UDC 514.18
DOI: 10.15587/1729-4061.2025.328825

MATHEMATICAL DESCRIPTION OF BENDING A SURFACE OF REVOLUTION INTO A HELICAL CONOID

Andrii Nesvidomin
PhD*
Serhii Pylypaka
Doctor of Technical Sciences, Professor, Head of Department*
Tetiana Volina
Corresponding author
PhD, Associate Professor*
E-mail: volina@nubip.edu.ua
Mykhailo Kalenyk
PhD, Professor, Dean
Department of Mathematics, Physics and Methods of Their Education**
Svitlana Botvinovska
Doctor of Technical Sciences, Professor
Department of Descriptive Geometry and Engineering Graphics
Kyiv National University of Construction and Architecture
Povitrianykh Syl ave., 31, Kyiv, Ukraine, 03037
Iryna Hryshchenko
PhD, Associate Professor*
Dmytro Spirintsev
PhD, Associate Professor, Head of Department
Department of Mathematics and Physics
Bogdan Khmelnitsky Melitopol State Pedagogical University
Naukovoho mistechka str., 59, Zaporizhzhia, Ukraine, 69097
Vitalii Kolodnenko
Senior Lecturer
Department of Transport Technologies***
Serhii Borodai
Senior Lecturer
Department of Architecture and Surveying Engineering***
Irina Zakharova
PhD, Associate Professor
Department of Management of Education and Pedagogy of the Higher School**
*Department of Descriptive Geometry, Computer Graphics and Design
National University of Life and Environmental Sciences of Ukraine
Heroyiv Oborony str., 15, Kyiv, Ukraine, 03041
**Sumy State Pedagogical University named after A. S. Makarenko
Romenska str., 87, Sumy, Ukraine, 40002
***Sumy National Agrarian University
Herasyma Kondratieva str., 160, Sumy, Ukraine, 40021

The object of this study is the process of theoretical gradual bending of a catenoid into a helical conoid coil. A helical conoid or a straight closed helicoid is formed by the helical motion of a segment around an axis, and this segment intersects the axis at a right angle during movement. It cannot be bent into a plane, but by gradually reducing the pitch it can be transformed into a known surface of revolution – a catenoid. With such deformation, the lengths of the lines and the area of the coil as a whole do not change, that is, the deformation occurs similarly to unfolded surfaces. Such deformation is based on the theory of bending surfaces of a separate section of differential geometry. According to it, any helical surface can be bent into a surface of revolution and vice versa. Bending the non-folded surface of a helical conoid into a catenoid is a classic example of differential geometry. This approach makes it possible to find an approximate flat workpiece for manufacturing a screw coil. This task is resolved by approximating the obtained catenoid by a truncated cone. The sweep of the truncated cone will be the approximate sweep of the screw turn. This is the peculiarity of finding the approximate sweep, which in engineering practice is calculated by other formulas. This is also the essence of the reported results.

In the work, parametric equations were derived that describe a one-parameter set of intermediate surfaces during bending of a screw conoid due to a gradual decrease in the surface pitch to zero. In the given example, one turn of the screw is considered, put on a shaft with a radius $r = 0.125$ m and limited by an external radius $R = 0.25$ m with a surface pitch $H = 0.5$ m. The dimensions of the truncated cone, which replaces the catenoid, are $r = 0.148$ m for the smaller base, $R = 0.262$ m for the larger base, and $H = 0.05$ m for the height of the cone. The specified dimensions of the cone are sufficient to find its exact sweep, which will be approximate for the turn of the screw conoid

Keywords: surface pitch, screw turn, truncated cone, approximate sweep, non-expandable surface

Received 10.02.2025
Received in revised form 31.03.2025
Accepted 22.04.2025
Published 27.06.2025

How to Cite: *Nesvidomin, A., Pylypaka, S., Volina, T., Kalenyk, M., Botvinovska, S., Hryshchenko, I., Spirintsev, D., Kolodnenko, V., Borodai, S., Zakharova, I. (2025). Mathematical description of bending a surface of revolution into a helical conoid. Eastern-European Journal of Enterprise Technologies, 3 (1 (135)), 30–37. https://doi.org/10.15587/1729-4061.2025.328825*

1. Introduction

Screw surfaces are widely used in various devices, mechanisms, and machines. Thus, in [1], the design of a self-propelled multifunctional combine harvester with a screw working surface is proposed. In [2], the issue of developing new designs of flexible sectional screw working bodies with extended technological properties is raised. In [3], the design of an auger working body for surface tillage of the soil from the compartment of the expanding helicoid is calculated. In [4], the synthesis of screw loaders is carried out using the morphological analysis method in order to obtain structures of mechanisms with the best technical and economic characteristics. In addition, the cited work substantiates and investigates the designs and methods for calculating the main structural and power parameters of the screw loader.

Many scientific works consider the issue of designing screw surfaces. In [5], the method of the contour point vector is proposed for calculating the forming profile of the cutter of the screw surface. The effectiveness of the method is demonstrated on a numerical and experimental example of grinding the shape of a helical gear. In [6], an analytical method for calculating a ramp in the form of a spiral helicoid is considered. In [7], technological processes for forming curvilinear profiles on screw blanks were devised and experimentally tested.

The technology of manufacturing helical surfaces is complicated by the fact that they are non-expandable (with the exception of the expandable helicoid [8]). The most common helical surface is the helical conoid, known in engineering practice as a screw. One of the technologies for its manufacture is the deformation of a flat workpiece into a coil of the required dimensions. During such deformation, the flat workpiece is stretched along the inner periphery of the coil and compressed along the outer one. This leads to significant plastic deformations and, as a result, is accompanied by resistance to the formation of the workpiece into a finished product. To minimize the resistance, it is necessary to have a flat workpiece with dimensions that will provide the lowest resistance and, accordingly, the energy intensity of the process.

