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The growth in the number of mobile users of electronic communications services leads to the exhaustion of the radio frequency resource, which has made it urgent to find ways to reuse it. The use of methods from the modern theory of multi-user detection allows simultaneous transmission of a certain number of mutually non-orthogonal signals with different power in a common channel resource. These methods provide for the possibility of algorithms development for detecting and separating two digital signals in the processes of information transmission in radio channels with mutually non-orthogonal digital signals, which are the object of this study. Here, we propose a solution to the problem of determining the potential noise immunity of separation-demodulation of two mutually non-orthogonal BPSK signals, which involves taking into account the intermittency of their radiation. For this purpose, the results of the synthesis of the algorithm for detecting-separating two mutually non-orthogonal intermittent BPSK signals, optimal according to the criterion of minimum probability of error in estimating their discrete information parameters, have been used as initial data. This criterion is a necessary condition for solving this problem. The characteristic differences of the represented methodology for analyzing potential interference immunity are the absence of restrictions on the energy characteristics of signals, the degree of their non-orthogonality, and the probability of radiation.

The results allow developers to determine in practice the minimum required differences in the instantaneous signal powers in order to achieve maximum energy efficiency of radio channels under the given requirements for the error probability. In particular, it has been shown that the difference in such powers should reach at least 5÷6 dB in the case when both signals are emitted continuously, and with optimal detection and separation of intermittent signals, this difference increases to 9÷10 dB, depending on the degree of mutual non-orthogonality between the signals

Keywords: multi-user detection, mutually non-orthogonal digital signals, discrete parameter, potential noise immunity

DETERMINING THE POTENTIAL NOISE IMMUNITY OF SEPARATION-DEMODULATION OF TWO INTERMITTENT MUTUALLY NON-ORTHOGONAL SIGNALS BY BINARY PHASE-SHIFT KEYING

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1. Introduction

One of the most important areas in the field of electronic communications at present, which has undergone intensive development over the past four decades, is mobile communications. The emergence of 5G mobile communications is, in fact, a new era in telecommunications, which currently has five generations and offers more and more unique services. At the same time, the dynamic multimedia needs of humanity, the revolutionary emergence of the Internet of Things, virtual tourism, and so on have inevitably led to a rapid, random in time loading of the radio resource by statistically independent users.

Ways to overcome the limitation of the radio resource are the application of methods from the theory of multi-user detection (MUD), which allow the transmission of a certain number of mutually non-orthogonal signals that differ in power in one channel resource. These methods make it possible to synthesize complex algorithms for detecting and separating digital signals.

The evolution of the development of algorithms for optimal separation of two mutually non-orthogonal digital signals with binary phase-shift keying (BPSK) is a gradual process of complicating the statement of problems for the synthesis of the corresponding detection-separation algorithms. A characteristic feature of the synthesis procedures of these algorithms is the use of the Bayesian approach, which is the basis for further analysis of their potential noise immunity. In this case, the scientific and methodological apparatus of the statistical theory of separation of digital signals is used.

Modern mobile electronic communications systems are characterized by random access of users to a common channel resource. Existing analysis procedures do not take into account the discontinuity of BPSK signals; therefore, devising a methodology for analyzing the potential noise immunity of separation-demodulation even for the simplest case of two discontinuous mutually non-orthogonal BPSK signals is a timely task. In theory, it is expected to obtain an answer about the limits of potential noise immunity of detection-separation of such signals, and for practical applications – an answer about the necessary energy ratios between the signals to be separated.

Therefore, it is a relevant task to carry out studies on the minimum required differences in the instantaneous powers of mutually non-orthogonal signals in 5G generation mobile electronic communications systems.

2. Literature review and problem statement

In [1], an adaptive suboptimal approach was proposed to multi-user detection under conditions of incomplete a priori information about non-information parameters of digital signals (DSs), which are mutually non-orthogonal in the length of the information clock interval. However, issues related to the analysis of potential noise immunity were not considered.

In [2], the basic concepts and warnings against misconceptions in the theory of BCD were reported. In subsequent works [3, 4], the foundations of branches in the mentioned theory were exhaustively outlined. An example of the synthesis of an algorithm for the separation-demodulation of two mutually non-orthogonal signals with BPSK modulation, optimal according to the criterion of minimum probability of error in the estimation of discrete parameters (DPs) of each of the signals, was proposed. However, no analysis of its potential noise immunity was performed. A likely reason is computational difficulties and the fact that developers of modern mobile electronic communications are primarily interested in algorithms for separation-demodulation of mutually non-orthogonal signals of many users. And such algorithms, if synthesized according to the criterion of minimum probability of error in the estimation of information DP of each signal, turn out to be too complex and currently technologically unacceptable. As a result of the complexity, calculations of potentially achievable error probabilities will contain procedures of multiple (by the number of mutually non-orthogonal signals) integration with mutually dependent limits. Therefore, the authors of the above publications focused their primary attention on the development of suboptimal, technologically acceptable proposals for processing mutually non-orthogonal signals, when there are significantly more than two users, and limited themselves to upper estimates of error probabilities. Estimates of the noise immunity losses caused by the non-optimality of the proposed algorithms in comparison with those synthesized according to the above-mentioned optimality criterion were not performed, presumably due to computational difficulties.

In [5], general relations for mutual information and the minimum achievable mean square error in a Gaussian channel are given. However, the specific case of the presence of two mutually non-orthogonal signals in the channel and the intermittent nature of their radiation, which is interesting for practice, was not taken into account.

