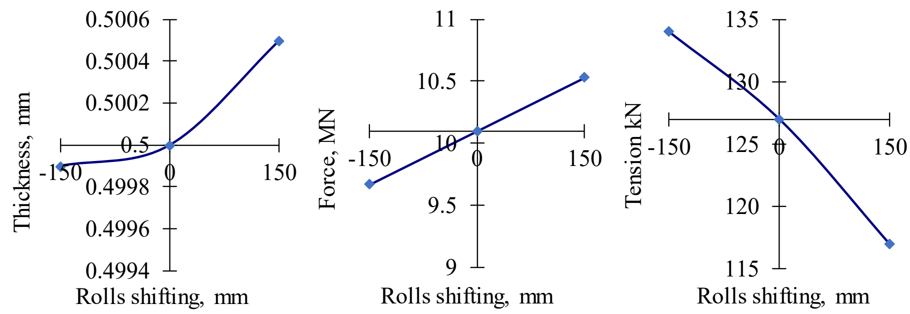


Fig. 4. Dependency graphs: *a* – effect of changing the rolls axial shift movement of the 4th stand working rolls on the final strip thickness; *b* – rolling force; *c* – tension in the last interstand gap

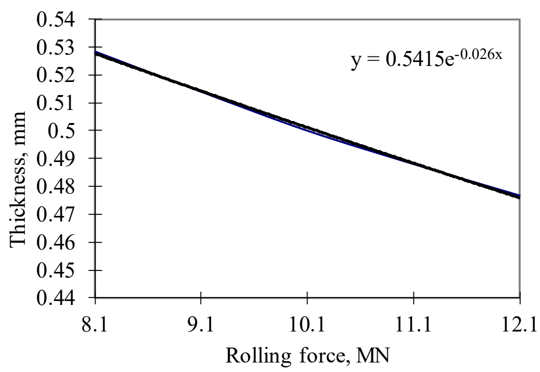


Fig. 5. Dependence of the final strip thickness on the rolling force

On this basis, strip thickness, rolling force and inter-cell tension were determined as a superposition of the values obtained by varying each control parameter separately. Non-flatness was also considered as a superposition of its values determined by changing the rolling force (Fig. 6, *a*), the force of rolls bending (Fig. 6, *b*) and direct change in the roll profile by changing the movement of their axial shift (Fig. 6, *c*).

The analysis has shown that from these dependencies it is possible not only to determine the influence of control actions on the controlled parameters, but also the degrees of mutual influence of these actions in order to compensate for the introduced disturbances. As a result of describing the dependencies by elementary functions, let's obtain a system of equations:

$$\delta h = f\left(\Delta v, \Delta Z, F, S\right);\ \delta T = f\left(\Delta v, \Delta Z, F, S\right);$$

$$\delta P = f\left(\Delta v, \Delta Z, F, S\right);\ \Pi = f\left(P, F, S\right). \quad (1)$$

These equations in a general form are nonlinear functions reflecting nonlinear regularities of changes in the regulated parameters depending on the control actions, shown in the graphs of Fig. 1–6. Nonlinearity of the functions allows to increase the accuracy of calculation of the optimal solution.

5. 2. Determination of transfer functions of each channel of influence on thickness, tension and flatness of the strip

The magnitude and change of each adjustable parameter were represented as a sum of transfer functions for each channel, taking into account probable perturbations (with a dash):

$$h = \frac{\varphi_{hv}\left(\Delta v\right) + \varphi_{hZ}\left(\Delta Z\right) + \varphi_{hF}\left(F\right) + \varphi_{hS}\left(S\right)}{4} + \delta h'; \quad (2)$$

$$T = \frac{\varphi_{Tv}\left(\Delta v\right) + \varphi_{TZ}\left(\Delta Z\right) + \varphi_{TF}\left(F\right) + \varphi_{TS}\left(S\right)}{4} + \delta T'; \quad (3)$$

$$P = \frac{\varphi_{Pv}\left(\Delta v\right) + \varphi_{PZ}\left(\Delta Z\right) + \varphi_{PF}\left(F\right) + \varphi_{TS}\left(S\right)}{4} + \Delta P'; \quad (4)$$

$$\delta h = h - h_s;\ \delta P = P - P_n;\ \delta T = T - T_s;\ \delta\Pi = \Pi - \Pi_s; \quad (5)$$

$$\Pi = \varphi_{\Pi}\ \left(P\right) + \varphi_{\Pi F}\left(F\right) + \varphi_{\Pi S}\left(S\right) + \delta\Pi', \quad (6)$$

where the index "*s*" corresponds to the set value of the parameter, and the index "*n*" – nominal; δ – parameter change.

