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CONSTRUCTION OF A MATHEMATICAL MODEL OF DYNAMIC LOADS IN A JIB SELF-PROPELLED CRANE WHEN PULLING A SHEET PILE OUT OF THE GROUND

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The object of this study is the process of extracting sheet piles from the ground using a jib self-propelled crane for their repeated use. The task addressed was the extraction of steel sheet piles by a jib self-propelled crane, its interaction with the vibratory pile driver, and the determination of dynamic loads.

The vibratory method significantly increases extraction efficiency; however, it also induces vibrational impacts on the crane, causing alternating stresses in the metal structure. This could lead to fatigue damage accumulation, cracks in weld seams, failure of base metal, and a decrease in the service life of the self-propelled crane. Furthermore, vibration negatively affects the working conditions of crane operators by causing fatigue, reducing performance, and compromising safety.

Mathematical modeling methods were used, with the construction of calculation schemes that reflect all stages of sheet pile extraction: preliminary insertion, taking up slack in the lifting system, tensioning the lifting ropes, extraction with vibration over 2/3 of the pile's length, and final extraction without vibration.

Numerical modeling has shown that during static extraction, the dynamic coefficient may reach 4.76, while with vibration it decreases to 1.47. This confirms the effectiveness of the vibratory method, provided its adverse effects on the crane are minimized. The results could be applied to improve crane design, devise protective measures against vibration, and enhance operational efficiency and safety. Additionally, the findings could become a basis for optimizing the parameters of elastic ties and the interaction scheme between the crane and the ground, thereby expanding the potential for model's practical application under more complex conditions

Keywords: vibratory pile driver, dynamic coefficient, loads in elastic ties, friction force, sheet pile

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1. Introduction

Modern construction puts forward high demands on the efficiency, economy, and environmental feasibility of assembly and dismantling operations. One of the common technological elements in temporary fencing structures is sheet piles, which, after the completion of construction work, are usually removed for reuse. This makes it possible to significantly reduce material costs and decrease the impact on the environment. However, such a routine operation, at first glance, is associated with a number of technically complex tasks that require in-depth engineering analysis.

The problem becomes acute when using heavy equipment, in particular jib self-propelled cranes, to remove sheet pile elements from dense soil. Under conditions of increased loads on the structural elements of the machine, there is a need to accurately predict the dynamic behavior of the "crane-pile-soil" system. Since dynamic effects can cause fatigue failure of metal structures, reduce the resource of equipment, and involve potentially dangerous emergencies, the issue of ensuring the reliability of equipment becomes particularly relevant.

From a practical point of view, the justification of the pulling modes, the distribution of loads in the elastic elements of the crane and the action of external forces make it possible not only to extend the service life of the equipment but also improve the efficiency of machine operation under difficult conditions. This, in turn, leads to reduced downtime, improved quality of work, reduced costs for repairs and maintenance. In addition, proper consideration of dynamic loads contributes to the improvement of working conditions for operators, reducing the risks of occupational fatigue and increasing the level of safety.

Therefore, scientific research in the field of dynamic analysis of the process of pulling sheet piles from the soil using a jib crane is extremely necessary. It provides a practical basis for improving the designs of equipment, increasing the reliability of construction processes, and devising modern methods for controlling dynamic processes in mechanical engineering.

2. Literature review and problem statement

It is known [1, 2] that for effective pulling of sheet piles it is advisable to use the vibration method, which greatly facilitates the pulling process, especially when the sheets are immersed to a considerable depth, or the soil is compacted and has a high resistance. Thus, in work [3], it is noted that the vibration method is the most common technique, and the most important parameters are the vibration frequency, amplitude, and static moment. In work [4], it is noted that the frequency of oscillations affects the reduction of soil resistance during pile movement.

When pulling sheet piles by vibration, the crane is exposed to vibration and alternating loads. And as is known, vibration contributes to the accumulation of damage in the material of its elements, the appearance of cracks both in welds and in the base metal, as well as fatigue failure. The most rapid destruction of the structure occurs under vibration effects under conditions of resonance phenomena, which lead to damage and shortening the service life of the crane [5]. Thus, in work [6], a vibration protection system for a crane equipped with vibration technological equipment (VTE) is proposed, in which vibration does not spread to the crane structure. In [7], the amplitude-frequency characteristics and frequencies of natural oscillations of a jib self-propelled crane (JSC) are reported.

Vibrations can cause both destruction of individual components and failure of the crane, as well as create unfavorable working conditions for drivers, worsen their functional state, contribute to fatigue, reduce labor productivity and quality of work. Prolonged exposure to vibration can lead to occupational diseases of drivers in the form of headaches, numbness of the fingers, pain in the hands and forearms, the occurrence of cramps, increased sensitivity to cooling, the appearance of insomnia, and others. When vibration sickness occurs in crane operators, pathological changes may occur in the spinal cord, cardiovascular system, bone tissue, and joints, and capillary blood circulation may change [5, 7].

The vibration parameters of the crane elements can be influenced by the characteristics of their elastic-damping ties. Therefore, it is very important to take into account the spectrum of their natural vibration frequencies since it determines the frequency of the source of the forcing force at which resonant vibrations occur [7–9], and to avoid them. Thus, in work [8], a multi-mass dynamic model is presented for determining the natural and forced vibrations of JSC under the action of the forced force of VTE on the hook. In [9], the issue of the action of vibration from a freely hanging VTE on the hook is considered.

In work [10], the parameters of the soil vibration velocity and the average effective stress are analyzed; a parametric analysis of the frequency of soil vibrations is performed. The results showed that with an increase in the vibration frequency, the soil resistance on the lateral surface of the pile can be significantly reduced. However, in [10], the issue of how the vibration frequency affects the soil resistance during pile extraction is not considered.

In [11], the development of the coupled Euler-Lagrange hydraulic method and its application for the inverse analysis of tests of the model of vibrational immersion of piles in water-saturated sand are reported. However, in [11], the behavior of the soil during the extraction of the pile by the vibration method is not considered.

In [12], the results of laboratory tests using a newly designed shaker for immersion of piles in sandy soil are reported. The results show that the improvement of pile driving occurs with high-frequency vibrations at low amplitude. However, in [12], the question of how the above-mentioned results affect the extraction of piles from the soil by the vibration method using lifting equipment is not considered.

In [13], issues are discussed regarding the assessment of the vibrational immersion of piles, but the modeling of the extraction of piles by the vibration method is not considered. At a bench described in [14], studies on the vibrational immersion of elements in saturated sand were performed. It was concluded that the vibration frequency is an important parameter for the effective immersion (extraction) of sheet piles. However, it was not stated that vibration has a negative effect on the crane design since the extraction process was not studied.

In [15], research was carried out on the vibrational immersion of a pile model in dry medium-compacted and dense sand. The results show that the speed of movement is affected by the vibration frequency, and it plays a significant role for the effective immersion of piles. However, the work does not consider the issue of modeling the process of extracting metal sheet piles from the soil and the vibration effect on the crane structure.

Thus, in [16], a study was conducted to assess the cyclic response of the soil during vibrational immersion of piles; the influence of the parameters of dynamic immersion of piles was shown. However, in [16], the process of extracting metal sheet piles from the soil by a jib crane and the influence of vibration on its structure are not considered.