Research in this area is important since reducing the energy intensity of the process leads not only to a decrease in energy costs but also to an increase in the accuracy of the finished product. There is reference literature to find the approximate expandable radius of the helical conoid coil. The calculation formulas given in it are derived from the assumption that the lengths of the peripheral outer and inner helical lines are equal to the lengths of the corresponding arcs of circles on a flat workpiece. However, these formulas do not reflect the essence of the gradual bending process. Thus, studies taking into account the gradual bending of a flat workpiece into a helical conoid turn are important for practice.

2. Literature review and problem statement

In [9], a screw electrothermomechanical converter capable of exerting a multiphysical effect on the technological environment through the distribution of various indicators on the rotor surface is examined. The disadvantage of the study is that the mathematical models used for numerical modeling showed deviations of up to 9.5% from the experimental data. This may be caused by simplifications in modeling or inaccuracies in experimental measurements. In [10], mathematical modeling of the oil extrusion process with preliminary grinding of raw materials in a twin-screw extruder is investigated. Among the limitations of the study, one can note the use of simplified mathematical models, which may lead to certain discrepancies between theoretical calculations and practical results. The likely reason is the difficulty of taking into account all parameters of the technological process in the mathematical model. This may be explained by the difficulties in including all aspects of the technological process in the mathematical model. In [11], a screw electromechanical hydrolyzer for processing poultry by-products is studied. The authors built a mathematical model for the analysis of thermal and electromagnetic processes in a hydrolyzer. Among the limitations of the study, one can note the limited shape of the proposed screw working body. This may be due to the fact that the authors focused on classical designs and their characteristics, without taking into account possible non-standard solutions. However, a detailed study and analysis of non-standard configurations of screw elements could contribute to a deeper understanding of their efficiency under various operating conditions and technological processes. In [12], a methodology for calculating the maximum torque when the screw of a screw conveyor jams was devised. The authors used a neural network that was trained on the basis of numerical integration of nonlinear differential equations. However, the assumptions adopted in the simplified mathematical model make it impossible to take into account different geometric shapes of the working body. The likely reason is the initial stage at which this promising research area is at present. In [13], new designs of articulated working bodies of screw conveyors were investigated, and their optimal parameters were substantiated to ensure the transportation of bulk materials along curvilinear routes. The authors derived analytical dependences for the stiffness conditions of a separate section with an articulated connection depending on the load and design parameters of the conveyor. A limitation of the study is that the resulting analytical dependences do not take into account possible variations in the geometry of the screw bodies and the features of their manufacture. The likely reason is the authors' desire for a more detailed study of specific structures instead of expanding the model to all possible options.

The simplified mathematical model proposed in those studies is based on a number of assumptions that do not take into account possible variations in the shape of the working body, which limits the overall versatility of the solution. One way to overcome this limitation is to build a more adaptive mathematical model that would take into account various configurations of the working body and their impact on the corresponding technological process.

In [14], the formation of helical surfaces using the running-in method is considered. A limitation of the study is that it considers only non-expandable helical surfaces, the sweep of which can only be approximate. This, in turn, increases the energy intensity of the process of their manufacture.

In [15], the influence of the main technological parameters of processing on the microgeometry, structure, and properties of electro erosion coatings is investigated; and in [16], the stress-strain state of the surface layer after plastic deformation of electro erosion coatings is analyzed. In [17], the development of energy-saving transport technological systems with screw working bodies is considered. In [18], the assessment of eddy currents and power losses in the rotor of a screw electrothermomechanical converter intended for additive manufacturing is reported. The cited studies are aimed at energy-efficient optimization of various technological processes; however, they do not take into account the possibility of a positive influence on such indicators of the geometric component at the design stage. The likely reason is objective difficulties associated with the complexity of taking into account the variability of geometric shapes.

That drawback was eliminated in [19], in which the design of the working body of the screw section of a combined soil tillage tool is considered; in particular, its main parameters and functional characteristics are substantiated. The authors consider a torso-helicoid with a horizontal axis of rotation. In [20], the movement of a soil particle along the surface of a deployed helicoid with a horizontal axis of rotation and a given angle of attack is analyzed. The authors built mathematical models and found the trajectories of particle motion. However, the issue of manufacturing such a working body remains out of consideration. This may be due to the fact that the authors' attention was focused precisely on finding the trajectories of material particle motion along surfaces, and not on the issues of their manufacturing. The wide use of a screw conoid as a working surface in various mechanisms and machines necessitates the search for improved technologies for its manufacturing. To this end, it is necessary to have an approximate sweep that would most accurately correspond to the manufacturing process. Existing calculation formulas are not related to the process of manufacturing a surface coil. In [21], the construction of the surface coil sweep is considered from the point of view of its bending into a finished product. However, it deals with finding the sweep of an expandable helicoid, for which the sweep is not approximate but exact. A similar approach is advisable to use for finding the approximate sweep of a helical conoid turn.

3. The aim and objectives of the study

The purpose of our study is a mathematical description of the gradual bending of the surface of revolution, which is a catenoid, into the turn of a helical conoid. This will make it possible to devise recommendations for finding an approximate sweep of the turn. Since there is no exact sweep, the one found on the basis of gradual bending will be more accurate among the approximate ones. In turn, this will reduce plastic deformations during the formation of the surface turn, and, accordingly, the energy consumption of the process.

