The object of this study is the process of receiving multidimensional signals formed on the basis of high-order phase-difference modulation of the data transmission system.

The development of mobile networks of the next generations is accompanied by increased requirements for the speed, reliability, and noise immunity of information transmission. Existing modulation methods provide an increase in the speed of information transmission by reducing noise immunity and increasing the spectral width of the signal. The general unsolved problem is the lack of an effective method for forming a multidimensional signal of the nth multiplicity and receiving it based on high-order phase-difference modulation, capable of improving the efficiency of these parameters.

The work proposes a method that makes it possible to form a three-dimensional multi-position signal 3D AFM-32, which uses three independent parameters – amplitude, phase, and time. A feature of the result is that noise immunity is ensured without increasing the spectral width of the signal, but due to three-dimensional formatting, which increases the distance between signal points by 50%.

A coherent reception algorithm has been developed that provides accurate signal recovery even in the presence of phase or frequency disturbances. It is shown that the reception efficiency is achieved at an averaging interval of not less than M = 20, at which the system demonstrates an error probability at the level of SER≈10^{-8} at Eb/N0 ≈ 17.4 dB. This makes it possible to obtain an energy gain of 2–3 dB compared to QAM-32 and classical AFM.

The proposed approach is invariant to phase shifts due to the first and second order phase differences, which eliminates ambiguities during reception. 3D AFM-32 demonstrated higher noise immunity compared to QAM-16/32 and AFM-16/32 under the same conditions. The results could be used in 5G/6G networks, in particular in adaptive OFDM systems, autonomous transport, and telemetry

Keywords: phase difference modulation, 32-position, 3D AFM-32, multidimensional, OFDM, noise immunity, coherent algorithm, energy efficiency, 5G/6G

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DEVISING A METHOD FOR RECEIVING A MULTIDIMENSIONAL SIGNAL USING HIGH-ORDER PHASE DIFFERENCE MODULATION IN NEXT-GENERATION MOBILE NETWORKS

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1. Introduction

The development of modern telecommunications technologies is rapidly moving towards the next generations of mobile networks, such as 5G and 6G, which imposes fundamentally new requirements on the quality, speed, and noise immunity of information transmission. Under the conditions of the mass distribution of devices that use large volumes of multimedia data, as well as the widespread implementation of the Internet of Things, the load on radio channels is increasing, which requires the implementation of highly efficient methods of transmitting and receiving signals.

One of the current scientific problems is the development of new methods for receiving multidimensional signals using high-order phase-difference modulation. Scientific research into this area is important due to its potential to provide increased resistance of signals to noise and interference, without compromising the spectral width of the signal. This is especially important for modern conditions, where the spectral resource is limited, and the requirements for the reliability and accuracy of signal transmission are constantly increasing.

The relevance of this topic is emphasized by the fact that classical modulation methods, such as QAM, demonstrate significant limitations when increasing the modulation order, in particular, reducing noise immunity. Therefore, the results of research aimed at devising and improving new methods of multidimensional phase-difference modulation are extremely necessary for practice. They can enable effective information transmission under conditions of high interference, characteristic of new generation mobile networks, in particular in the field of autonomous transport, telemetry, adaptive OFDM systems, and other promising areas of modern telecommunication technologies.

2. Literature review and problem statement

Work [1] reports the results of experimental measurements in the indoor environment for the frequency ranges of 0.6–1.9 GHz and 100–300 GHz, which are promising for 5G and 6G systems. However, from the point of view of multidimensional signal reception, the work does not present an analysis of the influence of communication channels on the accuracy of receiving a modulated signal. In particular, it does not examine how delays, phase shifts, and multipath effects affect the probability of error when using phase-difference schemes. There is no data on the interaction of signal propagation parameters with demodulation algorithms, which limits the possibility of using the obtained models for calculating the receiver under interference and unstable phase conditions. The reason for these limitations is the objective focus of the study on the physical transmission level, in particular, on the parameters of the environment, without involving digital signal processing problems.

Unlike the previous work, in [2], a thorough analysis of communication channel models and signal propagation conditions in fifth-generation networks was carried out. However, the paper does not investigate the reception of complex modulated signals. And no mathematical or algorithmic approaches are presented to reduce the probability of error or compensate for channels with high delay/dispersion, which is critically important for the reception of multidimensional signals. This limits the application of the research results in the tasks of ensuring interference-resistant communication at the physical level. The main reason is the focus of the work on macro-level aspects of the functioning of 5G networks, without a deep dive into the issues of digital signal processing and optimization of receiving circuits.

