The inlet region of a plane-parallel pressure flow is the subject of this study. The patterns of variations in the hydrodynamic inlet region under unsteady plane-parallel pressure flow of a viscous fluid are examined in this research. Based on the boundary layer equation and flow characteristics, the boundary conditions of the problem were determined and a boundary value problem was formulated. The problem's boundary conditions were established and a boundary value problem was developed based on the boundary layer equation and flow characteristics. In order to find patterns of velocity change over time and over the length of the inlet region under general boundary conditions, a method for integrating the boundary value conditions was created. Solutions for scenarios with a constant and parabolic velocity distribution in the inlet region were derived from the general solutions. Regularities of pressure and velocity change were found along the entire hydrodynamic inlet region.

Using computer analysis, graphs of velocity changes over time at various points along the entire length of the inlet region are constructed. The patterns of velocity distribution along the entire length of the inlet region depending on time can be seen using graphs. This allows one to estimate the length of the hydrodynamic inlet region and calculate the fluid flow velocity at any point in this region. The findings enable revealing the essence of the processes running in an hydropneumatic automation system's transition sections. Based on the revealed regularities of the hydrodynamic parameters of viscous incompressible liquid during unsteady flows, it is possible to correctly design of the automatic systems' channels of regulating units ensuring their smooth and accurate operation

Keywords: plane-parallel flow, inlet section, unsteady flow, viscous fluid, velocity distribution

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IDENTIFICATION OF PATTERNS OF NON-STATIONARY LAMINAR FLOW OF A VISCOUS FLUID AT THE INLET SECTION OF PLANE-PARALLEL PRESSURE FLOW

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1. Introduction

In transition region of closed section (inlet and outlet cross-sections, sudden expansion of the cross-section area, narrowing, etc.) occurs velocity distribution change. A transition zone is considered to be the section of velocity rearrangement where the patterns of velocity distribution become equivalent to the patterns of the stabilized zone of closed beds. Studies of the hydrodynamic parameters of the flow in transition areas are important from the point of view of the correct construction of transition units of mechanical equipment, since they determine the smooth operation of the given equipment.

In case of non-stationary motion, the change of velocities occurs depending on time and length coordinates.

The most important issue in the study of hydrodynamic processes running in the transition region is the construction of a mathematical model of the phenomenon taking place in it. This enables to identify the patterns of changes in hydrodynamic phenomena. It is important that the proposed model correctly describes the ongoing hydromechanical phenomena and provides the possibility of obtaining analytical solutions. The investigation of non-stationary movement in closed fluid channels is one of the main ways of improving the structures of mechanical equipment, because their results determine the design problems of control and regulation equipment. Therefore, the study of changes in hydrodynamic parameters of unsteady flow in transition sections is relevant in connection with their correct design.

2. Literary review and problem statement

Experimental studies were carried out on the determination of the distribution pattern of the laminar movement velocity of a viscous fluid in the round cross-section pipe's hydrodynamic entrance region [1]. The results of the experiments with sufficient accuracy coincide with the proposed theoretical solutions. However, the experimental studies were carried out under stationary motion conditions, which limits the applicability of the results.

In [2] the laminar movement of viscous fluid at the hydrodynamic entrance region of a cylindrical pipe was considered. Suddenly started laminar flow in entrance region of a circular tube, with a constant inlet velocity, is investigated analytically by using integral momentum approach. A closed form solution to the integral momentum equation is obtained by the method of characteristics to determine the boundary layer thickness, entrance length, velocity profile, and pressure gradient. However, the problem is solved for a special case when the initial velocity distribution at the inlet cross-section has a constant value. However, the problem is solved for a special case when the initial velocity distribution in the input section has a constant value.

The stationary axisymmetric flow of a viscous fluid in the inlet region of a round cylindrical pipe was studied with an arbitrary distribution of axial and radial velocities [3]. Numerical results are presented for a uniform distribution of inlet velocities in the 0–100 range of the Reynolds number. Therefore, the obtained results are not acceptable for unstable flows. Therefore, the obtained results are unacceptable for unsteady flows.

The authors of paper [4] divided the input region of laminar flow of a viscous fluid into two parts: the input region and the filled region, and in the input region the boundary layers meet on the axis, and in the filled region a velocity profile is formed. The length of the input region is the sum of the indicated regions, which is confirmed by experiments. The formation of the velocity profile takes place in the input region.

However, the formation of the velocity profile occurs throughout the entire input region. From this point of view, the results of the study provide satisfactory results in narrow ranges of flow.

The discrepancy between analytical and numerical solutions to the problem of the input region of a cylindrical pipe has been proven [5]. A comparative analysis of analytical methods using approximation was carried out and new approaches to studying the input region of pressure motion were proposed [6]. Recommendations for the proposed solutions are given. These studies relate to stationary conditions, therefore the obtained results are not applicable to non-stationary conditions.

In [7], a study was carried out on the development of flow in the inlet section during the accelerating movement of a viscous incompressible fluid. The patterns of changes in axial velocities in the direction of movement were obtained. The general solution of laminar flow in axisymmetric pipes under conditions of changes in viscosity and pressure gradient over time [8] makes it possible to identify patterns of changes in the hydrodynamic parameters of the flow. The proposed method is also acceptable for constant viscosity. Identification of patterns of changes in the hydrodynamic parameters of a viscous fluid in a cylindrical pipe during unsteady laminar motion was obtained in [9]. A similar problem for the inlet section of plane-parallel pressure motion is considered in [10]. However, these studies relate to unsteady flows outside the hydrodynamic entrance region, which limits their application.