In studies [6, 7], energy efficiency estimates of wireless electronic communications with broadband signals and the principles of factorized combination of spatiotemporal processing and detection of mutually non-orthogonal CSs were performed. The presence of several (at least more than two) mutually non-orthogonal signals in the observation was assumed and, as a result, the use of simplified suboptimal processing algorithms was used. Therefore, the issue of analyzing potential noise immunity was excluded, only top-down energy efficiency estimates were performed.

In [8], energy efficiency estimates were reported for electronic communications systems with non-orthogonal multiple access with heuristic separation of unequal-energy CSs. A sequential, cascade separation-demodulation was proposed – first the most powerful signal was restored and then its estimate was removed from the observation. Then the less powerful one was demodulated, and so on. The problem of calculating the limits of potential noise immunity in such a formulation could also not be solved. All this gives grounds for substantiating the expediency of solving the problem of analyzing the potential noise immunity of detection-separation of mutually non-orthogonal signals.

Along with this, in [9], a brief overview of the results of the development of the methodology for synthesizing optimal algorithms for separation-demodulation of two mutually non-orthogonal BPSK signals was presented. However, in [9], the problem of analyzing the potential noise immunity was not solved. A likely reason is objective difficulties associated with the complexity of the calculations.

In [10], an analysis of the potential noise immunity of separation-demodulation of two asynchronously emitted BPSK signals with an arbitrary degree of their mutual non-orthogonality, when only the interfering signal is intermittent in radiation, is reported.

In [11], the result of the synthesis of the algorithm for the separation-demodulation of two mutually non-orthogonal synchronous BPSK signals is reported, which is also optimal according to the criterion of the minimum probability of error in the estimation of the DP of each of them, when both signals are intermittent. However, the analysis of the noise immunity of the synthesized algorithm was not the task of that study.

Accordingly, one of the areas to build on the results reported in [1–10] is to solve the problem of determining the potential noise immunity of the algorithm [11] for the detection-separation of two mutually non-orthogonal intermittent BPSK signals, synthesized according to the criterion of the minimum probability of error in the estimation of the binary DP of the usable signal.

3. The aim and objectives of the study

The aim of our study is to solve the problem of analyzing the potential noise immunity of the algorithm for the separation-demodulation of two synchronous clock points intermittent mutually non-orthogonal BPSK signals.

To achieve the goal, the following tasks were set:

– to improve the form of the representation of the decision rule (DR), synthesized in [11] with respect to the components of its argument;

– to derive an expression for the unconditional probability of error in the estimation of DPs of the usable signal in the general form;

– to derive expressions for the conditional probabilities of error in the estimation of DPs of the usable signal in the explicit form;

– to represent the unconditional probability of error in the estimation of DPs of the usable signal through the conditional ones in the explicit form;

– to calculate and graphically illustrate the probabilities of error in the estimation of DPs of the usable signal.

4. The study materials and methods

The object of our research is the processes of information transmission in radio channels with mutually non-orthogonal digital signals.

The basic hypothesis of our study assumes that the optimality criterion in the synthesis of DR reported in [11] is the minimum probability of error in the estimation of the information discrete parameter of the usable signal, which determines the principle possibility of obtaining the limit of its potential noise immunity for the given parameters of the usable signal and the interfering signal.

The initial data for the analysis, which are determined during the synthesis in [11] and applied here:

– observation model:

$$y_t=\left[(-1)^{(r_1)}+r_1\frac{1-r_1}{2}\right]s_1(t)+ \\ +\left[(-1)^{(r_2)}+r_2\frac{1-r_2}{2}\right]s_2(t)+n(t); \\ t\in\left[t_{k-1},t_k\right];\ \ k=1,2,...;\ \ r_i\in\{0,1,2\}, \tag{1}$$

where $s_1(t)$ and $s_2(t)$ are known square-integrated functions that characterize in the general case carrier signals without information manipulation, mutually non-orthogonal in the length of the information clock interval; $r_1,r_2\in\{0,\ 1,\ 2\}$ are information DPs of signals, while according to (1) the states $r_1,r_2=2$ correspond to the absence of radiation of the corresponding signal; $n(t)$ is additive white Gaussian noise (AWGN); t_k are the moments of possible state change (clock points) of DPs r_1, r_2; k is the number of the information clock interval;

– algorithm (DR) for separation-demodulation of two mutually non-orthogonal intermittent signals with a description of the relations for its components in the explicit form given below, derived in [11];

– the ratio of the energy of the first usable signal over the length of the information packet to the one-way spectral density of the power of AWGN;

– the signal radiation probabilities;

– the coefficient of non-orthogonality between the carriers normalized to unity.

Limitations and assumptions for an example of calculations of potential noise immunity:

– a priori probabilities of active states (radiation) of discrete parameters of signals $s_1(t)$, $s_2(t)$ are equal to 0.5 or 0.25;

– the ratio of instantaneous signal/noise powers varies within (–15÷25) dB.