In [3], a number of OFDM multiplexing formats were analyzed, which were considered in the context of fifth-generation networks. The paper does not address the issue of coherent reception of multidimensional signals with high-order phase-difference modulation. The issue of invariance to frequency perturbations or phase jumps is not considered from the point of view of signal configuration formation in three-dimensional space, which is critically important for mobile networks with high dynamics. The main reason is the focus of the study on system-wide modulation formats approved by 3GPP, without detailing the reception algorithms or analyzing complex phase formats.

Work [4] considers the problem of synchronization in 5G under conditions of low signal-to-noise ratio. However, it leaves unresolved the applied issues related to the processing of high-order multidimensional signals in the context of resistance to phase shifts and the influence of interference on the probability of error. This requires further research, in particular in the context of coherent reception. The reason for this limitation is that the research was conducted without detailing the algorithms for receiving signals that go beyond the scope of classical modulations.

Despite the significant contribution to the formation of classical OFDM systems, study [5] leaves important questions open. A mathematical model for estimating interchannel interference in data transmission systems using OFDM signals was proposed, and the dependence of the interchannel interference magnitude on the parameters of the guard interval, the number of subcarriers, and the number of previous receptions was established. However, the problems of receiving a multidimensional signal remain unresolved. The main reason for this is that the model built does not cover the influence of phase distortions, which are characteristic of systems with high spectral efficiency and a complex multipath channel structure.

In [6], second-order phase-difference modulation is considered. However, despite the detailed study of reception algorithms, the paper does not consider the issue of multidimensional signal reception. In particular, in cases where signals are transmitted in a complex multichannel environment with a high level of interference and phase fluctuations. Also, criteria for minimizing the probability of error for complex phase spaces of signal states, which is relevant for 5G and 6G systems, are not presented. The reasons for this are the objective limitations of the study on classical two- and four-phase schemes, as well as the emphasis on autocorrelation demodulators with reduced implementation complexity.

As is known, the maximum possible speed is ensured by increasing the number of signal positions and reducing the duration of the packet.

It should be noted that increasing the signal positions leads to a decrease in noise immunity.

In the process of implementing the latest generation of mobile network technologies, it is necessary to create ultra-dense networks characterized by maximum information transfer speed, minimal delay, high reliability, and resistance to random signal disturbances.

Mobile networks with such properties enable the provision of high-quality broadband services.

Scientific sources, in particular works [7, 8] consider in detail methods for increasing the data transfer speed, which is achieved by increasing the signal modulation rate.

However, despite the theoretical novelty, work [7] does not investigate the issue of coherent reception of multidimensional signals. This is especially true for cases with high-order phase-difference modulations. This limits the practical application of the presented solutions in real mobile environments with unstable phase and frequency shifts. The main reason for this is the focus of the research on the generation of index structures and bandwidth analysis, without focusing on the mathematical aspects of reception and minimizing the probability of error in phase-sensitive environments. In [8], modern modulation methods are reported, among which special attention is paid to increasing spectral efficiency. This is achieved by using high-order modulation schemes (64-QAM, 256-QAM) in combination with pulse shaping and subband filtering methods. However, the issue of implementing such approaches in three-dimensional signals is not considered. No analytical models or criteria for receiving such signals are given, in particular from the point of view of minimizing the probability of error or compensating for phase fluctuations. The reasons for this are the objective focus of the study on macro-level parameters of modulation formats for 5G, as well as the subjective limitation of the selected topic without a detailed analysis of the receiving path for complex phase schemes.

In turn, studies [9, 10] proposed new approaches to improving the noise immunity of transmission systems, which are implemented through the use of multidimensional signals. These methods make it possible to achieve greater signal resistance to external interference while ensuring more efficient information transmission under difficult communication channel conditions.

In [9], the study was conducted on the basis of ideal coherent reception. No formal theoretical justification of the proposed metric is given – neither from the point of view of information theory, nor in comparison with ML/MAP solutions. Multipath effects, time dispersion, or receiver mobility are not taken into account, which reduces the practical significance of the results for the conditions of modern mobile communications. There are no analytical solutions to the criteria for minimizing the probability of error, as well as adaptive reception algorithms for receiving a multidimensional signal using high-order phase-difference modulation. The reason for this is the focus of the study on two-dimensional polar structures in the signal space without generalization to complex multidimensional and dynamic models taking into account temporal or spatial changes, as well as the limitations of the analytical apparatus regarding the combined effect of phase and multiplicative interference in the channel.

In [10], promising methods based on index modulation are implemented, which cover several domains (frequency, time, spatial, phase) in order to increase the efficiency of information transmission. The proposed methods demonstrate high spectral efficiency and the ability to adapt to channel conditions. At the same time, the work does not solve the issue of forming a multidimensional n-th multiplicity signal and receiving based on high-order phase-difference modulation, in particular in conditions of phase shifts and noise. No analysis of the probability of reception error is given, and no algorithms are presented that make it possible to implement noise-resistant reception. The reason for this is the authors' focus on the transmitter structure and modulation architecture, without delving into the complex problems of phase-resistant reception in a multidimensional signal space.