To achieve the goal, the following tasks were set:

– to construct parametric equations that describe the set of intermediate surfaces during the gradual bending of the catenoid into the turn of a helical conoid;

– to find an approximate sweep of the helical conoid turn and devise recommendations for its manufacture.

4. The study materials and methods

The object of our study is the process of gradual bending of the catenoid surface into a helical conoid turn. The hypothesis of the study assumes that, given the initial surface of rotation (catenoid) and the final surface (helical conoid), it is possible to construct a set of intermediate surfaces that reflect the process of gradual bending of the catenoid into a helical conoid. In this case, a simplification is introduced in which the surface thickness of all these surfaces is zero.

Our studies are based on the section of differential geometry on the theory of bending of surfaces. The surface belongs to two families of coordinate lines. A point on the surface is given by the values of two curvilinear coordinates, similar to a plane, where these coordinates (x and y) are rectilinear. If the curvilinear coordinates of the surface are related by a certain dependence, then a curve will be described on the surface. From the point of view of bending, it is necessary to find such analytical transformations of the surface that the length of the line on its surface does not change. According to the theorem of differential geometry, any surface of revolution can be bent into a helical one. If the meridian of the surface of revolution is given by the explicit equation $z=f(\rho)$, then it is described by the following parametric equations:

$$X=\rho\cos\alpha;$$

$$Y=\rho\sin\alpha;$$

$$Z=f(\rho), \tag{1}$$

where α and ρ are the curvilinear coordinates of the surface, and ρ is the distance from the vertical axis of rotation to a point on the surface, and α is the angular coordinate.

The length of a line on a surface can be found using a linear element of the surface, which is also called the first quadratic form of the surface

$$dS^2=Ed\alpha^2+2Fd\alpha d\rho+Gd\rho^2, \tag{2}$$

where the coefficients E, F, G are found through the partial derivatives of equations (1):

$$E=\left(\frac{\partial X}{\partial\alpha}\right)^2+\left(\frac{\partial Y}{\partial\alpha}\right)^2+\left(\frac{\partial Z}{\partial\alpha}\right)^2;$$

$$F=\frac{\partial X}{\partial\alpha}\cdot\frac{\partial X}{\partial\rho}+\frac{\partial Y}{\partial\alpha}\cdot\frac{\partial Y}{\partial\rho}+\frac{\partial Z}{\partial\alpha}\cdot\frac{\partial Z}{\partial\rho};$$

$$G=\left(\frac{\partial X}{\partial\rho}\right)^2+\left(\frac{\partial Y}{\partial\rho}\right)^2+\left(\frac{\partial Z}{\partial\rho}\right)^2. \tag{3}$$

For the surface of rotation (1), the first quadratic form (2) takes the form

$$dS^2=\left[1+f'^2(\rho)\right]d\rho^2+\rho^2d\alpha^2. \tag{4}$$

The average coefficient $F=0$, which indicates that the grid of coordinate lines, which are parallels and meridians, is rectangular. If we set the dependence $\alpha=\alpha(\rho)$ or $\rho=\rho(\alpha)$, then on the surface (1) these dependences will correspond to certain lines, the length of which can be found from expression (4). But in the process of bending the surface, the lengths of the lines do not change. This means that it is necessary to find other parametric equations for the surface (1), according to which the surface should transform into a helical one and at the same time the first quadratic form (4) should not change. If such equations are found, then they absolutely accurately describe the process of bending the surface, however, without taking into account its thickness. Taking it as the median, we can obtain quite accurate results. This served as the basis for choosing the research method.

5. Results of gradual theoretical bending of a catenoid into a helical conoid by increasing the step from zero to the final value

5. 1. Parametric equations of the set of helical surfaces that are intermediate between a catenoid and a helical conoid when they are bent into each other

The helical surface into which the surface of revolution (1) will be bent can be written in the form:

$$X=w\cos\alpha;$$

$$Y=w\sin\alpha;$$

$$Z=\varphi+h\alpha, \tag{5}$$

where α and w are independent surface variables, h is a helical parameter (a constant), $\varphi = \varphi(w)$ is the axial section of the helical surface (an unknown function to be found).

At $h = 0$, the helical surface (5) is transformed into a surface of revolution. It is required to find such a dependence $\varphi = \varphi(w)$ that at $h = 0$ it describes the same curve (meridian) as the dependence $f = f(\rho)$.

The coefficients of the first quadratic form of the surface (5) and its linear element can be found from formulas (3) and (2):

$$E = 1 + \varphi'^2;$$

$$F = h\varphi';$$

$$G = w^2 + h^2; \tag{6}$$

$$dS^2 = \left(1 + \varphi'^2\right)dw^2 + 2h\varphi' dw d\alpha + \left(w^2 + h^2\right)d\alpha^2. \tag{7}$$

From (7) it is clear that the grid of coordinate lines is not orthogonal since $F \neq 0$. According to the theorem of differential geometry, when a helical surface is bent into a surface of revolution, the helical lines are superimposed on the parallels, and their orthogonal trajectories are superimposed on the meridians. Therefore, it is necessary to switch to a rectangular grid of coordinate lines. One family of these lines is the family of helical lines, and the second is the family of orthogonal trajectories. To find this family of lines perpendicular to the helical lines, it is required to solve the following differential equation

$$Fdw + Gd\alpha = 0. \tag{8}$$

Substituting into equation (8) the values of coefficients F and G from (6), taking into account the fact that the variables are w and α, gives the result

$$\alpha = -h\int \frac{\varphi'}{w^2 + h^2}\mathrm{d}w + t, \tag{9}$$

where t is the constant of integration.