The resulting system of nonlinear equations (2)–(7) was solved by Newton's method taking into account the imposed restrictions on the control parameters and, accordingly, the areas of operability of the obtained approximation dependencies (transfer functions φ).

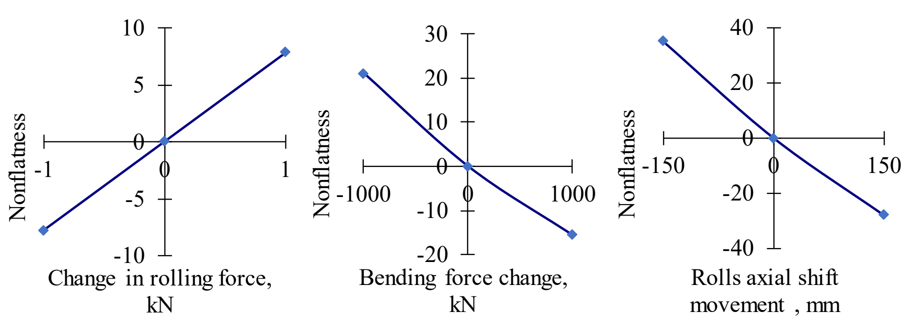


Fig. 6. Dependency graphs: *a* – influence of rolling force change; *b* – of bending force change; *c* – of roll axial shift movement on the flatness (mm/m) of rolled strips

5. 3. Criterion, selection of the method and solving the optimization problem on a specific example

As a criterion of combined effect on thickness and flatness of strips the minimum deviations of thickness, tension and flatness at the same time were adopted.

Combined control criterion:

$$F_{CC} = k_h \left|\delta h\right| + k_T \left|\delta T\right| + k_\Pi \Pi \to \min = 0, \tag{7}$$

where k_h, k_T, k_Π – weighting factors.

On concrete examples it has been established that within the limits of imposed restrictions on the values of control actions and force parameters of the process (T_{3-4} and P_4) the received system of equations has a solution at which it is provided simultaneously $\delta h_4 = \delta T_{3-4} = \delta \Pi_4 = F_{CC} = 0$ that means full compensation of deviation of strip thickness, strip tension from the set value and its shape from flat.

Let's examine this assertion with the first concrete example.

Example 1. Rolling process $2 \to 0.5 \times 1000$ mm; 08F steel; 4th stand of rolling mill 1700; $h_4 = 0.5$ mm; $T_{3-4} = 127$ kN; $\varepsilon_4 = 17$ %; $P_4 = 10.1$ MN; $\Pi_4 = 0$. Let's introduce a system of restrictions on possible changes in roll speed ±5 %, pressure device position ±0.5 mm, roll bending force (±1000 kN), axial roll shift movement (±150 mm) and rolling force not less than 5 MN and not more than 14 MN. This system of constraints meets not only the actual capabilities of the actuating control systems, but also the conditions of computational experiments, as a result of which the approximation functions of the transfer were determined.

The functional dependences are given below (obtained as a result of approximation of the computational experiment results, Fig. 1–4):

– thicknesses:

$$\varphi_{hv} = -0.0689 \ln\left(2 + \frac{\Delta v}{5}\right) + 0.545;$$

$$\varphi_{hZ} = 0.0064 \ln\left(2 + \frac{\Delta Z}{0.5}\right) + 0.4955;$$

$$\varphi_{hF} = 0.0012\left(2 + \frac{F}{1{,}000}\right) + 0.4976;$$

$$\varphi_{hS} = 0.0003\left(2 + \frac{S}{150}\right) + 0.4995;$$

– tensions:

$$\varphi_{Tv} = -0.291 \ln\left(2 + \frac{\Delta v}{5}\right) + 10.3;$$

$$\varphi_{TZ} = -35.5\left(2 + \frac{\Delta Z}{0.5}\right) + 197.67;$$

$$\varphi_{TF} = -17.5\left(2 + \frac{F}{1{,}000}\right) + 161;$$

$$\varphi_{TS} = -8.5\left(2 + \frac{S}{150}\right) + 143;$$

– forces:

$$\varphi_{Pv} = 88.506 \ln\left(2 + \frac{\Delta v}{5}\right) + 65.906;$$

$$\varphi_{PZ} = 2.135\left(2 + \frac{\Delta Z}{0.5}\right) + 5.8333;$$

$$\varphi_{PF} = \left(2 + \frac{F}{1{,}000}\right) + 8.1;$$

$$\varphi_{PS} = 0.43\left(2 + \frac{S}{150}\right) + 9.24;$$

– flatness:

$$\varphi_{\Pi P} = 7.8\left(2 + \frac{\delta P}{1}\right) - 15.6;$$

$$\varphi_{\Pi F} = -32.909\left(2 + \frac{F}{1{,}000}\right) + 21.488;$$

$$\varphi_{\Pi S} = -31.5\left(2 + \frac{S}{150}\right) + 65.333.$$

Let's adopt the values of weighting coefficients: $k_h = 0.7$; $k_T = 0.3$; $k_\Pi = 0.5$.