In the area of sudden expansion of the living cross-section of a round cylindrical pipe, a study of structural changes in a stationary laminar flow was carried out [11]. Graphs of changes in axial velocities were constructed and the length of the transition section was determined. Extensive studies have been carried out on the steady-state modes of laminar and turbulent flows of long pipelines that feed accelerating flows with given velocities and drops [12]. The obtained dependencies are acceptable for three modes of motion and provide an accuracy of 10 % over a wide range of Re and α. However, in the given problems, unsteady processes for special cases are studied.

Changes in the hydrodynamic parameters of the flow also occur in areas of sudden expansion of the living cross-section. A study was carried out [13] on the pattern of changes in hydrodynamic parameters in the area of sudden expansion of the living section of plane-parallel pressure motion based on the boundary layer equations.

Analysis of the stability of the flow in a pipe with a stepwise increase in flow rate leads to the identification of a non-stationary mode of movement in a certain time interval [14]. In [15], a three-dimensional analysis of potential flow was carried out with the aim of developing the design of water intake structures that prevent local erosion around structures. An experimental study of changes in the hydrodynamic parameters of laminar flow in the area of sudden expansion of a cylindrical pipe was carried out [16]. It was revealed that, depending on the Reynolds number and the relative average velocity, unstable turbulent spots appear on the axis of the expanded section.

Numerical studies of the oscillatory laminar motion of an incompressible fluid at the inlet section were carried out at Reynolds numbers $20 \leq Re \leq 200$ and an oscillation frequency of $1.08 \leq S \leq 2.8$. As a result, axial velocity profiles and the length of the transition section were identified [17].

From the above literature review [1–17] it follows that the study of unsteady flow of viscous fluid at the input section of plane-parallel flow was conducted under certain initial and boundary conditions, which limit the area of their acceptability. Based on the importance of the practical application of obtained results of this problem, it is advisable to conduct a study to identify the regularities of hydrodynamic parameters of non-stationary laminar plane-parallel flow at the input section under common initial and boundary conditions. The scientific results obtained as a result of the studies will allow the correct design of transition units of hydropneumatic automation.

3. The aim and objectives of the study

The aim of the study is to identify changes in the hydrodynamic parameters of a non-stationary flow at the inlet section of a plane-parallel pressure movement.

To achieve this aim, the following objectives are solved:

– formulating a boundary value problem and determining the initial and boundary conditions, developing a method for solving a boundary value problem and identifying change regularities in the hydrodynamic parameters of a viscous non-stationary flow at the inlet section of a plane-parallel pressure flow;

– plotting a graph of the axial velocity change depending on time and the Reynolds number, as well as identifying the condition for determining the length of the inlet section.

4. Materials and methods

4. 1. Object and hypothesis of the study

The object of the study is the outlet region of a plane-parallel pressure flow.

The main hypothesis of the study is the parallel flow of fluid particles and the formation of viscous friction between layers, depending on the velocity gradient normal to its direction.

The assumption made in the study is the adoption of pressure on a given effective cross-section dependent on time.

The simplification adopted in the study is the adopted laminar flow regime.

4. 2. Theoretical background

The hydrodynamic entrance region of plane-parallel pressure flow is the subject of the study. It is of theoretical and practical importance to study regularities of change in the hydrodynamic characteristics of a viscous incompressible fluid during unsteady laminar flow.

The study of the viscous fluid flow development in the hydrodynamic entrance region of a flat pipe was conducted using simplified Navier-Stokes equations. Through integration, approximate results were obtained, which provide adequate accuracy for engineering calculations. The findings from this study hold the potential for enhancing the design and construction of hydrodynamic entrance region of the hydraulic systems of various mechanisms and machines. Implementing the improvements suggested by these results can contribute to such system's enhanced reliability of operation.

Based on the relevance of the problem, further development is associated with clarifying the length of the hydrodynamic entrance region and related design changes in the inlet section and the hydrodynamic entrance region of the flat channel.

4. 3. Methodology

The study focuses on the laminar plane-parallel unsteady flow of a viscous fluid in the hydrodynamic entrance region. The origin of the z axis is the center of the entrance region (Fig. 1) and extends infinitely long in the direction of motion. The analysis takes into account the unsteady plane-parallel flow that runs between two stationary plates that are separated by $2h$ in a Cartesian coordinate system shown in Fig. 1.

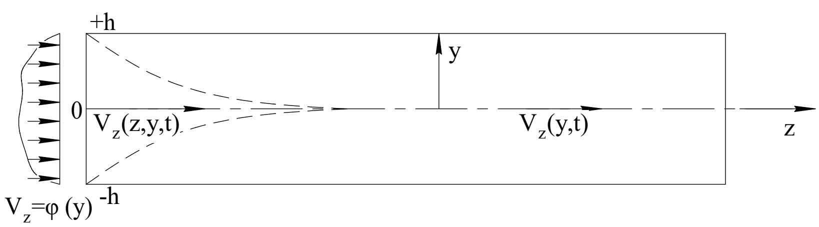

Fig. 1. On the study of a viscous incompressible fluid flow in the hydrodynamic entrance region of a plane-parallel pressure flow

It is assumed that the velocity in the plane-parallel pressure flow in the hydrodynamic entrance region $z=0$ varies in accordance with an arbitrary law. Considering the hydrodynamic characteristics of the viscous fluid to be axisymmetric and unstable, the goal is to identify the patterns of change in these parameters in the transition zone. Let's ignore mass forces.