The constant of integration t can be given different numerical values, which will correspond to the corresponding lines on the surface, and which will be perpendicular to the family of helical lines. Therefore, the constant t can be taken as a new independent variable. In this case, the differential $d\alpha$ will be written

$$d\alpha = dt - \frac{h\varphi'}{w^2 + h^2}dw. \tag{10}$$

Substituting the value of $d\alpha$ from (10) into the expression of the linear element (7) after simplifications gives the result

$$dS^2 = \left(1 + \frac{w^2\varphi'^2}{w^2 + h^2}\right)dw^2 + \left(w^2 + h^2\right)dt^2. \tag{11}$$

After such a replacement, the average coefficient is absent, i.e., the grid of coordinate lines is orthogonal. The next task is to reduce the linear element (11) to the form (4), i.e., in the linear element (11) it is necessary to go from the variables w and t to variables α and ρ in such a way that it remains unchanged. To do this, first their right-hand sides are equated at differentials dt^2 and $d\alpha^2$: $\rho^2 = w^2 + h^2$. The result is the following expressions:

$$\rho = \sqrt{w^2 + h^2};$$

$$d\rho = \frac{wdw}{\sqrt{w^2 + h^2}}. \tag{12}$$

Now the left-hand sides are equated taking into account the value of differential $d\rho$ from (12)

$$\left(1 + f'^2\right)\frac{w^2dw^2}{w^2 + h^2} = \left(1 + \frac{w^2\varphi'^2}{w^2 + h^2}\right)dw^2. \tag{13}$$

The solution to (13) relative to φ' has the form

$$\varphi'' = \frac{\sqrt{w^2 f'^2 - h^2}}{w}, \tag{14}$$

where φ' is the desired dependence of the curve of the axial section of the helical surface, f' is the derivative of the explicit equation of the curve (the equation of the meridian of the given surface of revolution).

For a catenoid, the equation of the meridian is a chain line. Its explicit equation and derivative take the following form:

$$f = a\arccos\left(\frac{\rho}{a}\right);$$

$$f' = \frac{a}{\sqrt{\rho^2 - a^2}}. \tag{15}$$

In the derivative (15), it is necessary to go to the variable w according to (12)

$$f' = \frac{a}{\sqrt{\rho^2 - a^2}} = \frac{a}{\sqrt{w^2 + h^2 - a^2}}. \tag{16}$$

Substitution (16) in (14) allows one to write

$$\frac{d\varphi}{dw} = \frac{\sqrt{w^2 f'^2 - h^2}}{w} = \frac{1}{w}\sqrt{\frac{a^2w^2}{w^2 + h^2 - a^2} - h^2}. \tag{17}$$

It is necessary to return in (17) to the variable ρ. According to (12):

$$w = \sqrt{\rho^2 - h^2};$$

$$dw = \frac{\rho d\rho}{\sqrt{\rho^2 - h^2}}. \tag{18}$$

Substitution of expressions (18) in (17) and after simplifications and transformations finally allows one to write

$$\frac{d\varphi}{d\rho} = \frac{\rho^2}{\rho^2 - h^2}\sqrt{\frac{a^2 - h^2}{\rho^2 - a^2}}. \tag{19}$$

Similarly, we can find an expression for the angle α. To do this, substituting (17) into (10) and after simplifications gives the result

$$d\alpha = dt - \frac{h}{w}\sqrt{\frac{a^2 - h^2}{\left(w^2 + h^2 - a^2\right)\left(w^2 + h^2\right)}}dw. \tag{20}$$

It is necessary to go in (20) to the variable ρ. Substitution in (20) of expressions (18) and after simplifications can obtain

$$\frac{d\alpha}{d\rho} = dt - \frac{h}{\rho^2 - h^2}\sqrt{\frac{a^2 - h^2}{\rho^2 - a^2}}. \tag{21}$$

The resulting expressions (18), (19), and (21) are substituted into the equation of the helical surface (5), with expressions (19) and (21) in the form of integrals. The result is the following equations:

$$X=\sqrt{\rho^2-h^2}\cos\left(t-h\int\frac{1}{\rho^2-h^2}\sqrt{\frac{a^2-h^2}{\rho^2-a^2}}\mathrm{d}\rho\right);$$

$$Y=\sqrt{\rho^2-h^2}\sin\left(t-h\int\frac{1}{\rho^2-h^2}\sqrt{\frac{a^2-h^2}{\rho^2-a^2}}\mathrm{d}\rho\right);$$

$$Z=\int\frac{\rho^2}{\rho^2-h^2}\sqrt{\frac{a^2-h^2}{\rho^2-a^2}}\mathrm{d}\rho-h^2\int\frac{1}{\rho^2-h^2}\sqrt{\frac{a^2-h^2}{\rho^2-a^2}}\mathrm{d}\rho. \quad (22)$$

If we take the partial derivatives of equations (22) and find the first quadratic form according to (3) and (4), then it takes the following form

$$dS^2=\left[\frac{\rho^2}{\rho^2-a^2}\right]d\rho^2+\rho^2dt^2. \quad (23)$$

If we substitute the derivative expression from (15) into the first quadratic form (4), it will exactly coincide with (23) except for the variables α and t. However, this is insignificant because the designation of the variable by a different symbol does not change anything. When $\alpha=t$, there will be a complete analogy.

