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# DEVELOPMENT OF AN ALGORITHM FOR ANALYTICAL MODELING OF THE DYNAMICS OF RANDOM PROCESSES IN AN ASYMMETRIC MARKOV CHAIN

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***The object of this study is the reliability of a military structure consisting of three separate autonomous units. The task to develop an algorithm has been solved by taking into account the exact analytical solution to Kolmogorov's differential equations, derived from the concept of harmonization of the mathematical description of models. The result of mathematical description harmonization manifests itself in the asymmetric structure of possible states of the system under study, consisting of three autonomous subsystems. The symmetric distribution of roots in the characteristic Kolmogorov equation on the complex plane in the ordered record of matrix tables and corresponding determinant tables has been revealed. The representation of the expanded formulas in the form of ordered tables makes it possible to adapt the algorithm to computer technologies and reduce computational costs by 2–3 times compared to conventional methods of numerical integration.***

***The results were verified by testing the algorithm on the example of assessing the reliability of a military structure consisting of three separate autonomous units. The probabilities of possible states of the military structure were determined depending on the intensity of the flow of losses and the restoration of combat units. The derived abstract, dimensionless results regarding the probabilities of states were interpreted through the physically significant time factor of the combat-ready state of combat units and the military structure as a whole. The results of calculations, as well as the algorithm and the mathematical model, have been validated by using a time-invariant condition that relates the probability of the system's states***

***Keywords: reliability of systems, Markov process, Kolmogorov equation, state probabilities, autonomous subsystems***

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## 1. Introduction

Analytical solution to Kolmogorov equations of the eighth order, describing an asymmetric, continuous Markov chain, reduces to solving the corresponding characteristic equation of the eighth power analytically, which, as is known, is problematic.

It is necessary to analytically model and control the Markov stochastic process in a system consisting of three independently functioning subsystems, varying the intensity of their development-restoration and degradation-destruction flows.

Computational experiment is the main tool for studying multidimensional, nonlinear, controlled dynamic systems [1, 2], including the dynamics of stochastic processes [3–5]. The theoretical basis of computational experiment is the methods for constructing mathematical models of dynamic systems [6], which include Markov models [7], as well as methods of their analysis, adapted to computer technologies [8]. The analysis methods are based on the development of approximate numerical methods for integrating systems of differential equations, as well as devising exact, analytical methods for solving systems of linear differential equations, which include the Kolmogorov equation [9]. Problems that allow for exact analytical solutions include the study of random dynamic processes occurring in asymmetric Markov chains with discrete states and continuous time, whose mathematical model is the Kolmogorov equation [10].

The arguments in favor of the relevance of this scientific and applied topic are the assessment of the reliability of complex systems consisting of a finite set of autonomous subsystems.

The functioning of such systems is associated with uncertainties, the causes of which are random factors that lead to changes in the failure flows and restorations of subsystems. For the military structure, the assessment of time of the combat-ready state and the dynamics of the probabilities of the states of combat units are especially important.

Modeling the functioning of such systems is almost the only way to understand the state of combat units and predict effective decisions for military operations. Such forecasting could make it possible to influence timely military decision-making. Therefore, research into this area is a relevant task.

## 2. Literature review and problem statement

The study of Markov random processes with discrete states and continuous time is reduced to the consideration of mathematical models in the form of Kolmogorov equations. Kolmogorov equations in a number of problems are systems of ordinary, linear, homogeneous differential equations with constant coefficients. Analytical solutions to such systems of $n$-th order differential equations are reduced to solving the corresponding characteristic equations of the $n$-th power. So far, only some analytical solutions to complete algebraic equations are known: Cardano, Ferrari, Descartes-Euler, trigonometric, solutions to biquadratic and inverse equations, Moivre's formula, and others.

In [11], nonstationary queues with losses are considered, which are a type of queue system where the arrival and service rates are not constant over time. The study does not completely solve the problem of analyzing nonstationary queues with losses since it gives only an approximation for the queue length and the probability of losses. The paper assumes that the arrival and service processes are independent and equally distributed, which may not be the case in many real-world scenarios. It does not provide a comprehensive analysis of a queuing system, as it focuses only on queue length and loss probability. Therefore, further research is needed to devise more accurate and robust methods for analyzing non-stationary queues with losses.

In [12], the possibility of using a Marxian model (HMM) to predict human mobility based on GPS tracking data is considered. The authors investigate whether the HMM can accurately capture the complex patterns of human mobility and predict future locations. The paper only compares the HMM with a simple Markov chain model but does not evaluate its performance compared to other state-of-the-art models. It uses a relatively small dataset and does not investigate the scalability of the HMM approach to larger datasets or real-world applications.

Although study [13] covers the basics of spectral theory of Markov chains, it does not delve into more advanced topics such as the connection between spectral theory and other branches of mathematics such as operator theory or functional analysis. The focus of the work on theoretical aspects means that it does not provide a comprehensive overview of the computational methods that are necessary to apply spectral theory to real-world problems.

In [14], the authors develop a stochastic modeling framework for the application of traffic flow dynamics and implement it in real-world traffic scenarios. Advanced modeling methods such as machine learning or deep learning approaches are advantageous. The study focuses on modeling and simulation but does not provide a comprehensive overview of real-time traffic forecasting methods and their applications.

A special role belongs to problems that allow analytical solutions in an ordered matrix form [15], which make it possible to analyze Markov random processes with discrete states and discrete time. The presented mathematical model of discrete Markov chains is constructed in algebraic form using asymmetric state graphs and is represented by ordered transition probability matrices. The methodology used is applied to solve a number of relevant problems but does not provide for the prediction of state probabilities.

Markov processes with discrete states and continuous time in a technical system, the state graph of which has a symmetric form, the fourth-order Kolmogorov equation for the state probabilities of a technical system consisting of two subsystems, are considered in [16]. In study [17], an analytical solution to the Kolmogorov equation for an asymmetric graph with four states was obtained. But the authors of these two works do not consider more complex multidimensional dynamic systems, which are more often used for the expedient control over random processes, taking into account the varied intensity of failure and recovery flows.

In the case of a system with two or three autonomous subsystems, analytical solutions to the eighth-order Kolmogorov equations were derived, reported in [18]. With an increase in the dimensionality of the problem, it becomes challenging to systematically describe large amounts of information in mathematical models and to represent analytical solutions in a form that is adapted to modern computer technologies.

Conventionally, high-order Kolmogorov differential equations are integrated by approximate numerical methods in a limited range of parameter changes. There are algorithms for analytical modeling of the dynamics of stochastic processes for solving a similar class of problems. At the same time, the authors of the above scientific works solve specific problems, limiting themselves to a certain set of situations. At the same time, the issue of analytical modeling of the dynamics of stochastic processes and the reliability of complex systems is not covered in those papers. This allows us to argue that it is advisable to conduct a study on developing an algorithm for conducting a computational experiment to assess the reliability of complex systems over time by the intensities of Poisson failure flows and restorations of autonomous subsystems, adapted to computer technology.

## 3. The aim and objectives of the study

The aim of our research is to develop an algorithm for analytical modeling of the dynamics of random processes in an asymmetric Markov chain with eight states, adapted to computer technology. This will make it possible to increase the efficiency of computational experiments and the probability of using combat tasks and reliability indicators of a military structure consisting of three separate military units.

To achieve the goal, the following tasks were set:

– to develop an algorithm for assessing the reliability of complex systems;

– to assess the probabilities of the states of a military structure;

– to determine the dynamics of the probabilities of the states of a military structure.

## 4. The study materials and methods

### 4. 1. The object and hypothesis of the study

The object of our study is the reliability of a military structure consisting of three separate autonomous units.

The method implements a comprehensive approach to increasing the reliability of a military structure with the possibility of predicting the probability of the state of autonomous units. The intensities of Poisson failure and recovery flows of each unit are considered known and constant.

Possible discrete states of the military structure are determined by the number $2^3=8$ and are connected by an asymmetric Markov chain as an asymmetric graph of eight states. Markov random processes of a military structure with discrete states and continuous time are described by the corresponding Kolmogorov equations.

A computational algorithm for assessing the reliability of a complex military structure over time by the intensity of failure and recovery flows of three autonomous units is developed based on the derived analytical solution to equations [19–23]. Kolmogorov equations model the dynamics of random processes in an asymmetric Markov chain with discrete states and continuous time. The given example of the military application of the developed algorithm is illustrative and methodical in nature, with the aim of attracting the attention of the military to this scientific area and improving the efficiency of military command.

It is not possible to quantitatively evaluate the calculations performed in comparison with known approaches because the real initial data are classified, and publications on a related topic are not available in the open press. The numerical example given in the paper does not use secret initial data and is illustrative in nature, demonstrating the qualitative capabilities of the proposed algorithm.

In this regard, it is proposed to use terminology similar to that adopted in technical tasks: technical system – military structure (division), subsystem – units (brigades), which does not fundamentally change the mathematical apparatus of the task. When stating the problem, it is assumed that the units function autonomously, the intensity of loss and recovery flows varies over time discretely with a given step. When performing calculations, standard software for operations with matrices and determinants was used.

A military structure is an organizationally defined grouping of troops operating as part of a single operational system to perform combat or operational tasks.

Brigades are separate components of a military structure that have functional autonomy and can be represented by separate units.