Between parallel stationary plates, there is an axisymmetric, isothermal flow of a viscous fluid. The velocity distribution diagram deforms in the entrance region based on the regularity $u=\varphi(r, t)$ when the uniformly moving fluid's velocity on the pipe wall drops to zero. This deformation occurs over a specific pipe length.

In the vicinity of the pipe walls, when the velocity gradient $\partial u/\partial n$ rises, a boundary layer emerges. Regardless of the viscosity coefficient, this causes the friction forces to increase noticeably. The boundary layer eventually fills the entire pipe after progressively growing from the area around the pipe walls. Consequently, research in the transition zone utilizing boundary layer equations are required.

For the boundary layer, Prandtl recommended using the Navier-Stokes equations given by [18], which may be simplified to provide the boundary layer equations. Prandtl simplifies the Navier-Stokes equations by excluding terms that are extremely minor in comparison to the viscous forces, since the viscous forces are the primary influencing forces in the boundary layer. This leads to boundary layer equations that are simplified. The boundary layer formulas will take the following form:

$$\frac{\partial V_z(y,z,t)}{\partial t}+V_z(y,z,t)\frac{\partial V_z(y,z,t)}{\partial z}= =-\frac{1}{\rho}\frac{\partial P(y,z,t)}{\partial z}+\nu\frac{\partial^2 V_z(y,z,t)}{\partial y^2}, \quad (1)$$

$$\frac{\partial V_z(y,z,t)}{\partial z}+\frac{\partial V_y(y,z,t)}{\partial y}=0. \quad (2)$$

Performing linearization of the above equations' system based on the method suggested by:

$$\frac{\partial V_z(y,z,t)}{\partial t}+U_0\frac{\partial V_z(y,z,t)}{\partial z}= =-\frac{1}{\rho}\frac{\partial P(y,z,t)}{\partial z}+\nu\frac{\partial^2 V_z(y,z,t)}{\partial y^2}, \quad (3)$$

$$\frac{\partial V_z(y,z,t)}{\partial z}+\frac{\partial V_y(y,z,t)}{\partial y}=0, \quad (4)$$

where U_0 is the characteristic velocity of the flow section, which is equal to the average velocity of the flow section:

$$U_0=\frac{1}{2h}\int_0^T\int_{-h}^{+h}\phi(y,t)\mathrm{d}y\mathrm{d}t. \quad (5)$$

There is $P(y, z, t)$ if to assume that the pressures at every point in the flow section have the same values.

The boundary conditions of the problem are defined in order to integrate equations (3) and (4):

$$V_z(y,z,t)=0,\ \text{when}\ y=\pm h; \quad (6)$$

$$V_z(y,0,0)=\phi(y),\ \text{when}\ z=0,\ -h<y<+h; \quad (7)$$

$$V_z(y,z,t)\to V'(y,t),\ \text{when}\ z\to\infty,\ \ -h<y<+h, \quad (8)$$

where $V'(y, t)$ is the velocity at the fully developed region, which is determined by the following equation:

$$\frac{\partial V_z'(y,t)}{\partial t}=-\frac{1}{\rho}\frac{\partial P(z,t)}{\partial z}+\nu\frac{\partial^2 V_z'(y,t)}{\partial y^2}. \quad (9)$$

The study assumes that the boundary layers forming near the stationary walls in the fully developed velocity region merge to form a symmetric parabolic velocity profile.

Let's introduce dimensionless variables:

$$x=\frac{y}{h},\ \tau=\frac{tU_0}{h},\ \sigma=\frac{z}{h},\ \bar{V}_z(x,\sigma,\tau)=\frac{V_z(y,z,t)}{U_0},\ \bar{P}=\frac{P}{P_0}. \quad (10)$$

(3) and (4) with dimensionless variables and boundary conditions (6)–(8) will take the following forms:

$$\frac{\partial\bar{V}_z(x,\sigma,\tau)}{\partial\tau}+\frac{\partial\bar{V}_z(x,\sigma,\tau)}{\partial\sigma}= =-\frac{P_0}{\rho U_0^2}\frac{\partial\bar{P}(\sigma,\tau)}{\partial\sigma}+\frac{\nu}{hU_0}\frac{\partial^2\bar{V}_z(x,\sigma,\tau)}{\partial x^2}, \quad (11)$$

$$\frac{\partial\bar{V}_z(x,\sigma,\tau)}{\partial\sigma}+\frac{\partial\bar{V}_y(x,\sigma,\tau)}{\partial x}=0. \quad (12)$$

$$\bar{V}_z(\pm1,\sigma,\tau)=0,\quad \bar{V}_y(\pm1,\sigma,\tau)=0, \quad (13)$$

$$\bar{V}_z(x,0,0)=\phi(hx)=\psi(x), \quad (14)$$

$$\bar{V}_z(x,\sigma,\tau)\to V'(x,\tau),$$

$$\text{when } \sigma\to\infty,\ \tau\to\infty,\ -h<y<+h. \quad (15)$$

In the case of boundary and beginning circumstances (13)–(15), the integration of (11) and (12) allows the velocity profile and pressure to be fully formed.