The integrands included in equations (22) can be integrated. After this and after replacing the symbol "t" with the symbol "α", equations (22) take on the final form:

$$X=\sqrt{\rho^2-h^2}\cos\left(\alpha-\operatorname{arctg}\frac{h}{\rho}\sqrt{\frac{\rho^2-a^2}{a^2-h^2}}\right);$$

$$Y=\sqrt{\rho^2-h^2}\sin\left(\alpha-\operatorname{arctg}\frac{h}{\rho}\sqrt{\frac{\rho^2-a^2}{a^2-h^2}}\right);$$

$$Z=\sqrt{a^2-h^2}\ln\left(\rho+\sqrt{\rho^2-a^2}\right)+h\alpha. \quad (24)$$

The first quadratic form (23) does not include the constant h while it affects the shape of the surface. It is a bending parameter. At $h=0$, equations (24) describe the surface of revolution – a catenoid (Fig. 1, a). The constant a is the radius of the smallest parallel. Thus, the variable ρ cannot be less than a. When constructing the surface, it varied within $\rho=0.1...0.2$ m, as can be seen from Fig. 1, a. When the catenoid is deformed into a conoid, the parallels turn into helical lines. This deformation is carried out by a gradual increase in the constant h. At the maximum value of $h=a$, the smallest parallel of radius a is stretched into a straight line – the axis of the helical conoid (Fig. 1, b). The length of this parallel is equal to $2\pi a$; accordingly, this value is equal to the pitch of the helical conoid. For a given maximum value of the parameter ρ (in the considered case, $\rho=0.2$ m) on the catenoid there will correspond a parallel with the value of radius R_c (Fig. 1, a), and on the helical conoid – a helical line on a cylinder of radius R_a (Fig. 1, b). Since at the maximum value $h=a=0.1$ in equations (24) there is a division by zero, the image of the helical conoid (Fig. 1, b) is obtained at $h=0.099$.

For other values of the constant h, any number of intermediate positions can be obtained from the interval $a>h\geq 0$. Some of them are plotted in Fig. 2.

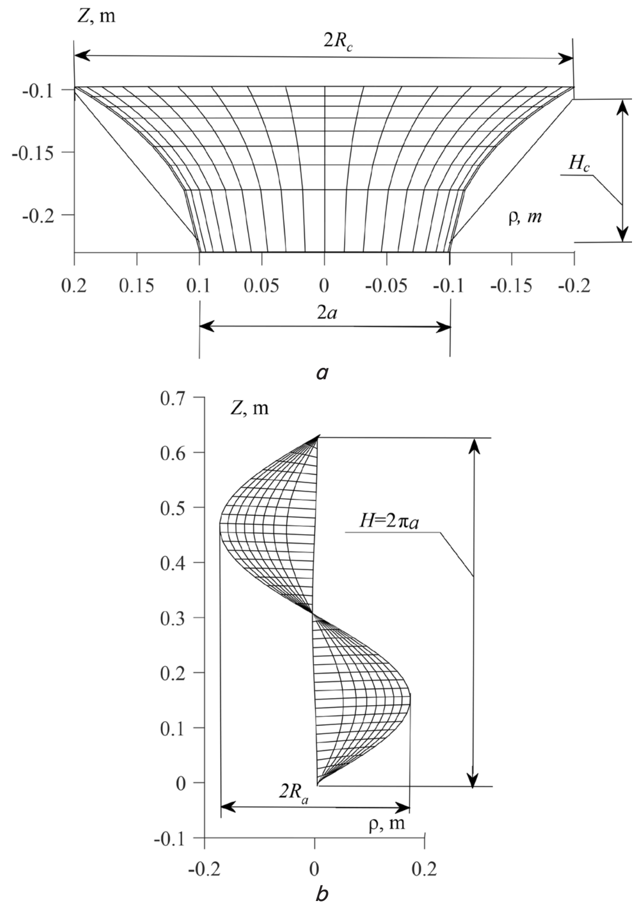

Fig. 1. Frontal projections of the surface described by equations (24) at $a=0.1$ m and different values of h: a – catenoid ($h=0$); b – helical conoid ($h=0.099$)

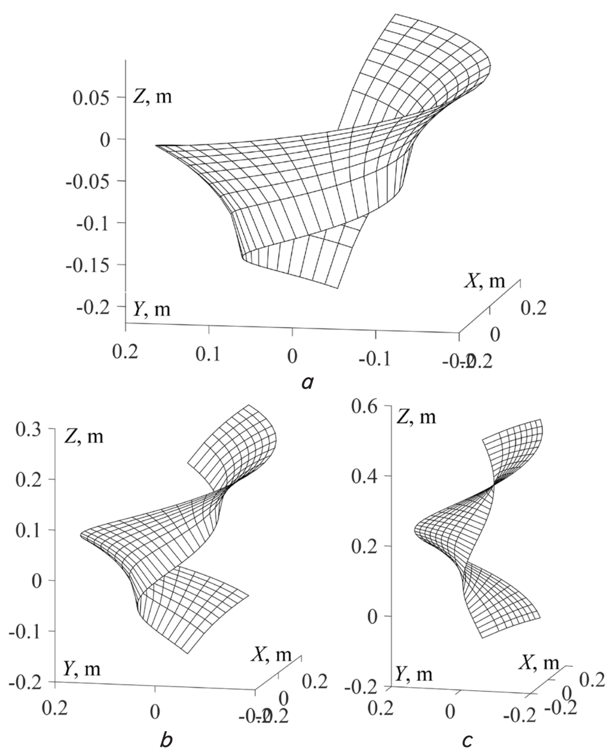

Fig. 2. Axonometric image of intermediate positions of the surface (24) when bending the catenoid into a helical conoid at $a=0.1$ m and different values of h: a – $h=0.03$; b – $h=0.06$; c – $h=0.09$

With such bending, the grid of curvilinear coordinate lines remains orthogonal, one family of which is the helical lines, and the other is the orthogonal trajectories to them. When the catenoid is transformed into a helical conoid (Fig. 1, *a*), one family is transformed into rectilinear generatrices (Fig. 1, *b*).