In order to numerically evaluate the results of control with automatic thickness control system; automatic tension control system; automatic strip thickness and tension control system, automatic strip thickness and tension control system+system of automatic regulation of profile and shape of strips, as well as the combined effects (by testing the respective techniques) introduced possible perturbations: $\delta h' = 1\ \% = 0.005$ mm; $\delta T' = 6\ \% = 7.1$ kN. Set it up that $\delta \Pi' = 0$; $\Pi' = 5$ mm/m.

As a result of performing a series of computational experiments, it is possible to obtain the necessary data on the control results using different methods corresponding to the regularities of operation of the analyzed automatic control systems. The obtained data are presented in Fig. 7.

Thus, the solution of the system (2)–(7) makes it possible to determine the necessary parameters of the actuators depending on the perturbations introduced into the process, which makes it possible to carry out the combined control of strip thickness and flatness.

As follows from the presented graphs, the combined effect on thickness, tension and flatness of the strip is more effective than the most modern complex of simultaneously operating local systems automatic strip thickness and tension control system+system of automatic regulation of profile and shape of strips and allows to reduce the optimization criterion to the minimum value (0) at each moment of time with the step determined by the cut-off frequency of the control contour.

This comparative analysis proves the validity of the approach to the combined control of strip thickness, tension and flatness, implemented on the basis of the method of solving the system of nonlinear equations (2)–(6) using the proposed optimization criterion (7).

At the same time, axial roll movement is an inertial control channel. The speed of axial roll movement in the systems used in the mills is 2–10 mm/s depending on the rolling speed. Therefore, in order to increase the speed of control, it is proposed to realize a fast compensating effect by roll bending at the initial moment of time when the rolling force changes, and then to change the axial position of the rolls, which causes 'readjustment' of the stand with simultaneous return to the initial value of the roll bending force. In this case, the proposed system of equations also has a solution.

Initially, it is necessary to determine the necessary direction and movement of the rolls axial, to predict the movement value for the next time interval (based on the current rolling speed) and to fix these values, and to find a solution only for 3 control parameters: roll speed, positions of pressure devices, roll bending force. If the change of strip flatness has a certain tendency to persist for a sufficiently long time, commensurate with the time of axial displacement of rolls by the initially determined value, the force of roll bending should be minimal, which is the most favorable condition from the point of view of readiness for subsequent control actions.

In general, due to the limited and different in magnitude velocity characteristics of actuators, it was of interest to take into account in the algorithm for synthesizing optimal control actions not only the constraints, but also their velocity characteristics and current position. Let's consider such an algorithm in more detail on the specific example 2.

Example 2. It is necessary to have quantitative dependences of the main characteristics of strip quality on the type and magnitude of control actions. Such dependencies were determined by mathematical modelling for the conditions of a 4-stand mill with a barrel length of 1700 mm, the last stand of which is equipped with hydraulic pressure devices, axial shift and bending systems of working rolls. As control effects mainly on the strip thickness it is possible to take the change of rolls rotation speed of the 4th stand υ (within ±5 %), tension in the last intercellular gap – the change of position of the 4th stand pressure devices $\pm Z$ (within ±0.5 mm). The force F of rolls bending (additional bending '–' and counter-bending '+'; within ±1,000 kN) and the axial roll shift movement (within ±150 mm) were taken as control actions on strip flatness. The changes in thickness h, tension in the last stand gap T, rolling force P and strip flatness index (nonflatness value Π) were considered as response functions to the control actions.

The process of rolling a strip of steel 08F with a width of 1000 mm from the initial thickness of 2.0 mm to the final thickness of 0.5 mm was considered. As in the previous example, the deformation mode of rolling, which provided uniform distribution of forces on the mill stands, was taken as the basic one. The initial (zero) values of bending force and axial roll shift movement were also set, assuming that the roll profiles were chosen correctly and there was nonflatness of the strip after rolling, the mill setting was performed accurately, and all adjustable parameters corresponded to the nominal ones. During the rolling process, the nominal values of the controlled parameters change due to perturbations of the rolling characteristics. To compensate for the influence of this factor it is necessary to know the nature and magnitude of deviation of controlled parameters from the basic values under the influence of regulating influences.