The relationship between the military structure and units is established through their functional interaction and exchange of resources. Substructures perform specialized tasks within a single operational system, and their state determines the overall combat capability of the military structure. Analytical modeling is based on considering the interdependent intensities of loss and recovery flows, which makes it possible to assess the impact of each substructure on the overall effectiveness of the military structure.

The following criteria are introduced to analyze the assessment of the states of the military structure:

– combat-ready state – the level of manning and functional readiness of the unit is not lower than 80 % of the staffing schedule;

– uncombatable state – a decrease in manning below 50 %, which makes it impossible to perform the tasks set.

Assumptions and software used:

– the intensities of loss and recovery flows are assumed to be constant over a given time interval ($T=7$ days);

– state probabilities are calculated assuming a Poisson distribution of events; the influence of external operational-tactical factors, such as enemy intervention or changing strategic conditions, is not taken into account;

– calculations were performed in the MATLAB and Mathcad environments using the Symbolic Math Toolbox and Statistics and Machine Learning Toolbox packages;

– computers with an Intel Core i7-8565U 1.9 GHz processor and 32 GB of RAM were used.

**4. 2. Matrix of intensities of failure flows $\lambda_j$ and restorations $\mu_j$ ($j=1,2,3$)**

The ordered matrix of intensities of failure flows and restorations of three autonomous subsystems is introduced in the following asymmetric form:

$$R=\left\| \begin{matrix} -\lambda_1-\lambda_2-\lambda_3 & 0 & \mu_1 & 0 & \mu_2 & 0 & \mu_3 & 0 \\ 0 & -\mu_1-\mu_2-\mu_3 & 0 & \lambda_1 & 0 & \lambda_2 & 0 & \lambda_3 \\ \lambda_1 & 0 & -\mu_1-\lambda_2-\lambda_3 & 0 & 0 & \mu_3 & 0 & \mu_2 \\ 0 & \mu_1 & 0 & -\lambda_1-\mu_2-\mu_3 & \lambda_3 & 0 & \lambda_2 & 0 \\ \lambda_2 & 0 & 0 & \mu_3 & -\lambda_1-\mu_2-\lambda_3 & 0 & 0 & \mu_1 \\ 0 & \mu_2 & \lambda_3 & 0 & 0 & -\mu_1-\lambda_2-\mu_3 & \lambda_1 & 0 \\ \lambda_3 & 0 & 0 & \mu_2 & 0 & \mu_1 & -\lambda_1-\lambda_2-\mu_3 & 0 \\ 0 & \mu_3 & \lambda_2 & 0 & \lambda_1 & 0 & 0 & -\mu_1-\mu_2-\lambda_3 \end{matrix} \right\|, \qquad (1)$$

This matrix is matched to by an ordered determinant:

$$\Delta=\det\left[R\right]. \qquad (2)$$

Due to the asymmetric structure of matrix $R$, it follows:

$$\Delta=0,$$

that is, matrix $R$ is singular. This property is a criterion for verifying mathematical models for the class of problems under consideration.

**4. 3. Roots $\nu_k$ of the characteristic equation ($k=1, 2, 3, 4, 5, 6, 7, 8$)**

The characteristic determinant for the ordered matrix $R$ is formed:

$$\Delta(\nu)=\begin{vmatrix} -\lambda_1-\lambda_2-\lambda_3-\nu & 0 & \mu_1 & 0 & \mu_2 & 0 & \mu_3 & 0 \\ 0 & -\mu_1-\mu_2-\mu_3-\nu & 0 & \lambda_1 & 0 & \lambda_2 & 0 & \lambda_3 \\ \lambda_1 & 0 & -\mu_1-\lambda_2-\lambda_3-\nu & 0 & 0 & \mu_3 & 0 & \mu_2 \\ 0 & \mu_1 & 0 & -\lambda_1-\mu_2-\mu_3-\nu & \lambda_3 & 0 & \lambda_2 & 0 \\ \lambda_2 & 0 & 0 & \mu_3 & -\lambda_1-\mu_2-\lambda_3-\nu & 0 & 0 & \mu_1 \\ 0 & \mu_2 & \lambda_3 & 0 & 0 & -\mu_1-\lambda_2-\mu_3-\nu & \lambda_1 & 0 \\ \lambda_3 & 0 & 0 & \mu_2 & 0 & \mu_1 & -\lambda_1-\lambda_2-\mu_3-\nu & 0 \\ 0 & \mu_3 & \lambda_2 & 0 & \lambda_1 & 0 & 0 & -\mu_1-\mu_2-\lambda_3-\nu \end{vmatrix}, \quad (3)$$

The characteristic equation is solved:

$$\Delta(\nu)=0.$$

There are truly conjugate roots of the characteristic equation with respect to the center of symmetry, which are expressed in terms of the intensities of the failure and recovery flows of three autonomous military units:

$$\nu_1=0,\ \nu_2=-\lambda_1-\mu_1-\lambda_2-\mu_2-\lambda_3-\mu_3,$$

$$\nu_3=-\lambda_1-\mu_1,\ \nu_4=-\lambda_2-\mu_2-\lambda_3-\mu_3,$$

$$\nu_5=-\lambda_2-\mu_2,\ \nu_6=-\lambda_1-\mu_1-\lambda_3-\mu_3,$$

$$\nu_7=-\lambda_3-\mu_3,\ \nu_8=-\lambda_1-\mu_1-\lambda_2+\mu_2. \quad (4)$$

Each of the eight roots is verified by directly calculating the characteristic determinant:

$$\Delta(\nu_k)=0,\ (k=1,2,3,\ldots,8). \quad (5)$$

or by applying Vieta's theorem to the characteristic equation [24].

**4. 4. Construction of a square matrix $[\Delta_i(\nu_k)]$ by roots and states $i$ ($i$=1, 2, 3, 4, 5, 6, 7, 8)**

A column matrix of given initial conditions for the probabilities of eight asymmetric states is introduced:

$$[P_i(0)],\ (i=1,2,3,\ldots,8). \quad (6)$$

In the eight determinants $\Delta(\nu_k)$, $k$=1,2,3,…,8 the $i$-th column is replaced by the column: $[-P_i(0)]$ and ordered determinants of the following form are built:

$$\Delta_i(\nu_k),\ (i,k=1,2,3,\ldots,8), \quad (7)$$

constructing a square matrix of the eighth order: $[\Delta_i(\nu_k)]$.

**4. 5. Construction of a matrix-column of exponents by roots: $[e^{\nu_k\cdot t}/\Pi_k]$**

Products of the following form are formed:

$$\Pi_k=\prod_{\substack{s=2\\ s\neq k}}^{8}(\nu_k-\nu_s), \quad (8)$$

for each $k$-th root $k$=1,2,3,…,8 and the column matrix is built:

$$\left[\frac{e^{\nu_k\cdot t}}{\Pi_k}\right], \quad (9)$$

where $t$ is the time of the Markov process.

**4. 6. Probability of states $[P_i(t)]$**

$$[P_i(t)]=[\Delta_i(\nu_k)]\cdot\left[\frac{e^{\nu_k\cdot t}}{\Pi_k}\right],\ (i,k=1,2,3,\ldots,8). \quad (10)$$

Here, the probability of each state of the system is determined by a superposition of exponents depending on the roots of the characteristic equation, initial conditions, and time.

**4. 7. Verification of the mathematical model**

The result is verified at $t$=0:

$$[P_i(0)]=[\Delta_i(\nu_k)]\cdot\left[\frac{1}{\Pi_k}\right], \quad (11)$$

where $[P_i(0)]$ is the column matrix of the given initial conditions.

The criterion for verifying the probabilities of states for an arbitrary time instant $t$ is the invariant condition:

$$\sum_{i=1}^{8}P_i(t)=1. \quad (12)$$

The established (stationary) mode of the Markov random process is found using the limit transition $t\to\infty$ by the matrix formula in the form of a column matrix:

$$[P_i(\infty)]=\left[\frac{\Delta_i(\nu_1)}{\Pi_1}\right]. \quad (13)$$

Here, the extreme probabilities of states $P_i(\infty)$ satisfy the condition:

$$\sum_{i=1}^{8}P_i(\infty)=1. \quad (14)$$

The extreme probabilities are also determined by the following algebraic method. Kolmogorov differential equations:

$$\frac{d}{dt}\left[P_i(t)\right]=R\cdot\left[P_i(t)\right], \tag{15}$$

in the established (stationary) regime of a continuous Markov process are simplified by:

$$\frac{d}{dt}\left[P_i(\infty)\right]=\left[0_i\right], \tag{16}$$

where $[0_i]$ is a zero-column matrix of the eighth order, i.e., the Kolmogorov equations degenerate into a system of eight linear, homogeneous algebraic equations:

$$R\cdot\left[P_i(\infty)\right]=\left[0_i\right]. \tag{17}$$

Solving this system is not possible because the square matrix of order eight is singular. The uncertainty is eliminated by introducing an invariant condition instead of one of the eight equations of the system:

$$\sum_{i=1}^{8}P_i(\infty)=1. \tag{18}$$

The eight constructed variants of equivalent systems of linear, inhomogeneous algebraic equations have a single, identically equal solution, which is a criterion for verifying the results and the mathematical model as a whole.

