The Capacitated Vehicle Routing Problem with Time-Dependent Demands (CVRPTD) is a significant optimization challenge in the logistics and transportation domain, characterized by dynamic customer demands, strict time windows, and heterogeneous vehicle fleets. This study focuses on urban parcel delivery operations as the primary object of research. The problem addressed involves the inefficiency of conventional vehicle routing strategies in adapting to time-varying customer demands and operational constraints, which often lead to increased costs and service delays. This study aims to minimize total operational costs while ensuring compliance with capacity constraints, service continuity, and demand fluctuations. A comprehensive mathematical model is developed based on a fully connected, directed acyclic graph G = (V, A), incorporating decision variables that represent vehicle routing sequences, timing, and vehicle type assignments. This study addresses the Capacitated Vehicle Routing Problem with Time-Dependent Demands (CVRPTD) in urban parcel delivery, where traditional routing methods struggle with dynamic demands and operational constraints. A mathematical model using a directed acyclic graph is developed, optimized via a gradient-based method with Hessian approximation, LU decomposition, and quasi-Newton techniques. Experiments on datasets with up to 200 customers and 20 vehicles with reductions ranging from 1.79% to 12.75%. The most significant improvement was observed in Sidorame Timur, where the optimization distance decreased by 12.75%, indicating high accuracy in route optimization. For the SCP, the proposed algorithm achieved a 6.46% improvement in solution quality over traditional greedy algorithms

Keywords: optimization, machine learning, set cover, CVRP, cost

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DEVELOPMENT OF OPTIMIZATION ALGORITHMS TO VEHICLE ROUTING PROBLEM

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1. Introduction

Optimization problems [1] in logistics and transportation are increasingly becoming crucial in today's fast-changing world, especially as demands for cost efficiency and timely delivery increase [2]. One of the most challenging problems in this area is the Capacitated Vehicle Routing Problem with Time-Dependent Demand (CVRPTD), which aims to optimize vehicle routes while considering capacity constraints, dynamic demand patterns, and specific service time windows. Addressing these issues is crucial to minimize operational costs and improve service quality, especially in industries that rely on complex supply chains and heterogeneous vehicle fleets [3].

Various studies have been conducted to address the vehicle routing problem (VRP). Classical approaches, such as those discussed in static demand scenarios, do not take into account real-time demand fluctuations or vehicle heterogeneity. More advanced methodologies, such as the integration of dynamic demand patterns, often ignore the complexity of different types of vehicles and interdependent routing schedules. These limitations highlight the need for more robust and comprehensive models to handle real-world logistics challenges [4].

CVRPTD is modeled on a fully directed acyclic graph, where nodes represent customers and depots, and arcs define possible routes. This problem introduces unique complexities, such as service window management, fluctuating demand accumulation, and vehicle scheduling over multiple periods. Current state-of-the-art methods often rely on simplified assumptions, leading to suboptimal solutions in heterogeneous environments [5, 6].

Therefore, research on the development of a robust and computationally efficient optimization model for CVRPTD is highly relevant, particularly in addressing the pressing challenges of dynamic logistics systems and ensuring service reliability in complex and heterogeneous operational environments.

2. Literature review and problem statement

This research [7] provides a comprehensive review and experimental study of learning-based optimization algorithms to solve the Vehicle Routing Problem (VRP). This research highlights recent advances in the use of learning methods, such as Machine Learning (ML) and Deep Learning (DL), to address challenges in vehicle route planning, including dynamic variations such as changing customer demand, traffic conditions, and capacity constraints. This study evaluates various learning-based approaches, such as reinforcement learning for adaptive decision-making and supervised learning for demand pattern prediction. Experiments show that learning-based algorithms can provide competitive results, especially in large-scale problems or with real-time data, compared with traditional optimization methods. This research identifies the great potential of integrating learning techniques in modern logistics solutions and the challenges that still need to be overcome, such as the need for big data and high computing times.

This research [8] presents the results of research on the multi-trip vehicle routing problem (MTVRP) in humanitarian logistics, particularly in disaster relief distribution. It is shown that while previous studies have addressed single-trip models, there is a significant gap in considering multi-trip vehicle usage, which could improve vehicle utilization and the speed of relief deliveries. However, unresolved issues remain, particularly the lack of models incorporating dynamic, multi-period planning that adjusts to real-time changes in disaster conditions. The reason for this may be the fundamental challenges associated with modeling dynamic environments in a way that remains computationally efficient. Additionally, cost constraints and the complexity of real-time data updates make implementing such models difficult. A potential way to overcome these difficulties could involve the development of more advanced heuristics and metaheuristics to handle computational load, as was done in the case of the GF3EA algorithm proposed in this paper. However, this approach still faces limitations, such as the need for further refinement of the solution approach to improve its scalability. All this suggests that it is advisable to conduct a study on integrating multi-period, multi-trip vehicle routing models that account for dynamic changes in disaster scenarios, with further investigation into hybrid algorithmic solutions to enhance computational performance.

This research [9] presents the results of research on improving disaster relief operations through a multi-depot, multi-trip vehicle routing model. The study demonstrates that considering multiple trips per vehicle and a multi-period approach improves the efficiency of the response operations. However, several unresolved issues were identified, such as optimizing the network structure, considering uncertainty in parameters, and addressing the lack of focus on routing reliability and social costs in disaster response. These challenges are partly due to the complex nature of disaster logistics, the need for real-time updates, and the dynamic environmental changes during relief efforts. A way to overcome these difficulties could be to integrate hybrid algorithms with heuristics, improving both solution quality and computational efficiency. This approach was successfully used in a study on vehicle routing in humanitarian logistics, though the need for further optimization remains. All of this suggests that it is advisable to conduct a study on enhancing the robustness and adaptability of routing models for disaster response, particularly in incorporating real-time data and developing more flexible network optimization strategies.

This research [10] presents the results of research on metaheuristic-based optimization approaches for the Vehicle Routing Problem (VRP) in waste management. It is shown that metaheuristics, such as Ant Colony Optimization (ACO) and Genetic Algorithms (GA), have proven effective in minimizing costs and improving service quality in waste collection operations. However, there are unresolved issues related to the dynamic nature of waste collection, particularly the lack of real-time optimization in response to changing conditions like traffic congestion, varying waste generation rates, and road closures. The reason for this may be the fundamental difficulty of incorporating real-time data into routing algorithms, as well as the computational complexity of solving such dynamic problems within time constraints, which makes relevant research in this area challenging. A way to overcome these difficulties can be through the integration of advanced technologies such as IoT, GPS, and GIS to enhance real-time data processing and decision-making. This approach was used in case studies, such as those conducted in Ho Chi Minh City, where Geographic Information Systems (GIS) were combined with optimization algorithms to improve route planning. However, this integration is still in its infancy and faces challenges in scaling and real-time adaptation. All of this suggests that it is advisable to conduct a study on developing more robust, scalable models for real-time waste collection optimization, incorporating both metaheuristic algorithms and advanced real-time data technologies.