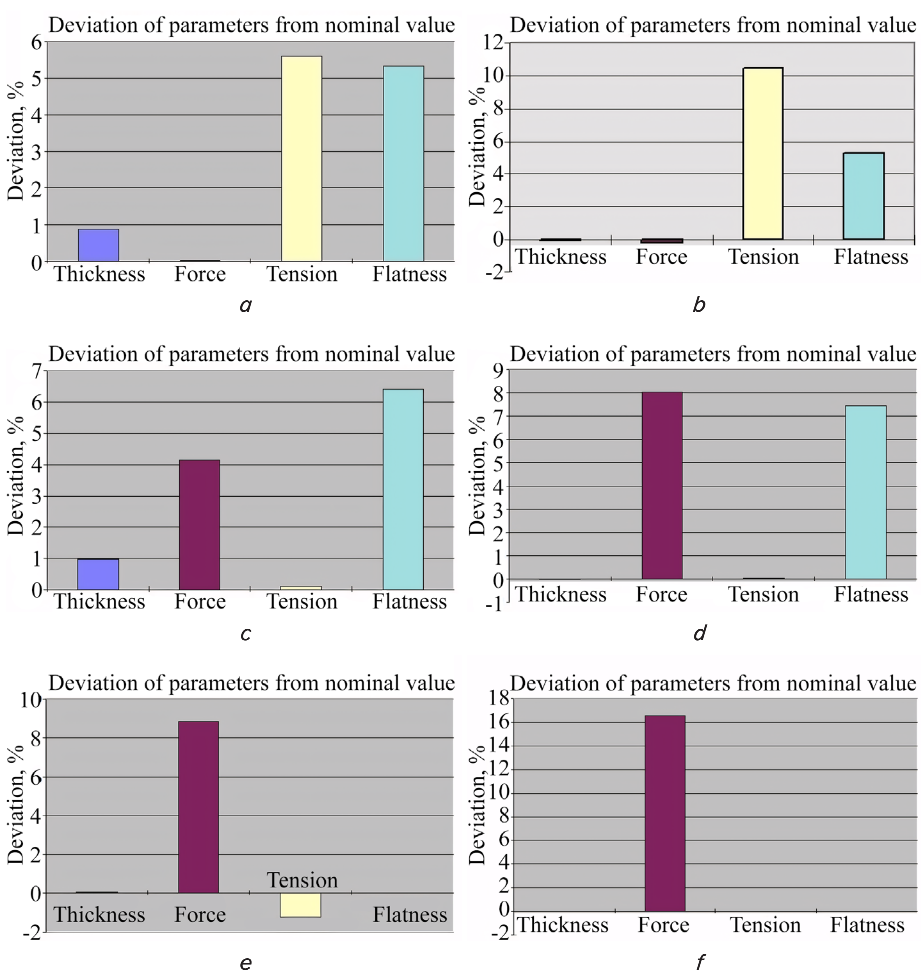


Fig. 7. Modelling results of different control methods: *a* – initial state; *b* – automatic thickness control system; *c* – automatic tension control system; *d* – automatic control system for strip thickness and tension; *e* – automatic strip thickness and tension control system + system of automatic regulation of profile and shape of strips; *f* – combined regulation

Let's consider the methodology and algorithm of combined control of three parameters (thickness, tension and flatness) by means of four channels (change of rolls rotation speed, position of pressure devices, force of rolls bending and change of axial rolls shift stroke of the last stand of the mill.

Nominal values of regulated parameters set (index "s"): h_s=0.5 mm; P_s=10.1 MN; T_s=127 kN; Π_s=0 mm/m.

Let's use the following measuring units: adjustable parameters: strip thickness h, mm; rolling force P, MN; tension T, kN; flatness Π, mm/m; regulating parameters: change of rolls rotation speed ΔV, %; change of position of pressure devices, ΔZ, mm; force of bending of rolls (counter-bending with '+' sign; additional bending with '–' sign), F, kN; movement of rolls axial shift (direction to increase conditional convexity of rolls with '+' sign, to decrease – with '–' sign), S, mm.

As an example, let's model a moment of time when the following disturbances (%) are in effect, which should be cancelled in the process of combined regulation: $\delta h_0=1$; $\delta P_0=0$; $\delta T_0=6$; $\delta\Pi_0=5$ %. In absolute units, these disturbances are defined as follows:

$$\delta h_0 := \delta h_0 \cdot \frac{h_s}{100};\ \delta P_0 := \delta P_0 \cdot \frac{P_s}{100};\ \delta T_0 := \delta T_0 \cdot \frac{T_s}{100}; \tag{8}$$

$\delta h_0=5\cdot10^{-3}$ mm; $\delta P_0=0$ MN; $\delta T_0=7.62$ kN, and the absolute values of the regulated parameters will be:

$$h := h_s + \delta h_0;\ P := P_s + \delta P_0;\ T := T_s + \delta T_0;\ \Pi := \Pi_s + \delta\Pi_0; \tag{9}$$

$h=0.505$ mm; $P=10.1$ MN; $T=134.62$ kN; $\Pi=5$ mm/m.