**4. 8. Interpretation of state probabilities**

The intensities of failure and recovery flows of three autonomous military units are found by statistical methods on the time interval [0, $T$], which is determined depending on the features of the applied problem being solved. On the time interval under consideration, the intensities of flows are considered to be given and constant. A continuous Markov process on the time interval $T$ has two qualitatively different modes [25]:

– transient (dynamic) on the time interval $T_n$;

– constant (stationary) on the time interval $T_y$, which are connected by an obvious condition:

$$T_n+T_y=T. \tag{19}$$

Qualitatively, the nature of a continuous Markov process on the time interval $T$ is mainly determined by the given initial conditions and the sum of exponents, the indices of which are equal to the roots of the characteristic equation. The transient process on the time interval $T_n$ is determined by the sum of decaying exponents corresponding to the negative roots of the characteristic equation [26–30]. The steady process on the time interval $T_y$ is determined asymptotically at $t\to\infty$ by the dominant zero roots of the characteristic equation. On the following time intervals, the intensities of the failure and recovery flows of the three military units can change discretely. Then the dynamics of the Markov random process will be determined by the corresponding new roots of the characteristic equation, and the extreme probabilities of states $P_i(\infty)$ on the previous time step are taken as the initial conditions.

In order to interpret the probabilities of the states of military units in time, the operation of integration over time on the interval [0, $T$] of the invariant condition is performed:

$$\int_0^T\sum_{i=1}^{8}P_i(t)\cdot\mathrm{d}t=\int_0^T 1\cdot\mathrm{d}t. \tag{20}$$

Hence:

$$\sum_{i=1}^{8}\int_0^T P_i(t)\mathrm{d}t=T, \tag{21}$$

or:

$$\sum_{i=1}^{8}\int_0^{T_n} P_i(t)\mathrm{d}t+\sum_{i=1}^{8}\int_{T_n}^{T} P_i(\infty)\mathrm{d}t=T, \tag{22}$$

where:

$$\int_0^{T_n} P_i(t)\mathrm{d}t=T_{in},\ \int_{T_n}^{T} P_i(\infty)\mathrm{d}t=T_{iy}, \tag{23}$$

$T_{in}$ – time spent in the $i$-th state under a transient mode; $T_{iy}$ – time spent in the $i$-th state under a steady-state mode.

That is:

$$T_{in}+T_{iy}=T_i, \tag{24}$$

$T_i$ – the time of the system being in the $i$-th state.

Here:

$$T_{iy}=P_i(\infty)\int_{T_n}^{T}\mathrm{d}t=P_i(\infty)\cdot(T-T_n)=P_i(\infty)\cdot T_y; \tag{25}$$

$$T_{in}=\sum_{k=1}^{8}\frac{\Delta_i(\nu_k)}{\Pi_k}\int_0^{T_n}e^{\nu_k\cdot t}\mathrm{d}t=$$

$$=\frac{\Delta_i(\nu_1)}{\Pi_1}T_n+\sum_{k=2}^{8}\frac{\Delta_i(\nu_k)}{\Pi_k}\cdot\frac{1}{\nu_k}\left(e^{\nu_k\cdot T_n}-1\right). \tag{26}$$

Thus, the verification condition in time takes the form:

$$\sum_{i=1}^{8}T_i=T, \tag{27}$$

That is, the total time spent by the military structure in each of the states determines the specified time interval under consideration.

## 5. Results of developing an algorithm for analytical modeling of dynamics of stochastic processes

**5. 1. Development of algorithm for reliability assessment of complex systems**

The algorithm for conducting a computational experiment to assess the reliability of complex systems with three subsystems over time with a known initial state and given intensities of failure and recovery flows is considered on a methodological example of a problem for assessing the probabilities of states of a military structure (Fig. 1), consisting of three autonomous substructures – military units. During combat operations, the intensity of loss flows of military units is considered statistically known. The intensity of recovery flows of military units is considered given and is regulated depending on the available reserves. As a result of analytical modeling, by limiting the intensity of loss flows and varying the intensity of recovery flows, it is possible to effectively manage random processes and make balanced, well-founded decisions with limited resources.

Indexing is introduced for the considered military units: brigade No. 1 – 1, brigade No. 2 – 2, brigade No. 3 – 3. The possible states of each of the substructures are defined as combat-capable – ⊕ or non-combat-capable – ⊖. Then the military structure as a whole has eight possible states:

| | | | |
|---|---|---|---|
| 1. | ⊕⊕⊕, | 2. | ⊖⊖⊖, |
| 3. | ⊕⊕⊖, | 4. | ⊖⊖⊕, |
| 5. | ⊕⊖⊕, | 6. | ⊖⊕⊖, |
| 7. | ⊖⊕⊕, | 8. | ⊕⊖⊖. |

Fig. 1. Possible states of the military structure

At the time interval of combat operations $T$ adopted in this problem, equal to a week (7 days), the intensities of Poisson flows of losses ($\lambda_1$, $\lambda_2$, $\lambda_3$) and recoveries ($\mu_1$, $\mu_2$, $\mu_3$) of military units are considered to be statistically established on average and constant:

$$\lambda_1 = 2;\ \lambda_2 = 1;\ \lambda_3 = 0.5;$$

$$\mu_1 = 1.5;\ \mu_2 = 0.5;\ \mu_3 = 2.$$

The initial state of the military structure ($t=0$) is considered known and corresponds to possible state No. 1, i.e., brigade No. 1, brigade No. 2, and brigade No. 3 of the military structure are combat-ready or using the probabilities of the states of the military structure, it follows:

$$P_1(0)=1;\ P_3(0)=0;\ P_5(0)=0;\ P_7(0)=0;$$

$$P_2(0)=0;\ P_4(0)=0;\ P_6(0)=0;\ P_8(0)=0.$$

It is necessary to determine the possible states of the military structure at the current point in time:

$$P_i(t)\ \ (i=1,2,3,4,5,6,7,8), \tag{28}$$

and the expected states in the future ($t\to\infty$), i.e., the extreme probabilities of the states:

$$P_i(\infty)\ \ (i=1,2,3,4,5,6,7,8). \tag{29}$$

Based on the given intensities of the flows of losses and restorations of three military units, eight roots of the characteristic equation are calculated using analytical formulas:

$$\nu_1 = 0,\ \nu_2 = -7.5,$$

$$\nu_3 = -3.5,\ \nu_4 = -4,$$

$$\nu_5 = -1.5,\ \nu_6 = -6,$$

$$\nu_7 = -2.5,\ \nu_8 = -5. \tag{30}$$

Verification of each of the eight roots $\nu_k$ ($k=1, 2, 3, \ldots, 8$) is carried out using the characteristic determinant $\Delta(\nu)$ by substituting the $k$th root into the characteristic determinant and expanding it $\Delta(\nu_k)$ under the condition:

$$\Delta(\nu_k)=0\ \ (k=1,2,3,4,5,6,7,8). \tag{31}$$

The proposed calculation formulas for probabilities of states of the military structure in time are as follows:

$$P_1(t)=\sum_{k=1}^{8}\frac{\Delta_1(\nu_k)}{\Pi_k}\cdot e^{\nu_k\cdot t};\ P_2(t)=\sum_{k=1}^{8}\frac{\Delta_2(\nu_k)}{\Pi_k}\cdot e^{\nu_k\cdot t};\ \ldots \tag{32}$$

The determinations of $\Pi_1$–$\Pi_8$ are carried out according to the formulas:

$$\begin{aligned}
\Pi_1 &= (\nu_1-\nu_2)(\nu_1-\nu_3)(\nu_1-\nu_4)(\nu_1-\nu_5)(\nu_1-\nu_6)\times\\
&\times(\nu_1-\nu_7)(\nu_1-\nu_8),\\
\Pi_2 &= (\nu_2-\nu_1)(\nu_2-\nu_3)(\nu_2-\nu_4)(\nu_2-\nu_5)(\nu_2-\nu_6)\times\\
&\times(\nu_2-\nu_7)(\nu_2-\nu_8),\\
\Pi_3 &= (\nu_3-\nu_1)(\nu_3-\nu_2)(\nu_3-\nu_4)(\nu_3-\nu_5)(\nu_3-\nu_6)\times\\
&\times(\nu_3-\nu_7)(\nu_3-\nu_8),\\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\
\Pi_8 &= (\nu_8-\nu_1)(\nu_8-\nu_2)(\nu_8-\nu_3)(\nu_8-\nu_4)(\nu_8-\nu_5)\times\\
&\times(\nu_8-\nu_6)(\nu_8-\nu_7).
\end{aligned} \tag{33}$$

$$\Delta_1(\nu_k) = \begin{vmatrix}
-P_1(0) & 0 & \mu_1 & 0 & \mu_2 & 0 & \mu_3 & 0\\
-P_2(0) & -\mu_1-\mu_2-\mu_3-\nu_k & 0 & \lambda_1 & 0 & \lambda_2 & 0 & \lambda_3\\
-P_3(0) & 0 & -\mu_1-\lambda_2-\lambda_3-\nu_k & 0 & 0 & \mu_3 & 0 & \mu_2\\
-P_4(0) & \mu_1 & 0 & -\lambda_1-\mu_2-\mu_3-\nu_k & \lambda_3 & 0 & \lambda_2 & 0\\
-P_5(0) & 0 & 0 & \mu_3 & -\lambda_1-\mu_2-\lambda_3-\nu_k & 0 & 0 & \mu_1\\
-P_6(0) & \mu_2 & \lambda_3 & 0 & 0 & -\mu_1-\lambda_2-\mu_3-\nu_k & \lambda_1 & 0\\
-P_7(0) & 0 & 0 & \mu_2 & 0 & \mu_1 & -\lambda_1-\lambda_2-\mu_3-\nu_k & 0\\
-P_8(0) & \mu_3 & \lambda_2 & 0 & \lambda_1 & 0 & 0 & -\mu_1-\mu_2-\lambda_3-\nu_k
\end{vmatrix}, \tag{34}$$