This research [11] presents research on applying machine learning (ML) to vehicle routing problems (VRPs), highlighting the benefits of reinforcement learning (RL). However, unresolved issues include the high computational cost, insufficient training data, and difficulties in handling dynamic, real-world scenarios. These challenges stem from objective difficulties such as the extensive resources required for training large RL models and the lack of comprehensive datasets for large-scale VRPs. Overcoming these issues may involve combining ML-based methods with traditional optimization techniques, as demonstrated in some studies. This approach could be further explored to improve scalability and effectiveness in real-world applications, making it advisable to conduct a study on hybrid algorithms that integrate classical and ML methods.

This research [12] presents research on the Multi-trip Time-dependent Vehicle Routing Problem with Time Windows (MT-TDVRPTW). It shows that while the algorithms developed are effective, challenges remain in optimizing trip scheduling due to time-dependent travel times and maximum trip durations. These challenges stem from the complexity of real-world constraints, such as varying travel speeds and the synchronization of multiple trips for each vehicle, making the research costly and difficult to implement at scale. To address these difficulties, a potential solution could be the development of more efficient meta-heuristic algorithms, like adaptive large neighborhood search (ALNS), which has been applied successfully to similar problems. However, further refinement is needed to make it applicable to MT-TDVRPTW in large-scale scenarios. This suggests that future research should focus on improving ALNS methods and exploring their practical applications in urban logistics with time-dependent constraints.

This research [13] presents research on optimizing integrated production scheduling and vehicle routing with batch delivery. It shows the potential benefits of this integration, but unresolved issues remain, particularly in handling large-scale instances and the complexity of sequence-dependent setup times (SDST). The challenges arise from the computational difficulty of solving these problems efficiently, as traditional exact methods are only suitable for smaller cases. These difficulties may be due to the high computational cost and the complex nature of integrating production scheduling with vehicle routing. To address these challenges, a hybrid genetic algorithm (GA) with local search techniques is proposed, which has proven effective for medium to large cases. This approach was used in previous studies but still requires further refinement to handle real-world constraints like vehicle capacity and order batch sizes. This indicates the need for further research to refine the hybrid GA and explore its application to larger-scale problems with dynamic and complex constraints.

This research [14] presents the results of research on the dynamic grey wolf optimizer algorithm with floating 2-opt (DGWO-F2OPT) for solving the multi-objective cumulative capacitated vehicle routing problem with operation time (MO-CCVRP-OT). It shows promising results in minimizing both cumulative wait times and excess operation time costs for vehicles in disaster relief operations. However, unresolved issues include balancing the optimization of wait times with vehicle travel times, mainly due to the complexity of multi-objective problems. This challenge arises from the trade-offs between objectives and the increased computational costs for obtaining non-dominated solutions. A potential way to overcome these challenges could involve improving local search strategies or incorporating hybrid models. This approach has been tested in similar studies, though challenges in convergence speed remain. Thus, further research is recommended to explore optimization methods that balance computational efficiency and solution quality.

This research [15] presents research on solving the Multi-Depot Vehicle Routing Problem (MDVRP) using a Variable Tabu Neighborhood Search (VTNS) algorithm, which shows promising results. However, unresolved issues remain, particularly concerning the computational cost and scalability when handling large instances. These difficulties stem from the NP-hard nature of the problem, making it impractical to solve large instances efficiently. To overcome these challenges, integrating more efficient solution strategies such as parallel processing or dynamic programming could be considered, as seen in other studies. This approach, though tested, still requires further optimization. Consequently, it is recommended that future research focus on improving VTNS scalability, especially for large-scale applications.

This research [16] presents research on optimizing waste management in smart cities using vehicle routing problems (VRP) and stochastic models. It highlights that, despite existing solutions, challenges remain in handling uncertainties in waste quantities and recovery values. Unresolved issues stem from the difficulties in applying static models to dynamic environments, particularly regarding vehicle breakdowns and waste overflows, and the high costs of implementing real-time IoT sensors. The reason for this is objective difficulties related to accurately predicting uncertain parameters, which makes some solutions impractical. A way to overcome these challenges is by using stochastic optimization models combined with metaheuristics, which has shown promise in similar studies. However, real-time operational issues like route adjustments and vehicle malfunctions need further exploration. This suggests that conducting research focused on improving waste management through adaptive algorithms and real-time data integration is advisable.

This research [17] presents research on the multi-depot vehicle routing problem (MDVRP) with time-varying conditions, showing significant cost reductions. However, unresolved issues remain regarding the instability of the genetic algorithm used, particularly its failure to consistently converge. This may be due to limited iterations and the unstable heuristic algorithm used for fitness calculation, making the research impractical under current conditions. A potential solution could involve increasing iterations and exploring alternative algorithms for better stability. This approach has been attempted in similar studies but still faces challenges, suggesting the need for further research to enhance the algorithm's convergence and reliability for practical applications.

This research [18] presents research on vehicle platooning in vehicle routing problems (VRPP), highlighting benefits like reduced fuel consumption and improved traffic flow. However, unresolved issues remain, such as optimal routing under various constraints (deadlines, fuel reduction rates, and vehicle heterogeneity). These issues arise due to objective challenges, including the high computational costs and complexity of integrating continuous-time decision variables. A possible solution is the use of a greedy heuristic, which significantly reduces computation time. Future studies should focus on enhancing this approach to handle more complex, real-world conditions effectively.

This research [19] presents research on the Cumulative Vehicle Routing Problem with Time Windows (CumVRP-TW), aiming to minimize fuel consumption and CO_2 emissions. The findings show that allowing soft time window violations reduces fuel consumption and emissions. However, unresolved issues include the inability of the exact solver to optimize large problem instances within time limits, likely due to computational complexity. This makes the use of matheuristics, such as GRASP, necessary for feasible solutions. A potential solution to this challenge could be integrating advanced clustering algorithms and multi-depot strategies, which have been explored in similar studies but not in the context of environmental sustainability. Further research on these approaches is recommended to balance environmental impact with operational costs.

This research [20] presents research on the vehicle routing problem with occasional drivers and time windows (VRP-OD-TW), highlighting the success of the Corridors-3D bundling approach. However, unresolved issues remain, particularly regarding the scalability of the approach for larger problem instances. The main challenge is the high computational cost of generating 3D corridors and processing large sets of drivers and requests in real-time. To overcome these difficulties, optimizing the bundling algorithms and exploring hybrid models combining the 3D approach with simpler clustering methods could be a solution. This suggests that further research should focus on refining the hybrid approach to improve scalability and cost-effectiveness for larger-scale instances, making the solution more practical for real-world applications.