Applied limits that take into account the speed capabilities of the actuators and their current position. As mentioned earlier, axial roll movement is a more inertial control channel compared to hydraulic roll bending. The speed of axial roll movement in systems used in cold rolling mills depends on the rolling speed. This must be taken into account in the algorithm of combined control. In general, it is necessary to take into account the speed capabilities of all actuators, which means that an appropriate set of constraints on the system solution should be formed. For this purpose the rolling speed, for example, $\nu_r=10$ m/s, basic speed characteristics of actuators ($\nu_{\Delta V}=2$ m/s; $\nu_{\Delta Z}=40$ mm/s; $\nu_{\Delta F}=10{,}000$ kN/s) and time interval for decision making and movement of actuators $\Delta\tau=0.05$ s were introduced into the solution. The relations of these characteristics with the parameters of the rolling process were also introduced. For the speed of axial movement of rolls there is its dependence on the speed of rotation of rolls (rolling speed), rolling force and force of axial shifting of rolls [8].

Thus, at the force developed by the devices of axial roll movement, equal to $F_c=100$ kN, the degree of deformation of the strip $\varepsilon=0.15$, the coefficient of friction in the center of deformation $f=0.08$, the coefficient of sliding friction between the working and supporting rolls $f_c=0.1$ and the coefficient of rolling friction $f_k=0.0009$ the speed of axial roll movement (ν_S) is determined by the expression:

$$\nu_S = \frac{\nu_r \cdot f_c}{P}\left[\frac{\frac{\varepsilon}{1-\varepsilon}}{f\left(3-\frac{\varepsilon}{1-\varepsilon}\right)} + \frac{f_k}{f_c^2}\right], \tag{10}$$

and is $\nu_S=86.26$ mm/s.

As it is possible to see, the channel of influence on strip flatness by axial roll shifting is the slowest. The entire range of axial displacement from one extreme position –50 mm to the other +150 mm takes 3.5 sec. Therefore, when searching for an optimal solution, it is important to take into account the limited channel utilization per one control cycle.

5. 4. Boundary conditions that take into account the capabilities of the actuators and their current position

Knowing the functional dependences of changes in the parameters of the actuators in time, it is easy to determine the maximum possible new 'position' of the actuators at the next time step. For this purpose, the second set of imposed constraints reflecting the speed possibilities of changing the control parameters at the next time step was introduced into the solution of the system of equations.

Let's introduce possible ranges of change of control variables, conditioned by the speed capabilities of actuators and control systems (micro-level constraints):

$$-\frac{\nu_{\Delta V}\cdot\Delta\tau\cdot100}{\nu_r} \le \Delta V \le \frac{\nu_{\Delta V}\cdot\Delta\tau\cdot100}{\nu_r};$$

$$-\nu_{\Delta Z}\cdot\Delta\tau \le \Delta Z \le \nu_{\Delta Z}\cdot\Delta\tau\ ;\ -\nu_F\cdot F \le \nu_F\cdot\Delta\tau;$$

$$-\nu_S\cdot\Delta \le S \le \nu_S\cdot\Delta\tau. \tag{11}$$

The given inequalities satisfy the following limit values of the control parameters:

$$\delta Z_{\max} := \nu_{\Delta Z}\cdot\Delta\tau;\ \delta F_{\max} := \nu_F\cdot\Delta\tau;\ \delta S_{\max} := \nu_S\cdot\Delta\tau;$$

$$\delta V_{\max} := \frac{\nu_{\Delta V}\cdot\Delta\tau\cdot100}{\nu_r}. \tag{12}$$

Let's introduce the previously obtained matrices of transfer function coefficients for our rolling case:

$$Ko_h = \begin{bmatrix} -0.0689 & 0.545 \\ 0.0064 & 0.4955 \\ 0.0012 & 0.4976 \\ 0.0003 & 0.4995 \end{bmatrix};\ Ko_T := \begin{bmatrix} 88.506 & 65.90 \\ -35.5 & 197.67 \\ -17.5 & 161 \\ -8.5 & 143 \end{bmatrix};$$

$$Ko_P := \begin{bmatrix} -0.291 & 10.3 \\ 2.135 & 5.8333 \\ 1 & 8.1 \\ 0.43 & 9.24 \end{bmatrix};\ Ko_\Pi := \begin{bmatrix} 7.8 & -15.6 \\ -32.909 & 21.488 \\ -31.5 & 65.333 \end{bmatrix}. \tag{13}$$

The matrices contain influence coefficients for thickness (Ko_h), tension (Ko_T), force (Ko_P) and strip flatness (Ko_Π), respectively. Columns of matrices contain two influence coefficients of control parameters each, and rows contain influence coefficients of roll speed, position of pressure devices, bending force and axial shift movement of rolls, respectively.