5. 2. Finding the approximate sweep of the helical conoid turn and recommendations for its deformation into a finished article

If it were necessary to obtain a helical conoid coil shown in Fig. 1, *b*, when the inner edge is the axis of the conoid, then the catenoid shown in Fig. 1, *a* would need to be deformed. Conditionally, it could be replaced by a truncated cone, as shown in this figure. In this case, the exact sweep of the truncated cone would be an approximate sweep of the conoid turn. However, a significant deviation of the catenoid surface from the cone surface indicates significant plastic deformations that will occur when forming a turn from a flat workpiece. It would be possible to first form the catenoid surface and then stretch it into a turn, but this requires special equipment, such as a stamp. Modern machines can forcefully form a flat workpiece into a turn, while performing the necessary plastic deformations. From the visual image of the process of bending a catenoid into a helical conoid, which is carried out on the basis of accurate calculations, certain conclusions and recommendations can be drawn.

In the manufacture of screws, the inner edge of the surface is not a straight line but a helical line, along which the turn is welded to the shaft (Fig. 3).

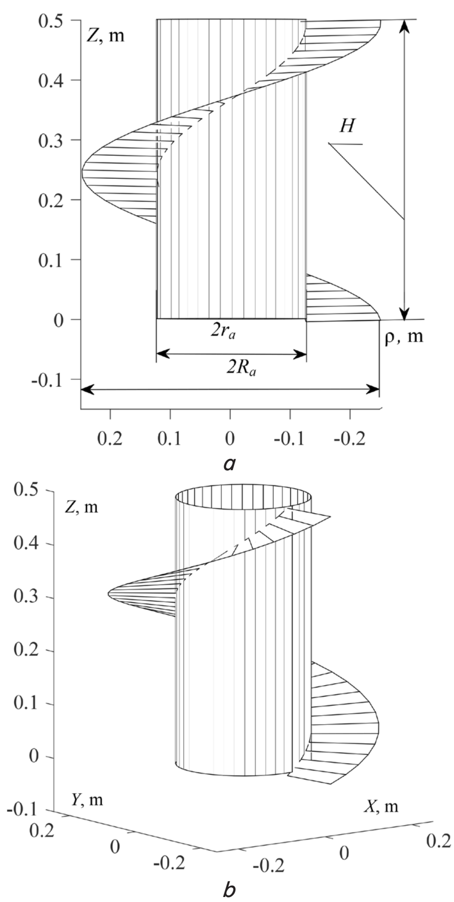

Fig. 3. One turn of the screw:
a — frontal projection with the indicated dimensions;
b — axonometry

In this case, the surface inside the shaft is removed, that is, that part of the catenoid surface that is in its lower part. With such a restriction of the catenoid, the remaining part is much better approximated by a truncated cone. In Fig. 4, the deviation of the cone surface from the catenoid surface is insignificant (shown by the arrow). This means that when forming the cone sweep into the spiral of the helical conoid, the plastic deformations will also be insignificant.

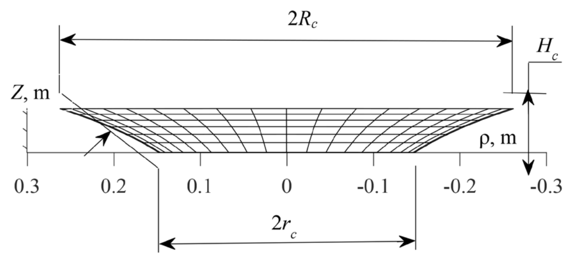

Fig. 4. The part of the catenoid that corresponds to the turn of the helical conoid in Fig. 3

In general, the smaller the difference between the outer radius R_a and the inner radius r_a, the more accurate the approximation of the catenoid section by a truncated cone will be and, accordingly, the smaller the plastic deformations. The approximation will also improve if the difference between the radii R_a and r_a is unchanged, but the radius r_a of the shaft increases.

In small-scale production of screw turns, there is a practice when flat rings welded together are put on the shaft and then stretched using a winch. According to the results obtained, it is not the flat ring that needs to be stretched but the truncated cone formed from it, or, at least, the stretching of the flat ring should be alternated with its twisting around the shaft axis.

It is necessary to find the dimensions of the truncated cone, which approximates the catenoid, according to the given dimensions of the screw turn. Let its pitch $H = 0.5$ m. From here one can find the constant $a = H/(2\pi) = 0.08$ m. The variable ρ is the real value of the radius of a certain parallel of the catenoid. When the helical parameter h increases from zero to a, this parallel turns into a helical line, the radius ρ_a of which decreases. Conversely, it is necessary to determine ρ from the radius ρ_a. The relationship between these quantities is established by the dependence that enters equations (24) as the radius

$$\rho_a = \sqrt{\rho^2 - h^2}. \quad (25)$$

Dependence (25) applies to all surfaces in the bending process. The final result of bending is of interest; that is, a helical conoid, which is obtained at $h = a$. Therefore, in dependence (25) it is necessary to replace h with a and find ρ

$$\rho = \sqrt{\rho_a^2 + a^2}. \quad (26)$$

Let the turn be limited by two helical lines at $r_a = 0.125$ m and $R_a = 0.25$ m (Fig. 3, *a*). Substituting these two values alternately into expression (26) at the previously found value $a = 0.08$ allows us to find the radii of the two bases of the truncated cone: $r_c = 0.148$ m and $R_c = 0.262$ m (Fig. 4). It remains to find the height H_c. It is determined from the last equation (24) at $h = 0$, which corresponds to the surface of the catenoid. H_c will be the difference in heights for the smaller (r_c)

and larger (R_c) bases of the catenoid. The result of the transformations is the expression

$$H_c = a\ln\frac{R_c+\sqrt{R_c^2-a^2}}{r_c+\sqrt{r_c^2-a^2}}. \quad (27)$$

Using the found radii of the bases r_c and R_c from formula (27) we can obtain $H_c = 0.05$ m. Using the found dimensions of the truncated cone, we can determine its exact sweep, which will be approximate for the turn of the helical conoid.