$$\Delta_2(v_k)=\begin{vmatrix} -\lambda_1-\lambda_2-\lambda_3-v_k & -P_1(0) & \mu_1 & 0 & \mu_2 & 0 & \mu_3 & 0 \\ 0 & -P_2(0) & 0 & \lambda_1 & 0 & \lambda_2 & 0 & \lambda_3 \\ \lambda_1 & -P_3(0) & -\mu_1-\lambda_2-\lambda_3-v_k & 0 & 0 & \mu_3 & 0 & \mu_2 \\ 0 & -P_4(0) & 0 & -\lambda_1-\mu_2-\mu_3-v_k & \lambda_3 & 0 & \lambda_2 & 0 \\ \lambda_2 & -P_5(0) & 0 & \mu_3 & -\lambda_1-\mu_2-\lambda_3-v_k & 0 & 0 & \mu_1 \\ 0 & -P_6(0) & \lambda_3 & 0 & 0 & -\mu_1-\lambda_2-\mu_3-v_k & \lambda_1 & 0 \\ \lambda_3 & -P_7(0) & 0 & \mu_2 & 0 & \mu_1 & -\lambda_1-\lambda_2-\mu_3-v_k & 0 \\ 0 & -P_8(0) & \lambda_2 & 0 & \lambda_1 & 0 & 0 & -\mu_1-\mu_2-\lambda_3-v_k \end{vmatrix}, \quad (35)$$

$$\Delta_3(v_k)=\begin{vmatrix} -\lambda_1-\lambda_2-\lambda_3-v_k & 0 & -P_1(0) & 0 & \mu_2 & 0 & \mu_3 & 0 \\ 0 & -\mu_1-\mu_2-\mu_3-v_k & -P_2(0) & \lambda_1 & 0 & \lambda_2 & 0 & \lambda_3 \\ \lambda_1 & 0 & -P_3(0) & 0 & 0 & \mu_3 & 0 & \mu_2 \\ 0 & \mu_1 & -P_4(0) & -\lambda_1-\mu_2-\mu_3-v_k & \lambda_3 & 0 & \lambda_2 & 0 \\ \lambda_2 & 0 & -P_5(0) & \mu_3 & -\lambda_1-\mu_2-\lambda_3-v_k & 0 & 0 & \mu_1 \\ 0 & \mu_2 & -P_6(0) & 0 & 0 & -\mu_1-\lambda_2-\mu_3-v_k & \lambda_1 & 0 \\ \lambda_3 & 0 & -P_7(0) & \mu_2 & 0 & \mu_1 & -\lambda_1-\lambda_2-\mu_3-v_k & 0 \\ 0 & \mu_3 & -P_8(0) & 0 & \lambda_1 & 0 & 0 & -\mu_1-\mu_2-\lambda_3-v_k \end{vmatrix}, \quad (36)$$

$$\Delta_8(v_k)=\begin{vmatrix} -\lambda_1-\lambda_2-\lambda_3-v_k & 0 & \mu_1 & 0 & \mu_2 & 0 & \mu_3 & -P_1(0) \\ 0 & -\mu_1-\mu_2-\mu_3-v_k & 0 & \lambda_1 & 0 & \lambda_2 & 0 & -P_2(0) \\ \lambda_1 & 0 & -\mu_1-\lambda_2-\lambda_3-v_k & 0 & 0 & \mu_3 & 0 & -P_3(0) \\ 0 & \mu_1 & 0 & -\lambda_1-\mu_2-\mu_3-v_k & \lambda_3 & 0 & \lambda_2 & -P_4(0) \\ \lambda_2 & 0 & 0 & \mu_3 & -\lambda_1-\mu_2-\lambda_3-v_k & 0 & 0 & -P_5(0) \\ 0 & \mu_2 & \lambda_3 & 0 & 0 & -\mu_1-\lambda_2-\mu_3-v_k & \lambda_1 & -P_6(0) \\ \lambda_3 & 0 & 0 & \mu_2 & 0 & \mu_1 & -\lambda_1-\lambda_2-\mu_3-v_k & -P_7(0) \\ 0 & \mu_3 & \lambda_2 & 0 & \lambda_1 & 0 & 0 & -P_8(0) \end{vmatrix}, \quad (37)$$

or otherwise:

$$\Delta_1(v_1)=\begin{vmatrix} -P_1(0) & 0 & \mu_1 & 0 & \mu_2 & 0 & \mu_3 & 0 \\ -P_2(0) & -\mu_1-\mu_2-\mu_3 & 0 & \lambda_1 & 0 & \lambda_2 & 0 & \lambda_3 \\ -P_3(0) & 0 & -\mu_1-\lambda_2-\lambda_3 & 0 & 0 & \mu_3 & 0 & \mu_2 \\ -P_4(0) & \mu_1 & 0 & -\lambda_1-\mu_2-\mu_3 & \lambda_3 & 0 & \lambda_2 & 0 \\ -P_5(0) & 0 & 0 & \mu_3 & -\lambda_1-\mu_2-\lambda_3 & 0 & 0 & \mu_1 \\ -P_6(0) & \mu_2 & \lambda_3 & 0 & 0 & -\mu_1-\lambda_2-\mu_3 & \lambda_1 & 0 \\ -P_7(0) & 0 & 0 & \mu_2 & 0 & \mu_1 & -\lambda_1-\lambda_2-\mu_3 & 0 \\ -P_8(0) & \mu_3 & \lambda_2 & 0 & \lambda_1 & 0 & 0 & -\mu_1-\mu_2-\lambda_3 \end{vmatrix}, \quad (38)$$

$$\Pi_1=(\lambda_1+\lambda_2+\lambda_3+\mu_1+\mu_2+\mu_3)(\lambda_1+\mu_1)\times$$
$$\times(\lambda_2+\lambda_3+\mu_2+\mu_3)(\lambda_2+\mu_2)(\lambda_1+\lambda_3+\mu_1+\mu_3)\times$$
$$\times(\lambda_3+\mu_3)(\lambda_1+\lambda_2+\mu_1+\mu_2);$$

$$\Delta_1(v_2)=$$

$$=\begin{vmatrix} -P_1(0) & 0 & \mu_1 & 0 & \mu_2 & 0 & \mu_3 & 0 \\ -P_2(0) & \lambda_1+\lambda_2+\lambda_3 & 0 & \lambda_1 & 0 & \lambda_2 & 0 & \lambda_3 \\ -P_3(0) & 0 & \lambda_1+\mu_2+\mu_3 & 0 & 0 & \mu_3 & 0 & \mu_2 \\ -P_4(0) & \mu_1 & 0 & \mu_1+\lambda_2+\lambda_3 & \lambda_3 & 0 & \lambda_2 & 0 \\ -P_5(0) & 0 & 0 & \mu_3 & \mu_1+\lambda_2+\mu_3 & 0 & 0 & \mu_1 \\ -P_6(0) & \mu_2 & \lambda_3 & 0 & 0 & \lambda_1+\mu_2+\lambda_3 & \lambda_1 & 0 \\ -P_7(0) & 0 & 0 & \mu_2 & 0 & \mu_1 & \mu_1+\mu_2+\lambda_3 & 0 \\ -P_8(0) & \mu_3 & \lambda_2 & 0 & \lambda_1 & 0 & 0 & \lambda_1+\lambda_2+\mu_3 \end{vmatrix}, \quad (39)$$

$$\Pi_2=(-\lambda_1-\lambda_2-\lambda_3-\mu_1-\mu_2-\mu_3)(-\lambda_1-\mu_1)\times$$
$$\times(-\lambda_2-\lambda_3-\mu_2-\mu_3)(-\lambda_2-\mu_2)(-\lambda_1-\lambda_3-\mu_1-\mu_3)\times$$
$$\times(-\lambda_3-\mu_3)(-\lambda_1-\lambda_2-\mu_1-\mu_2);$$

$$\Delta_1(v_3)=$$

$$=\begin{vmatrix} -P_1(0) & 0 & \mu_1 & 0 & \mu_2 & 0 & \mu_3 & 0 \\ -P_2(0) & \lambda_1-\mu_2-\mu_3 & 0 & \lambda_1 & 0 & \lambda_2 & 0 & \lambda_3 \\ -P_3(0) & 0 & \lambda_1-\lambda_2-\lambda_3 & 0 & 0 & \mu_3 & 0 & \mu_2 \\ -P_4(0) & \mu_1 & 0 & \mu_1-\mu_2-\mu_3 & \lambda_3 & 0 & \lambda_2 & 0 \\ -P_5(0) & 0 & 0 & \mu_3 & \mu_1-\mu_2-\lambda_3 & 0 & 0 & \mu_1 \\ -P_6(0) & \mu_2 & \lambda_3 & 0 & 0 & \lambda_1-\lambda_2-\mu_3 & \lambda_1 & 0 \\ -P_7(0) & 0 & 0 & \mu_2 & 0 & \mu_1 & \mu_1-\lambda_2-\mu_3 & 0 \\ -P_8(0) & \mu_3 & \lambda_2 & 0 & \lambda_1 & 0 & 0 & \lambda_1-\mu_2-\lambda_3 \end{vmatrix}, \quad (40)$$

$$\Pi_3=(-\lambda_1+\lambda_2+\lambda_3-\mu_1+\mu_2+\mu_3)(-\lambda_1-\mu_1)\times$$
$$\times(\lambda_2+\lambda_3+\mu_2+\mu_3)(\lambda_2+\mu_2)(-\lambda_1+\lambda_3-\mu_1+\mu_3)\times$$
$$\times(\lambda_3+\mu_3)(-\lambda_1+\lambda_2-\mu_1+\mu_2).$$

The time interval in days is considered: $0 \le t \le 7$.