5. 5. Implementation of the solution of the optimization problem by the Levenberg-Marquardt method

Let's introduce initial approximations for the control variables (ΔV=0 %; ΔZ=0 mm; F=0 kN; S=0 mm).

Let's express the target parameters of regulation through transfer functions and write down the solver block:

$$\left(\begin{aligned}&\left(Ko_{h_{0,0}}\cdot\ln\left(2+\frac{\Delta V}{\Delta V_{\max}}\right)+Ko_{h_{0,1}}\right)+\\&+\left(Ko_{h_{1,0}}\cdot\ln\left(2+\frac{\Delta Z}{\Delta Z_{\max}}\right)+Ko_{h_{1,1}}\right)+\\&+\left[Ko_{h_{2,0}}\cdot\left(2+\frac{F}{F_{\max}}\right)+Ko_{h_{2,1}}\right]+\\&+\left[Ko_{h_{3,0}}\cdot\left(2+\frac{S}{S_{\max}}\right)+Ko_{h_{3,1}}\right]\end{aligned}\right)/4+\delta h_0=h_s;$$

$$\left(\begin{aligned}&\left(Ko_{T_{0,0}}\cdot\ln\left(2+\frac{\Delta V}{\Delta V_{\max}}\right)+Ko_{T_{0,1}}\right)+\\&+\left(Ko_{T_{1,0}}\cdot\ln\left(2+\frac{\Delta Z}{\Delta Z_{\max}}\right)+Ko_{T_{1,1}}\right)+\\&+\left[Ko_{T_{2,0}}\cdot\left(2+\frac{F}{F_{\max}}\right)+Ko_{T_{2,1}}\right]+\\&+\left[Ko_{T_{3,0}}\cdot\left(2+\frac{S}{S_{\max}}\right)+Ko_{T_{3,1}}\right]\end{aligned}\right)/4+\delta T_0=T_s; \qquad (14)$$

$$\left(\begin{aligned}&\left(Ko_{P_{0,0}}\cdot\ln\left(2+\frac{\Delta V}{\Delta V_{\max}}\right)+Ko_{P_{0,1}}\right)+\\&+\left(Ko_{P_{1,0}}\cdot\ln\left(2+\frac{\Delta Z}{\Delta Z_{\max}}\right)+Ko_{P_{1,1}}\right)+\\&+\left[Ko_{P_{2,0}}\cdot\left(2+\frac{F}{F_{\max}}\right)+Ko_{P_{2,1}}\right]+\\&+\left[Ko_{P_{3,0}}\cdot\left(2+\frac{S}{S_{\max}}\right)+Ko_{P_{3,1}}\right]\end{aligned}\right)/4+\delta P_0=P;$$

$$\begin{aligned}&\left(Ko_{\Pi_{0,0}}\cdot\ln\left(2+\left(P-P_s\right)\right)+Ko_{\Pi_{0,1}}\right)+\\&+\left(Ko_{\Pi_{1,0}}\cdot\ln\left(2+\frac{F}{F_{\max}}\right)+Ko_{\Pi_{1,1}}\right)+\\&+\left[Ko_{\Pi_{2,0}}\cdot\left(2+\frac{S}{S_{\max}}\right)+Ko_{\Pi_{2,1}}\right]+\delta\Pi_0=\Pi_s.\end{aligned}$$

After introducing macro- and micro-level constraints, the solving function of the Mathcad system implementing the Levenberg-Marquardt solution method:

$$\begin{vmatrix}\Delta V\\ \Delta Z\\ F\\ S\end{vmatrix}:=\mathrm{MinErr}\left(\Delta V,\,\Delta Z,\,F,\,S\right). \qquad (15)$$

The solution results (ΔV=–1.04 %; ΔZ=0.15 mm; F=467.99 kN; S=–4.31 mm) are the closest in satisfying the equations and inequalities in the solver block.

In addition to the speed capabilities of the actuators, it is also necessary to take into account their current position within the permissible range (macro-level of constraints). In this connection, at the end of each calculation and execution cycle, it is possible to redefine the current (index 'c') parameters of the actuators and the corresponding initial values of the control variables: $\Delta V_c=\Delta V$; $\Delta Z_c=\Delta Z$; $F_c=F$; $S_c=S$.

The current position of the actuators shifts the micro-level constraints within the macro-level constraints.