Thus, the cited works consider the theory and experimental modeling of the interaction of a pile with the soil under vibration. Although those studies focus on the pile driving process, they do not address the effects of vibration on the crane design. Furthermore, they do not determine the dynamic loads on the jib self-propelled crane structure.

3. The aim and objectives of the study

To build a mathematical model of the process of extracting metal sheet piles from the soil, to determine the dynamic loads on the structure of a jib self-propelled crane as a "crane-vibratory pile driver" system with the subsequent development of means for protecting the crane and the operator from the effects of vibration. This will make it possible to determine the dynamic loads and vibration effects on the crane structure when extracting a sheet pile from the soil.

To study the goal, the following tasks were set:

– to develop calculation schemes for extracting a sheet pile from the soil by a jib self-propelled crane;

– to define stages in the process of extracting a sheet pile from the soil;

– to determine the values of forces in elastic ties and to calculate the dynamic coefficients in the ropes of the cargo hoist when extracting a sheet pile from the soil, under different options for external influence.

4. The study materials and methods

4. 1. The object and hypothesis of the study

The object of our study is the process of extracting sheet piles from the soil by a self-propelled jib crane for their reuse using the vibration method.

The principal hypothesis of the study assumes that the use of the vibration method significantly reduces static loads on the crane structure when extracting a sheet pile.

The following simplifications and assumptions are accepted:

– the sheet pile is a solid body;

– the soil covering the sheet pile is immobile;

– a dry friction force acts between the lateral surfaces of the pile and the soil;

– the force of the vibrator on the pile is described by the harmonic function $P(t)=P_0\sin\omega t$.

4. 2. Methodology of investigating the process of sheet pile extraction

4. 2. 1. Justification of the extraction technique adopted in the study

The method of extracting sheet piles using vibration [1, 3, 17], caused by the action of a vibratory pile driver rigidly connected to the piles using a jib self-propelled crane, was chosen for our study. This approach is justified as the most effective for extracting piles that have been in the soil for a long time, without metal damage or deformations that would make their reuse impossible.

Static methods require significantly greater efforts (over 1,000–1,500 kN), which may exceed the crane's lifting capacity, therefore they were not considered as the main ones within the framework of this study.

4. 2. 2. Equipment and conditions for pulling sheet piles out of the ground

This paper investigates the process of pulling a metal sheet pile of the Larsen L-IV type with a length of 12 m. Lifting is carried out by a crawler crane MKG-25.01A. The vibration source is a VPP-2A vibratory pile driver with a spring-loaded load, with an electric motor (Fig. 1).

The procedure for pulling a sheet pile is that first, when the lifting ropes are pulled, the vibratory pile driver is turned on. When the sheet pile, vibrating, sinks into the ground by 2–4 cm, its lifting begins. With this technique, the separation of the sheet pile from the ground occurs under the action of its natural weight with a vibratory pile driver. Therefore, it is sufficient to apply an effort equal to twice the weight of the vibratory pile driver with the sheet pile so that the latter begins to move upwards from the ground; in this case, the speed of movement should be approximately the same as during the downward movement. If the speed exceeds that which occurred during the immersion, a correspondingly greater effort will be required.

Before the first stage, when the ropes of the cargo hoist are in a free position, when the system of ropes of the cargo hoist hangs over the vibratory pile driver rigidly connected to the sheet pile, the vibratory pile driver is turned on. When the sheet pile vibrates and sinks into the ground by 2–4 cm, the pile is separated from the ground under the action of the natural weight of the sheet pile with the vibratory pile driver.

Therefore, it is enough to apply a force on the crane hook equal to twice the weight of the vibratory pile driver with the pile so that it moves up from the ground.

When lifting the pile, the condition of the lower springs of the vibratory pile driver is monitored. At those moments when the spring coils are completely closed under the action of the traction force, the lifting of the hook is suspended. This is a sign that the sheet pile has not yet been torn from the ground, and the lift force is not enough. The lifting is temporarily stopped for a while, until the pile is lifted due to the elastic forces of the spring system of the vibratory pile driver.

After the springs are straightened, the lifting of the hook is restored under the action of vibration, to a height equal to approximately 2/3 of the length of the sheet pile located in the soil, and further lifting is carried out without vibration.

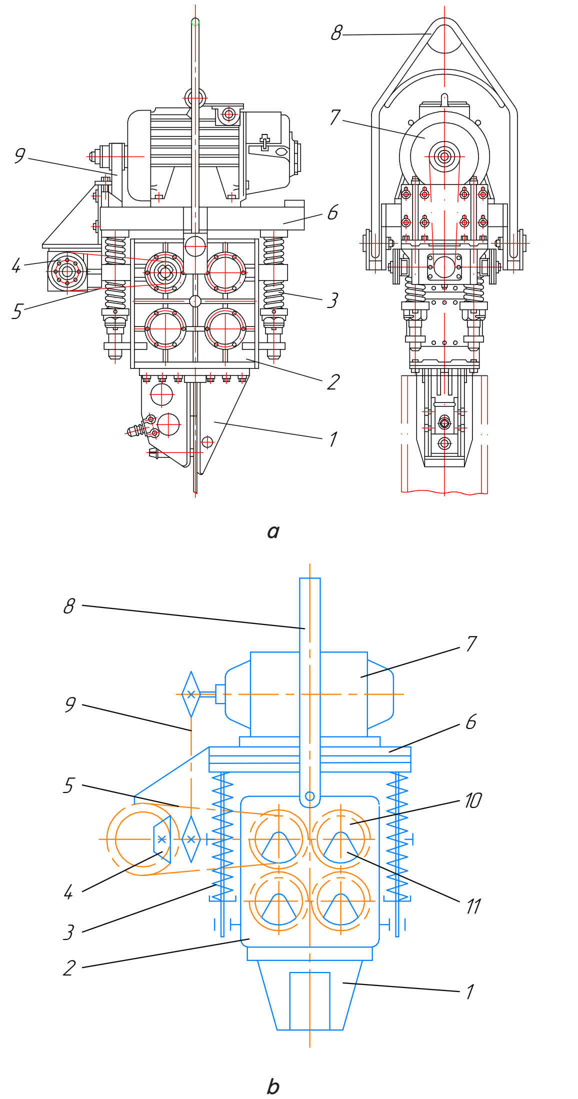


Fig. 1. General view of a vibratory pile driver with sprung loading: *a* – general view of the vibratory pile driver; *b* – schematic representation of the vibratory pile driver; 1 – pile head; 2 – vibrator; 3 – springs; 4 – bevel gearbox; 5 – horizontal chain drive; 6 – loading plate; 7 – electric motor; 8 – suspension; 9 – vertical chain drive; 10 – gears; 11 – unbalance shafts

4. 2. 3. Mathematical modeling of vibrations of the mechanical subsystem of a crane during sheet pile extraction

The vibrations of the mechanical subsystem of a jib crane, which arise as a result of the extraction of a sheet pile from the soil under the action of vibration, have been studied. The sources of the forced force are the drive of the load lifting mechanism and the vibratory pile driver with a loading plate rigidly connected to the sheet pile.