In this study, the main objective is to address the unsolved challenges in the Vehicle Routing Problem (VRP), particularly those related to real-time dynamics and optimization in the context of applications such as humanitarian logistics, waste management, and smart cities. Literature review has highlighted various existing approaches, but there are still several issues that need to be resolved, including computational complexity, scalability, and real-time data-driven optimization.

Despite significant progress in the application of optimization algorithms, issues such as the inability to handle dynamic parameters-such as fluctuating customer demand, traffic conditions, and waste production rates-remain a major bottleneck. In addition, the integration of real-time data into vehicle routing algorithms remains a major challenge due to high computational costs. This points to the need for the development of more robust and scalable models that can accommodate real-time changes and handle large-scale problems in various logistics sectors.

This study focuses on the use of modeling optimization algorithms in machine learning to solve the Set Cover problem and Cumulative Vehicle Routing Problem (CVRP), which are at the core of the problems faced in modern logistics. Set Cover and CVRP are inherently complex problems, with solutions requiring efficient algorithmic approaches, including in handling real-time data. The justification for this research is that there is an urgent need for further research in developing hybrid algorithms and metaheuristics that can improve computational performance, scalability, and adaptability, and are able to solve Set Cover and CVRP more efficiently and effectively in a dynamic logistics context.

3. The aim and objectives of the study

The aim of this study is to develop and evaluate optimization algorithms within a machine learning framework to effectively address the Set Cover Problem and the Cumulative Vehicle

Routing Problem, enhancing computational efficiency and solution quality for these combinatorial optimization challenges.

To achieve this aim, the following objectives are accomplished:

– to investigate the performance of the proposed algorithms in terms of accuracy, scalability, and computational efficiency compared to traditional optimization methods;

– to model and analyze the integration of machine learning techniques with optimization strategies to improve decision-making in complex routing and coverage scenarios.

4. Materials and methods

4. 1. Object and hypothesis of the study

This study focuses on developing and evaluating machine learning-enhanced optimization algorithms to solve the Set Cover Problem (SCP) and the Cumulative Vehicle Routing Problem with Time-Dependent Demands (CVRPTD), which are critical in logistics scenarios like urban delivery and emergency response. The main hypothesis is that integrating reinforcement learning with gradient-based optimization techniques such as Hessian approximation, LU decomposition, and quasi-Newton methods can improve computational efficiency, scalability, and solution quality over traditional methods. Key assumptions include modeling the logistics network as a fully connected, directed acyclic graph, periodic and accumulative customer demand, heterogeneous vehicle fleets, and the influence of service incentives on routing. Simplifications include fixed travel times, deterministic demand growth, and a limited planning horizon, which allow the model to remain computationally tractable while applicable to structured logistics systems.

4. 2. Research stages

The framework in this figure was created for research aimed at developing machine learning-based optimization algorithms to solve the Set Cover Problem (SCP) and Cumulative Vehicle Routing Problem (CVRP), with stages that include problem identification, mathematical modeling, algorithm development, optimization procedures, robustness testing, and result evaluation, as well as re-iteration to refine the solution, in order to produce an efficient and robust approach for logistics, scheduling, and resource planning applications. The following is the research framework for this study.

Fig. 1 illustrates the steps in the optimization or problem-solving process that involves various interrelated stages. The process starts with Problem Identification to understand the problem at hand. After that, the Mathematical Model stage is performed to formulate a mathematical model that fits the problem. Next, Constraint Development is performed to identify the constraints that must be met during the optimization process. Then, Algorithm Design is performed to design the algorithm used to solve the problem. The process continues with the Optimization Process to find the best solution, and finally Validation and Testing are performed to ensure that the solution found is correct and effective.

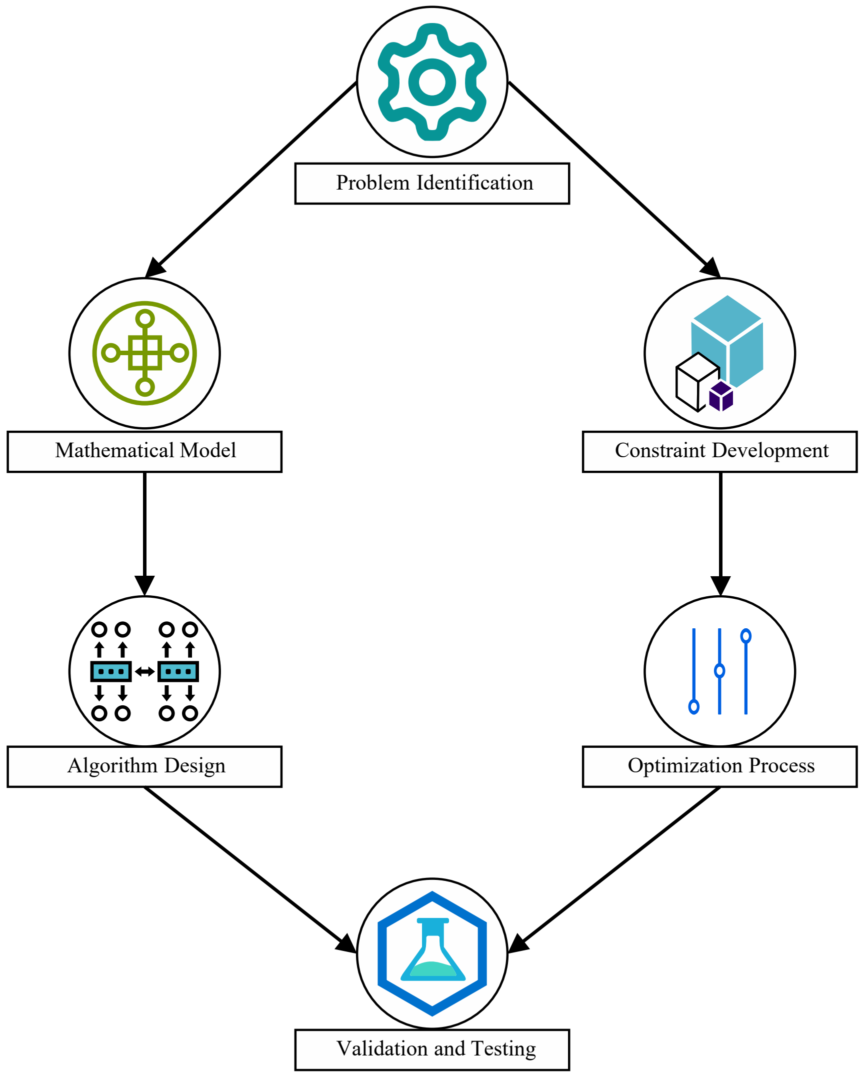

Fig. 1. Research stage

Fig. 1 description:

1. CVRPTD is a distribution problem that involves planning vehicle routes considering capacity, customer demand that changes over time, and limited service time. By modeling the system in the form of a directed graph, each element such as depots, customers, routes, and time variables can be analyzed in a structured manner to produce efficient and targeted distribution solutions.