5. 6. Estimation of the probable practical result from the implementation of the proposed method

According to the experimental evaluations based on the achieved performance of the best local automatic strip flatness control systems and automatic strip thickness control systems, a simultaneous improvement of flatness and reduction of the strip thickness limit deviations depending on the width and thickness as well as the current steel limit according to EN 10131 with index S (special tolerance) can be expected. On the basis of actual data obtained on a 5 stand endless cold rolling mill with a local automatic strip thickness control system in operation, as a result of optimization of the mill dynamic adjustment system, the specified limit lengths of thickened (according to EN 10131 (S)) near seam sections (50 m) were achieved in 97 % of cases at all grade transitions. In 80 % of the cases, the length of the thickened sections did not exceed 8–17 meters. This result refers to the case when the local system of automatic strip flatness control had practically no influence on strip thickness, as the last 5th stand of the mill realized the mode of operation with constant rolling force.

The achieved result with respect to flatness of cold-rolled strips is well illustrated by the example of a dressing mill [13], where the impact on strip thickness is minimal. As a result of operation of the developed system of automatic control of strip flatness in the test mode, when alternately strips were rolled in the manual and automatic control modes, the parameters of its efficiency, presented in Table 1, were achieved.

As it follows from the table, the system improved strip flatness by 20–30 % in comparison with the most careful manual control by the mill operator, providing flatness values within the limits set in EN 10131 (S) for steels with yield strength up to 260 MPa.

Table 1

Comparison of strip flatness in manual and automatic operation modes of AFC system

Strip thickness ranges, mm	Numerical values of flatness and its improvements in the automatic control mode in comparison with the manual one with a strip width (mm)											
	900–1,200				1,201–1,500				1,501–1,850			
	H	*A*	*M*	*I*	*H*	*A*	*M*	*I*	*H*	*A*	*M*	*I*
0.35–0.69	5	2.41	2.82	15	6	2.65	3.12	15	8	3.21	–	–
		49	33			180	97			2		
0.70–1. 2	4	2.37	2.88	18	5	2.87	3.4	16	7	3.91	4.51	13
		61	36			228	114			26	19	
1.21–1.5	3	2.73	3.12	13	4	2.73	3.58	24	6	3.29	4.54	28
		14	3			83	47			10	7	
1.51–2.5	3	2.84	3.31	14	4	3.18	3.75	15	6	3.95	–	–
		40	17			301	219			20	–	
2.51–3.5	3	2.44	2.59	6	4	3.28	3.69	11	6	3.8	–	–
		22	165			121	165			13	–	

Notes: numerator: H – limiting value of the wave height (mm) established by the terms of reference for the AFC numerically corresponding to the limited flatness tolerance according to EN 10131 (S); M – average height of the wave of flatness with manual control (mm); A – average wave height with automatic mode of operation of AFC (mm); I – improvement of flatness, %. Denominator: the number of coils when determining the mean values of M and A.

6. Discussion of the results of the developed combined thickness, tension and flatness control method

A classical approach was chosen to solve the problem of multifactor optimization of the main variable control variables of the rolling process, affecting simultaneously several key quality indicators – thickness accuracy and flatness of strips, ensuring the stability of the process by maintaining a stable tension in the last interstand gap. Realization of the approach taking into account the imposed constraints allows to use the solution in fast real-time systems, finding optimal solutions at each time step. In contrast to the commonly accepted linearization of control models in order to simplify solutions and improve the speed of algorithms in controllers, nonlinear dependencies (Fig. 1–4, 6) and corresponding nonlinear transfer function equations (1)–(6) are chosen to improve the accuracy of control actions.

Unlike the known solutions, when the control of thickness, tension and flatness of strips is realized as separate local systems that do not interact with each other [14, 15], the developed method eliminates this drawback. And makes it possible to simultaneously improve the accuracy in thickness and flatness of strips.

The formulated optimization criterion (7) allows the adjustment of the search algorithm by correcting the weight coefficients of the importance of providing individual targets. Concrete results simulating the operation of various local automatic control systems, as well as the proposed algorithm of combined regulation show convincing results (Fig. 7), where Newton's method was used as one of the possible solution methods to solve the system of equations. Further mathematical formalization of the idea into a solution method and algorithm was related to the consideration of a set of macro- and micro-level constraints and the verification of the alternative Levenberg-Marquardt optimization method (14), (15). The macro-level constraints (12) are related to the limiting characteristics of the actuators, and the micro-level constraints (11) are related to their significantly different speed capabilities.

The optimal solution is found on a concrete example.

The results of the work are not a part of the structure and ready program code for automatic control systems, but they give an opportunity to create them on this basis. Since this idea realizes an important advantage of simultaneous optimal control of two most important quality indicators of finished thin sheet products.