For the studies conducted below, the following notations are adopted (Fig. 2): m_0 – mass of the rotating drive parts of the load lifting mechanism reduced to the ropes, kg; m_{41} – mass of the hook suspension, kg; m_{42} – mass of the sprung load with an electric motor, kg; $m_4=m_{41}+m_{42}$ – reduced mass of the hook suspension with a sprung load, kg; m_5 – mass of the vibrating system, kg; J_3 – reduced moment of inertia of the crane boom relative to the root joint, kg·m^2; Q – gravitational force of the vibrating system, (weight of the load) N; $P(t)=P_0\sin\omega t$ – forced force of the vibratory pile driver, (longitudinal dynamic effect on the element being immersed or extracted), N; P_0 – amplitude value of the forced force, N; P_{k3} – force in the ropes of the boom support system at the third stage of pulling, kN; P_{k4} – force in the ropes of the load-lifting system at the fourth stage of pulling; M_{st} – static moment of mass of unbalances, kg·m; F – soil resistance along the lateral surface of the pile, N; A – amplitude of oscillations, m; α – angle of rotation of unbalances, rad; α_3 – angle of inclination of the boom to the horizontal, rad; c_{31} – stiffness of the rope branch of the boom lifting system from the axis of the blocks of the boom system to the drum, c_{32} – stiffness of the ropes of the boom hoist, N/m; c_{33} – stiffness of the ropes of the boom stay cables, N/m; c_{41} – stiffness of the cargo rope branch from the axis of the blocks on the boom head to the drum, c_{42} – stiffness of the ropes of the cargo hoist, N/m; c_5 – total stiffness of the springs of the vibratory loader, N/m; β_{32} – damping coefficient in the ropes of the boom hoist, N·s/m; β_{33} – damping coefficient in the ropes of the boom stays, N·s/m; β_{41} – damping coefficient in the load rope branch from the axis of the blocks on the boom head to the drum, N·s/m; β_{42} – damping coefficient in the load ropes of the boom hoist, N·s/m; z_4, z_5 and φ_3 – generalized coordinates, m; $\dot{z}_4$, $\dot{z}_5$ – generalized linear speeds, m/s; $\dot{\varphi}_3$ – generalized angular velocity of the crane boom, 1/s; Δ_k – total gap in the drive with the overlap of the ropes, m; γ_3 – angle between the ropes of the boom support system and the longitudinal axis of the boom, rad; γ_4 – angle between the branch of the rope of the lifting system above the boom and the longitudinal axis of the boom, rad; l_3 – boom length, m; l_{31} – length of the force arm, P_{k3}.

The mechanical subsystem of the jib crane under consideration consists of five rigid bodies and has four degrees of freedom.

It should be noted that the mass of the load plate with the electric motor m_{42} at the first stage is found together with the mass of the hook suspension m_{41}, that is, as one mass $m_4=m_{41}+m_{42}$, and at the fourth stage the mass m_{42} is combined with the mass of the sheet pile with the vibrator $m_{51}=m_{45}+m_{42}$.

To study the motion of the jib crane subsystem in the process of pulling the sheet pile out of the soil, the Lagrange equations of the second kind for mechanical systems were applied:

$$\frac{d}{dt}\left(\frac{\partial \mathrm{T}_i}{\partial \dot{q}_i}\right)-\frac{\partial \mathrm{T}_i}{\partial q_i}+\frac{\partial \Phi_i}{\partial \dot{q}_i}+\frac{\partial \Pi_i}{\partial q_i}=Q_i, \qquad (1)$$

where T_i is the kinetic energy of the mechanical subsystem of the jib crane; Π_i is the potential energy of the mechanical subsystem of the jib crane; Φ_i is the dissipative function (dissipation function); Q_i is the generalized external forces corresponding to the generalized coordinates; $\dot{q}_i$ is the generalized velocities.

The following values are taken as generalized coordinates:

$$q_1=z_0,\quad q_2=z_4,\quad q_3=\varphi_3,\quad q_4=z_5,$$

$$\dot{q}_1=\dot{z}_0,\quad \dot{q}_2=\dot{z}_4,\quad \dot{q}_3=\dot{\varphi}_3.$$

Based on Lagrange equations of the second kind, systems of differential equations were compiled that describe the entire process of pulling a sheet pile out of the soil at all four stages without taking into account dissipative functions.

5. Results of investigating the action of loads on elastic ties in the process of pulling a sheet pile out of the soil

5. 1. Calculation schemes for pulling a sheet pile out of the soil by a jib self-propelled crane

The constructed calculation schemes are shown in Fig. 2.

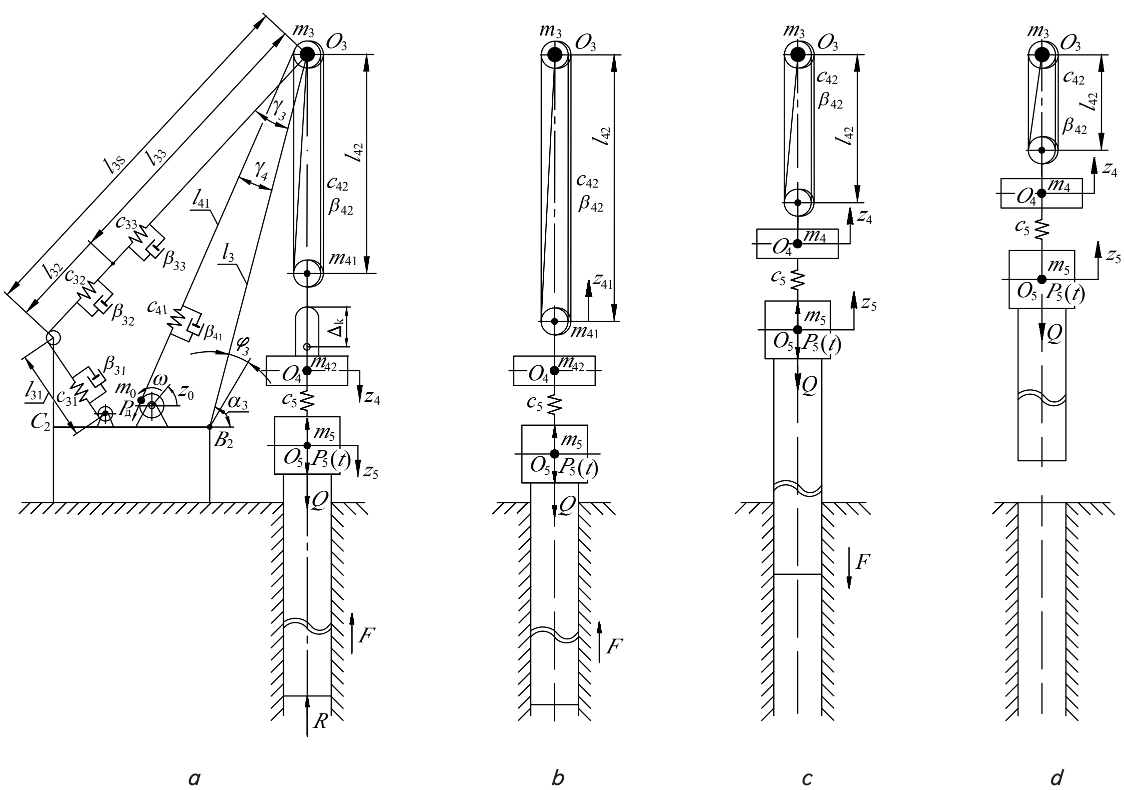


Fig. 2. Calculated equivalent scheme for pulling a sheet pile out of the ground: *a* — immersion of the sheet pile by 2—4 cm and selection of the total clearance Δ_k in the drive of the lifting mechanism and the overhang of the load ropes; *b* — tension of the load ropes to double their load from the weight of the sheet pile with a vibrator; *c* — pulling the sheet pile with vibration by 2/3 of its length; *d* — final extraction without vibration

5. 2. Stages in the process of pulling a sheet pile out of the ground

There are two techniques of pulling. The more effective one is the technique in which, first, when the ropes of the cargo hoist are pulled, the vibratory pile driver is turned on (Fig. 2, *a*). When the sheet pile, vibrating, is immersed in the ground by 2...4 cm, its lifting begins. In this way, the pile is separated from the ground under the action of the pile's natural weight and the vibratory pile driver. Therefore, it is enough to apply a force equal to twice the weight of the vibratory pile driver with the pile so that the latter starts moving upwards; in this case, the lifting speed should be approximately the same as during immersion. If it exceeds the immersion speed, then more effort will be required for pulling. The maximum pulling speed of a metal sheet pile is 4...5 m/min. As the pulling speed increases, the force applied to the vibratory pile driver suspension also increases, potentially damaging the vibratory pile driver or pile.