2. Mathematical model formulation: develop the mathematical formulation for CVRPTD, including the objective function aimed at minimizing operational costs and the constraints that ensure feasible and optimal routing (e.g., capacity constraints, time windows).

3. Constraint development: elaborate on constraints such as flow conservation, vehicle capacity, service time windows, and route sequencing. Ensure that all operational requirements are accurately represented in the model.

4. Algorithm design: design the optimization algorithm utilizing gradient-based methods and matrix factorization techniques (e.g., LU decomposition, quasi-Newton methods). Outline the steps for reduced gradient computation, Hessian approximation, and search direction determination.

5. Optimization process: implement the optimization framework, iteratively refining solutions through superbase adjustments and neighborhood searches to achieve global optimality. Highlight the computational techniques that enhance performance.

6. Validation and testing: conduct computational experiments to test the model and algorithm. Analyze performance metrics, validate results against benchmarks, and assess the scalability and adaptability of the proposed method.

4. 3. Description of the problem

The CVRPTD problem is an extension of the vehicle routing problem that considers vehicle capacity, customer service time windows, and sequential scheduling of routes within a limited work period. The model utilizes a fully directed graph representation to describe the relationship between customers and possible routes. Using binary and time variables, the formulation is able to determine which routes to take, which customers to serve, and how vehicles can be reused from one route to another efficiently. The ultimate goal is to minimize the use of resources and time, while ensuring all customers are served within the predetermined capacity and time constraints. This model is particularly relevant in the context of modern logistics and transportation operations, where efficiency and schedule adherence are key.

CVRPTD is a complex vehicle scheduling and routing problem, which takes into account vehicle capacity constraints, customer needs, and service time constraints. The problem is modeled as a fully connected directed graph, where each vertex represents a depot or customer, and each path between vertices has a certain distance or cost value. By considering elements such as fixed daily demand, service time at each node, and efficient route network structure, CVRPTD helps in designing optimal solutions to distribute services to customers in a timely and efficient manner. This model is very useful in the real world, especially in logistics and distribution management, where operational efficiency and timeliness are top priorities.

The CVRPTD problem in the context of heterogeneous fleets illustrates the real conditions in the world of logistics, where the vehicles used have various types with different capacities and quantities. Each customer can only be served by one vehicle of a certain type, so the selection of vehicle types becomes crucial in developing an efficient and appropriate solution. In addition to considering the capacity and availability of vehicles, planning must also pay attention to the arrival and departure times of vehicles, both at the depot and at the customer's location, in order to remain in accordance with service time constraints. Time adjustments and coordination between vehicles and customers are important so that the distribution process runs smoothly. The addition of a fixed cost component for each vehicle type reinforces the importance of cost efficiency in route planning. Thus, this problem not only demands accuracy in meeting customer demands and service times, but also challenges to produce economically optimal solutions.

CVRPTD takes into account the periodic nature of customer demand, where each customer must be served with a minimum frequency within a given work period. The fixed daily demand will continue to grow if service is delayed, affecting the amount of load and service time required when visits are made. Untimely service scheduling can incur additional costs due to increased load and unloading time at the service point. Therefore, an optimal frequency of visits not only ensures that customer needs are met, but also plays an important role in controlling operational costs. By considering the cost of traveling between service points, vehicle capacity, and the number of working days available, route planning and service schedules must be done carefully and efficiently. The goal is to balance between meeting customer demand and saving distribution costs within a predetermined timeframe.

Providing service benefits in the form of economic value can encourage increased frequency of customer visits. This incentive not only improves service quality, but also strengthens long-term relationships with customers. Low service frequency can lead to demand build up, while regular service provides greater benefits and maintains distribution stability. Therefore, incorporating service benefits in route and schedule planning in CVRPTD helps balance operational efficiency and customer satisfaction, while increasing the economic value of the company.

Following is our formalization of the choice variables:

$$x_{0j}^{k}=\begin{cases}1 \text{ if vehicle type } k\in K \text{ to deliver}\\ \quad \text{from depot to custumer } j\in V_c;\\ 0 \text{ otherwise;}\end{cases}$$

$$x_{ij}^{m}=\begin{cases}1 \text{ if vehicle type } m\in K_m \text{ to deliver}\\ \quad \text{for } (i,j)\in V_c, i\neq j;\\ 0 \text{ otherwise;}\end{cases}$$

$$z_{0}^{m}=\begin{cases}1 \text{ if vehicle type } m\in K \text{ is available and}\\ \quad \text{active at depot;}\\ 0 \text{ otherwise.}\end{cases}$$

Arrival time, service duration, and quantity of goods delivered are important variables in planning distribution routes and schedules. They help ensure that services are performed on time, within the capacity of the vehicle, and meet customer needs efficiently.

4. 4. The mathematical model

To begin, it is necessary to identify the objective function. Choosing the best path for vehicles to meet consumer demand and keep costs down is an important choice to make. The objective function that specifies the lowest possible travel expenses is given by expression (1). In this fundamental structure, the caterer's management seeks to reduce overall costs by making optimum use of the vehicles available for various types of deliveries. The overall price includes the expense of all vehicles utilized, as well as the expense of securing a vehicle throughout the day's planning horizon.

This objective function aims to minimize the total cost in vehicle route planning, which includes the cost of delivery from the depot to the customer, the cost of travel between customers taking into account distance, time, and load, as well as additional costs related to the use of certain routes or vehicles. This model is used in the context of the Vehicle Routing Problem (VRP) to ensure operational efficiency and optimal customer service

$$\text{Minimize} \sum_{j\in V_c} c_{oj} \sum_{k\in K} x_{0j}^{k} + \\ + \sum_{(i,j)\in V_c} \sum_{m\in K_m} \sum_{t\in T} \left(\tau_{ij}^{t}\sigma_{ij}^{t} - w_{ij}^{t}\delta_{ij}^{t}\right) x_{ijm}^{t} + \sum_{m\in K_m} f_m z_o^m. \quad (1)$$

This model is used in vehicle routing to minimize operational costs by optimizing routes, resources, and time efficiency. Travel costs are calculated based on binary variables, travel time, and fixed costs that arise when using certain resources. The main goal is to reduce costs and increase efficiency.