6. Discussion of results based on the theoretical bending of a catenoid into a helical conoid and finding their approximate sweep

Our results of theoretical bending of a catenoid into a helical conoid turn can be transferred to a real process provided that the thickness of the surfaces is not taken into account. However, for a relatively small thickness of the surfaces, the calculations can be quite accurate. They are explained by the fact that when bending a catenoid into a helical conoid, the lengths of the lines and the surface area do not change during the theoretical bending process. To this end, the blank for bending must be a section of the catenoid, which does not have an exact sweep. However, it can be replaced by a truncated cone and this cone gives an idea of how much the section of the catenoid differs from the cone, which is demonstrated in Fig. 4. Owing to this approach, the advantages of finding a blank for deforming it into a helical conoid turn are provided. In [7, 14], various options for calculating a flat workpiece are proposed but they are not related to the bending process.

The derived parametric equations (24) make it possible to construct both a helical conoid turn and the corresponding catenoid section. The reliability of our results is evidenced by the first quadratic form (23). This is confirmation that the catenoid can be deformed into a helical conoid turn by increasing the pitch, that is, by stretching along the axis. In this case, the lengths of the lines and the area of the section as a whole do not change during the deformation process, that is, bending occurs similarly to unfolding surfaces (as an example, bending a sheet of paper into a cylinder). However, the catenoid itself is an unfolding surface, it cannot be formed from a flat sheet. However, it can be replaced by a truncated cone. And this is very important because such a replacement or approximation gives an idea of how much they differ from each other. For example, a comparison of the catenoids in Fig. 1 and Fig. 4 gives a visual picture of such an approximation. The more accurate it is, the less plastic deformations of the metal workpiece in the form of a truncated cone will occur and, accordingly, there will be less resistance to deformation when forming a turn. The resulting formula (26) allows one to find the radii of the smaller and larger bases of the cone by substituting the radii of the screw shaft and the outer edge into it, and formula (27) – the height of the truncated cone. These dimensions are sufficient to find the cone sweep. It is important that these dimensions are obtained on the basis of the theoretically described bending process. Our equations (24) make it possible to assess the accuracy of the approximation of the catenoid by a truncated cone for coils of different pitches and given diameters of the limiting cylinders, which is impossible for alternative existing methods of finding the approximate sweep.

The disadvantage of the study is that the thickness of the sheet material is not taken into account in the described surface bending. There is also a minor caveat. It relates to the fact that the screw turn according to formulas (24) is constructed at $h = a$, however, in this case division by zero occurs. However, when h is reduced by a fraction of a percent, a practically accurate result is obtained, as shown in Fig. 1, *b*.

This study in the future will address bending other surfaces of rotation into helical ones.

7. Conclusions

1. Using differential geometry methods, parametric equations for the gradual bending of the surface of revolution, which is a catenoid, into a helical conoid turn have been derived. Bending is carried out without stretching or compressing the surface by gradually increasing the step from zero to a given value. The final and intermediate surfaces were constructed using our equations, which gives a visual representation of the bending process.

2. Our results have made it possible to estimate the degree of plastic deformations during the formation of an approximate flat workpiece into a helical conoid turn. Their magnitude depends on how much the catenoid section differs from the truncated cone that approximates it. The approximation accuracy increases with distance from the catenoid axis. Calculation formulas for obtaining the dimensions of the approximating truncated cone according to the given parameters of the helical conoid turn have been derived.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study, as well as the results reported in this paper.

Funding

The study was conducted without financial support.

Data availability

All data are available, either in numerical or graphical form, in the main text of the manuscript.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

Acknowledgments

The authors express their gratitude to soldiers in the Armed Forces of Ukraine, whose courage allows scientists to engage in science in Ukraine.