Now let's search for the solution to (11) as a sum:

$$\bar{V}_z(x,\sigma,\tau)=U(x,\sigma,\tau)+\bar{V}'(\sigma,\tau). \quad (16)$$

where $U(x, \sigma, \tau)$ is the solution of the homogeneous equation for inhomogeneous boundary conditions:

$$\frac{\partial U(x,\sigma,\tau)}{\partial\tau}+\frac{\partial U(x,\sigma,\tau)}{\partial\sigma}=\frac{1}{\mathrm{Re}}\frac{\partial^2 U(x,\sigma,\tau)}{\partial x^2}, \quad (17)$$

and $\bar{V}'(\sigma,\tau)$ is the solution of the inhomogeneous equation under homogeneous boundary conditions:

$$\frac{\partial\bar{V}'(x,\tau)}{\partial\tau}=\frac{1}{\mathrm{Re}}\frac{\partial^2\bar{V}'(x,\tau)}{\partial x^2}+f(\sigma,\tau), \quad (18)$$

where:

$$\mathrm{Re}=\frac{hU_0}{\nu},\quad f(\sigma,\tau)=-\frac{P_0}{\rho U_0^2}\frac{\partial\bar{P}(\sigma,\tau)}{\partial\sigma}.$$

5. Results of integration of the formulated boundary value problem

5. 1. Formulating a boundary value problem and developing a method for solving

5. 1. 1. Integration of the formulated boundary value problem

The development of a methodology for solving a boundary value problem should pursue the goal of identifying regularities of change in the hydrodynamic parameters of an unsteady laminar flow in the inlet section of a plane-parallel pressure flow.

Let's look for the solution of (17) in the form developed by [19]:

$$U(x,\sigma,\tau)=\sum_{k=1}^{\infty}C_k(\sigma,\tau)\cos\lambda_k x. \quad (19)$$

From the boundary conditions there is $U(\pm1, \sigma, \tau)=0$ from where:

$$\sum_{k=1}^{\infty}C_k(\sigma,\tau)\cos\lambda_k=0,$$

from which it follows that $\cos\lambda_k=0,\ \lambda_k=(2k-1)\frac{\pi}{2}$ are the positive roots of the eigenfunction of the problem.

The eigenfunction ($\cos\lambda_\kappa$) is orthogonal in the ($-1\leq x\leq 1$) interval:

$$\int_{-1}^{1}\cos\lambda_n\cos\lambda_k\mathrm{d}x=\begin{cases}0,\ \lambda_k\neq\lambda_n,\\1,\ \lambda_k=\lambda_n.\end{cases} \quad (20)$$

Inserting the solution of eq. (19) into eq. (17):

$$\frac{\partial C_k(\sigma,\tau)}{\partial\tau}+\frac{\partial C_k(\sigma,\tau)}{\partial\sigma}=-\frac{\lambda_k^2}{\mathrm{Re}}C_k. \quad (21)$$

Let's look for the solution of (21) in the form of:

$$C_k(\sigma,\tau)=\exp\left(-\frac{\lambda_k^2\sigma}{\mathrm{Re}}\right)T_k(\tau),$$

so $T_k'(\tau)=0$, from where it follows that, $T_k(\tau)=D_k=\text{const}$ so:

$$C_k(\sigma,\tau)=\exp\left(-\frac{\lambda_k^2\sigma}{\mathrm{Re}}\right)D_k.$$

Inserting this value of the coefficient $C_k(\sigma, \tau)$ in (19):

$$U(x,\sigma,\tau)=\sum_{k=1}^{\infty}D_k\exp\left(-\frac{\lambda_k^2\sigma}{\mathrm{Re}}\right)\cos\lambda_k x. \quad (22)$$

Let's look for the solution of the inhomogeneous (18) in the form of eigenfunctions:

$$\bar{V}'(x,\tau)=\sum_{k=1}^{\infty}\phi_k(\tau)\cos\lambda_k x. \quad (23)$$

Considering condition (23), (18) will take the following form:

$$\sum_{k=1}^{\infty}\phi_k'(\tau)\cos\lambda_k x=-\frac{1}{\mathrm{Re}}\sum_{k=1}^{\infty}\lambda_k^2\phi_k(\tau)\cos\lambda_k x+ +\sum_{k=1}^{\infty}a_k(\sigma,\tau)\cos\lambda_k x, \quad (24)$$

where:

$$a_k(\sigma,\tau)=\int_{-1}^{1}\bar{f}(\sigma,\tau)\cos\lambda_k x\mathrm{d}x=\frac{2(-1)^{k+1}}{\lambda_k}\bar{f}(\sigma,\tau). \quad (25)$$

The solution to (24) will be:

$$\phi_k(\tau)=\exp\left(-\frac{\lambda_k^2\tau}{\mathrm{Re}}\right)\left[\int_0^{\tau}\exp\left(\frac{\lambda_k^2\xi}{\mathrm{Re}}\right)a_k(\sigma,\xi)\mathrm{d}\xi+\phi_k(0)\right].$$