Given that the process of sinking the pile and its separation from the soil is carried out without the use of a lifting device, it is not considered within the scope of this work.

The process of pulling a sheet pile out of the soil is divided into four stages. Let us consider each of them.

At the first stage, after the sheet pile is separated from the soil, the combined movement of masses m_0 and m_{41}begins: the reduced mass of the rotating parts of the lifting mechanism drive m_0, and the mass of the hook suspension m_{41}, which will be accompanied by a decrease in the gap Δ_k.

At this stage, there will be three masses m_0, m_3 and m_{41} in motion (and it is described by a system of two inhomogeneous second-order differential equations, which takes the following form:

$$\begin{cases} m_0\ddot{z}_0 + c_4\left(z_0 - l_3\sin\gamma_4\varphi_3 - z_{41}\right) = P_{dq}; \\ m_{41}\ddot{z}_{41} - c_4\left(z_0 - l_3\sin\gamma_4\varphi_3 - z_{41}\right) = -G_4. \end{cases} \quad (1)$$

From the system of equations (1), expressions can be extracted for determining the force in the cargo ropes P_{k41} and the driving force of the lifting mechanism P_{dq}, which is determined from the Kloss equations.

$$\begin{cases} P_{k41} = c_4\left(z_0 - l_3\sin\gamma_4\varphi_3 - z_{41}\right); \\ P_{dq} = K_q\left(v_0 - \dot{z}_0\right) \Big/ \left[B_q + \left(v_0 - \dot{z}_0\right)^2\right]. \end{cases} \quad (2)$$

After the transformation, the system of equations (1) taking into account expressions (2) took on the following form:

$$\begin{cases} \ddot{z}_0 = \left(P_{dq} - P_{k41}\right) \big/ m_0; \\ \ddot{z}_4 = \left(P_{k41} - G_{41}\right) \big/ m_{41}. \end{cases} \quad (3)$$

The first stage of movement ends with the selection of the gap Δ_k.

At the second stage (Fig. 2, *b*) without turning on the vibrator, the joint movement of masses m_0, m_3 and m_4 begins, during which the load increases in all elastic ties – the ropes of the boom support system P_{k32}, the lifting ropes P_{k42} and the spring system of the vibratory pile driver P_{52} (Fig. 2, *b*).

The equation of mass movement is described by a system of three inhomogeneous differential equations of the second order, which take the following form:

$$\begin{cases} m_0\ddot{z}_0 + c_4\left(z_0 - l_3\sin\gamma_4\varphi_3 - z_4\right) = P_{dq}; \\ J_3\ddot{\varphi}_3 + c_3 l_3^2\sin^2\gamma_3\varphi_3 - \\ -c_4 l_3\sin\gamma_4\left(z_0 - l_3\sin\gamma_4\varphi_3 - z_4\right) = 0; \\ m_4\ddot{z}_4 - c_4\left(z_0 - l_3\sin\gamma_4\varphi_3 - z_4\right) = -G_4. \end{cases} \quad (4)$$

The expressions for determining the forces in elastic ties and the moments acting on the boom are as follows:

$$\begin{cases} P_{k42} = c_4\left(z_0 - l_3\sin\gamma_4\varphi_3 - z_4\right); \\ M_{32} = c_3 l_3^2\sin^2\gamma_3\varphi_3; \\ M_{42} = c_4 l_3\sin\gamma_4\left(z_0 - l_3\sin\gamma_4\varphi_3 - z_4\right); \\ P_{dq} = K_q\left(v_0 - \dot{z}_0\right) \Big/ \left[B_q + \left(v_0 - \dot{z}_0\right)^2\right]. \end{cases} \quad (5)$$

After the transformation, the system of equations (4) taking into account the notations of expressions (5) took the following form:

$$\begin{cases} \ddot{z}_0 = \left(P_{dq} - P_{k42}\right) \big/ m_0; \\ \ddot{\varphi}_3 = \left(M_{42} - M_{32}\right) \big/ J_3; \\ \ddot{z}_4 = \left(P_{k42} - G_4\right) \big/ m_{42}. \end{cases} \quad (6)$$

The second stage of movement ends with the achievement of tensioning the load ropes to a force equal to the sum of the weight of the hook suspension with the load plate and the double weight of the sheet pile with the vibrator, i.e., $P_{k42} = \left(m_4 + 2m_5\right)g$.

At the third stage (Fig. 2, *c*) due to the separation of the sheet pile from the soil, its extraction with a vibration effect begins. In this case, all four masses, m_0, m_3, m_4 and m_5, will be in motion.

It should be noted that the force of pulling the pile should not exceed 120 kN since with such a load, the coils of the vibratory pile driver springs will close, which will lead to an increase in the extraction force, as well as to the transfer of the vibration effect to the crane structure.

The equation of motion of the third stage is described by a system of four inhomogeneous differential equations of the second order, which takes the following form:

$$\begin{cases} m_0\ddot{z}_0 + c_4\left(z_0 - z_4 - l_3\sin\gamma_4\varphi_3\right) = P_{dq}; \\ J_3\ddot{\varphi}_3 + c_3 l_3^2\sin^2\gamma_3\varphi_3 - \\ -c_4 l_3\sin\gamma_4\left(z_0 - z_4 - l_3\sin\gamma_4\varphi_3\right). = 0. \\ m_4\ddot{z}_4 - c_4\left(z_0 - z_4 - l_3\sin\gamma_4\varphi_3\right) - c_5\left(z_5 - z_4\right) = 0; \\ m_5\ddot{z}_5 + c_5\left(z_5 - z_4\right) = P_0\sin\omega t - Q. \end{cases} \quad (7)$$

The expressions for determining the forces in elastic ties and the moments acting on the boom are as follows:

$$\begin{cases} P_{k43} = c_4\left(z_0 - z_4 - l_3\sin\gamma_4\varphi_3\right); \\ P_{53} = c_5\left(z_5 - z_4\right); \\ M_{33} = c_3 l_3^2\sin^2\gamma_3\varphi_3; \\ M_{43} = c_4 l_3\sin\gamma_4\left(z_0 - z_4 - l_3\sin\gamma_4\varphi_3\right); \\ P_{dq} = K_q\left(v_0 - \dot{z}_0\right) \Big/ \left[B_q + \left(v_0 - \dot{z}_0\right)^2\right]. \end{cases} \quad (8)$$

After the transformation, the system of equations (7), taking into account the notations of expressions (8), took on the following form:

$$\begin{cases} \ddot{z}_0 = \left(P_{dq} - P_{k43}\right)/m_0; \\ \ddot{\varphi}_3 = \left(M_{43} - M_{32}\right)/J_3; \\ \ddot{z}_4 = \left(P_{k43} + P_{53}\right)/m_4; \\ \ddot{z}_5 = \left(P(t) - P_{53} - Q\right)/m_5. \end{cases} \tag{9}$$

The third stage of pile movement ends with its extraction under the action of vibration by 2/3 of the immersed length with the transition to the fourth stage.