The results of the calculations are summarized in Table 1 and illustrated in Fig. 2.

Table 1

Calculation results

| $t$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| $P_1$ | 1 | 0.175 | 0.126 | 0.117 | 0.115 | 0.114 | 0.114 | 0.114 |
| $P_2$ | 0 | 0.053 | 0.072 | 0.075 | 0.076 | 0.076 | 0.076 | 0.076 |
| $P_3$ | 0 | 0.218 | 0.168 | 0.156 | 0.153 | 0.153 | 0.152 | 0.152 |
| $P_4$ | 0 | 0.042 | 0.054 | 0.056 | 0.057 | 0.057 | 0.057 | 0.057 |
| $P_5$ | 0 | 0.189 | 0.218 | 0.226 | 0.228 | 0.228 | 0.229 | 0.229 |
| $P_6$ | 0 | 0.049 | 0.042 | 0.039 | 0.038 | 0.038 | 0.038 | 0.038 |
| $P_7$ | 0 | 0.039 | 0.031 | 0.029 | 0.029 | 0.029 | 0.029 | 0.029 |
| $P_8$ | 0 | 0.234 | 0.290 | 0.301 | 0.304 | 0.305 | 0.305 | 0.305 |

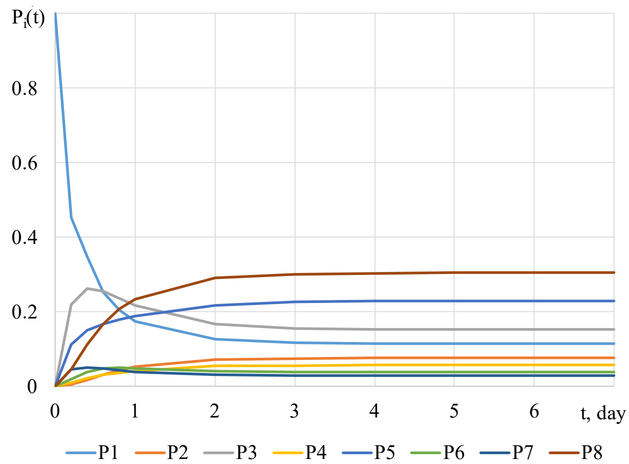


Fig. 2. Change in the probabilities of states of a military structure over time

*Note: [0, 5] – time interval of the transition process; (5,7] – time interval of the stationary (steady) regime*

At each time point $t$, the verification of calculations is carried out according to the invariant condition:

$$\sum_{i=1}^{8} P_i(t)=1.$$

It is not difficult to show using a table that the sum of the elements of each column is 1. It is worth noting, in particular, that at $t=0$ and at $t\to\infty$ the general formulas for the probabilities of states are simplified and take the following form:

$$P_1(0)=\sum_{k=1}^{8}\frac{\Delta_1(v_k)}{\Pi_k};\; P_2(0)=\sum_{k=1}^{8}\frac{\Delta_2(v_k)}{\Pi_k};\; \ldots,$$

$$P_1(\infty)=\frac{\Delta_1(v_1)}{\Pi_1};\; P_2(\infty)=\frac{\Delta_2(v_1)}{\Pi_1};\; \ldots \quad (41)$$

where the given initial conditions $P_1(0)$, $P_2(0)$, ... and the extreme probabilities of states $P_1(\infty)$, $P_2(\infty)$, ... satisfy the equations:

$$\sum_{i=1}^{8} P_i(0)=1;\; \sum_{i=1}^{8} P_i(\infty)=1,$$

that is, the invariant condition is fulfilled at the initial time $t=0$ and in the future $t\to\infty$.

**5. 2. Estimating the probabilities of states of a military structure**

Eight variants of equivalent systems of algebraic equations regarding the extreme probabilities of states are compiled in expanded form. The extreme probabilities of states are calculated using Cramer's formulas.

Variant No. 1:

$$\begin{Vmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & -\mu_1-\mu_2-\mu_3 & 0 & \lambda_1 & 0 & \lambda_2 & 0 & \lambda_3 \\ \lambda_1 & 0 & -\mu_1-\lambda_2-\lambda_3 & 0 & 0 & \mu_3 & 0 & \mu_2 \\ 0 & \mu_1 & 0 & -\lambda_1-\mu_2-\mu_3 & \lambda_3 & 0 & \lambda_2 & 0 \\ \lambda_2 & 0 & 0 & \mu_3 & -\lambda_1-\mu_2-\lambda_3 & 0 & 0 & \mu_1 \\ 0 & \mu_2 & \lambda_3 & 0 & 0 & -\mu_1-\lambda_2-\mu_3 & \lambda_1 & 0 \\ \lambda_3 & 0 & 0 & \mu_2 & 0 & \mu_1 & -\lambda_1-\lambda_2-\mu_3 & 0 \\ 0 & \mu_3 & \lambda_2 & 0 & \lambda_1 & 0 & 0 & -\mu_1-\mu_2-\lambda_3 \end{Vmatrix} \begin{Vmatrix} P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \\ P_4(\infty) \\ P_5(\infty) \\ P_6(\infty) \\ P_7(\infty) \\ P_8(\infty) \end{Vmatrix} = \begin{Vmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Vmatrix}. \quad (42)$$

Solution:

$$P_i(\infty) = \frac{\Delta_i}{\Delta}, (i = 1,2,3,...,8), \quad (43)$$

where:

$$\Delta = \begin{vmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & -\mu_1-\mu_2-\mu_3 & 0 & \lambda_1 & 0 & \lambda_2 & 0 & \lambda_3 \\ \lambda_1 & 0 & -\mu_1-\lambda_2-\lambda_3 & 0 & 0 & \mu_3 & 0 & \mu_2 \\ 0 & \mu_1 & 0 & -\lambda_1-\mu_2-\mu_3 & \lambda_3 & 0 & \lambda_2 & 0 \\ \lambda_2 & 0 & 0 & \mu_3 & -\lambda_1-\mu_2-\lambda_3 & 0 & 0 & \mu_1 \\ 0 & \mu_2 & \lambda_3 & 0 & 0 & -\mu_1-\lambda_2-\mu_3 & \lambda_1 & 0 \\ \lambda_3 & 0 & 0 & \mu_2 & 0 & \mu_1 & -\lambda_1-\lambda_2-\mu_3 & 0 \\ 0 & \mu_3 & \lambda_2 & 0 & \lambda_1 & 0 & 0 & -\mu_1-\mu_2-\lambda_3 \end{vmatrix}, \quad (44)$$

$$\Delta_1 = \begin{vmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & -\mu_1-\mu_2-\mu_3 & 0 & \lambda_1 & 0 & \lambda_2 & 0 & \lambda_3 \\ 0 & 0 & -\mu_1-\lambda_2-\lambda_3 & 0 & 0 & \mu_3 & 0 & \mu_2 \\ 0 & \mu_1 & 0 & -\lambda_1-\mu_2-\mu_3 & \lambda_3 & 0 & \lambda_2 & 0 \\ 0 & 0 & 0 & \mu_3 & -\lambda_1-\mu_2-\lambda_3 & 0 & 0 & \mu_1 \\ 0 & \mu_2 & \lambda_3 & 0 & 0 & -\mu_1-\lambda_2-\mu_3 & \lambda_1 & 0 \\ 0 & 0 & 0 & \mu_2 & 0 & \mu_1 & -\lambda_1-\lambda_2-\mu_3 & 0 \\ 0 & \mu_3 & \lambda_2 & 0 & \lambda_1 & 0 & 0 & -\mu_1-\mu_2-\lambda_3 \end{vmatrix}, \quad (45)$$