Inserting this value $\varphi_\kappa(\tau)$ into (23):

$$\bar{V}'(x,\tau)= =\sum_{k=1}^{\infty}\exp\left(-\frac{\lambda_k^2\tau}{\text{Re}}\right)\left[\begin{array}{l}\int_0^{\tau}\exp\left(\frac{\lambda_k^2\xi}{\text{Re}}\right)a_k(\sigma,\xi)\,\text{d}\xi+\\+\phi_k(0)\end{array}\right]\cos\lambda_k x. \quad (26)$$

From the boundary condition of the problem: $\bar{V}'(x,0)=0$, it is possible to obtain $\varphi_\kappa(0)=0$. Taking into account $\varphi_\kappa(0)$ and from (25) the $a_k(\sigma,\tau)$ there is the value of the functions:

$$\bar{V}'(x,\tau)=\sum_{k=1}^{\infty}\frac{2(-1)^{k+1}}{\lambda_k}\exp\left(-\frac{\lambda_k^2\tau}{\text{Re}}\right)L_k(\sigma,\tau)\cos\lambda_k x, \quad (27)$$

where:

$$L_k(\sigma,\tau)=\int_0^{\tau}\bar{f}(\sigma,\tau)\exp\left(\frac{\lambda_k^2\xi}{\text{Re}}\right)\text{d}\xi. \quad (28)$$

Substituting (22) and (27) into (16):

$$\bar{V}_z(x,\sigma,\tau)=\sum_{k=1}^{\infty}D_k\exp\left(-\frac{\lambda_k^2\sigma}{\text{Re}}\right)\cos\lambda_k x+ +\sum_{k=1}^{\infty}\frac{2(-1)^{k+1}}{\lambda_k}\exp\left(-\frac{\lambda_k^2\tau}{\text{Re}}\right)L_k(\sigma,\tau)\cos\lambda_k x. \quad (29)$$

$\bar{V}_z(x,\sigma,\tau)$ inserting this value of the function into equation (11) let's get $\frac{\partial\bar{f}(\sigma,\tau)}{\partial\sigma}=0$, so $\bar{f}(\sigma,\tau)=B_0=\text{const}$.

From the above condition:

$$a_k(\tau)=\frac{2(-1)^{k+1}B_0}{\lambda_k},$$

$$\int_0^{\tau}\exp\left(\frac{\lambda_k^2\xi}{\text{Re}}\right)a_k(\xi)\,\text{d}\xi= =\frac{2(-1)^{k+1}B_0}{\lambda_k^3}\text{Re}\left[\exp\left(\frac{\lambda_k^2\tau}{\text{Re}}\right)-1\right].$$

Inserting these values into (28) and (29):

$$\bar{V}_z(x,\sigma,\tau)=\sum_{k=1}^{\infty}D_k\exp\left(-\frac{\lambda_k^2\sigma}{\text{Re}}\right)\cos\lambda_k x+ +2B_0\text{Re}\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{\lambda_k^3}\left[1-\exp\left(-\frac{\lambda_k^2\tau}{\text{Re}}\right)\right]\cos\lambda_k x. \quad (30)$$

The value of the d_k coefficient is determined from the boundary condition (14), so:

$$\psi(x)=\sum_{k=1}^{\infty}D_k\cos\lambda_k x. \quad (31)$$

Multiplying both parts of (31) by eigenfunctions, integrating in the range of (–1; 1) and taking into account condition (20):

$$D_k=\int_{-1}^{1}\psi(x)\cos\lambda_k x\text{d}x. \quad (32)$$

Given that $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}\cos\lambda_k x}{\lambda_k^3}=\left(\frac{1-x^2}{4}\right)$ eq.(30) will take the following form:

$$\bar{V}_z(x,\sigma,\tau)= =\sum_{k=1}^{\infty}\left\{\begin{array}{l}D_k\exp\left(-\frac{\lambda_k^2\sigma}{\text{Re}}\right)-\\-2B_0\text{Re}\frac{(-1)^{k+1}}{\lambda_k^3}\exp\left(-\frac{\lambda_k^2\tau}{\text{Re}}\right)\end{array}\right\}\cos\lambda_k x+ +\frac{B_0\text{Re}}{2}(1-x^2). \quad (33)$$

The solutions that were found correspond the problem's general boundaries and initial conditions. These general solutions can be used to derive specific solutions that fit the given circumstances in each unique instance. For the purpose of additional analysis, let's look at two particular examples.

5. 1. 2. Investigation of axial velocity change in case of constant distribution of initial velocity in a plane-parallel flow

Assuming a constant velocity of the incoming fluid there is: $\phi(y)=u_0^*=\text{const}$, $-h\leq y<h$, correspondingly, $\psi(x)=A_0$.