5. Results of analyzing the potential noise immunity of the separation-demodulation of two intermittent mutually non-orthogonal binary phase-shift keying signals

5. 1. Improving the form of representing the decision rule synthesized in [11] with respect to the components of its argument

In the general case, DR [11] about the state of DPs $r_1\in\{0,\ 1,\ 2\}$ of the usable first intermittent BPSK signal based on the input observation (1) can be represented in an equivalent form convenient for calculations:

$$r_1^*=2rect\left[p(r_1=2/y_t)-p(r_1=0/y_t)\right]\times \\ \times rect\left[p(r_1=2/y_t)-p(r_1=1/y_t)\right]+ \\ +rect\left[p(r_1=1/y_t)-p(r_1=0/y_t)\right]\times \\ \times\left\{\begin{matrix}1-rect\left[p(r_1=2/y_t)-p(r_1=0/y_t)\right]\times \\ \times rect\left[p(r_1=2/y_t)-p(r_1=1/y_t)\right]\end{matrix}\right\}. \tag{2}$$

Here, $p(r_1/y_t)$ are the posterior probabilities of the states of DPs $r_1\in\{0,1,2\}$ of the first (usable) signal, $rect(x\geqslant 0)=1$; $\mathrm{rect}(x<0)=0$; $r_1^*\in\{0,1,2\}$ are the decisions about the state of this DP.

DR (2) can be represented in a compact form:

$$r_1^*=2r_{11}^*r_{12}^*+r_{13}^*\left(1-r_{11}^*r_{12}^*\right), \tag{3}$$

where decisions on auxiliary $r_{11}^*,\ r_{12}^*,\ r_{13}^*$ are made according to the rules:

$$r_{11}^*=rect\left[p(r_1=2/y_t)-p(r_1=0/y_t)\right]; \\ r_{12}^*=rect\left[p(r_1=2/y_t)-p(r_1=1/y_t)\right]; \\ r_{13}^*=rect\left[p(r_1=1/y_t)-p(r_1=0/y_t)\right]; \\ r_{11}^*,r_{12}^*,r_{13}^*\in\{0,1\}.$$

Let us represent the components of DR (3) in accordance with [11] in explicit form:

$$r_{11}^*=rect\left\{\begin{matrix}-b_1+ \\ +\ln\left[\dfrac{2(1-P_{\mathrm{tr1}})(1-P_{\mathrm{tr2}})\sqrt{(1-th^2b_2)(1-th^2 2R)\exp 2(h_1^2+h_2^2)}+(1-P_{\mathrm{tr1}})P_{\mathrm{tr2}}\sqrt{(1-th^2 2R)\exp 2h_1^2}}{P_{\mathrm{tr1}}P_{\mathrm{tr2}}(1-thb_2th2R)+P_{\mathrm{tr1}}(1-P_{\mathrm{tr2}})\sqrt{(1-th^2 2R)\exp 2h_2^2}}\right]\end{matrix}\right\}= \\ =rect(-b_1+\alpha). \tag{4}$$

$$r_{12}^*=rect\left\{\begin{matrix}b_1+ \\ +\ln\left[\dfrac{2(1-P_{\mathrm{tr1}})(1-P_{\mathrm{tr2}})\sqrt{(1-th^2b_2)(1-th^2 2R)\exp 2(h_1^2+h_2^2)}+(1-P_{\mathrm{tr1}})P_{\mathrm{tr2}}\sqrt{(1-th^2 2R)\exp 2h_1^2}}{P_{\mathrm{tr1}}P_{\mathrm{tr2}}(1+thb_2th2R)+P_{\mathrm{tr1}}(1-P_{\mathrm{tr2}})\sqrt{(1-th^2 2R)\exp 2h_2^2}}\right]\end{matrix}\right\}= \\ =rect(b_1+\beta). \tag{5}$$

$$r_{13}^*=rect\left[\begin{matrix}-b_1+ \\ +Arth\left(thb_2th2R\dfrac{P_{\mathrm{tr2}}}{P_{\mathrm{tr1}}+(1-P_{\mathrm{tr2}})\sqrt{(1-th^2b_2)(1-th^2 2R)\exp 2h_2^2}}\right)\end{matrix}\right]= \\ =rect(-b_1+\gamma). \tag{6}$$

Using the notations (4)÷(6), DR (2), (3) can be represented in the following form:

$$\begin{aligned} r_1^* &= 2rect(-b_1+\alpha)rect(b_1+\beta)+ \\ &+rect(-b_1+\gamma)\left[1-rect(-b_1+\alpha)rect(b_1+\beta)\right]= \\ &=2rect(-b_1+\alpha)rect(b_1+\beta)+ \\ &+rect(-b_1+\gamma)rect(b_1-\alpha)rect(-b_1-\beta). \end{aligned} \tag{7}$$

In components (4)÷(6) of DR (7), the following notations are used:

– P_{tr1}, P_{tr2} – the radiation probabilities of the first (usable) and second (interfering) signals, while assuming that the a priori probabilities of the active states of DPs of signals are:

$$p(r_{1,2}=0)=p(r_{1,2}=1)=\frac{P_{tr1,tr2}}{2};$$

– $r_1, r_2 \in \{0, 1, 2\}$ – states of DPs of the first and second signals;

$$b_{1,2}=\frac{2}{N_0}\int_{t_{k-1}}^{t_k} y_t s_{1,2}(t)\mathrm{d}t \tag{8}$$

– correlation integrals;

$t_k-t_{k-1}=T$ – information clock interval;

y_t – observation model (1);

$s_{1,2}(t)$ – square-integrated functions – carrier oscillations of the first and second signals;

$n(t)$ – AWGN;

N_0 – one-sided power spectral density of AWGN;

$R=\frac{1}{N_0}\int_{t_{k-1}}^{t_k} s_1(t)s_2(t)\mathrm{d}t=\rho\sqrt{h_1^2h_2^2}$ – mutual energy of the first and second signals on length T of the clock interval, referred to N_0;

$0<\rho<1$ – non-orthogonality coefficient between the first and second signals normalized to unity;

$h_{1,2}^2=\frac{1}{N_0}\int_{t_{k-1}}^{t_k} s_{1,2}(t)\mathrm{d}t$ – energies of signals on length T of the clock interval, referred to N_0.