$$\Delta_2 = \begin{vmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & \lambda_1 & 0 & \lambda_2 & 0 & \lambda_3 \\ \lambda_1 & 0 & -\mu_1-\lambda_2-\lambda_3 & 0 & 0 & \mu_3 & 0 & \mu_2 \\ 0 & 0 & 0 & -\lambda_1-\mu_2-\mu_3 & \lambda_3 & 0 & \lambda_2 & 0 \\ \lambda_2 & 0 & 0 & \mu_3 & -\lambda_1-\mu_2-\lambda_3 & 0 & 0 & \mu_1 \\ 0 & 0 & \lambda_3 & 0 & 0 & -\mu_1-\lambda_2-\mu_3 & \lambda_1 & 0 \\ \lambda_3 & 0 & 0 & \mu_2 & 0 & \mu_1 & -\lambda_1-\lambda_2-\mu_3 & 0 \\ 0 & 0 & \lambda_2 & 0 & \lambda_1 & 0 & 0 & -\mu_1-\mu_2-\lambda_3 \end{vmatrix}, \quad (46)$$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & -\mu_1-\mu_2-\mu_3 & 0 & \lambda_1 & 0 & \lambda_2 & 0 & \lambda_3 \\ \lambda_1 & 0 & 0 & 0 & 0 & \mu_3 & 0 & \mu_2 \\ 0 & \mu_1 & 0 & -\lambda_1-\mu_2-\mu_3 & \lambda_3 & 0 & \lambda_2 & 0 \\ \lambda_2 & 0 & 0 & \mu_3 & -\lambda_1-\mu_2-\lambda_3 & 0 & 0 & \mu_1 \\ 0 & \mu_2 & 0 & 0 & 0 & -\mu_1-\lambda_2-\mu_3 & \lambda_1 & 0 \\ \lambda_3 & 0 & 0 & \mu_2 & 0 & \mu_1 & -\lambda_1-\lambda_2-\mu_3 & 0 \\ 0 & \mu_3 & 0 & 0 & \lambda_1 & 0 & 0 & -\mu_1-\mu_2-\lambda_3 \end{vmatrix}, \quad (47)$$

$$\Delta_8=\begin{vmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1\\
0 & -\mu_1-\mu_2-\mu_3 & 0 & \lambda_1 & 0 & \lambda_2 & 0 & 0\\
\lambda_1 & 0 & -\mu_1-\lambda_2-\lambda_3 & 0 & 0 & \mu_3 & 0 & 0\\
0 & \mu_1 & 0 & -\lambda_1-\mu_2-\mu_3 & \lambda_3 & 0 & \lambda_2 & 0\\
\lambda_2 & 0 & 0 & \mu_3 & -\lambda_1-\mu_2-\lambda_3 & 0 & 0 & 0\\
0 & \mu_2 & \lambda_3 & 0 & 0 & -\mu_1-\lambda_2-\mu_3 & \lambda_1 & 0\\
\lambda_3 & 0 & 0 & \mu_2 & 0 & \mu_1 & -\lambda_1-\lambda_2-\mu_3 & 0\\
0 & \mu_3 & \lambda_2 & 0 & \lambda_1 & 0 & 0 & 0
\end{vmatrix}. \quad (48)$$

Variant No. 2:

$$\begin{Vmatrix}
-\lambda_1-\lambda_2-\lambda_3 & 0 & \mu_1 & 0 & \mu_2 & 0 & \mu_3 & 0\\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1\\
\lambda_1 & 0 & -\mu_1-\lambda_2-\lambda_3 & 0 & 0 & \mu_3 & 0 & \mu_2\\
0 & \mu_1 & 0 & -\lambda_1-\mu_2-\mu_3 & \lambda_3 & 0 & \lambda_2 & 0\\
\lambda_2 & 0 & 0 & \mu_3 & -\lambda_1-\mu_2-\lambda_3 & 0 & 0 & \mu_1\\
0 & \mu_2 & \lambda_3 & 0 & 0 & -\mu_1-\lambda_2-\mu_3 & \lambda_1 & 0\\
\lambda_3 & 0 & 0 & \mu_2 & 0 & \mu_1 & -\lambda_1-\lambda_2-\mu_3 & 0\\
0 & \mu_3 & \lambda_2 & 0 & \lambda_1 & 0 & 0 & -\mu_1-\mu_2-\lambda_3
\end{Vmatrix}
\begin{Vmatrix} P_1(\infty)\\ P_2(\infty)\\ P_3(\infty)\\ P_4(\infty)\\ P_5(\infty)\\ P_6(\infty)\\ P_7(\infty)\\ P_8(\infty)\end{Vmatrix}
=\begin{Vmatrix}0\\1\\0\\0\\0\\0\\0\\0\end{Vmatrix}. \quad (49)$$

Solution:

$$P_i(\infty)=\frac{\Delta_i}{\Delta},\ (i=1,2,3,\ldots,8), \quad (50)$$

where:

$$\Delta=\begin{vmatrix}
-\lambda_1-\lambda_2-\lambda_3 & 0 & \mu_1 & 0 & \mu_2 & 0 & \mu_3 & 0\\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1\\
\lambda_1 & 0 & -\mu_1-\lambda_2-\lambda_3 & 0 & 0 & \mu_3 & 0 & \mu_2\\
0 & \mu_1 & 0 & -\lambda_1-\mu_2-\mu_3 & \lambda_3 & 0 & \lambda_2 & 0\\
\lambda_2 & 0 & 0 & \mu_3 & -\lambda_1-\mu_2-\lambda_3 & 0 & 0 & \mu_1\\
0 & \mu_2 & \lambda_3 & 0 & 0 & -\mu_1-\lambda_2-\mu_3 & \lambda_1 & 0\\
\lambda_3 & 0 & 0 & \mu_2 & 0 & \mu_1 & -\lambda_1-\lambda_2-\mu_3 & 0\\
0 & \mu_3 & \lambda_2 & 0 & \lambda_1 & 0 & 0 & -\mu_1-\mu_2-\lambda_3
\end{vmatrix}, \quad (51)$$

$$\Delta_1=\begin{vmatrix}
0 & 0 & \mu_1 & 0 & \mu_2 & 0 & \mu_3 & 0\\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1\\
0 & 0 & -\mu_1-\lambda_2-\lambda_3 & 0 & 0 & \mu_3 & 0 & \mu_2\\
0 & \mu_1 & 0 & -\lambda_1-\mu_2-\mu_3 & \lambda_3 & 0 & \lambda_2 & 0\\
0 & 0 & 0 & \mu_3 & -\lambda_1-\mu_2-\lambda_3 & 0 & 0 & \mu_1\\
0 & \mu_2 & \lambda_3 & 0 & 0 & -\mu_1-\lambda_2-\mu_3 & \lambda_1 & 0\\
0 & 0 & 0 & \mu_2 & 0 & \mu_1 & -\lambda_1-\lambda_2-\mu_3 & 0\\
0 & \mu_3 & \lambda_2 & 0 & \lambda_1 & 0 & 0 & -\mu_1-\mu_2-\lambda_3
\end{vmatrix}, \quad (52)$$

$$\Delta_2=\begin{vmatrix}
-\lambda_1-\lambda_2-\lambda_3 & 0 & \mu_1 & 0 & \mu_2 & 0 & \mu_3 & 0\\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1\\
\lambda_1 & 0 & -\mu_1-\lambda_2-\lambda_3 & 0 & 0 & \mu_3 & 0 & \mu_2\\
0 & 0 & 0 & -\lambda_1-\mu_2-\mu_3 & \lambda_3 & 0 & \lambda_2 & 0\\
\lambda_2 & 0 & 0 & \mu_3 & -\lambda_1-\mu_2-\lambda_3 & 0 & 0 & \mu_1\\
0 & 0 & \lambda_3 & 0 & 0 & -\mu_1-\lambda_2-\mu_3 & \lambda_1 & 0\\
\lambda_3 & 0 & 0 & \mu_2 & 0 & \mu_1 & -\lambda_1-\lambda_2-\mu_3 & 0\\
0 & 0 & \lambda_2 & 0 & \lambda_1 & 0 & 0 & -\mu_1-\mu_2-\lambda_3
\end{vmatrix}, \quad (53)$$

$$\Delta_3=\begin{vmatrix} -\lambda_1-\lambda_2-\lambda_3 & 0 & 0 & 0 & \mu_2 & 0 & \mu_3 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ \lambda_1 & 0 & 0 & 0 & 0 & \mu_3 & 0 & \mu_2 \\ 0 & \mu_1 & 0 & -\lambda_1-\mu_2-\mu_3 & \lambda_3 & 0 & \lambda_2 & 0 \\ \lambda_2 & 0 & 0 & \mu_3 & -\lambda_1-\mu_2-\lambda_3 & 0 & 0 & \mu_1 \\ 0 & \mu_2 & 0 & 0 & 0 & -\mu_1-\lambda_2-\mu_3 & \lambda_1 & 0 \\ \lambda_3 & 0 & 0 & \mu_2 & 0 & \mu_1 & -\lambda_1-\lambda_2-\mu_3 & 0 \\ 0 & \mu_3 & 0 & 0 & \lambda_1 & 0 & 0 & -\mu_1-\mu_2-\lambda_3 \end{vmatrix}, \quad (54)$$

$$\Delta_8=\begin{vmatrix} -\lambda_1-\lambda_2-\lambda_3 & 0 & \mu_1 & 0 & \mu_2 & 0 & \mu_3 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ \lambda_1 & 0 & -\mu_1-\lambda_2-\lambda_3 & 0 & 0 & \mu_3 & 0 & 0 \\ 0 & \mu_1 & 0 & -\lambda_1-\mu_2-\mu_3 & \lambda_3 & 0 & \lambda_2 & 0 \\ \lambda_2 & 0 & 0 & \mu_3 & -\lambda_1-\mu_2-\lambda_3 & 0 & 0 & 0 \\ 0 & \mu_2 & \lambda_3 & 0 & 0 & -\mu_1-\lambda_2-\mu_3 & \lambda_1 & 0 \\ \lambda_3 & 0 & 0 & \mu_2 & 0 & \mu_1 & -\lambda_1-\lambda_2-\mu_3 & 0 \\ 0 & \mu_3 & \lambda_2 & 0 & \lambda_1 & 0 & 0 & 0 \end{vmatrix}. \quad (55)$$

The following options 3, 4, 5, 6, 7 are composed and solved similarly.