The fourth stage (Fig. 2, *d*) is the final one and is characterized by the final extraction of the remaining part of the sheet pile from the soil in the absence of vibration influence (with the vibrator turned off). In this case, four masses of the crane subsystem m_0, m_3, m_4 and m_5 will be in motion.

A distinctive feature of the differential equation systems describing the process of sheet pile extraction at this stage is the absence of the expression of the forced vibrator force $P(t)=P_0\sin\omega t=0$ in the right-hand side of the third equation.

Taking this into account, the system of equations takes the following form:

$$\begin{cases} m_0\ddot{z}_0 + c_4\left(z_0 - l_3\sin\gamma_4\varphi_3 - z_4\right) = P_{dq}; \\ J_3\ddot{\varphi}_3 - c_4 l_3 \sin\gamma_4\left(z_0 - l_3\sin\gamma_4\varphi_3 - z_4\right) + \\ + c_3 l_3^2 \sin^2\gamma_3\varphi_3 = 0. \\ m_4\ddot{z}_4 - c_4\left(z_0 - l_3\sin\gamma_4\varphi_3 - z_4\right) - c_5\left(z_5 - z_4\right) = 0; \\ m_5\ddot{z}_5 + c_5\left(z_5 - z_4\right) = -Q. \end{cases} \tag{10}$$

From the system of differential equations (10), expressions can be extracted that make it possible to determine the forces in elastic ties and the moments acting on the boom from the ropes of the lifting and boom-supporting systems,

$$\begin{cases} P_{k44} = c_4\left(z_0 - l_3\sin\gamma_4\varphi_3 - z_4\right); \\ P_{k54} = c_5\left(z_5 - z_4\right); \\ M_{34} = c_3 l_3^2 \sin^2\gamma_3\varphi_3; \\ M_{44} = c_4 l_3 \sin\gamma_4\left(z_0 - l_3\sin\gamma_4\varphi_3 - z_4\right); \\ P_{dq} = K_q\left(v_0 - \dot{z}_0\right)/\left[B_q + \left(v_0 - \dot{z}_0\right)^2\right]. \end{cases} \tag{11}$$

After transforming the system of differential equations (10) taking into account the accepted notations of expressions in (11), a new system of differential equations is obtained with the right-hand side in the form:

$$\begin{cases} \ddot{z}_0 = \left(P_{dq} - P_{k44}\right)/m_0; \\ \ddot{\varphi}_3 = \left(M_{44} - M_{34}\right)/J_3; \\ \ddot{z}_4 = \left(P_{k44} + P_{54}\right)/m_4; \\ \ddot{z}_5 = \left(-P_{54} - Q\right)/m_5. \end{cases} \tag{12}$$

The fourth stage of movement ends with the final pulling of the pile without vibration.

The modeling was carried out under the assumption of elastic work of the ropes and linear behavior of the soil, which may slightly underestimate the influence of complex nonlinearities characteristic of the real soil environment. The study was also performed for specific values of mass parameters and stiffnesses.

A numerical solution to the equations of mass movement during pulling a sheet pile from the soil in 4 stages was obtained for a jib crane equipped with a vibratory pile driver, under the following parameters: m_0=CM0=954,236 kg; CM3=1,154 kg; CM41=370 kg; CM42=1,500 kg; CM5=1,588 kg; C3=2.419·10^6 N/m; C4=1.464 10^6 N/m; C5=2,090,707.2 N/m; CL3=14.4 m; γ_3=0.4524 rad; γ_4=0.1071 rad; SM5=7 kg·m; OM5=157 1/s=25 Hz.

Within these parameters, the model demonstrates adequacy. However, the expansion of research involves taking into account a wider range of input data and including experimental verification of the model.

5. 3. The value of forces in the elastic ties and the dynamic coefficients of the lifting system when pulling a sheet pile out of the soil

5. 3. 1. Pulling a sheet pile out of the soil under the action of vibration with a friction force of 138 kN

Fig. 3–7 show plots of the full cycle of the dependence of forces in the elastic ties on the time of pulling the sheet pile. On the left, the plots indicate PK4 – forces in the ropes of the lifting system, PK3 – forces in the ropes of the boom support system, P5 – forces when compressing the lower springs of the vibratory pile driver. On the right, the ordinate axes of the plots indicate Pd – forces of the lifting mechanism drive.

Fig. 3 shows the value of forces in the elastic ties when pulling a sheet pile out of the soil with a jib crane for t=100 s under the action of vibration and a friction force equal to 138 kN on its lateral surface.

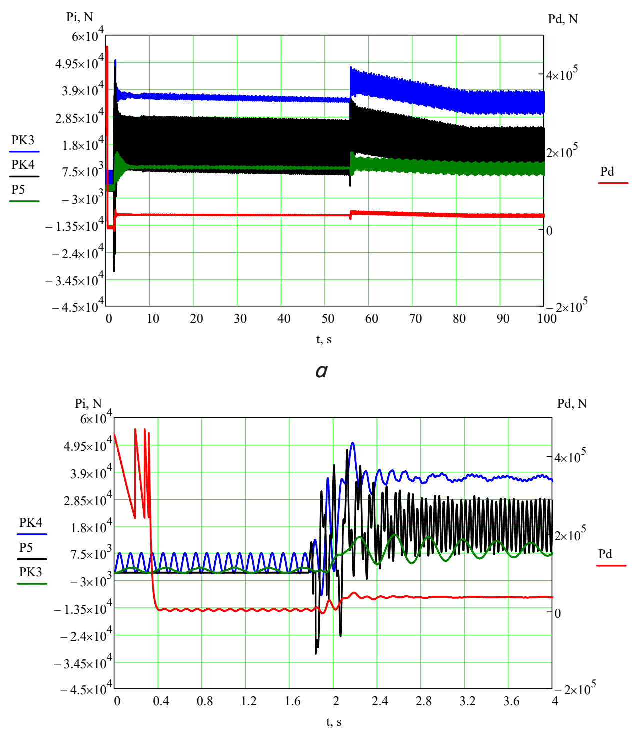


Fig. 3. The value of forces in the elastic ties when pulling a sheet pile out of the soil during four stages under the action of vibration and a friction force equal to 138 kN: *a* – from zero to 100 seconds; *b* – a fragment from zero to four seconds

The electric motor transits from artificial characteristics to natural ones in 0.38 s, Fig. 3. From 0 to 1.8 s, the total gap in the drive of the lifting mechanism and the loading ropes are selected. From 1.8 s to 2.9 s, there is a period of increasing load in the elastic ties of the lifting ropes. The force in the ropes of the lifting system (PK4) reaches a maximum value of 50 kN in 2.18 s. From 2.9 s to 55.8 s, the third stage of sheet pile extraction occurs with a maximum PK4 force of 39 kN. From 55.8 s to 57 s, there is a period of transition from the action of vibration to the beginning of sheet pile extraction without the action of vibration. Next, the final pulling of the sheet pile from the soil occurs without the action of vibration, at 56 s PK4 is 35 kN and decreases to 25 kN at 82 s, and then does not change for up to 100 s.