We note here that under the condition of a continuously emitted usable signal $P_{tr1}=1$ it turns out that $\alpha\to-\infty$, $\beta\to-\infty$, and DR (7) degenerates into the one obtained earlier [10].

To calculate the probability of errors $r_1^*\neq r_1$ in estimating DPs of the usable (first) CS, it is necessary to calculate the probability of errors in each of the three components – parts of DR of the total DR in the form (7):

$$r_{11}^*=rect(-b_1+\alpha);\quad r_{12}^*=rect(b_1+\beta);\quad r_{13}^*=rect(-b_1+\gamma). \tag{9}$$

To represent the unconditional error probability in an explicit form, it is necessary to derive expressions for the conditional error probabilities as functions of the mathematical expectations, variances, and non-orthogonality coefficients. These parameters depend on the ratio of the first signal/second signal (noise) and on the mutual energy over length T of the information packet to the power density N_0 of AWGN.

5. 2. Deriving an expression for the probability of error in estimating a discrete parameter of a usable signal in the general form

The expression for calculating the unconditional probability of error r_1^* in estimating DP of the first signal must contain all possible conditional error probabilities listed below.

If both signals are present in the observation (1):

$$p(r_1^*=1-r_1/r_1,r_2) \text{ and } p(r_1^*=2/r_1,r_2),\quad r_{1,2}\in\{0,1\}.$$

If there is only a second interfering signal in the observation:

$$p(r_1^*=0/r_1=2,r_2) \text{ and } p(r_1^*=1/r_1=2,r_2),\quad r_2\in\{0,1\}.$$

If only the first, usable signal is observed:

$$p(r_1^*=1-r_1/r_1,r_2=2) \text{ and } p(r_1^*=2/r_1,r_2=2),\quad r_1\in\{0,1\}.$$

In the absence of both signals in the observation:

$$p(r_1^*=0/r_1=r_2=2) \text{ and } p(r_1^*=1/r_1=r_2=2).$$

Then the expression for the probability of error r_1^* in the estimation of DP of the usable first signal based on reviewing the complete group of events (all possible error variants $r_1^*\neq r_1$) taking into account the a priori probabilities of the above situations takes the following form:

$$\begin{aligned} P_2^1 &= \frac{P_{tr1}P_{tr2}}{4}\left[\begin{aligned}&\sum_{r_1=0}^{1}\sum_{r_2=0}^{1}p(r_1^*=1-r_1/r_1,r_2)+\\&+\sum_{r_1=0}^{1}\sum_{r_2=0}^{1}p(r_1^*=2/r_1,r_2)\end{aligned}\right]+ \\ &+\frac{(1-P_{tr1})P_{tr2}}{2}\sum_{r_2=0}^{1}\left[\begin{aligned}&p(r_1^*=0/r_1=2,r_2)+\\&+p(r_1^*=1/r_1=2,r_2)\end{aligned}\right]+ \\ &+\frac{P_{tr1}(1-P_{tr2})}{2}\sum_{r_1=0}^{1}\left[\begin{aligned}&p(r_1^*=1-r_1/r_1,r_2=2)+\\&+p(r_1^*=2/r_1,r_2=2)\end{aligned}\right]+ \\ &+(1-P_{tr1})(1-P_{tr2})\left[\begin{aligned}&p(r_1^*=0/r_1=r_2=2)+\\&+p(r_1^*=1/r_1=r_2=2)\end{aligned}\right]. \end{aligned} \tag{10}$$

Expression (10) for the probability of error is clear and transparent, but it is somewhat inconvenient to use – to calculate conditional probabilities of error $p(r_1^*\neq r_1/r_1,r_2)$, it is necessary to find out which errors $r_{11}^*=1-r_{11}$; $r_{12}^*=1-r_{12}$; $r_{13}^*=1-r_{13}$ (combinations of errors) in partial DRs (9) included in general DR (7) lead to this.

5. 3. Deriving expressions for conditional error probabilities in estimating a discrete parameter of a usable signal in explicit form

Let us now write expressions for conditional error probabilities of the form:

$$\begin{aligned} &p(r_{11}^*=1/r_1=0,r_2); \\ &p(r_{11}^*=1/r_1=1,r_2);\ p(r_{11}^*=0/r_1=2,r_2); \\ &p(r_{12}^*=1/r_1=0,r_2); \\ &p(r_{12}^*=1/r_1=1,r_2); \\ &p(r_{12}^*=0/r_1=2,r_2); \\ &p(r_{13}^*=1-r_1/r_1=\overline{0,1},r_2). \end{aligned} \tag{11}$$

First, we note that under the condition of a priori equal probability of the active states of DPs r_1, r_2:

$$\left(p\left(r_{1,2}=0\right)=p\left(r_{1,2}=1\right)=\frac{P_{\text{tr1,tr2}}}{2}\right),$$

the resulting error $p\left(r_1^*=0/r_1=1\right)=p\left(r_1^*=1/r_1=0\right)$ probabilities will be the same, which promises to halve the calculations of at least the last conditional probability in (11).