Variant No. 8:

$$\begin{Vmatrix} -\lambda_1-\lambda_2-\lambda_3 & 0 & \mu_1 & 0 & \mu_2 & 0 & \mu_3 & 0 \\ 0 & -\mu_1-\mu_2-\mu_3 & 0 & \lambda_1 & 0 & \lambda_2 & 0 & \lambda_3 \\ \lambda_1 & 0 & -\mu_1-\lambda_2-\lambda_3 & 0 & 0 & \mu_3 & 0 & \mu_2 \\ 0 & \mu_1 & 0 & -\lambda_1-\mu_2-\mu_3 & \lambda_3 & 0 & \lambda_2 & 0 \\ \lambda_2 & 0 & 0 & \mu_3 & -\lambda_1-\mu_2-\lambda_3 & 0 & 0 & \mu_1 \\ 0 & \mu_2 & \lambda_3 & 0 & 0 & -\mu_1-\lambda_2-\mu_3 & \lambda_1 & 0 \\ \lambda_3 & 0 & 0 & \mu_2 & 0 & \mu_1 & -\lambda_1-\lambda_2-\mu_3 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{Vmatrix} \begin{Vmatrix} P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \\ P_4(\infty) \\ P_5(\infty) \\ P_6(\infty) \\ P_7(\infty) \\ P_8(\infty) \end{Vmatrix} = \begin{Vmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{Vmatrix}. \quad (56)$$

Solution:

$$P_i(\infty)=\frac{\Delta_i}{\Delta}, \ (i=1,2,3,...,8), \quad (57)$$

where:

$$\Delta=\begin{vmatrix} -\lambda_1-\lambda_2-\lambda_3 & 0 & \mu_1 & 0 & \mu_2 & 0 & \mu_3 & 0 \\ 0 & -\mu_1-\mu_2-\mu_3 & 0 & \lambda_1 & 0 & \lambda_2 & 0 & \lambda_3 \\ \lambda_1 & 0 & -\mu_1-\lambda_2-\lambda_3 & 0 & 0 & \mu_3 & 0 & \mu_2 \\ 0 & \mu_1 & 0 & -\lambda_1-\mu_2-\mu_3 & \lambda_3 & 0 & \lambda_2 & 0 \\ \lambda_2 & 0 & 0 & \mu_3 & -\lambda_1-\mu_2-\lambda_3 & 0 & 0 & \mu_1 \\ 0 & \mu_2 & \lambda_3 & 0 & 0 & -\mu_1-\lambda_2-\mu_3 & \lambda_1 & 0 \\ \lambda_3 & 0 & 0 & \mu_2 & 0 & \mu_1 & -\lambda_1-\lambda_2-\mu_3 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{vmatrix}, \quad (58)$$

$$\Delta_1=\begin{vmatrix} 0 & 0 & \mu_1 & 0 & \mu_2 & 0 & \mu_3 & 0 \\ 0 & -\mu_1-\mu_2-\mu_3 & 0 & \lambda_1 & 0 & \lambda_2 & 0 & \lambda_3 \\ 0 & 0 & -\mu_1-\lambda_2-\lambda_3 & 0 & 0 & \mu_3 & 0 & \mu_2 \\ 0 & \mu_1 & 0 & -\lambda_1-\mu_2-\mu_3 & \lambda_3 & 0 & \lambda_2 & 0 \\ 0 & 0 & 0 & \mu_3 & -\lambda_1-\mu_2-\lambda_3 & 0 & 0 & \mu_1 \\ 0 & \mu_2 & \lambda_3 & 0 & 0 & -\mu_1-\lambda_2-\mu_3 & \lambda_1 & 0 \\ 0 & 0 & 0 & \mu_2 & 0 & \mu_1 & -\lambda_1-\lambda_2-\mu_3 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{vmatrix}, \quad (59)$$

$$\Delta_2=\begin{vmatrix} -\lambda_1-\lambda_2-\lambda_3 & 0 & \mu_1 & 0 & \mu_2 & 0 & \mu_3 & 0 \\ 0 & 0 & 0 & \lambda_1 & 0 & \lambda_2 & 0 & \lambda_3 \\ \lambda_1 & 0 & -\mu_1-\lambda_2-\lambda_3 & 0 & 0 & \mu_3 & 0 & \mu_2 \\ 0 & 0 & 0 & -\lambda_1-\mu_2-\mu_3 & \lambda_3 & 0 & \lambda_2 & 0 \\ \lambda_2 & 0 & 0 & \mu_3 & -\lambda_1-\mu_2-\lambda_3 & 0 & 0 & \mu_1 \\ 0 & 0 & \lambda_3 & 0 & 0 & -\mu_1-\lambda_2-\mu_3 & \lambda_1 & 0 \\ \lambda_3 & 0 & 0 & \mu_2 & 0 & \mu_1 & -\lambda_1-\lambda_2-\mu_3 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{vmatrix}, \quad (60)$$

$$\Delta_3=\begin{vmatrix} -\lambda_1-\lambda_2-\lambda_3 & 0 & 0 & 0 & \mu_2 & 0 & \mu_3 & 0 \\ 0 & -\mu_1-\mu_2-\mu_3 & 0 & \lambda_1 & 0 & \lambda_2 & 0 & \lambda_3 \\ \lambda_1 & 0 & 0 & 0 & 0 & \mu_3 & 0 & \mu_2 \\ 0 & \mu_1 & 0 & -\lambda_1-\mu_2-\mu_3 & \lambda_3 & 0 & \lambda_2 & 0 \\ \lambda_2 & 0 & 0 & \mu_3 & -\lambda_1-\mu_2-\lambda_3 & 0 & 0 & \mu_1 \\ 0 & \mu_2 & 0 & 0 & 0 & -\mu_1-\lambda_2-\mu_3 & \lambda_1 & 0 \\ \lambda_3 & 0 & 0 & \mu_2 & 0 & \mu_1 & -\lambda_1-\lambda_2-\mu_3 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{vmatrix}, \quad (61)$$

$$\Delta_8=\begin{vmatrix} -\lambda_1-\lambda_2-\lambda_3 & 0 & \mu_1 & 0 & \mu_2 & 0 & \mu_3 & 0 \\ 0 & -\mu_1-\mu_2-\mu_3 & 0 & \lambda_1 & 0 & \lambda_2 & 0 & 0 \\ \lambda_1 & 0 & -\mu_1-\lambda_2-\lambda_3 & 0 & 0 & \mu_3 & 0 & 0 \\ 0 & \mu_1 & 0 & -\lambda_1-\mu_2-\mu_3 & \lambda_3 & 0 & \lambda_2 & 0 \\ \lambda_2 & 0 & 0 & \mu_3 & -\lambda_1-\mu_2-\lambda_3 & 0 & 0 & 0 \\ 0 & \mu_2 & \lambda_3 & 0 & 0 & -\mu_1-\lambda_2-\mu_3 & \lambda_1 & 0 \\ \lambda_3 & 0 & 0 & \mu_2 & 0 & \mu_1 & -\lambda_1-\lambda_2-\mu_3 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{vmatrix}. \quad (62)$$

The criterion for verifying the results of calculating the extreme probabilities of states and the initial mathematical model of an asymmetric Markov chain with eight states and continuous time is the equality of solutions to eight variants of compound systems of algebraic equations.

**5. 3. Dynamics of probabilities of states of the military structure**

As a result of calculating the probabilities of states of the military structure, it was established that in the time interval under consideration ($T$=7) the transitional process corresponds to $T_n$=5, and under the established stationary regime, respectively, $T_y$=2. Then the possible time of the military structure being in the $i$-th state under the stationary regime is determined as:

$$T_{iy}=P_i(\infty)\cdot 2 \ \ (i=1,2,3,4,5,6,7,8), \quad (63)$$

where $P_1(\infty)$=0.1145; $P_2(\infty)$=0.076; $P_3(\infty)$=0.1525; $P_4(\infty)$= =0.057; $P_5(\infty)$=0.2285; $P_6(\infty)$=0.038; $P_7(\infty)$=0.0285; $P_8(\infty)$=0.305.

That is, $T_{1y}$=0.229; $T_{2y}$=0.152; $T_{3y}$=0.305; $T_{4y}$=0.114; $T_{5y}$=0.457; $T_{6y}$=0.076; $T_{7y}$=0.057; $T_{8y}$=0.610.

The verification criterion is equality:

$$\sum_{i=1}^{8} T_{iy}=2. \quad (64)$$

In the transition process, the possible time of a military structure in the $i$-th state is determined from the formula:

$$T_{in}=\frac{\Delta_i(\nu_1)}{\Pi_1}\cdot 5+\sum_{k=2}^{8}\frac{\Delta_i(\nu_k)}{\Pi_k}\cdot\frac{1}{\nu_k}\left(e^{\nu_k\cdot 5}-1\right), \quad (65)$$

that is, as a result of calculations, we find: $T_{1n}$=0.87; $T_{2n}$=0.315; $T_{3n}$=0.878; $T_{4n}$=0.243; $T_{5n}$=1.028; $T_{6n}$=0.197; $T_{7n}$=0.165; $T_{8n}$=1.304.