At the stage of system realization it is necessary to create on the second level of the mill automation system the procedures for determination of nonlinear transfer functions for the current pass plan, which makes such a system of combined control largely dependent on the second level of the mill automation system.

The potentially expected effect of using the combined control system depends on the rolling conditions, the actual patterns of changes in parameter deviations over time, requires experimental verification at a real facility and adjustment of the weighting factors in the optimization criterion (7).

The features of the proposed method consists in the interconnectedness of control actions on the thickness and flatness of the strip, when at each control cycle the deviation of one of the regulated parameters does not increase, as it happens in the known solutions, when two separate systems work:

1) automatic regulation of the thickness;

2) flatness of the strip.

This problem is especially acute when at the last stand of a continuous mill a significant degree of deformation (15–20 %) and thickness and flatness control by local control systems is implemented in it. Then, as a result of influencing the strip thickness by changing the ratio of roll speeds of the last stand and the group of stands preceding the last stand, there is a change of tension in the last interstand gap, which is compensated by changing the interroll gap in the last stand by changing the position of hydraulic pressure devices. As a result of such influences there is a significant change in the rolling force. And change of rolling force causes change of elastic deflection of rolls, which in turn worsens flatness of strip.

It should be noted that during the periods of filling of the continuous mill and release of the rear end of the strip at reduced speed, as well as during the skipping of welds and dynamic reconfiguration of the endless rolling mill for a new product mix, it is especially important to ensure optimal combined control of strip thickness and flatness, as the main problem is to ensure the specified thickness, as well as flatness precisely at the near seam areas at the ends of the strip. This is due to both inherited thickenings at the strip ends from the previous hot rolling process and the algorithms for passing welds at reduced speed in the cold rolling mill to avoid the risk of strip tearing.

The proposed method of interconnected regulation predicts changes of both regulated parameters from each influence and allows to find the optimal combination of them so that simultaneously to minimize deviations of both main regulated parameters.

Summarizing the above mentioned, the practical implementation of the proposed in this research method of combined control in the last stand of the 4-stand mill should be expected to achieve the best indicators of strip thickness deviation stipulated by the mentioned standard. In 80 % of cases the length of thickened near seam sections will not exceed 8–17 meters. It should be noted that, according to the standard, the limit deviations in the area of welded seams can be increased by 50 % for a length of 10 meters. At the same time, it is expected that the flatness of rolled strips will be reduced by 20–30 %.

It makes sense to develop further research by simulating probabilistic modeling of the algorithm of combined control of thickness, tension and flatness of strips. The probabilistic model implemented in the computer system WinColdRolling [12], where the algorithm of the local system of automatic control of strip thickness and tension is currently modeled, can be taken as a basis. It should be supplemented with modules simulating the formation of strip flatness and realize the developed algorithm of combined regulation of strip thickness, tension and flatness.

7. Conclusions

1. Numerical nonlinear dependences of regulated rolling parameters on the type and magnitude of regulating influences have been obtained on the example of a four-stand cold rolling mill, the last stand of which is equipped with axial shifting and hydraulic bending systems of working rolls, as well as hydraulic pressure devices and roll speed control system. In contrast to typical linearized expressions of real-time control systems, nonlinear functions are used.

2. Transfer functions of each channel of influence on thickness, tension and flatness of the strip are determined on concrete examples.

3. The criterion and method for solving the optimization problem on a concrete example are proposed, where the

possibility of ensuring the minimum deviation of regulated parameters – the main quality indicators of thin strips (thickness and flatness) is shown. The criterion includes weight coefficients of importance of separate indicators, which allows to purposefully manage the course of iterative procedures when searching for an optimal solution.

4. Boundary conditions, taking into account the speed capabilities of the actuators and the time of the control cycle, where the inertia of the roll's axial shifting channel is emphasized and evaluated, have been justified and established. This allows realistic influences to be applied at each control cycle.

5. On a concrete example of rolling process numerically confirmed the possibilities of realization of combined control of thickness, tension and flatness of strips taking into account the imposed restrictions associated with the speed capabilities of the channels of influence. Possibilities of realization of the combined control of thickness, tension and flatness of strips are confirmed (system of equations has a solution).

6. Numerical estimations of probable practical results achieved when using the proposed method of combined control of thickness, tension and flatness of strips are performed. To demonstrate such possibilities, two cases are specially selected when the automatic control systems of strip thickness and flatness do not interfere with each other. A simultaneous reduction in strip flatness of 20–30 % and the provision of limited thickness limit deviations to EN 10131(S) in at least 80 % of the inter-grade transitions are expected.

Conflict of interest

The authors declare that they have no conflict of interest in relation to this study, including financial, personal, authorship, or other, that could affect the study and its results presented in this article.

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Data availability

Data will be made available on reasonable request.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

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