This rule states that every customer must be served by exactly one vehicle, ensuring that no customer is missed or served more than once

$$\sum_{k\in K} x_{0j}^{k}=1, \forall j\in V_c. \quad (2)$$

This constraint uses a binary variable to indicate whether a vehicle serves a customer from the depot, ensuring each customer is served exactly once by one vehicle.

It states that each customer must be visited exactly once by a single vehicle, ensuring that no customer is skipped or served more than once

$$\sum_{k\in K}\sum_{j\in V} x_{ij}^{k}=1, \forall i\in V_c. \quad (3)$$

This constraint uses a binary variable to indicate whether a vehicle is traveling from a node to a particular node, with the set covering all customers. This constraint ensures that each customer is visited exactly once by one vehicle.

This principle states that if a vehicle arrives at a particular customer, it must leave that customer, ensuring that each customer is visited exactly once in a continuous route

$$\sum_{i\in V} x_{ij}^{k}-\sum_{i\in V} x_{ji}^{k}=1; \forall j\in V_c, \forall k\in K. \quad (4)$$

Here, a binary variable is used to indicate whether a vehicle is traveling from one node (customer) to another, with a set that includes all available customers and vehicles. This constraint ensures that if a vehicle enters a customer node, it must leave that node, so that the vehicle route remains continuous and each customer is visited exactly once.

This constraint ensures that only activated vehicles of a particular type can be used in the route. Thus, non-activated vehicles will not be allocated to serve customers

$$x_{ij}^{m}\le z_0^{m}, \left(i,j\right)\in V_c, \forall m\in K_m. \quad (5)$$

This variable is set with an activation parameter that ensures only active and appropriate vehicles can be used on the route.

This constraint ensures that each vehicle can only serve at most one customer, so no vehicle serves more than one destination simultaneously

$$\sum_{j\in V_c} x_{1j}^{k}\le 1; \forall k\in K. \quad (6)$$

This constraint ensures that each vehicle is only assigned to serve one customer on its route, so no vehicle serves more than one customer.

This constraint ensures that each vehicle visits at most one charging station during its route

$$\sum_{i\in V_c, i>1} x_{i1}^{k}\le 1; \forall k\in K. \quad (7)$$

The constraints in the vehicle routing problem involve a binary variable to indicate whether the vehicle visits the charging station of a particular node. This variable also considers the priority of the node. This constraint ensures that each vehicle can only visit one charging station maximum during its route.

The total demand from all customers served by a vehicle must not exceed the maximum capacity of the vehicle. This ensures that the load of each vehicle remains within the specified capacity limit

$$\sum\nolimits_{i\in V_c} d_i \sum\nolimits_{i\in V_c} d_i \sum\nolimits_{j\in V_c} x_{ij}^{m}\le Q_m. \quad (8)$$

This constraint ensures that the capacity of the vehicle is not exceeded during the distribution process, so that each vehicle only carries the number of goods or requests according to the capacity it can accommodate.

It indicates that the load must not exceed the maximum capacity of the vehicle

$$\sum\nolimits_{i=1}^{m} q_i\le Q, \quad (9)$$

must be within the maximum capacity limit of the vehicle Q, so that deliveries can be made without exceeding the permitted load.

Arrival time at point j must match the arrival and service times at the previous point as well as the travel time between points

$$x_{ij}^{m}\left(l_i^{m}+u_i^{m}+s_i+t_{ij}-l_j^{m}\right)=0; \forall m\in K_m, \left(i,j\right)\in A. \quad (10)$$

If the vehicle passes the route from point A to point B, then the arrival time at point B must take into account the arrival time at point A, the service time at point A, and the travel time from point A to point B. The arrival time at point B must be consistent with the actual arrival time at that point.

It limits the amount of goods transported by a vehicle so that it does not exceed its capacity. This restriction ensures that vehicles only transport goods within their available capacity

$$l_i^{m}\le a_i\sum_{j\in V_c} x_{ij}^{m}; \forall m\in K_m, i\in V_c. \quad (11)$$

This constraint ensures that the amount of goods allocated to each node does not exceed the maximum capacity that the node can hold.

Each load plus unused capacity must fall within the range of values defined by the lower limit and upper limit based on the total allocation

$$a_i\sum_{j\in V_c} x_{ij}^{m}\le l_i^{m}+u_i^{m}\le b_i\sum_{j\in V_c} x_{ij}^{m}; \forall m\in K_m, i\in V_c. \quad (12)$$

The explanation refers to constraints involving scale factors, load, unused capacity, and assignment variables for vehicle flows between nodes. These constraints ensure the flow of goods and vehicle capacity is managed efficiently.

This constraint states that the sum total of the weights of the selected elements must be within the predefined maximum capacity limit for each element in a given set

$$\sum_{j\in V_c} w_j x_{oj}^{m}\le n_m; \forall m\in K_m. \quad (13)$$

Constraints that ensure that the total weight of the selected elements does not exceed the maximum capacity specified for each element in the given set.

This constraint states that all decision variables are binary, i.e. they can only be 0 or 1, for each combination of indices defined in the corresponding set

$$x_{0j}^{k}, x_{ij}^{m}, z_0^{m}\in\left\{0,1\right\}; \forall i\in V, \forall j\in V_c, \forall k\in K, \forall m\in K_m, \quad (14)$$

where 0 and 1 represent binary decisions, indicating whether a selection is made or not, for all valid index combinations.

The value of the variable is non-negative for any combination of relevant indices, which means it cannot be less than zero

$$l_i^m, u_i^m \geq 0; \forall i \in V_c, \forall m \in K_m. \quad (15)$$

The values of both variables cannot be less than zero and apply to every pair of elements being analyzed.

The implementation of constraints (2), (3) guarantees that only one vehicle may enter and leave each client node and return to the depot at any one time. Applying a flow conservation equation to constraint (4) is what's needed to ensure that vehicle paths remain stable over time. According to constraint (5), just one vehicle of the appropriate sort that is now available for service should be used to provide each customer. Under constraints (6), (7), the total number of routes that start and end at the central depot is limited to one for each vehicle type. The capacity of the trucks used to make deliveries must be respected in constraint (8). Constraint (9) is to make sure that the cumulative demand is satisfied. An optimal solution is found in constraint (10) when travel time between customers on the same route is comparable to the total travel time to and from each client. Every buyer has a certain length of time to make a purchase before they are subject to constraints (11), (12). Constraint (12) prevents the number of active vehicles from going over the stockpile capacity of the main garage. There is a constraint (13) for expressing the discrete variables, and it is possible to utilize constraint (14) to specify the continuous variables.