Our review of the literature [1–10] reveals the limitations of known methods of receiving multidimensional signals in mobile networks of the next generations. Thus, the general unsolved problem is the lack of an effective method of forming a multidimensional signal of n-th multiplicity and receiving it based on high-order phase difference modulation (the information is the phase difference between the packets, which ensures invariance to phase jumps). Such a method should provide high noise immunity by increasing the distance between the signal points without expanding the spectral width of the signal.

3. The aim and objectives of the study

The purpose of our study is to devise a method for generating a multidimensional n-fold signal and receiving it based on high-order phase-difference modulation. This allows for significant improvement in noise immunity characteristics in next-generation data transmission systems without increasing the spectral width of the signal.

To achieve this goal, it is necessary to solve the following problems:

– to synthesize a multidimensional signal formed using three information parameters – amplitude, phase, and time of n-fold multiplicity, in order to ensure an increase in the distance between signal points and reduce the probability of signal transmission errors without expanding the spectral width of the signal based on the minimization criterion. The components are information parameters – amplitude, phase, and 32-position phase-difference modulation;

– to develop an algorithm for receiving a multidimensional signal using high-order phase-difference modulation;

– to assess the noise immunity of the method for receiving a multidimensional signal using high-order phase-difference modulation, which would make it possible to obtain accurate indicators of its reliability under conditions of various noise and interference influences.

4. The study materials and methods

The object of our study is the process of receiving multidimensional signals formed on the basis of high-order phase-difference modulation of the data transmission system.

The principal hypothesis of the study assumes that the use of high-order phase-difference modulation could significantly reduce the probability of signal reception errors. Thus, it is possible to ensure increased efficiency and reliability of information transmission in channels with a limited signal-to-noise ratio.

The following conditions and simplifications were adopted in the study:

– modeling was performed with the assumption of the presence of additive white Gaussian noise;

– the study was limited to the most common orders of phase-difference modulation;

– idealized signal propagation conditions were considered, which make it possible to isolate the influence of modulation characteristics.

The theoretical basis of the study was the methods of invariance theory, information theory, optimization theory, and potential noise immunity theory. These methods make it possible to develop a signal reception algorithm, determine the theoretical limits of modulation efficiency, and establish the relationship between the energy parameters of signals and the probability of error.

When forming a three-dimensional signal, a minimization criterion was used, which makes it possible to ensure the efficiency of symbol recognition in the presence of interference.

To experimentally confirm the proposed theoretical propositions, the simulation modeling method was used. The experimental part of the study was implemented using the MATLAB mathematical modeling package (MathWorks, USA), which made it possible to conduct a series of simulation experiments and construct the corresponding graphical dependences.

The experimental data processing procedure included a series of simulation experiments, during which the process of transmitting signals with high-order phase-difference modulation in a channel with additive white Gaussian noise was simulated. The experiments were carried out at different values of the averaging interval ($M = 1$, 5, 10, 20, 100) to determine the influence of this parameter on the accuracy of signal reception. The error probability was determined statistically based on independent implementations of experiments for each value of the signal/noise ratio.

The reliability of the results obtained was assessed by calculating confidence intervals with a significance level of 0.9, which made it possible to determine the required sample size to achieve the specified accuracy indicators. Plots of the dependence of error probability on the signal/noise ratio for different modeling conditions were constructed. A comparison was made with known modulation schemes (QAM-16, QAM-32, AFM-16, AFM-32), which made it possible to clearly demonstrate the advantages of the proposed method.

The key indicators of the effectiveness of the devised method were determined as the probability of error in signal reception, the reliability of information transmission, and the energy characteristics of signals depending on the level of interference in the communication channel. It was these indicators that allowed for a comprehensive assessment of the effectiveness of the proposed methods and made it possible to draw conclusions about the feasibility of their practical application.

5. Results of devising a method for forming a multidimensional signal of *n*-th multiplicity and receiving it based on high-order phase-difference modulation

5. 1. Synthesis of a three-dimensional signal with thirty-two-position high-order phase-difference modulation

In the context of increasing requirements for noise immunity and data transmission efficiency in mobile networks of the next generations, there is a need to synthesize a new type of multidimensional signal. The results of such synthesis should provide invariance to phase and frequency disturbances without expanding the signal spectrum. To increase the speed, it is necessary to increase the modulation multiplicity, but at the same time the noise immunity decreases. The task is to devise a method for forming a three-dimensional signal using high-order phase-difference modulation, where information is encoded through three independent parameters – amplitude, phase, and time. It is necessary to ensure an increase in the distance between signal points in three-dimensional space in order to increase the equivalent signal energy and achieve maximum noise immunity when using 32-position modulation. The synthesis of a multidimensional (three-dimensional) signal with thirty-two-position phase-difference modulation of high orders is proposed.