Let us start with the last error probability in the list (11), the expression for which was derived by us in [10]. Let us represent it in an explicit form as follows:

$$\begin{aligned}&p\left(r_{13}^*=1-r_1/r_1\in\{0,1\};r_2\in\{0,1,2\}\right)=\\&=p\left(r_1^*=1-r_1/r_1\in\{0,1\},r_2=\{0,1,2\}\right)=\\&=\frac{1+(-1)^{r_1^*}}{2}-(-1)^{r_1^*}\int_{-\infty}^{\infty}\int_{-\infty}^{X_3}\omega\left(x_1,x_2,\rho\right)\mathrm{d}x_1\mathrm{d}x_2.\end{aligned}\tag{12}$$

Here:

$$X_3=\frac{1}{\sigma_1}\left(\gamma-m_1\right);\;\;\gamma=Arth\left[K\left(b_2\right)thb_2th2R\right];$$

$$x_1=\left(b_1-m_1\right)/\sigma_1;\;\;x_2=\left(b_2-m_2\right)/\sigma_2;$$

$$m_1=2h_1^2\left\{(-1)^{r_1}+\left[(-1)^{r_2}+r_2\frac{1-r_2}{2}\right]\frac{\rho}{l^2}\right\};$$

$$m_2=2h_1^2\left\{(-1)^{r_1}\frac{\rho}{\sqrt{l^2}}+\left[(-1)^{r_2}+r_2\frac{1-r_2}{2}\right]/l^2\right\};$$

$\sigma_1=\sqrt{2h_1^2};\quad\sigma_2=\sqrt{2h_2^2}=\sqrt{\frac{2h_1^2}{l^2}};\quad l^2=\frac{h_1^2}{h_2^2}=\frac{l_{\text{ave}}}{P_{\text{tr2}}}$ – math expectation and variance of Gaussian random variables b_1, b_2, respectively;

$\omega\left(x_1,x_2,\rho\right)=\frac{1}{2\pi\sqrt{1-\rho^2}}\exp\left[-\frac{x_1^2+x_2^2-2\rho x_1x_2}{2\left(1-\rho^2\right)}\right]$ – two-dimensional Gaussian probability density;

$$K\left(b_2\right)=\frac{P_{\hat{a}2}}{P_{\hat{a}2}+\left(1-P_{\hat{a}2}\right)\sqrt{\left(1-th^2b_2\right)\left(1-th^2 2R\right)\exp 2h_2^2}};$$

$$b_1=x_1\sigma_1+m_1;\;\;b_2=x_2\sigma_2+m_2.$$

In order to match the ratio for calculating the probability of an error of the form (10), $r_1^*\neq r_1,\;\;r_1,r_1^*\in\{0,1\}$, we shall compile Table 1 of the variants of correspondence of possible errors in partial DRs (9) and in the general rule (3). The relationship between them is obtained from the specified DR (3).

Now, using Table 1, we shall write the components of ratio (10) of the form $p\left(r_1^*/r_1,r_2\right)$ in terms of the conditional probabilities of errors in partial DRs in the form $p\left(r_{11}^*/r_1,r_2\right)$, $p\left(r_{12}^*/r_1,r_2\right)$, $p\left(r_{13}^*/r_1,r_2\right)$:

$$\begin{aligned}&\sum_{r_1=0}^{1}\sum_{r_2=0}^{1}p\left(r_1^*=1-r_1/r_1,r_2\right)=\\&=\sum_{r_1=0}^{1}\sum_{r_2=0}^{1}\begin{bmatrix}p\left(r_{11}^*=0/r_1,r_2\right)p\left(r_{12}^*=1/r_1,r_2\right)+\\+p\left(r_{11}^*=1/r_1,r_2\right)p\left(r_{12}^*=0/r_1,r_2\right)+\\+p\left(r_{11}^*=0/r_1,r_2\right)p\left(r_{12}^*=0/r_1,r_2\right)\end{bmatrix}\times\\&\times p\left(r_{13}^*=1-r_1/r_1,r_2\right);\end{aligned}\tag{13}$$

$$\begin{aligned}&\sum_{r_1=0}^{1}\sum_{r_2=0}^{1}p\left(r_1^*=2/r_1,r_2\right)=\\&=\sum_{r_1=0}^{1}\sum_{r_2=0}^{1}\left[p\left(r_{11}^*=1/r_1,r_2\right)p\left(r_{12}^*=1/r_1,r_2\right)\right];\end{aligned}\tag{14}$$

$$\begin{aligned}&\sum_{r_2=0}^{1}\left[p\left(r_1^*=0/r_1=2,r_2\right)+p\left(r_1^*=1/r_1=2,r_2\right)\right]=\\&=\sum_{r_2=0}^{1}\begin{bmatrix}p\left(r_{11}^*=0/r_1=2,r_2\right)p\left(r_{12}^*=1/r_1=2,r_2\right)+\\+p\left(r_{11}^*=1/r_1=2,r_2\right)p\left(r_{12}^*=0/r_1=2,r_2\right)+\\+p\left(r_{11}^*=0/r_1=2,r_2\right)p\left(r_{12}^*=0/r_1=2,r_2\right)\end{bmatrix};\end{aligned}\tag{15}$$