In the case of constant velocity distribution in the entrance region, to obtain the velocity distribution profile in the entrance region, let's determine the values of the D_k coefficients:

$$D_k=\int_{-1}^{1}\psi(x)\cos\lambda_k x\text{d}x=\frac{2A_0\sin\lambda_k}{\lambda_k}=\frac{2A_0(-1)^{k+1}}{\lambda_k}. \quad (34)$$

Inserting this value of the D_k coefficient into eq. (33):

$$\bar{V}_z(x,\sigma,\tau)= =\sum_{k=1}^{\infty}\left\{\begin{array}{l}\frac{2A_0}{\lambda_k}\exp\left(-\frac{\lambda_k^2\sigma}{\text{Re}}\right)-\\-\frac{2B_0\text{Re}}{\lambda_k^3}\exp\left(-\frac{\lambda_k^2\tau}{\text{Re}}\right)\end{array}\right\}(-1)^{k+1}\cos\lambda_k x+ +\frac{B_0\text{Re}}{2}(1-x^2). \quad (35)$$

Based on the velocity change, it is possible to deduce the pressure change pattern from (11). Assuming that the pressures at any fixed point are equal, from (11) it is possible to obtain the pressure change function along the axis where $x=0$. This function will depend on the (σ, $\bar{t}$) variables, so:

$$\frac{\partial\bar{V}_z(0,\sigma,\tau)}{\partial\tau}+\frac{\partial\bar{V}_z(0,\sigma,\tau)}{\partial\sigma}=-\alpha\frac{\partial\bar{P}(\sigma,\tau)}{\partial\sigma}. \quad (36)$$

Inserting the value of the function $\bar{V}_z(x,\sigma,\tau)$ into (36), let's get the pressure change function:

$$\bar{P}(\sigma,\tau)=\bar{P}_0(0,\tau)-\frac{\rho U_0^2B_0}{P_0}\sigma. \quad (37)$$

Thus, the pattern of the axial velocity change, in case of constant values at the input cross-section, is determined by equation (35), and the pattern of change in pressure by equation (37).

5. 1. 3. Revealing patterns of changes in axial velocities and pressure with parabolic distributions of initial velocities

Let's consider the initial distribution of velocities of the incoming fluid in the inlet section of a plane-parallel flow as parabolic. Consequently, $\varphi(y)=A_0(1-y^2)$, $-h\leq y<h$, so $\psi(x)=A_0(1-y^2)$.

To obtain velocity profile in the hydrodynamic entrance region, it is necessary to determine the values of the coefficient D_k:

$$D_k=\int_{-1}^{1}\psi(x)\cos\lambda_k x\mathrm{d}x=$$
$$=2A_0\int(1-x^2)\cos\lambda_k x\mathrm{d}x=\frac{4A_0(-1)^{k+1}}{\lambda_k^3}.$$

Inserting this value of D_k coefficient into equation (33):

$$\bar{V}_z(x,\sigma,\tau)=$$
$$=\sum_{k=1}^{\infty}\left\{\frac{4A_0}{\lambda_k^3}\exp\left(-\frac{\lambda_k^2\sigma}{\mathrm{Re}}\right)-\frac{2B_0\mathrm{Re}}{\lambda_k^3}\exp\left(-\frac{\lambda_k^2\tau}{\mathrm{Re}}\right)\right\}\times$$
$$\times(-1)^{k+1}\cos\lambda_k x+\frac{B_0\mathrm{Re}}{2}(1-x^2). \quad (38)$$

By considering the regularity of the change in axial velocity as defined by eq. (38), it is possible to apply a similar approach to calculate the regularity of the change in pressure. Consequently, let's obtain (37).

5. 2. Graphs of changes in the hydrodynamic parameters of a viscous fluid at the inlet section of a plane-parallel pressure flow

Based on the obtained solutions, it is possible to analyze the nature of flow characteristics in the transition zone of the inlet section in a plane-parallel flow. By integrating the differential equations of viscous fluid flow, it is possible to derive patterns describing the changes in the distribution of axial velocities $\bar{V}_z(x,\sigma,\tau)$.

To visualize the patterns of changes in axial velocity $\bar{V}_z(x,\sigma,\tau)$ across the transverse section and along the length of the transition zone, graphs were constructed based on the initial velocity distribution $\bar{V}_z(x,0,0)=\psi(x)$ and Re=20, 40, 60, 80, 100. In the Fig. 2–7 the graphs for different flow conditions $\bar{V}_z(x,0,0)=A_0=1$ at Re=100 and $\bar{V}_z(x,0,0)=A_0(1-x^2)=(1-x^2)$ at Re=100.

During the unsteady flow, hydrodynamic parameters of the effective cross-section change over time, accordingly, the length of the entrance region undergo changes (Fig. 8).

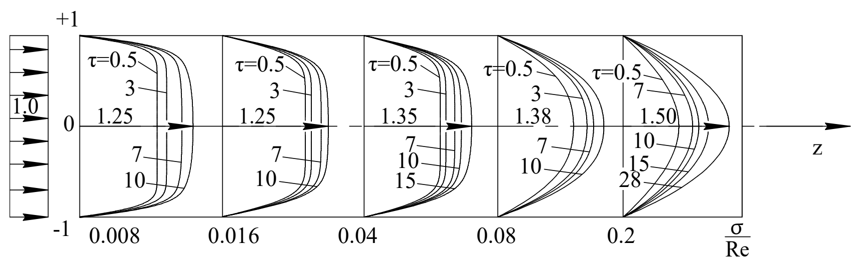

Fig. 2. Graphs of axial velocity $\bar{V}_z(x,\sigma,\tau)$ change at cross-sections of the plane-parallel pressure flow along the entrance region at A_0=1, Re=100 and on the parameter $\frac{\sigma}{\mathrm{Re}}$: 1 – $\frac{\sigma}{\mathrm{Re}}=0.008$; 2 – $\frac{\sigma}{\mathrm{Re}}=0.016$; 3 – $\frac{\sigma}{\mathrm{Re}}=0.04$; 4 – $\frac{\sigma}{\mathrm{Re}}=0.08$; 5 – $\frac{\sigma}{\mathrm{Re}}=0.2$