In modern mobile networks, two-dimensional 32-position OFDM signals with phase-difference modulation are used. In these signals, the information parameters are amplitude and phase, and the phase difference is determined between two packets. An example of such signals is shown in Fig. 1.

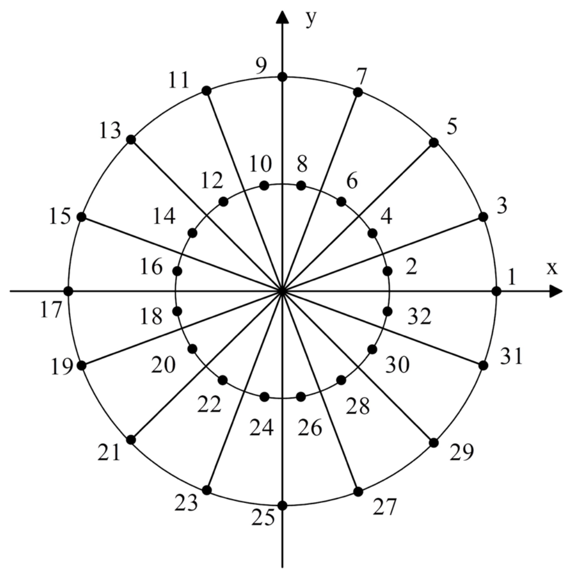

Fig. 1. 32-bit phase-difference modulation (PDM) signal

The signal points are placed on the two circles in such a way as to provide maximum equivalent energy.

The concept of the order of signal phase differences is considered as follows: suppose there are a series of signals transmitted through a harmonic FM with different initial phases

$$\phi_0, \phi_1, \phi_2 \ldots \phi_{n-1}, \phi, \phi_{n+1}. \tag{1}$$

To conduct the analysis, we form phase differences between each pair of neighboring signals, which makes it possible to identify patterns and characteristics of changes in phase differences during signal transmission. This makes it possible to assess signal stability, determine the impact of interference on the phase characteristic, and ensure more accurate synchronization in information transmission systems

$$\begin{cases} \Delta_1^1\phi = \phi_1 - \phi_0, \\ \Delta_2^1\phi = \phi_2 - \phi_1, \\ \Delta_n^1\phi = \phi_n - \phi_{n-1}, \\ \Delta_{n+1}^1\phi = \phi_{n+1} - \phi_n. \end{cases} \tag{2}$$

The phase differences specified in expression (2) are first-order differences since they are obtained by subtracting the phases of adjacent packets from the original sequence (1) once. This is explained by the index 1 in the notation of the difference operator Δ. The expression $\Delta_n^1\phi$ denotes the first-order phase difference between n and $(n-1)$ packets of the signal. Like the original phases, the sequence of these differences changes in time during signal transmission, reflecting their dynamic nature

$$\Delta_0^1\phi, \Delta_1^1\phi, \Delta_2^1\phi, \ldots, \Delta_{n-1}^1\phi, \Delta_n^1\phi, \Delta_{n+1}^1\phi. \tag{3}$$

Using this sequence of numbers, new differences can be formed by applying the same rule that was used to obtain differences (2) from the original phase sequence (1)

$$\begin{cases} \Delta_1^2\phi = \Delta_1^1\phi - \Delta_0^1\phi, \\ \Delta_2^2\phi = \Delta_2^1\phi - \Delta_1^1\phi, \\ \Delta_n^2\phi = \Delta_n^1\phi - \Delta_{n-1}^1\phi, \\ \Delta_{n+1}^2\phi = \Delta_{n+1}\phi - \Delta_n^1\phi. \end{cases} \tag{4}$$

The phase differences given in expression (4) are second-order differences since they are calculated from the original phase sequence (1) by double subtraction. This means that each such difference is the result of applying the phase subtraction operation of adjacent packets twice, which makes it possible to derive a more complex relationship between the signal phases. This process is reflected through index 2 in the notation of the difference calculation operator *Δ*. The sequence of such second-order phase differences is a time sequence similar to sequences (1) and (3), which changes during signal transmission

$$\Delta_0^2\phi, \Delta_1^2\phi, \Delta_2^2\phi, \ldots, \Delta_{n-1}^2\phi, \Delta_n^2\phi, \Delta_{n+1}^2\phi. \tag{5}$$

The process of calculating phase differences can be continued for the third and subsequent orders, which makes it possible to obtain more complex dependences between the signal phases. Each subsequent phase difference is defined as the difference between the two previous phase differences, which adds another level of abstraction and makes it possible to more accurately model the behavior of the signal under transmission conditions through channels with interference or noise. This approach can be used for a deeper analysis of synchronization and noise immunity of signals. All stages of the formation of phase differences of higher orders are illustrated in Table 1, where each element of the table is defined as the difference between two neighboring elements from the row above [11].