$$\begin{aligned}&\sum_{r_1=0}^{1}\begin{bmatrix}p\left(r_1^*=1-r_1/r_1,r_2=2\right)+\\+p\left(r_1^*=2/r_1,r_2=2\right)\end{bmatrix}=\\&=\sum_{r_1=0}^{1}\left\{\begin{array}{l}\begin{bmatrix}p\left(r_{11}^*=0/r_1,r_2=2\right)\times\\\times p\left(r_{12}^*=1/r_1,r_2=2\right)+\\+p\left(r_{11}^*=1/r_1,r_2=2\right)\times\\\times p\left(r_{12}^*=0/r_1,r_2=2\right)+\\+p\left(r_{11}^*=0/r_1,r_2=2\right)\times\\\times p\left(r_{12}^*=0/r_1,r_2=2\right)\end{bmatrix}\times\\\times p\left(r_{13}^*=1-r_1/r_1,r_2=2\right)+\\+p\left(r_{11}^*=1/r_1,r_2=2\right)\times\\\times p\left(r_{12}^*=1/r_1,r_2=2\right)\end{array}\right\};\end{aligned}\tag{16}$$

$$\begin{aligned}&p\left(r_1^*=0/r_1=r_2=2\right)+p\left(r_1^*=1/r_1=r_2=2\right)=\\&=p\left(r_{11}^*=0/r_1=r_2=2\right)p\left(r_{12}^*=1/r_1=r_2=2\right)+\\&+p\left(r_{11}^*=1/r_1=r_2=2\right)p\left(r_{12}^*=0/r_1=r_2=2\right)+\\&+p\left(r_{11}^*=0/r_1=r_2=2\right)p\left(r_{12}^*=0/r_1=r_2=2\right).\end{aligned}\tag{17}$$

Table 1

Variants of possible errors in partial DRs r_{11}^*, r_{12}^*, r_{13}^*

r_1	r_{11}^*	r_{12}^*	r_{13}^*	$r_1^*\neq r_1$
0	1	1	0;1	2
1	1	1	0;1	2
0	0	1	1	1
0	1	0	1	1
0	0	0	1	1
1	0	1	0	0
1	1	0	0	0
1	0	0	0	0
2	0	1	0;1	0;1
2	1	0	0;1	0;1
2	0	0	0;1	0;1

The derived relations allow us to further represent the expression for the unconditional probability of error in estimating the discrete parameters of the usable signal in a form convenient for calculations.

5. 4. Representation of the unconditional probability of error in the estimation of discrete parameters of the usable signal through conditional ones in explicit form

Now expression (9) for the probability of error $r_1^*\neq r_1$ taking into account (13)÷(17) will be written in the form:

$$
\begin{aligned}
&P_2^1=\frac{P_{\mathrm{tr}1}P_{\mathrm{tr}2}}{4}\left[\begin{array}{l}\displaystyle\sum_{r_1=0}^{1}\sum_{r_2=0}^{1}\left[\begin{array}{l}p\left(r_{11}^*=0/r_1,r_2\right)\times\\ \times p\left(r_{12}^*=1/r_1,r_2\right)+\\ +p\left(r_{11}^*=1/r_1,r_2\right)\times\\ \times p\left(r_{12}^*=0/r_1,r_2\right)+\\ +p\left(r_{11}^*=0/r_1,r_2\right)\times\\ \times p\left(r_{12}^*=0/r_1,r_2\right)\end{array}\right]\times\\ \times p\left(r_{13}^*=1-r_1/r_1,r_2\right)+\\ +p\left(r_{11}^*=1/r_1,r_2\right)p\left(r_{12}^*=1/r_1,r_2\right)\end{array}\right]+\\
&+\frac{\left(1-P_{\mathrm{tr}1}\right)P_{\mathrm{tr}2}}{2}\sum_{r_2=0}^{1}\left[\begin{array}{l}p\left(r_{11}^*=0/r_1=2,r_2\right)\times\\ \times p\left(r_{12}^*=1/r_1=2,r_2\right)+\\ +p\left(r_{11}^*=1/r_1=2,r_2\right)\times\\ \times p\left(r_{12}^*=0/r_1=2,r_2\right)+\\ +p\left(r_{11}^*=0/r_1=2,r_2\right)\times\\ \times p\left(r_{12}^*=0/r_1=2,r_2\right)\end{array}\right]+\\
&+\frac{P_{\mathrm{tr}1}\left(1-P_{\mathrm{tr}2}\right)}{2}\sum_{r_1=0}^{1}\left\{\begin{array}{l}\left[\begin{array}{l}p\left(r_{11}^*=0/r_1,r_2=2\right)\times\\ \times p\left(r_{12}^*=1/r_1,r_2=2\right)+\\ +p\left(r_{11}^*=1/r_1,r_2=2\right)\times\\ \times p\left(r_{12}^*=0/r_1,r_2=2\right)+\\ +p\left(r_{11}^*=0/r_1,r_2=2\right)\times\\ \times p\left(r_{12}^*=0/r_1,r_2=2\right)\end{array}\right]\times\\ \times p\left(r_{13}^*=1-r_1/r_1,r_2=2\right)+\\ +p\left(r_{11}^*=1/r_1,r_2=2\right)\times\\ \times p\left(r_{12}^*=1/r_1,r_2=2\right)\end{array}\right\}+\\
&+\left(1-P_{\mathrm{tr}1}\right)\left(1-P_{\mathrm{tr}2}\right)p\left(r_{11}^*=0/r_1=r_2=2\right)\times\\
&\times p\left(r_{12}^*=1/r_1=r_2=2\right)+p\left(r_{11}^*=1/r_1=r_2=2\right)\times\\
&\times p\left(r_{12}^*=0/r_1=r_2=2\right)+\\
&+p\left(r_{11}^*=0/r_1=r_2=2\right)p\left(r_{12}^*=0/r_1=r_2=2\right).
\end{aligned}
\tag{18}
$$