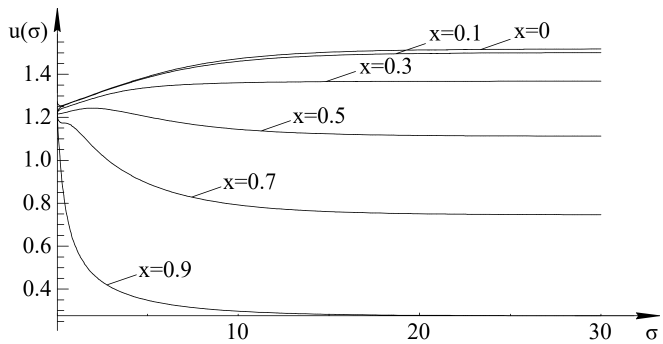

Fig. 3. Graphs of axial velocities $\bar{V}_z(x,\sigma,\tau)$ change depending on x at A0=1; Re=100

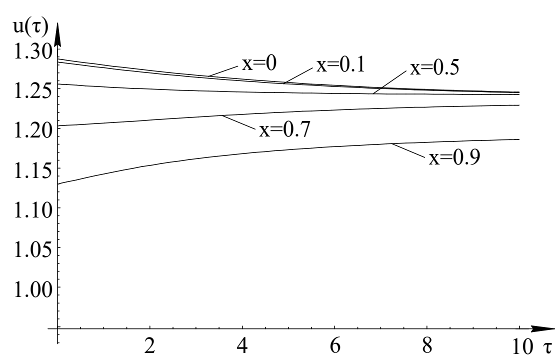

Fig. 4. Graphs of axial velocities' $\bar{V}_z(x,\sigma,\tau)$ change along the hydrodynamic entrance region of a round pipe depending on x at σ=0.4, A_0=1, Re=100

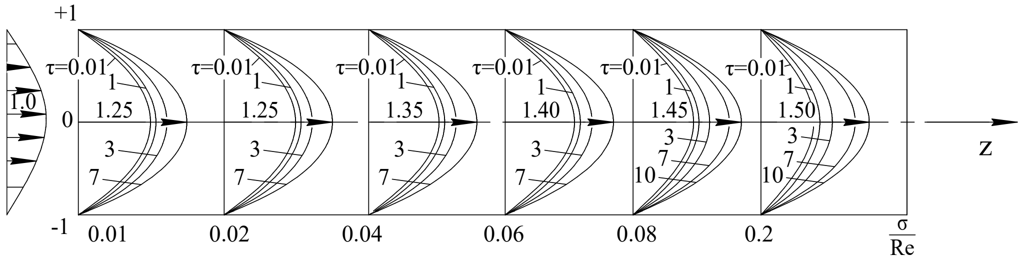

Fig. 5. Graphs of axial velocity $\bar{V}_z(x,\sigma,\tau)$ change at cross-sections along the entrance region in the case of a round pipe at $A_0(1-y^2)=1-y^2$, $b_0=10$, Re=40 and on the parameter $\frac{\sigma}{\mathrm{Re}}$

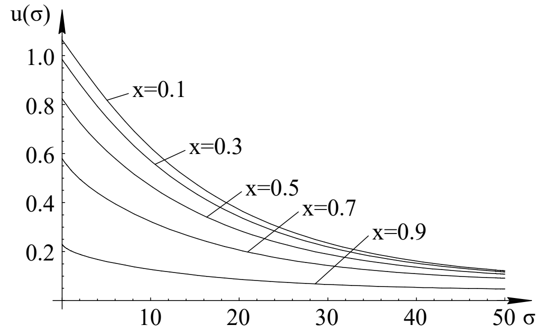

Fig. 6. Graphs of axial velocity` $\bar{V}_z(x,\sigma,\tau)$ change in the entrance region of a round pipe depending on x at $\tau=1$, $A_0(1+y^2)=1+y^2$; $b_0=10$, Re=40

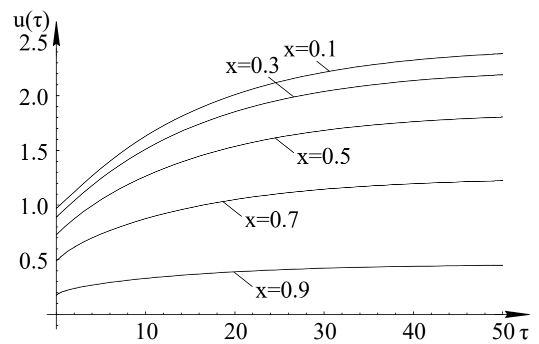

Fig. 7. Graphs of axial velocities $\bar{V}_z(x,\sigma,\tau)$ change along the entrance region of a round pipe depending on x at $\sigma=0.4$; $A_0(1+y^2)=1+y^2$; $b_0=10$, Re=40