Table 1

Formation of high-order phase differences

Order k	$\Delta_1^k\phi$	$\Delta_2^k\phi$	$\Delta_3^k\phi$	$\Delta_4^k\phi$	$\Delta_5^k\phi$	$\Delta_6^k\phi$
$k=1$	$\varphi_1-\varphi_0$	$\varphi_2-\varphi_1$	$\varphi_3-\varphi_2$	$\varphi_4-\varphi_3$	$\varphi_5-\varphi_4$	$\varphi_6-\varphi_5$
$k=2$	$\Delta_2^1\phi-\Delta_1^1\phi$	$\Delta_3^1\phi-\Delta_2^1\phi$	$\Delta_4^1\phi-\Delta_3^1\phi$	$\Delta_5^1\phi-\Delta_4^1\phi$	$\Delta_6^1\phi-\Delta_5^1\phi$	–
$k=3$	$\Delta_3^2\phi-\Delta_2^2\phi$	$\Delta_4^2\phi-\Delta_3^2\phi$	$\Delta_5^2\phi-\Delta_4^2\phi$	$\Delta_6^2\phi-\Delta_5^2\phi$	–	–
$k=4$	$\Delta_4^3\phi-\Delta_3^3\phi$	$\Delta_5^3\phi-\Delta_4^3\phi$	$\Delta_6^3\phi-\Delta_5^3\phi$	–	–	–
$k=5$	$\Delta_5^4\phi-\Delta_4^4\phi$	$\Delta_6^4\phi-\Delta_5^4\phi$	–	–	–	–
$k=6$	$\Delta_6^5\phi-\Delta_5^5\phi$	–	–	–	–	–

First-order phase-difference modulation (FDM-1) is a method of forming a phase-modulated signal in which information is transmitted through first-order phase differences between the initial phases of adjacent packets. In this method, the information parameter is the phase difference between two consecutive signal packets, which allows information to be modulated by changing the phase between them, thereby providing the necessary variability for data transmission in communication systems

$$\Delta_n^1\phi=\phi_n-\phi_{n-1}, \tag{6}$$

which means that the initial phase of each subsequent n-th packet entering the communication channel is determined using the first-order phase differences calculated between the previous packets. That is, the initial phase of the n-th packet is calculated based on the phase of the previous packet and the value of the phase difference between them, which ensures the correct recovery of the transmitted signal and maintenance of the phase structure during transmission through the communication channel

$$\phi_n=\phi_{n-1}+\Delta_n^1\phi. \tag{7}$$

Expressions (6), (7) describe the general algorithm for forming and processing a phase-modulated signal in the FRM-1 system, which is illustrated by the diagram in Fig. 2, *a*. On the transmitting side, each discrete symbol J_n corresponds to a certain value of the first-order phase shift$\Delta_n^1\phi$. Next, using a delay device for one packet and a summing block, according to formula (7), the initial phase of the next n-th packet transmitted through the channel in the period T_S is determined. On the receiver side, after fixing the initial phases of two consecutive packets, using a delay element and a subtraction device, the phase difference is calculated $\Delta_n^1\phi$, which is then identified as the transmitted symbol.

Second-order phase-difference modulation (2nd-order phase-difference modulation) is a method of forming a phase-modulated signal when information is transmitted through the values of the differences in the initial phases of the second order. In this method, the information parameter is the second-order phase difference, which is calculated from three signal packets. Owing to this approach, the system becomes insensitive to changes in the initial phase and frequency shifts, since for 2nd-order phase-difference modulation only the difference between the phases is important, and not their absolute values. This makes it possible to significantly increase the noise immunity of the system since changes in the initial phase and frequency shifts do not affect the transmission of information

$$\begin{aligned}\Delta_n^2\phi&=\Delta_n^1\phi-\Delta_{n-1}^1\phi=\left(\phi_n-\phi_{n-1}\right)-\left(\phi_{n-1}-\phi_{n-2}\right)=\\&=\phi_n-2\phi_{n-1}+\phi_{n-2}.\end{aligned} \tag{8}$$