The verification criterion is equality:

$$\sum_{i=1}^{8} T_{in}=5. \quad (66)$$

Thus, the possible time of the military structure being in the $i$-th state in the considered period $T$=7 is determined as:

$$T_i=T_{in}+T_{iy} \ \ (i=1,2,3,4,5,6,7,8), \quad (67)$$

that is: $T_1$=1.099; $T_2$=0.467; $T_3$=1.183; $T_4$=0.357; $T_5$=1.485; $T_6$=0.274; $T_7$=0.222; $T_8$=1.913.

Accordingly, the verification criterion takes the form:

$$\sum_{i=1}^{8} T_i=7. \quad (68)$$

As our calculations show, with the given initial data, the longest time by the military structure corresponds to the eighth state: $T_8$=1.913 days, where the troops of brigade No. 1 are combat-ready, and brigades No. 2 and No. 3 are non-combat-ready. The shortest time by the military structure corresponds to the seventh state: $T_7$=0.222 days, where the troops of brigade No. 1 are non-combat-ready, and brigades No. 2 and No. 3 are combat-ready. The time spent by each of the three military units in the combat-ready state $T(+)$ or non-combat-ready $T(-)$ states is introduced, respectively, for:

– brigade No. 1 $T_1(+)$ or $T_1(-)$;
– brigade No. 2 $T_2(+)$ or $T_2(-)$;
– brigade No. 3 $T_3(+)$ or $T_3(-)$.

This is determined from the following formulas:

$$\begin{aligned} &T_1(+)=T_1+T_3+T_5+T_8,\ T_1(-)=T_2+T_4+T_6+T_7; \\ &T_2(+)=T_1+T_3+T_6+T_7,\ T_2(-)=T_2+T_4+T_5+T_8; \\ &T_3(+)=T_1+T_4+T_5+T_7,\ T_3(-)=T_2+T_3+T_6+T_8. \end{aligned} \quad (69)$$

At the weekly time interval under consideration, we get:

$$\begin{aligned} &T_1(+)=5.680,\ T_1(-)=1.320; \\ &T_2(+)=2.778,\ T_2(-)=4.222; \\ &T_3(+)=3.163,\ T_3(-)=3.837. \end{aligned} \quad (70)$$

The conditions for verification of calculations are the following equalities:

$$T_1(+)+T_1(-)=7;$$
$$T_2(+)+T_2(-)=7; \quad (71)$$
$$T_3(+)+T_3(-)=7.$$

Our interpretation of the probabilities of states of the military structure makes it possible to predict the combat situation over time and make well-informed, balanced decisions for the appropriate management of random processes, varying the intensities of the flows of restorations and losses in military units.

## 6. Discussion of results based on investigating the reliability of a system over time by the intensity of failure flows and restoration of three autonomous subsystems

The results of the possible states of the military structure (Fig. 1) were obtained, which are explained by the ordered form of the special intensity matrix (1) and the symmetric distribution of roots in characteristic equation (4) and the ordered structure of matrix formula (10) of the probabilities of the system states.

An algorithm for conducting a computational experiment to assess the reliability of a multidimensional dynamic military structure over time with a known initial state and given intensities of failure flows and restorations of three autonomous military units over the time interval under consideration has been developed.

A methodology for determining the time of the combat-ready state of military units, as well as an assessment of the general state of the military structure over time under conditions of intensive flows of losses and restorations (Table 1, Fig. 2), has been proposed.

The algorithm is based on the results of the proposed analytical solution to the eighth-order Kolmogorov equations for an asymmetric Markov chain, derived on the basis of the concept of harmonization of the mathematical description of models [31–33]. The harmonization of the mathematical description is represented in the form of ordered eighth-order matrix tables and the corresponding determinant tables [10, 18].

Unlike conventional methods of reliability assessment (statistical modeling, Monte Carlo methods), the proposed algorithm is based on the analytical solution to the Kolmogorov equations, which provides a 2–3-fold reduction in computation time due to the elimination of iterative numerical methods, increased accuracy due to the absence of approximate calculations, the possibility of expansion to systems with a larger number of subsystems.

The proposed mathematical statement of the problem considers asymmetric Markov chains and the corresponding Kolmogorov differential equations, and the results reported in the paper are unique and have no analogs since they are based on the analytical solution to the eighth-power algebraic equation of partial form that we found. Adapting this scientific area to military topics would require the involvement of a qualified military specialist as a co-author, in the future, in the conceptual concepts of military management, typical troop structure, terminology, combat readiness and non-combat readiness limits, etc. Under such conditions, it might be possible to conduct a comparative analysis of the proposed algorithm with approaches used in closed sources.

The use of tabular notation of expanded calculation formulas provides systematization of a large amount of initial data, the possibility of direct implementation of the algorithm on computers, reduction of the probability of program errors due to the order of the tables, as well as facilitation of the algorithm verification process.

Conventionally, high-order Kolmogorov differential equations are integrated by approximate numerical methods in a limited range of parameter changes. Our paper considers a special class of Kolmogorov equations that correspond to asymmetric Markov chains and are linear, homogeneous differential equations with constant coefficients.

When considering high-order asymmetric Markov chains, methodological difficulties arise, which are solved by increasing the power of computational tools.

The limitations of this study are associated with high indicators of the size of the problem and the order of the Kolmogorov equations and the power of the characteristic equation, as well as the possibility of its analytical solution.

The disadvantage of our study relates to the hypothesis of the constancy of the intensities of failure and recovery flows of autonomous subsystems in the considered time interval. In the case of variable intensity of failure and recovery flows, the reported results may prove incorrect, so in the future it will be necessary to devise a different statement of the problem and conduct additional research.

Future development of this study involves the consideration of more complex systems containing a finite set of autonomous subsystems ($m$=1,2,3,4...). Here, the dimensionality of problems increases according to the exponential law $2^m$, and its order can be 8, 16, 32, etc. The difficulties are associated with the analytical solution to algebraic equations of 8, 16, 32, etc. powers, the disclosure of determinants of order 8, 16, 32, etc., which can be overcome by improving the mathematical support to computers and increasing their computational power.

## 7. Conclusions

1. An algorithm for determining the probabilities of states of a complex system based on the intensities of failure and recovery flows of an arbitrary number of autonomous subsystems has been proposed. The sequence of operations of the algorithm is as follows:

1) construction of a set of asymmetric states of the system as a whole, equal to $n=2^m$, where $m$ is the number of autonomous subsystems;

2) construction of a square matrix of intensities $R$ of order $n$ based on the given and constant in the considered time interval $[0, T)$ of failure flows $\lambda_j$ and recovery $\mu_j$ ($j$=1,2,3,..., $m$) of autonomous subsystems;

3) construction of the characteristic determinant $\Delta(\nu)$ of the intensity matrix $R$; determination of the set of roots $\nu_k$ ($k$=1,2,3,..., $n$) in the characteristic equation $\Delta(\nu)=0$ based on the intensities of failure and recovery flows of subsystems;

4) determination of all possible products of differences of roots in characteristic equation $\Pi_k$ ($k$=1,2,3,..., $n$);

5) construction of square matrix $[\Delta_i(\nu_k)]$ ($i, k$=1,2,3,..., $n$) by characteristic determinant $\Delta(\nu)$ depending on $k$-roots and $i$-states of the system, using column matrix of known initial conditions of the system state;

6) construction of a matrix-column of exponents $\left[e^{\nu_k \cdot t}/\Pi_k\right]$ by the set of roots $n_k$ and time $t$ from a limited interval $[0, T)$, where the intensities of failure and restoration flows of subsystems are considered known and constant.

2. We have determined the probabilities of system states in time $P_i(t)$ ($i$=1,2,3,..., $n$) on the considered inter-

val $[0,T)$ as a result of the product of the composite matrices $[\Delta_i(\nu_k)]\cdot[e^{\nu_k\cdot t}/\Pi_k]$. The solution to the problem of determining the probabilities of system states at the following time intervals with a discrete change in the failure and restoration flows is found in a cycle according to the proposed algorithm. The extreme probabilities of states at the previous step are taken as the initial conditions at the next interval-step. The process of verification and interpretation of the calculation results is based on the equality $\sum_{i=1}^{n} P_i(t)=1$, invariant with respect to time.

3. The dynamics of changes in the probabilities of states of a military structure makes it possible to predict the combat situation over time and make reasonable, balanced decisions for the appropriate management of random processes, varying the intensities of the flows of restorations and losses in military units. It was established that with the given initial data, the longest time by the military structure corresponds to the eighth state: $T_8=1.913$ days, where the troops of brigade No. 1 are combat-ready, and brigades No. 2 and No. 3 are non-combat-ready. The shortest time of the military structure corresponds to the seventh state: $T_7=0.222$ days, where the troops of brigade No. 1 are non-combat-ready, and brigades No. 2 and No. 3 are combat-ready.

## Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study, as well as the results reported in this paper.

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## Data availability

All data are available, either in numerical or graphical form, in the main text of the manuscript.

## Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

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