References

1. Tian, F., Xia, K., Wang, J., Song, Z., Yan, Y., Li, F., Wang, F. (2021). Design and experiment of self-propelled straw forage crop harvester. Advances in Mechanical Engineering, 13 (7). https://doi.org/10.1177/16878140211024455
2. Trokhaniak, O. (2022). Estimation of eddy currents and power losses in the rotor of a screw electrothermomechanical converter for additive manufacturing. Machinery & Energetics, 13 (3). https://doi.org/10.31548/machenergy.13(3).2022.92-98
3. Kresan, T., Ahmed, A. K., Pylypaka, S., Volina, T., Voloshko, T. (2024). Construction of the working surfaces of the tillage screw body from the compartments of the developable helicoid. Machinery & Energetics, 15 (3), 9–21. https://doi.org/10.31548/machinery/3.2024.09
4. Klendii, M., Logusch, I., Dragan, A., Tsvartazkii, I., Grabar, A. (2022). Justification and calculation of design and strength parameters of screw loaders. Machinery & Energetics, 13 (4). https://doi.org/10.31548/machenergy.13(4).2022.48-59
5. He, K., Li, G., Du, Y., Tang, Y. (2019). A digital method for calculation the forming cutter profile in machining helical surface. International Journal of Mechanical Sciences, 155, 370–380. https://doi.org/10.1016/j.ijmecsci.2019.03.018
6. Rynkovskaya, M. (2018). Support Draft Calculation for a Ramp in the Form of Developable Helicoid. IOP Conference Series: Materials Science and Engineering, 371, 012041. https://doi.org/10.1088/1757-899x/371/1/012041
7. Lyashuk, O. L., Gypka, A. B., Pundys, Y. I., Gypka, V. V. (2019). Development of design and study of screw working surfaces of auger mechanisms of agricultural machines. Machinery & Energetics, 10 (4), 71–78. Available at: https://technicalscience.com.ua/en/journals/t-10-4-2019/rozrobka-konstruktsiyi-ta-doslidzhyennya-gvintovikh-robochikh-povyerkhon-shnyekovikh-myekhanizmiv-silskogospodarskikh-mashin
8. Pylypaka, S., Hropost, V., Nesvidomin, V., Volina, T., Kalenyk, M., Volokha, M. et al. (2024). Designing a helical knife for a shredding drum using a sweep surface. Engineering Technological Systems, 4 (1 (130)), 37–44. https://doi.org/10.15587/1729-4061.2024.308195
9. Junge, S., Zablodskiy, M., Zaiets, N., Chuenko, R., Kovalchuk, S. (2023). The screw-type electrothermomechanical converter as a source of multiphysical influence on the technological environment. Machinery & Energetics, 14 (3), 34–46. https://doi.org/10.31548/machinery/3.2023.34
10. Mushtruk, M., Gudzenko, M., Palamarchuk, I., Vasyliv, V., Slobodyanyuk, N., Kuts, A. et al. (2020). Mathematical modeling of the oil extrusion process with pre-grinding of raw materials in a twin-screw extruder. Potravinarstvo Slovak Journal of Food Sciences, 14, 937–944. https://doi.org/10.5219/1436
11. Zablodskiy, M., Kovalchuk, S., Gritsyuk, V., Subramanian, P. (2023). Screw electromechanical hydrolyzer for processing poultry by-products. Machinery & Energetics, 14 (1). https://doi.org/10.31548/machinery/1.2023.36
12. Romasevych, Y., Loveikin, V., Malinevsky, O. (2022). The method of calculating the maximum torque when jamming the auger of the screw conveyor. Machinery & Energetics, 13 (2). https://doi.org/10.31548/machenergy.13(2).2022.83-90
13. Trokhaniak, O. (2023). Determination of optimal parameters of hinged operating elements of screw conveyers. Machinery & Energetics, 14 (1). https://doi.org/10.31548/machinery/1.2023.79
14. Nieszporek, T., Gołębski, R., Boral, P. (2017). Shaping the Helical Surface by the Hobbing Method. Procedia Engineering, 177, 49–56. https://doi.org/10.1016/j.proeng.2017.02.181
15. Tarelnyk, V. B., Gaponova, O. P., Konoplianchenko, Ye. V., Martsynkovskyy, V. S., Tarelnyk, N. V., Vasylenko, O. O. (2019). Improvement of Quality of the Surface Electroerosive Alloyed Layers by the Combined Coatings and the Surface Plastic Deformation. III. The Influence of the Main Technological Parameters on Microgeometry, Structure and Properties of Electrolytic Erosion Coatings. Metallofizika I Noveishie Tekhnologii, 41 (3), 313–335. https://doi.org/10.15407/mfint.41.03.0313
16. Tarelnyk, V. B., Gaponova, O. P., Konoplianchenko, Ye. V., Martsynkovskyy, V. S., Tarelnyk, N. V., Vasylenko, O. O. (2019). Improvement of Quality of the Surface Electroerosive Alloyed Layers by the Combined Coatings and the Surface Plastic Deformation. II. The Analysis of a Stressedly-Deformed State of Surface Layer after a Surface Plastic Deformation of Electroerosive Coatings. Metallofizika I Noveishie Tekhnologii, 41 (2), 173–192. https://doi.org/10.15407/mfint.41.02.0173
17. Chvartatskiy, I., Flonts, I., Grabar, A., Shatrov, R. (2021). Synthesis of energy-saving transport-technological systems with screw working bodies. Machinery & Energetics, 12 (4). https://doi.org/10.31548/machenergy2021.04.077
18. Gritsyuk, V., Nevliudov, I., Zablodskiy, M., Subramanian, P. (2022). Estimation of eddy currents and power losses in the rotor of a screw electrothermomechanical converter for additive manufacturing. Machinery & Energetics, 13 (2). https://doi.org/10.31548/machenergy.13(2).2022.41-49
19. Klendiy, M. B., Drahan, A. P. (2021). Substantiation of the design of the working body of the screw section of the combined tillage tool. Perspective technologies and devices, 18, 66–72. https://doi.org/10.36910/6775-2313-5352-2021-18-10
20. Kresan, T. (2021). Movement of soil particles on surface of developable helicoid with horizontal axis of rotation with given angle of attack. Machinery & Energetics, 12 (2). https://doi.org/10.31548/machenergy2021.02.067
21. Pylypaka, S., Kresan, T., Hropost, V., Babka, V., Hryshchenko, I. (2022). Calculation of the bending parameters of a flat workpiece into a twist of a helicoid torso. Machinery & Energetics, 13 (4). https://doi.org/10.31548/machenergy.13(4).2022.81-88