From equation (8) it follows that the initial phase of the next n-th signal packet transmitted through the communication channel is determined by the phase difference between the two previous packets. This expression is the result of using the first or second order phase difference modulation algorithm, depending on the specific modification of the system, where the initial phase of the next packet depends on the values of the previous phases and information parameters. This approach makes it possible to effectively transmit information through the signal phase, minimizing the impact of noise and external interference

$$\phi_n=\Delta_n^2\phi+2\phi_{n-1}-\phi_{n-2}. \tag{9}$$

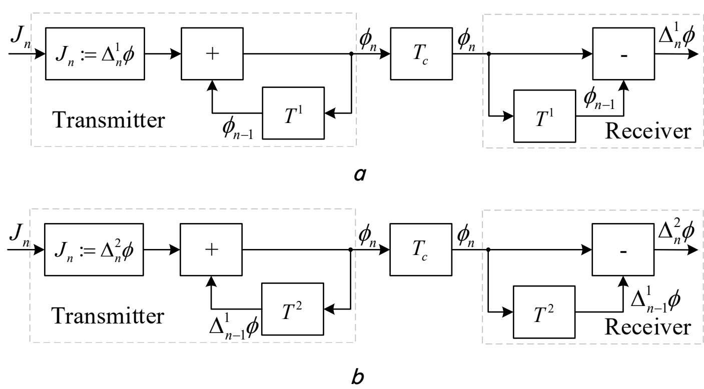

Fig. 2. Block diagram of the formation and processing of a phase-modulated signal: *a* — first order; *b* — second order

The initial phase of the next n-th packet can be represented in a similar form as in equation (7), using two recurrence relations

$$\left\{\begin{aligned}\phi_n&=\phi_{n-1}+\Delta_n^1\phi;\\\Delta_n^1\phi&=\Delta_{n-1}^1\phi+\Delta_n^2\phi\end{aligned}\right\}. \tag{10}$$

The device for forming the initial phases of signal packets at FDM-2, implementing algorithm (10), includes two series-connected phase shapers according to FDM-1. Each of these shapers is responsible for generating the initial phase for its corresponding packet, using the previous phase and the first-order phase difference. The first shaper calculates the initial phase for the first packet based on the initial phase and the phase difference, while the second shaper uses the calculated phase of the previous packet to determine the phase of the next one. Such a structure allows for modulation with high-order phase differences, increasing the noise immunity and efficiency of information transmission in systems with FDM-2.

The system operation algorithm assumes that at each stage of signal formation, the calculation of phase differences occurs, which are connected to the previous phases through the corresponding operations. This makes it possible to take into account both the initial phases and their changes during the transmission process.

The signal processing device consists of two sequential first-order phase difference calculators (Fig. 2, *b*). This makes it possible to correctly calculate the difference between the phases of the previous and current packets, using predefined initial phases to obtain second-order differences.

Such a processing structure allows for coherent reception of signals with second-order phase-difference modulation, ensuring accuracy in restoring the transmitted signal.

Invariance is one of the main properties of FDM-1 and FDM-2, which means the ability of the system to maintain stability under the influence of external factors, such as noise or changes in the parameters of the communication channel. This suggests that certain characteristics of the system remain unchanged even when changes occur under external conditions, for example, in the initial phase or frequency of the signal.

In the case of FDM-1, invariance consists in preserving the phase differences between neighboring packets, despite possible changes in the initial phase of the signal. This allows for efficient information transmission even if the initial phase of the signal is unknown at the receiving end.

For FDM-2, invariance extends not only to the initial phase, but also to frequency shifts of the carrier signal. This allows the system to remain insensitive to frequency changes, which makes it even more resistant to interference [12, 13], maintaining reliability and transmission accuracy even under conditions of frequency fluctuations.

Due to these properties, FDM-1 and FDM-2 operate effectively under complex and variable communication channel conditions, which makes them ideal for use in systems with high requirements for noise immunity.

The FDM-1 avoids ambiguity at the demodulator output that may arise due to uncertainty in the initial phase of the received signal. This is achieved by keeping the first-order phase difference invariant during the transformation, which adds an arbitrary common initial phase to the signal phases corresponding to the $(n-1)$-th n-th packets. For example, if an arbitrary unknown value φ_{Π} is added to the phases $\varphi_{\Pi-1}$ and φ_{Π}, then the phase difference between these $(n-1)$-th and n-th packets will remain unchanged from such a transformation

$$\Delta_n \phi = \left(\phi_n + \phi_0\right) - \left(\phi_{n-1} + \phi_0\right) = \phi_n - \phi_{n-1} = in\,\mathrm{var}\,\phi_0. \tag{11}$$

Thus, the transformation that adds the same fixed phase to each packet does not change the first-order phase difference, which allows for accurate identification of information even in the case of uncertainty in the initial phase of the signal.