5. 3. 2. Pulling a sheet pile out of the ground without vibration and with a friction force of 138 kN

Fig. 4 shows the values of forces in the elastic ties when pulling a sheet pile out of the ground with a jib crane for t=100 s without vibration and with a friction force of 138 kN.

The electric motor transits from artificial characteristics to natural ones occurs in 0.38 s, Fig. 4. From 0 to 1.8 s, the total gap in the drive of the lifting mechanism and the loading ropes are selected; from 1.8 s to 4 s, there is a period of increasing load in the elastic ties of the lifting ropes. At 3.2 s, the forces PK4=P5 reached a maximum value of 161.5 kN at 3.4 s, and from this time and throughout the entire period of pile pulling, oscillations with beating occur in the ropes of the lifting and boom-supporting systems, the force decreases to 37.7 kN at 84 s. Then the final pulling of the sheet pile from the soil occurs with a force in the ropes of the lifting system equal to 37.7 kN.

5. 3. 3. Pulling a sheet pile out of the soil under the action of vibration without taking into account the friction force

Fig. 5 shows the values of forces in the elastic ties when pulling a sheet pile out of the soil with a jib crane for t=100 s without the action of vibration and the friction force on the lateral surface of the sheet pile equal to 0.

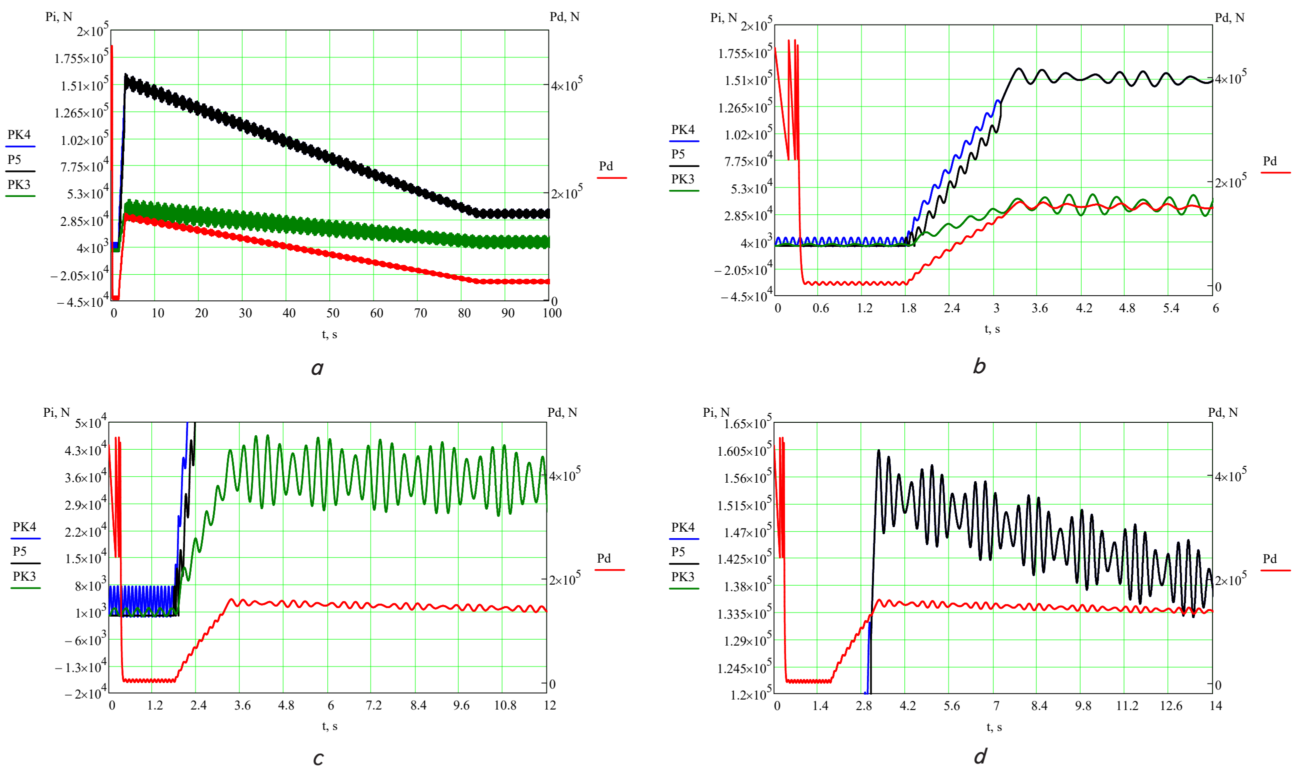


Fig. 4. The value of forces in the elastic ties when pulling a sheet pile out of the soil during four stages without the action of vibration and a friction force equal to 138 kN: *a* – from zero to 100 seconds; *b* – a fragment from zero to six seconds; *c* – PK3 on an enlarged scale; *d* – PK4 and P5 on an enlarged scale

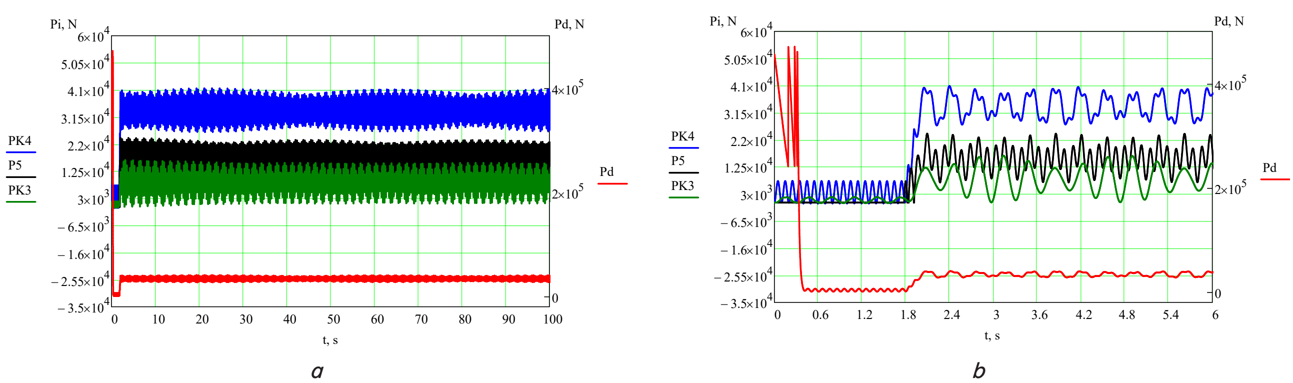


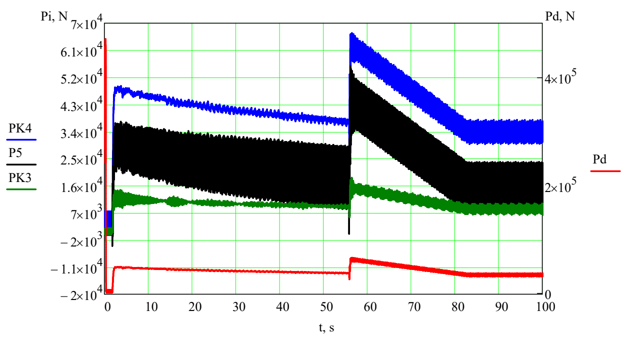
Fig. 5. The value of forces in the elastic ties when pulling a sheet pile out of the soil during four stages without the action of vibration and the friction force equal to 0: *a* – from zero to 100 seconds; *b* – a fragment from zero to six seconds

The electric motor transits from artificial characteristics to natural ones in 0.38 s, Fig. 5. From 0 to 1.8 s, the total gap in the drive of the lifting mechanism and the loading ropes is selected. At 2 s, the force in the loading ropes (PK4) increases and reaches a maximum value of 40.5 kN; it does not change until the final extraction of the sheet pile from the soil.