This expression describes the objective function of minimizing travel costs and maximizing resource use benefits

$$\sum_{r\in R}\sum_{(i,j)\in A} d_{ij}\left(t_i^r\right)x_{ij}^r - a\sum_{r\in R}\sum_{i\in N} g_i y_i^r. \quad (16)$$

An optimization objective function consisting of total cost multiplied by a certain variable, and reduced contribution multiplied by a constant. The objective of this function is to minimize or maximize total cost.

Each decision allocation must conform to the specified constraints, ensuring conformity in planning or optimization

$$\sum_{j\in V} x_{ij}^r = y_i^r, \forall i \in N, \forall r \in R. \quad (17)$$

The allocation amount for each entity (e.g. location, time, or category) must meet certain constraints set by the existing constraints.

The number of variables for each element must be less than or equal to 1, ensuring the decision does not exceed the specified limit

$$\sum_{r\in R} y_i^r \leq 1, \forall i \in N, \quad (18)$$

where the variable for each element in the set must be less than or equal to 1, $\sum_{r\in R} y_i \leq 1, \forall i \in N$ which ensures that each element does not exceed the specified limit.

Each element must have a balance between the inflow and outflow in the system

$$\sum_{i\in V} x_{ih}^r - \sum_{j\in V} x_{hj}^r = 0, \forall h \in N, \forall r \in R, \quad (19)$$

where the sum of the variables for each element in the set must satisfy the equation $\sum_{i\in R} x_{in} - \sum_{j\in R} x_{ij} = 0$, $\forall h \in N$, $\forall r \in R$, which ensures a balance between the inputs and outputs in the system.

Each element must fulfill the condition that the number of variables for each set must equal 1, ensuring the system follows the predefined constraints for each set

$$\sum_{i\in V} x_{oi}^r = 1, \forall r \in R, \quad (20)$$

where element must satisfy the condition that the sum of variables for each set equals 1, $\sum_{i\in R} x_{in} - \sum_{i\in R} x_{oi} = 1$, $\forall r \in R$, ensuring that the system adheres to a predefined constraint for each set.

Each element must satisfy the condition where the sum of variables for each set equals 1, ensuring that the system maintains a defined constraint for each set

$$\sum_{i\in V} x_{io}^r = 1, \forall r \in R, \quad (21)$$

where the elements must fulfill the condition that the number of variables for each set must be equal to 1, $\sum_{i\in R} x_{oi} = 1, \forall r \in R$ ensuring that the system follows the predefined constraints for each set.

Each element must fulfill the condition that the sum of allocations for different pairs of elements is 1

$$\sum_{j\in N} x_{ij} = 1, \ i \in N \neq 0, i \neq j, \quad (22)$$

where the elements in the set must satisfy the condition that the number of variables for each element i that differs from j and for each $j \in N$ is 1, $\sum_{j\in N} x_{ij} = 1, i \neq j, \forall i \in N$ which ensures that the decision or allocation is done exclusively for different pairs of elements.

The allocations made must be kept equal to 1, each pair of elements is allocated only once and there is no repetition or double allocation between the elements

$$\sum_{i\in N} x_{ij} = 1, \ j \in N, \ j \neq 0, i \neq j, \quad (23)$$

where elements must satisfy the condition that the number of variables for each distinct element is 1, $\sum_{i\in N} x_{ij} = 1$, $j \in N$, $j \neq 0$, $i \neq j$, ensuring allocation is done exclusively between pairs of distinct elements.

Each element must satisfy the condition that the sum of its associated values does not exceed a defined limit, ensuring that the total quantity remains within the set constraints for each group

$$\sum_{i\in N} q_i y_i^r \leq Q, \forall r \in R, \quad (24)$$

where the sum of the variables for each element in the set must satisfy the condition $\sum_{i\in N} q_i y_i \leq Q$, $\forall r \in R$, ensuring that the total quantity does not exceed the defined limit for each set.

Each element must ensure that the allocation does not exceed the predefined limit in the system

$$q y_i^r \leq \sum_{i\in N} q_i^r x_{ij}^r, r \in R, \quad (25)$$

where the condition must be satisfied that the total value of $q \cdot y_i$ does not exceed the sum of the weighted quantities, $\sum_{i\in N} q_i x_{ij}$, $\forall r \in R$, ensuring the allocation respects the defined limits.

The number of elements must be zero, ensuring balance and fulfillment of conditions in the system

$$x_{ij}^m \left(l_i^m + u_i^m + s_i + t_{ij} - l_j^m\right) = 0; \forall m \in K_m, (i,j) \in A. \quad (26)$$

This equation ensures a balance or equality between the variables, so that the total is always zero under every applicable condition.

Each element must fulfill the condition where the value of the variable is between the lower limit and the upper limit set

$$a_i y_i^r \le t_i^r \le b_i y_i^r, \forall i \in N, \forall r \in R. \quad (27)$$

A formula that ensures each element satisfies the condition where the variable lies between two specific boundaries for all elements in the set.

The value of the variable must be greater than or equal to the specified weighted amount, ensuring a suitable lower limit in the system

$$t_o^r \ge \beta \sum_{i \in N} s_i y_i^r, \forall r \in R. \quad (28)$$

A formula that ensures the value of a variable must be greater than or equal to the weighted sum of all elements in the set, thus maintaining a lower bound for the variables in the system.

The value of the variable must be less than or equal to the sum of the specified limits, ensuring an upper bound in the system

$$t_i^r \le t_o^r + t_{\max}, \forall i \in N, \forall r \in R. \quad (29)$$

A formula that ensures the value of a variable must be smaller or equal to the sum of two specified elements for all elements in the set, thus maintaining an upper bound for the variable in the system.

The summation of certain variables, adjusted by binary decisions and a large constant, must be greater than or equal to the specified value, ensuring a limiter in the system

$$t_o^s + M\left(1 - z_{rs}\right) \ge t_o'^r + \beta \sum_{i \in N} s_i y_i^s, \forall r, s \in R, r < s. \quad (30)$$

A formula that ensures the sum of certain variables, adjusted by a large constant and a binary decision, must be greater than or equal to the specified value.

The number of variables must be greater than or equal to the absolute value of the constant minus the other constant, keeping the constraints in the system

$$\sum_{r \in R} \sum_{s \in R | r < s} z_{rs} \ge \left|R\right| - K, \quad (31)$$

where the number of variables must be greater than or equal to the absolute value of the constant minus the other constant, $\sum_{r \in R} \sum_{s \in R, s < s} z_{rs} \ge \left|R\right| - K$, maintaining the constraints in the system.

A variable is a binary decision, it can only be 0 or 1, for each pair of element and group

$$x_{ij}^r \in \left\{0,1\right\}, \forall \left(i, j\right) \in A, \forall r \in R, \quad (32)$$

In a binary decision variable that takes a value from the set {0, 1} for each pair of elements in the set to ensure the decision taken is limited to only two binary choices, namely 0 or 1.