FDM-1 has the important property of invariance to the initial phase of the signal. One of the main advantages of this method is that the invariance is preserved regardless of the technique for receiving the signal. For incoherent and autocorrelation reception, information about the initial phase is not needed at all, while coherent reception requires only knowledge of it with a certain accuracy that depends on the modulation rate. For example, for single-shot FDM 1, an accuracy of within 180° is sufficient, which is easily achieved. The implementation of modems using FDM-1 and providing invariance to the initial phase is relatively simple, as demonstrated earlier. Incoherent demodulators for signals with FDM-1 can be considered absolutely invariant since they are not sensitive to the initial phase of the signal. Unlike absolute phase modulation, which requires accurate information about the initial phase, FDM-2 allows for efficient reception of signals even in the absence of this information. In this context, FDM-1 is less versatile, since its invariance is limited only to the initial phase, while FDM-2 offers additional resistance to changes in the carrier frequency. This makes FDM-2 more reliable for use under conditions of frequency instability in the communication channel, providing a high level of noise immunity and accuracy of information transmission.

FDM-2 signals, like FDM-1, can be received by various methods, such as coherent, incoherent, or autocorrelation. However, the frequency invariance property is realized only in autocorrelation reception, since coherent and incoherent methods require an exact carrier frequency for correct operation. One of the key advantages of FDM-2 is its higher noise immunity compared to FDM-1 during incoherent reception. In addition, during coherent reception at a known frequency, the noise immunity of FDM-2 is equivalent to FDM-1, which makes it more stable and versatile under various communication conditions.

The algorithm for receiving a multidimensional signal with a high-order phase difference takes into account the guard interval duration parameter, which adds a time dimension and converts the signal into a three-dimensional one. The location of the 32-position signal in multidimensional space doubles the distance between the signal points, which in equivalent form corresponds to an increase in the efficiency of signal transmission under noise conditions.

For the general case of digital transmission with m-position signals of arbitrary amplitudes a_1, a_2, ..., a_m and initial phases φ_1, φ_2, ..., φ_m, the phase difference between the three packets is determined by the differential coding function. The information parameter is precisely this phase difference. The reception of the three-dimensional signal $Si(t)$ is carried out by selecting the i-th signal variant if for all $j \neq i$ the following inequality holds

$$\int_0^T \left[x(t) - S_i(t)\right]^2 \mathrm{d}t < \int_0^T x(t) - S_i(t)\,\mathrm{d}t, \tag{12}$$

where $x(t)$ is the received signal, T is the duration of the integration interval.

Note that for each signal variant, the value of the guard interval in time Δt_1 or Δt_2 is taken into account. Thus, a third parameter is introduced, which complements the other two: the amplitude of the signal variant and the phase.

Thus

$$i = \arg\min \int_0^T \left[x(t) - S_i(t)\right]^2 \mathrm{d}t. \tag{13}$$

For further calculations, the third parameter z is introduced into the reception algorithm, which is determined by changing the guard interval time. Let us consider the system in the three-dimensional space of a multi-position signal. The coordinates of the thirty-two-position signal are determined through the amplitude z, which in three-dimensional space corresponds to the distance from the initial reference point to a specific signal position. In such a coordinate system, the full hemispherical angle corresponds to the angle between the z axis and the vector directed from the origin to the signal point P, and the same range describes the angle between the x axis and this same vector, which lies on the xy plane and connects the origin with the point P. Thus, a 32-position signal is formed in three-dimensional space. To further determine the algorithm for receiving a multi-dimensional 32-position signal, a transition is made from spherical coordinates to Cartesian coordinates.

Then

$$\begin{Bmatrix} x_{0i} = r_{0i}\cos\varphi_0 \sin\theta_i \\ y_{0i} = r_{0i}\sin\varphi_0 \sin\theta_i \\ z_{0i} = r_{0i}\cos\varphi_0 \mathrm{tg}\theta_i \end{Bmatrix}. \quad (14)$$

Then the acceptance algorithm will be determined by the amplitude

$$i = \arg\min \int_0^T \left[\left(\left(x_0 - x_j\right) + \left(y_0 - y_j\right) + \left(z_0 - z_j\right) \right)^2 \right] \mathrm{d}t. \quad (15)$$

The input values x_0, y_0, z_0 are determined by processing the current packet. Let the coordinates of the received signal on the nth packet be denoted as x_0, y_0, z_0.

A plot is constructed with 32 evenly spaced signal points in a spherical coordinate system (Fig. 3). This corresponds to a 32-position multidimensional signal in modulation systems.