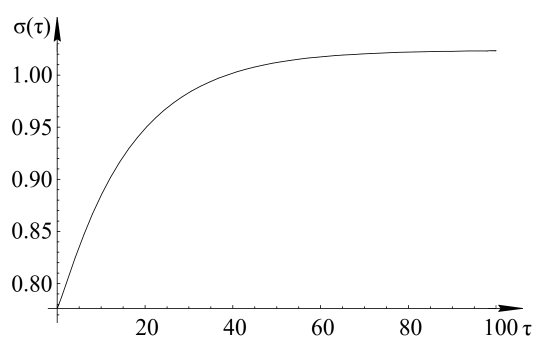

Fig. 8. Graph of the change in the length of the entrance region at the Reynolds number of Re=1000

Analysis of the results of numerical calculations and the resulting graphs determined the dynamics of the axial velocity change along the hydrodynamics entrance region depending on the Reynolds number. It can be seen that at the start of the process, the length of the entrance region is equal to $z=0.00345r\cdot\mathrm{Re}$ (Fig. 8). In the process of unsteady development, the length of the entrance region practically does not change, at $\tau=5$, $z=0.0035r\cdot\mathrm{Re}$. Therefore, with practical accuracy, the length of the entrance region can be taken to be equal to $z=0.0035r\cdot\mathrm{Re}$.

6. Discussion of the study findings on the development of a viscous fluid unsteady flow at the flat pipe inlet section

The Navier-Stokes equations (1), (2), (3), and (4) were used to examine this problem, and characteristic boundary conditions (6)–(8) were created for these equations. The specified boundary value issue (11), (12) under arbitrary boundary conditions (12)–(14) was integrated using a technique developed within this study. Axial velocity change regularities along the input hydrodynamic area (33) were identified. Based on the obtained general solutions, two special cases were considered. In the first case, the velocity of the incoming liquid is constant; in the second case, the distribution of velocities in the inlet section changes according to a parabolic law. For the first case, a pattern of velocity change along the length of the input region depending on time is obtained (35), and for the second case (38). The regularity of pressure change along the length of the input region is obtained in the form (37).

As distinct from [9], where a study on identifying regularities of change in the hydrodynamic parameters of an unsteady flow in the inlet section of a round cylindrical pipe is presented, this paper studies a similar problem for plane-parallel pressure flow. The study was performed based on the Navier-Stokes equations, which describe the unsteady flow of a viscous fluid in the hydrodynamic entrance region, were used to study the unsteady plane-parallel flow. Thanks to the developed method of the boundary value problem integration, the regularities of the change in hydrodynamic parameters of a unsteady viscous fluid in the input region of a flat pipe.

To visualize the process running in the input region of a flat pipe, computerized studies were carried out and graphs of changes in the hydrodynamic parameters of the viscous flow in the input region of a flat pipe were plotted. An analysis of numerical calculations and the corresponding graphs (Fig. 2–8) revealed that the extent of process development is influenced by the pressure gradient, the initial velocity distribution in the entrance region, and the Reynolds number.

The shape of the velocity diagrams in each fixed section, in the entrance region, changes over time (Fig. 2–8) due to the deformation of the diagrams and the influence of the pressure gradient. Outside the transition section, the change in velocity diagrams occurs due to the pressure gradient. With an increase in the Reynolds number, the length of the transition section decreases, which is explained by the intense dissipation of flow energy.

By analyzing the results of numerical calculations and the corresponding graphs, the length of the entrance region can be determined based on the Reynolds number. It is crucial to ensure that the deviation of the axial velocity in the entrance region at $y=0$ does not exceed 1 % of the unsteady velocity in the fully developed region. This condition led to the derivation of a calculation formula to determine the length of the entrance region. This formula has important practical applications in the design of various hydraulic automation systems.

Unlike [10–12], where the results of similar studies are presented for round cylindrical pipes under limited boundary conditions, in this work the problem is solved for a flat pipe under general boundary conditions. This allows to identify a complete picture of the processes occurring in the inlet section, which is extremely necessary when designing transition sections of liquid channels. This study has been carried out for laminar flow, which limits the application of the results obtained to transient or turbulent regimes.

Further improvement of the proposed study is due to the use of more complex models of the fluid flow regime at the inlet section and the corresponding accounting of the shear stress between the fluid layers. When accounting for complex models of shear stress, insurmountable mathematical complications arise during integration.

7. Conclusions

1. On the basis of the approximating Navier-Stokes equations, a boundary value problem is formulated to study non-stationary laminar motion at the hydrodynamic entrance region of a flat pipe. A method for integrating a boundary value problem is developed that makes it possible to identify patterns of change in non-stationary hydrodynamic flow parameters along the length, with uniform and parabolic distributions of initial velocities. The formulae for the change in time of unsteady axial velocities and pressure along the length of the inlet hydrodynamic region are obtained.

2. Graphs of the change in the dimensionless hydrodynamic flow parameters for uniform and parabolic distributions of the initial velocities at the flat pipe inlet, depending on the Reynolds number for different values of the dimensionless time, are plotted. The results obtained make it possible to reveal the influence of the flat pipe and liquid parameters on the change in the parameters of the initial section. Due to the universality of the obtained graphs, it is possible to conclude the nature of the unsteadiness. Conditions have been established for determining the length of the entrance region depending on the dimensionless time, with uniform and parabolic distributions of initial velocities, which is important information in the design of various mechanisms and machines of hydropneumoautomatics.

Conflict of interest

The authors declare that they have no conflict of interest in relation to this research, whether financial, personal, authorship or otherwise, that could affect the research and its results presented in this paper.

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Data availability

The manuscript has no associated data.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

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