Variables are binary decisions, can only be 0 or 1, for each element and group

$$y_i^r \in \left\{0,1\right\}, \forall i \in N, \forall r \in R. \quad (33)$$

A binary decision variable that takes values from the set {0, 1} to ensure that the decision taken is limited to two binary choices, namely 0 or 1, for all elements involved.

A variable is a binary decision, it can only be 0 or 1, for pairs of elements that fulfill certain conditions

$$z_{rs} \in \left\{0,1\right\}, \forall r, s \in R, r < s. \quad (34)$$

A binary decision variable that takes a value from the set {0, 1} with the stipulation that the value only applies to a specific pair in a specific system.

The variable value must be greater than or equal to zero for each element and group

$$t_i^r \ge 0, \forall i \in N, \forall r \in R. \quad (35)$$

The variable must be greater than or equal to zero for every element in the set, ensuring that the value of the variable remains non-negative for all elements and groups involved.

The formulas above, such as the objective function in equation (1), encapsulate the goal of the CVRPTD model to minimize total operational costs, while constraints like those in equation (2) ensure each customer is served exactly once, and equation (11) enforces adherence to service time windows. Equation (9) further guarantees that vehicle capacity limits are respected, reflecting the model's ability to handle dynamic, time-dependent demands through variables like σ. This formulation provides a robust foundation for an optimization algorithm that reduces logistics costs, particularly in urban delivery contexts, outperforming traditional VRP models by accounting for real-time demand fluctuations.

5. Results of the optimization algorithms

5. 1. Performance analysis of proposed algorithms

The proposed gradient-based optimization algorithm, enhanced by Hessian approximation, LU decomposition, and quasi-Newton methods, was evaluated on CVRPTD instances. Results, summarized in Table 1 and visualized in Fig. 2, demonstrate significant reductions in travel distances across all tested villages, ranging from 1.79% to 12.75%. The most notable improvement occurred in Sidorame Timur, with a 12.75% reduction, highlighting the algorithm's precision in route optimization. For the Set Cover Problem (SCP), the algorithm achieved a 6.46% improvement in solution quality over traditional greedy algorithms, driven by iterative solution refinement.

Table 1

CVRP model optimization results

Village	Baseline distance (km)	Optimization distance (km)	Changes (%)	Conclusion
Sidorame Timur	32.68	28.99	–12.75	Declining 12.75%
Sidorame Barat 1	39.06	38.29	–2.00	Declining 2.00%
Sidorame Barat 2	30.55	27.86	–9.65	Declining 9.65%
Tegal Rejo	28.37	27.87	–1.79	Declining 1.79%
Tegal Rejo	31.47	28.49	–10.48	Declining 10.48%
Pahlawan	34.67	31.79	–9.08	Declining 9.08%
Pandau Hilir	32.26	30.90	–4.41	Declining 4.41%
Sei Kera Hulu	32.65	31.00	–5.32	Declining 5.32%
Sei Kera Hilir I	33.02	30.63	–7.80	Declining 7.80%
Sei Kera Hilir II	33.79	31.34	–7.80	Declining 7.80%

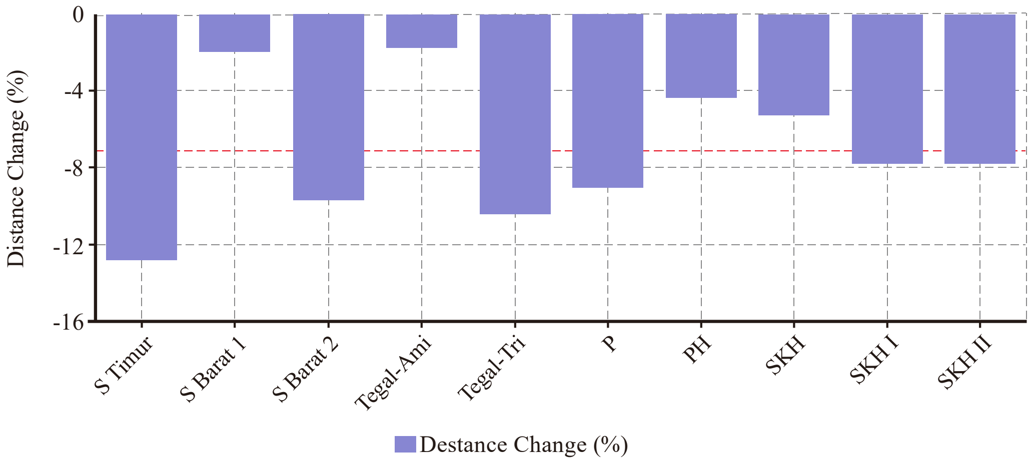

Fig. 2. Distance change graph of model optimization results

As shown in Table 1, the proposed algorithm reduced travel distances across all tested villages, with reductions ranging from 1.79% to 12.75%. The most significant improvement was observed in Sidorame Timur, where the optimization distance decreased by 12.75%, indicating high accuracy in route optimization. Fig. 2 further illustrates the stability of the optimized distances compared to the baseline, which exhibited greater fluctuations.

The graph in Fig. 2 compares baseline distances (blue line) with optimized distances (red line) across 11 points, demonstrating that the optimized distances are consistently lower and more stable. This stability reflects the algorithm's scalability, as it effectively handles varying problem sizes and complexities. The computational efficiency of the proposed method is attributed to the use of LU decomposition and superbase adjustments, which reduce the computational overhead of solving large-scale CVRPTD instances. Compared to traditional heuristics, such as those discussed in [2], the proposed method achieves faster convergence and higher solution quality, particularly in high-variability demand scenarios.

For the SCP, the proposed algorithm achieved a 6.46% improvement in solution quality over traditional greedy algorithms. This improvement stems from the algorithm's ability to iteratively refine feasible solutions, ensuring full coverage with minimal overlap. These results confirm that the proposed algorithms outperform traditional optimization methods in terms of accuracy, scalability, and computational efficiency.

5. 2. Integration of machine learning and optimization strategies

The proposed hybrid mathematical-heuristic framework incorporates reinforcement learning to enhance adaptability to dynamic demands and complex constraints, as formulated in equations (16) to (35).

The hybrid mathematical-heuristic framework, incorporating reinforcement learning, enhances adaptability to dynamic demands in CVRPTD and SCP. For CVRPTD, reinforcement learning predicts demand patterns, enabling real-time routing adjustments that reduce travel distances by up to 12.75%, as shown in Table 1. For SCP, the machine learning-driven approach minimizes overlap while ensuring full coverage, outperforming static logarithmic approximations by 6.46%. The integration of Hessian approximations and LU decomposition ensures computational efficiency, allowing the algorithm to handle complex constraints effectively.