In (18):

$$
p\left(r_{11}^*=1/r_1,r_2\right)=\int_{-\infty}^{\infty}\int_{-\infty}^{X_1}\omega\left(x_1,x_2,\rho\right)\mathrm{d}x_1\mathrm{d}x_2; \tag{19}
$$

$$
p\left(r_{12}^*=1/r_1,r_2\right)=\int_{-\infty}^{\infty}\int_{X_2}^{\infty}\omega\left(x_1,x_2,\rho\right)\mathrm{d}x_1\mathrm{d}x_2;
$$

$$
p\left(r_{11}^*=0/r_1,r_2\right)=\int_{-\infty}^{\infty}\int_{X_1}^{\infty}\omega\left(x_1,x_2,\rho\right)\mathrm{d}x_1\mathrm{d}x_2;
$$

$$
p\left(r_{12}^*=0/r_1,r_2\right)=\int_{-\infty}^{\infty}\int_{-\infty}^{X_2}\omega\left(x_1,x_2,\rho\right)\mathrm{d}x_1\mathrm{d}x_2;
$$

$$
X_1=\frac{1}{\sigma_1}\left(\alpha-m_1\right);
$$

$$
X_2=\frac{1}{\sigma_2}\left(\beta-m_2\right).
$$

Centering and normalizing the Gaussian two-dimensional probability density under the integrals in (19) unifies the calculation procedures.

5. 5. Calculations and graphic illustration of error probabilities in estimating a discrete parameter of a usable signal

The results of calculating the unconditional error probability in the form $r_1^*\neq r_1$ according to relations (12), (18), (19) for the case $h_1^2=10.53$ dB, ρ=0.9 are represented by the plots in Fig. 1.

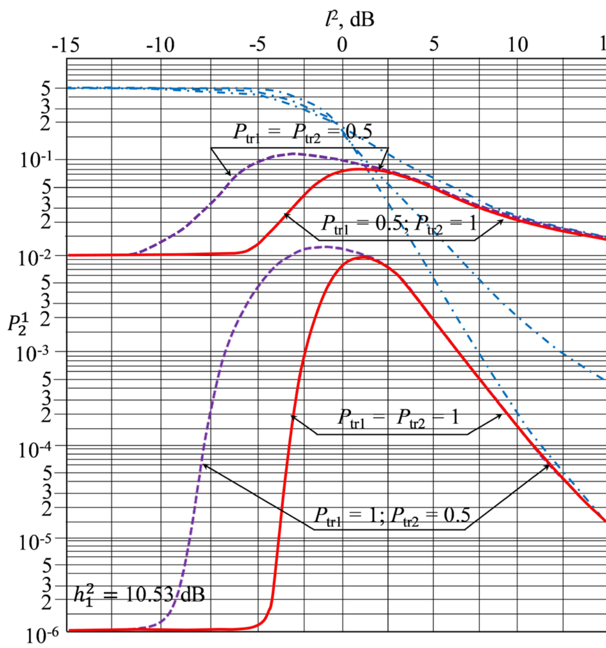

Fig. 1. Potential noise immunity of separating 2 intermittent signals at $h_1^2=10.53$dB, ρ=0.9, $P_{tr1,2}\in\{0.5;\ 1\}$

The basic result that follows from our calculations is the following. It turned out that the noise immunity of separation-demodulation depends on the cancellation in the instantaneous powers of the signals to be separated. A significant deterioration in noise immunity is observed in cases where such cancellation does not exceed 5–9 dB, depending on the radiation probabilities.

6. Discussion of results based on the analysis of potential noise immunity of the detection-separation of two mutually non-orthogonal binary manipulation signals

The results of calculations using formulas (3)–(7), (9), (12), (18), (19) are shown in Fig. 1, where continuous unimodal curves correspond to continuous radiation of both signals, dotted unimodal curves correspond to intermittent radiation of the second interfering signal. For comparison, the dependences of the noise immunity of the reception of the first signal are also given there, when the compensation of the second according to DR (7) is not performed (dashed-dotted monotone curves in blue).

Although in [11] the principle possibility of separating two intermittent BPSK signals was proven, there is a significant deterioration in the noise immunity of the reception of the first signal by approximately 5÷6 dB compared to the condition when it is emitted continuously. The fact is that the intermittent nature of the emission of the usable signal is equivalent to amplitude manipulation, which is worse in terms of noise immunity than binary manipulation of opposite signals by approximately 6 dB.