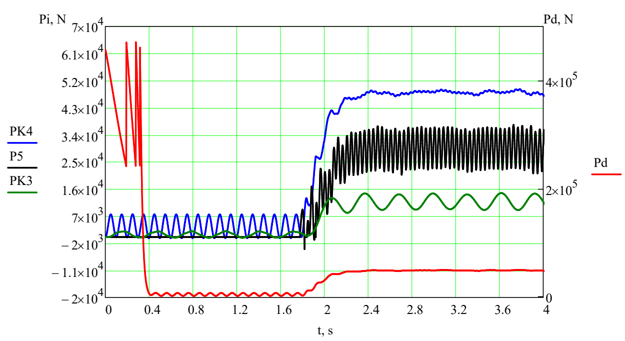
5. 3. 4. Extraction of a sheet pile from the soil under the action of vibration with a friction force of 100 kN

Fig. 6 shows the value of forces in the elastic ties when extracting a sheet pile from the soil by a jib crane for *t*=100 s under the action of vibration and the friction force on the lateral surface of the sheet pile equal to 100 kN.

The electric motor transits from artificial characteristics to natural ones in 0.38 s, Fig. 6. From 0 to 1.8 s, the total gap in the drive of the lifting mechanism and the loading ropes are selected. From 1.8 s to 2.4 s, there is a period of increasing load in the lifting ropes, the force in the ropes of the lifting system (PK4) reaches a maximum value of 49 kN at 2.4 s and decreases to 38 kN at 55.8 s. From 2.4 s to 55.8 s, the third stage of sheet pile extraction occurs. From 55.8 s to 57 s, there is a period of transition from the action of vibration to the beginning of sheet pile extraction without the action of vibration. Then the final extraction of the sheet pile from the soil without the action of vibration occurs. At 56.2 s, PK4 reaches a maximum value of 67 kN, followed by a decrease to 37 kN at 83 s, and then does not change until 100 s.



a

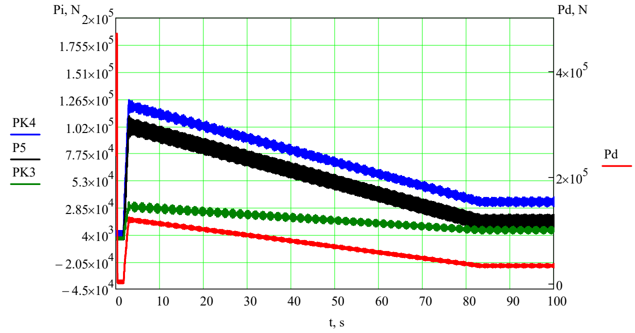


b

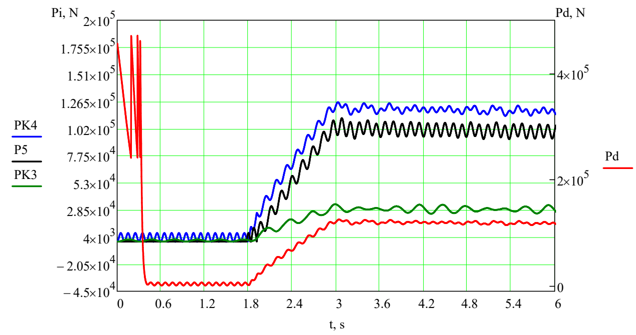
Fig. 6. The value of forces in the elastic ties when pulling a sheet pile out of the soil during four stages under the action of vibration and a friction force equal to 100 kN: *a* – from zero to 100 seconds; *b* – a fragment from zero to six seconds

5. 3. 5. Pulling a sheet pile out of the ground without vibration and with a friction force of 100 kN

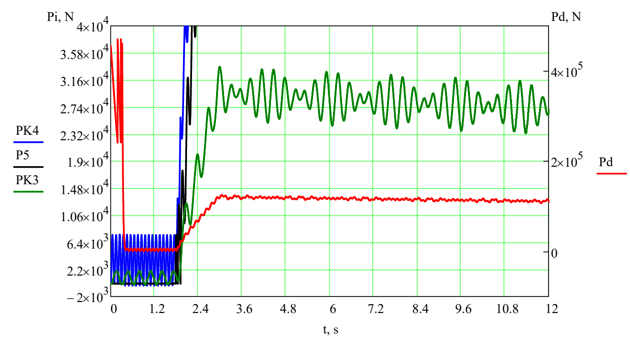
Fig. 7 shows the values of forces in the elastic ties when pulling a sheet pile out of the ground with a jib crane for 100 s without vibration and with a friction force on the side surface of the sheet pile equal to 100 kN.



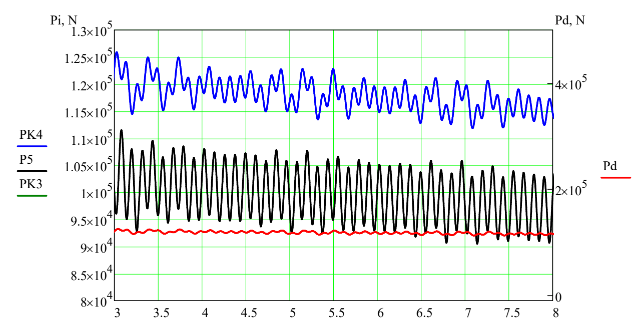
a



b



c



d

Fig. 7. The value of forces in the elastic ties when pulling a sheet pile out of the soil during four stages without the action of vibration and a friction force equal to 100 kN: *a* – from zero to 100 seconds; *b* – a fragment from zero to six seconds; *c* – PK3 on an enlarged scale; *d* – PK4 and P5 on an enlarged scale

The electric motor transits from artificial characteristics to natural ones in 0.38 s, Fig. 7. From 0 to 1.8 s, the total gap in the drive of the lifting mechanism and the loading of the load ropes is selected. From 1.8 s to 3 s, there is a period of increasing the load in the load-lifting ropes. The force in the ropes of the load-lifting system (PK4) reaches a maximum value of 126.5 kN at 3 s and decreases to 38 kN at 83 s. From 3 s and throughout the entire period of pile pulling, oscillations with beating occur in the ropes of the boom-supporting system. From 3 s to 83 s, the third stage of sheet pile pulling occurs, then the final pulling of the pile from the soil occurs at a force in the ropes of the load-lifting system of 38 kN and does not change up to 100 s.

5. 3. 6. Values of dynamic coefficients of the lifting system for various conditions of external influence on the mechanical system

The principal results of our calculations of the values of dynamic coefficients of the lifting system are summarized in Table 1.