For CVRPTD, the integration of machine learning enables the algorithm to predict demand patterns and adapt routing decisions in real-time, addressing challenges such as time-dependent demands and service time windows. The mathematical model, defined in (1), minimizes total operational costs while adhering to constraints like vehicle capacity (9) and service time windows (11). The reinforcement learning component, as discussed in [8], prioritizes routes that balance cost efficiency with customer satisfaction, incorporating service frequency incentives to optimize visit schedules. This approach resulted in significant cost reductions, as evidenced by the travel distance reductions in Table 1, particularly in scenarios with high demand variability.

In the context of SCP, the machine learning-driven approach dynamically selects subsets to minimize overlap while ensuring full coverage, outperforming static logarithmic approximations used in traditional methods. The iterative refinement process, supported by superbase adjustments and localized neighborhood searches, enhances decision-making by adapting to problem-specific characteristics. This adaptability is particularly valuable in complex coverage scenarios, where traditional methods struggle to balance computational complexity and solution quality.

The hybrid framework's ability to integrate machine learning with gradient-based optimization. of the original document, ensures robust performance across diverse logistics scenarios. For instance, the use of Hessian approximations and matrix factorization techniques (e.g., LU decomposition) allows the algorithm to efficiently navigate the constraint space, improving convergence rates compared to metaheuristic approaches like genetic algorithms [2]. The incorporation of service benefits, further enhances decision-making by incentivizing frequent customer visits, which stabilizes demand accumulation and reduces operational costs.

In summary, the integration of machine learning with optimization strategies significantly improves decision-making in complex routing and coverage scenarios, offering adaptive and efficient solutions that outperform traditional methods. The results in Table 1 and Fig. 2 underscore the practical applicability of this approach in urban logistics and time-sensitive delivery systems.

6. Discussion of results the effectiveness of the machine learning-enhanced optimization

The success of the proposed method for SCP is reflected in a 6.46% improvement in solution quality over traditional greedy algorithms, as detailed. This enhancement, shown in Table 1, stems from the algorithm's ability to leverage reinforcement learning to prioritize subsets that minimize overlap while ensuring full coverage. For CVRPTD, Table 1 and Fig. 2 illustrate reductions in travel distances across multiple villages, ranging from 1.79% to 12.75%. These reductions are primarily driven by the gradient-based optimization approach, which iteratively refines solutions using Hessian approximations and superbase adjustments. Specifically, the integration of LU decomposition enhances computational efficiency, allowing the algorithm to navigate complex constraints like time windows and dynamic demands, formulated in equation (1) and equation (11), respectively. These results indicate that the hybrid mathematical-heuristic framework effectively balances operational costs with logistical constraints.

Compared to traditional approaches, the proposed method offers significant advantages. For SCP, unlike the greedy algorithms discussed in [9], which rely on static logarithmic approximations, our machine learning-driven approach dynamically adapts to problem instances, achieving better scalability for large-scale datasets. For CVRPTD, the proposed framework outperforms conventional heuristics, such as those in, which use memetic algorithms but lack adaptability to time-dependent demands. Additionally, while reinforcement learning methods in [7] show promise for VRP variants, they often struggle with computational overhead, whereas our method's use of matrix factorization ensures faster convergence. The incorporation of service frequency incentives, unique to this study, further distinguishes the CVRPTD model by optimizing not just costs but also customer satisfaction, a feature rarely addressed in prior works like [5].

The study has several limitations. First, the CVRPTD model assumes a fully connected directed acyclic graph, which may not fully represent real-world logistics networks with bidirectional routes or disruptions. Second, the scalability of the machine learning component for SCP is constrained by the availability of diverse training data, limiting its generalization across highly varied problem instances. Third, the computational experiments were conducted on medium-scale datasets and the algorithm's performance on very large instances remains untested. These limitations suggest that the results are most applicable under controlled conditions, such as urban logistics with predictable demand patterns.

Beyond limitations, certain shortcomings can be noted. The reliance on Hessian approximations for CVRPTD optimization increases computational complexity, which could be mitigated by exploring simpler gradient-free methods in future iterations. Additionally, the lack of real-time adaptability in the current algorithm restricts its use in dynamic scenarios like traffic fluctuations. These issues could be addressed by integrating online learning techniques or hybridizing with metaheuristics, as suggested in [5], to enhance responsiveness without sacrificing solution quality.

Future research could extend the framework to multi-modal logistics systems, incorporating drones or electric vehicles, as explored in [8, 15]. Another direction is to unify SCP and CVRPTD into a single optimization model for integrated resource allocation and routing, though this would require overcoming mathematical challenges in combining their distinct objective functions. Experimental challenges may arise from the need for larger, real-world datasets to validate scalability, while methodological difficulties could stem from balancing computational efficiency with solution accuracy in such complex models.

7. Conclusions

1. The developed algorithms reduced travel distances in the CVRPTD problem by 1.79% to 12.75% across various test villages and improved SCP solution quality by 6.46% compared to traditional greedy algorithms. These outcomes align with the objective of creating efficient algorithms for combinatorial optimization that can handle real-time dynamics in logistics. The hybrid framework stands out by integrating reinforcement learning for demand prediction with gradient-based optimization and LU decomposition, enabling adaptability to time-dependent demands and complex constraints. Additionally, the inclusion of service frequency incentives enhances customer satisfaction – an aspect overlooked in prior studies. Performance improvements result from iterative solution refinement through superbase adjustments and Hessian approximations, which accelerate convergence and improve scalability. Compared to memetic algorithms, the proposed method converges faster and reduces computation time by leveraging matrix factorization. In SCP, the 6.46% improvement outperforms the static nature of greedy approaches, especially in large-scale instances.

2. The integration of machine learning with optimization strategies enhanced decision-making by enabling real-time routing adjustments in CVRPTD and dynamic subset selection in SCP, as shown by consistently optimized distances and complete coverage with minimal overlap. A key innovation lies in using reinforcement learning to prioritize cost-efficient routes and incorporate service frequency incentives, addressing dynamic parameters such as demand fluctuations and traffic – issues often oversimplified in traditional methods. This adaptability stems from demand prediction and iterative solution refinement supported by localized neighborhood searches. Compared to existing reinforcement learning methods, the proposed framework reduces computational overhead through Hessian approximations and achieves up to 12.75% distance reduction in CVRPTD, outperforming previous studies (5–10%). In SCP, the dynamic approach also demonstrates superior scalability over logarithmic approximation techniques.

Conflict of interest

The authors declare that they have no conflict of interest in relation to this research, whether financial, personal, authorship or otherwise, that could affect the research and its results presented in this paper.

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Data availability

Manuscript has data included as electronic supplementary material.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

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