There is a zone of reduced noise immunity, located within the limits where the instantaneous signal powers differ within (–9 ÷16) dB. In this case, the noise immunity of the demodulation of the usable (first) signal when the instantaneous power of the second interfering signal exceeds by 9 dB or more (l^2<–9 dB in the plots of Fig. 1) approaches the potential noise immunity in the channel without such interference. If the interfering signal has a continuous radiation mode (P_{tr2}=1), then the zone of reduced noise immunity narrows to the left along l^2 by approximately 3 dB and is approximately – 6÷16 dB (continuous red curves in Fig. 1). This can be explained by the fact that the probability of error $r_2^* = 1 - r_2$ in estimating DPs of the interfering signal at $l^2 = h_1^2 / h_2^2 < 6$ dB for the case P_{tr2}=1 and at l^2<9 dB for the case P_{tr2}<1 turns out to be significantly lower than the probability of error $r_1^* = 1 - r_1$ in estimating DPs of the usable signal in the channel without such interference. Note that only the information parameters r_1, r_2 are unknown in observation (1).

Comparison of the reported results [10] with the findings described here regarding analysis of the potential noise immunity of two mutually non-orthogonal BPSK signals allows us to state the following. The best potential noise immunity is characterized by the situation when both signals are emitted continuously (P_{tr1}=P_{tr2}=1), and the shift between the clock points of the signals is equal to half the clock interval: τ=T/2. If the interfering signal is intermittent (P_{tr2}<1), then the best noise immunity is also observed under the same condition τ=T/2, but there is an increase in the zone of reduced noise immunity by – (5÷6) dB to the left along l^2 (unimodal dashed curves in Fig. 1). At P_{tr2}→1 this increase gradually disappears. In general, the potential noise immunity (error probability) of demodulation of two mutually non-orthogonal BPSK signals – signals in the zone of reduced noise immunity improves approximately twice when the shift between the clock points of the signals changes from τ=0 to τ=T/2. At τ=T/2 and $r_2^k \neq r_2^{k+1}$, that is, when the states of DPs of the second interfering signal do not coincide on adjacent clock intervals that overlap in time with the first usable signal. There is a mutual compensation of the influence of the second signal on the first.

Our results make it possible in practice to apply the optimal values of the ratio of instantaneous signal powers to achieve maximum energy efficiency of radio channels under the given requirements for the probability of error in estimating the binary discrete parameter of the usable signal.

The current study is strictly limited by the idealized statement, inherent exclusively to the problems of analyzing potential noise immunity. Here, only the informational discrete parameters of mutually non-orthogonal signals are considered unknown, and the non-informational ones are assumed to be exactly known. However, only with such an idealization could it be possible to obtain an answer about the potentially achievable noise immunity of the separation-demodulation of two intermittent mutually non-orthogonal BPSK signals.

The analyzed DR (3) has a rather complex analytical form. Answers to the questions about the influence of technologically acceptable simplifications on its noise immunity are of unconditional practical interest. For example, these may be linear-polynomial approximations of nonlinear transfer characteristics of compensation components in (4) to (6).

Beyond the scope of our study are issues regarding the influence of the accuracy of estimates of non-informational parameters of signals on losses in noise immunity of separation-demodulation. Resolving them seems problematic due to the need for further development of the known theory of compatible optimal nonlinear filtering of discrete-continuous Markov parameters for cases when mutually non-orthogonal digital signals are present in the observation. The results of such estimates could be of interest to developers as a basis for choosing certain technical solutions.

7. Conclusions

1. The procedure for analyzing the noise immunity of the algorithm for separating-demodulating two intermittent BPSK signals is reduced to the form when DR is represented by a superposition of binary solutions in the form $rect(x \geqslant 0)=1$; $rect(x<0)=0$, convenient from the point of view of calculations. That allowed us to factor the calculations of the probability of error in estimating the information DP of the usable signal into a series of calculations of conditional error probabilities, which simplified the task.

2. By employing expressions for all conditional error probabilities in estimating DPs of the usable signal, taking into account the a priori radiation probabilities of each of the signals, an expression for the unconditional error probability has been derived. This expression contains all variants of incorrect decisions about the state of DPs of the usable signal.

3. Taking into account the possible states of DPs of both mutually non-orthogonal signals, their instantaneous powers, and an arbitrary degree of their non-orthogonality, explicit expressions for all possible conditional error probabilities in estimating the DP of the usable signal have been derived. The expressions are double integrals of the two-dimensional Gaussian probability density with mutually dependent integration limits. The dependence of integration limits on the mathematical expectations of random variables at the outputs of the correlators of mutually non-orthogonal signals in the decision-making rule was taken into account.

4. Applying convenient expressions for conditional error probabilities, an expression has been derived for calculating the potential noise immunity of separation-demodulation of two mutually non-orthogonal signals with BPSK. The integration limits were separately determined when calculating the conditional error probabilities for cases when the usable first signal is absent. It is possible to specify the signal-to-noise ratio of the first and second signals, their emission probabilities, and the degree of their mutual non-orthogonality.

5. Our calculation examples confirm the assumption of a significant negative impact on the potential noise immunity of the detection-separation-demodulation procedures of CS of the intermittent nature of their emission. This is explained by the known difference in the noise immunity of the coherent demodulation procedures of CSs with a passive pause and with opposite signals (approximately 6 dB in instantaneous power). The best results in solving the problem of detection-separation-of two intermittent mutually non-orthogonal digital signals on one clock interval should be expected with differences in their instantaneous powers of no less than 9 dB. Under the condition of their continuous emission, a cancellation of 6 dB is sufficient.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study, as well as the results reported in this paper.

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Data availability

All data are available, either in numerical or graphical form, in the main text of the manuscript.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

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