Table 1

Dynamic load in elastic ties during sheet pile pulling with vibration and friction force 138 kN

Elements of the crane subsystem	Parameter	Stages of pulling out a sheet pile		
		Stage 2	Stage 3	Stage 4
The value of forces in elastic ties under the action of vibration and friction force equal to 138 kN				
Block and tackle suspension of cargo ropes	PK4, kN	50	39	35
Boom support system ropes	PK3, kN	15	15	15
Vibratory pile driver elastic ties	P5, kN	49	30	35
Dynamic coefficient (maximum)	Cd	1.47	1.15	1.03
The value of forces in elastic ties without vibration and friction force is equal to 138 kN				
Block and tackle suspension of cargo ropes	PK4, kN	161.5	37.5	37.5
Boom support system ropes	PK3, kN	30	47	14
Vibratory pile driver elastic ties	P5, kN	161.5	37.5	37.5
Dynamic coefficient (maximum)	Cd	4.76	1.1	1.1
The value of forces in elastic ties without the action of vibration and friction force is 0				
Block and tackle suspension of cargo ropes	PK4, kN	40.5	40.5	40.5
Boom support system ropes	PK3, kN	40	40	40
Vibratory pile driver elastic ties	P5, kN	15	15	15
Dynamic coefficient (maximum)	Cd	1.2	1.2	1.2
The value of forces in elastic ties under the action of vibration and friction force is equal to 100 kN				
Block and tackle suspension of cargo ropes	PK4, kN	49	67	37
Boom support system ropes	PK3, kN	15	18.5	10
Vibratory pile driver elastic ties	P5, kN	34	36	52
Dynamic coefficient (maximum)	Cd	1.44	1.98	1.1
The value of forces in elastic ties without vibration and friction force is equal to 100 kN				
Block and tackle suspension of cargo ropes	PK4, kN	80	126.5	38
Boom support system ropes	PK3, kN	27	33	22
Vibratory pile driver elastic ties	P5, kN	70	110	12
Dynamic coefficient (maximum)	Cd	2.36	3.73	1.12

Analysis of our results of calculating the dynamic coefficients given in Table 1 reveals that they differ significantly depending on the conditions of pulling out the sheet pile. Thus, when pulling out the sheet pile under the action of vibration and the calculated friction force equal to 138 kN, the value of the dynamic coefficient is 1.47. Therefore, it is advisable to use means to reduce dynamic loads.

6. Discussion of results based on modeling the process of pulling a sheet pile out of the soil

The calculation schemes built (Fig. 2) allowed us to determine the sequence of stages in the pile pulling process, the number of masses involved in the movement and the number of active and passive actions of external influences at each stage. Based on them, systems of second-order inhomogeneous differential equations were compiled, describing the 4-stage process of pulling a pile out of the soil.

The results of our numerical modeling of the process of pulling a sheet pile out of the soil were obtained according to the four stages considered within the framework of the tasks set in chapter 3. All stages were studied on the basis of a mathematical model implemented in the form of a system of second-order differential equations (Lagrange equations), which takes into account the dynamic features of the mechanical subsystem of the jib crane at each stage.

Fig. 3–7 show changes in the forces in the ropes of the boom-supporting and load-lifting systems during the full cycle of pile pulling. Analysis of these plots allows us to establish the following.

At the third stage, the dominant role in the system loading is played by the friction force of the pile against the soil, which causes peak values of the forces in the ropes. These forces are due to the simultaneous influence of the pile's natural weight, the vibratory pile driver, and the drive force.

At the fourth stage, a gradual decrease in the contact interaction of the pile with the soil is observed. This is confirmed by the decrease in forces in the elastic ties, which is reflected in Fig. 4–6.

Table 1 gives dynamic coefficients in the ropes for various combinations of mass and stiffness parameters. The values demonstrate a decrease in dynamic overloads in the system with a decrease in the soil friction force on the lateral surface of the pile, which confirms the correctness of the selected model.

Our work takes into account the dynamic interaction between the vibratory pile driver, hook suspension, rope system, and crane boom system. This makes it possible to model not only quasi-stationary loads, but also oscillatory processes caused by the influence of vibration loading on the pile.

In particular, the original statement of the problem with four stages of pulling out is that it allowed us to study the change in the behavior of the system depending on the depth of pile immersion. A variable mass structure is also introduced, where at the fourth stage the mass of the loading is taken into account as part of the pile system. The proposed model allows us to fully describe the entire cycle of pile movement taking into account the forces in the elastic ties of the "crane-vibratory pile driver-pile-soil" system.

The current study has the following limitations.

The modeling was carried out under the assumption of elastic work of the ropes and linear behavior of the soil, which may slightly underestimate the influence of complex nonlinearities characteristic of the real soil environment.

The study was also performed for specific values of mass parameters and stiffnesses: within these parameters, the model demonstrates adequacy. At the same time, further research involves taking into account a wider range of input data and including experimental verification of the model.

In the future, to increase the adequacy of the model, it is planned to implement the following improvements:

– to increase the number of masses of the system;

– to take into account the action of dissipative forces;

– to apply the nonlinear law of change of the soil friction force on the lateral surface of the sheet pile depending on the length of the pile immersion during its extraction;

– to consider the operation of the vibratory pile driver under transient modes;

– to determine the load on the ropes of the load-lifting system when the lower springs of the vibratory pile driver are compressed in the case of reaching the maximum load on the ropes, when all three masses are combined – the hook suspension, the load with the electric motor, and the vibrator with the sheet pile, which is accompanied by the maximum vibration effect on the crane structure;

– to design means to reduce the vibration effect and dynamic loads.

7. Conclusions

1. Original calculation schemes have been proposed for each of the four stages of extracting a metal sheet pile from the soil by a self-propelled jib crane, which take into account the variability of external influences, sequential switching of the operating modes of the vibration device, and the presence of gaps in the drives. The schemes reflect the specificity of interaction among the elements of the "pile – soil – crane" system and allow for an adequate assessment of the dynamic loads on the crane structural elements.

2. Four consecutive stages in the process of extracting a sheet pile from the soil after preliminary immersion of the pile by 2–4 cm have been defined: selection of the total gap in the drive of the lifting mechanism and the introduction of cargo ropes; tensioning of lifting ropes to double load (weight of pile with vibrator; pulling out the pile under vibration by 2/3 of its length; final pulling out of the pile without vibration. Identifying these stages allowed us to detail the dynamic pattern of loads on the elastic ties of the jib crane, which formed the basis of the mathematical model of the process.

3. Numerical modeling allowed us to determine the forces in the lifting ropes of a jib self-propelled crane and the corresponding dynamic coefficients for different modes of pulling a sheet pile out of the ground. The maximum value of the dynamic coefficient is 4.76 when moving from the second to the third vibration-free pulling stage, which indicates significant overloads of the system. Under vibration conditions, the coefficient value decreases to 1.47, which significantly reduces the dynamic impact on the crane elements. The presence of vibrations with beating was established, which indicates the presence of adding different frequencies, which can cause alternating stresses in the welded joints of the metal structure of the jib, which in turn can cause fatigue failure. This emphasizes the feasibility of using the vibration method provided that measures are taken to reduce the vibration impact and increase the reliability of the crane structure.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study, as well as the results reported in this paper.

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Data availability

All data are available, either in numerical or graphical form, in the main text of the manuscript.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

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