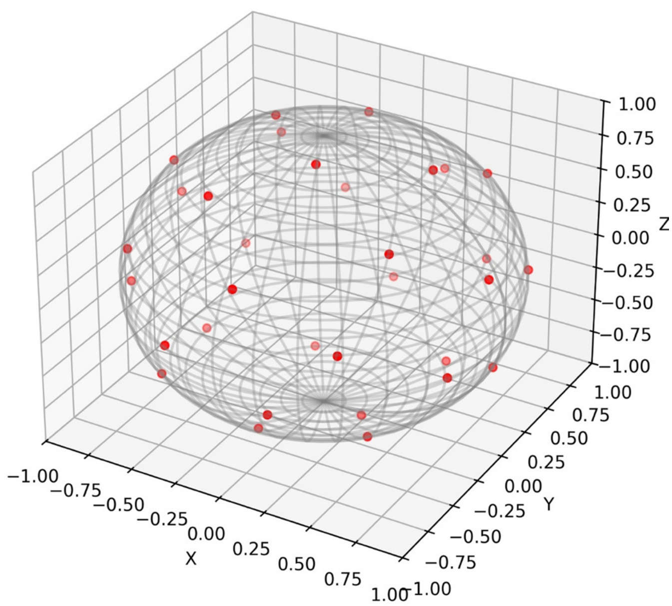

Fig. 3. Arrangement of 32 signal points in a spherical coordinate system

To ensure the calculation of projections of all variants of a multi-position three-dimensional signal at the reception, it is necessary to synchronize the variants of the received signal between transmission and reception. To determine the estimates of projections of a multi-position signal at the reception x_j, y_j, z_j, the method of reduction and averaging of projections of the received signal is used.

The first variant of the signal x_j, y_j, z_j is chosen as the averaged values, to which other variants of the received signal will be converted during its correction in accordance with the information message. In the case when the signal $x(t)$, received within N parcels, includes the component $S_i(t)$ superimposed on Gaussian noise, then the plausible estimates of these quantities x_j, y_j, z_j are determined as follows

$$\begin{Bmatrix} \tilde{x}_1 = \dfrac{1}{N}\sum\limits_{n=1}^{N} x_{0n}, \\ \tilde{y}_1 = \dfrac{1}{N}\sum\limits_{n=1}^{N} y_{0n}, \\ \tilde{z}_1 = \dfrac{1}{N}\sum\limits_{n=1}^{N} z_{0n}, \end{Bmatrix}, \quad (16)$$

where x_{on}, y_{on}, z_{on} are determined by the values of projections on the interval of the nth packet, and the tilde above x_j, y_j, z_j indicates that these are estimates. The estimates determined from formula (16) are unbiased and effective.

To form the projections necessary for constructing algorithm (15), effective and unbiased estimates were used, which ensure the accuracy and correspondence of the signal parameters. In this context, a notation was introduced that makes it possible to create projections for any signal variants that are part of the algorithm, as follows

$$\begin{aligned} x_j &= \int_0^T a_j \,\mathrm{in}\left(wt + \phi_j\right) a_0 \sin\left(wt + \phi_0\right) \mathrm{d}t = \\ &= \frac{a_j}{a_1}\int_0^T a_1 \sin\left(wt + \phi_1 + \Delta\phi_j\right) a_0 \sin\left(wt + \phi_0\right) \mathrm{d}t = \\ &= \frac{a_j}{a_1}\left[\begin{aligned} &\cos\Delta\phi_j \int_0^T a_1 \sin\left(wt + \phi_1\right) Q_0 \sin\left(wt + \phi_0\right) \mathrm{d}t + \\ &+ \sin\Delta\phi_j \int_0^T Q_1 \sin\left(wt + \phi_1\right) a_0 \sin\left(wt + \phi_0\right) \mathrm{d}t \end{aligned} \right] = \\ &= \frac{a_j}{a_1}\left[\begin{aligned} &\cos\Delta\phi_j \int_0^T a_1 \sin\left(wt + \phi_1\right) a_0 \sin\left(wt + \phi_0\right) \mathrm{d}t - \\ &- \sin\Delta\phi_j \int_0^T a_1 \sin\left(wt + \phi_1\right) a_0 \cos\left(wt + \phi_0\right) \mathrm{d}t \end{aligned} \right] = \\ &= \frac{a_j}{a_1}\left(x_1 \cos\Delta\phi_j - y_1 \sin\Delta\phi_j\right). \end{aligned} \quad (17)$$

Similarly, the projections y_j and z_j are determined.

To ensure coherent reception of a multi-position AFM signal, a special synchronization signal is used. This synchronization signal precedes the transmission of information packets and serves to determine the projections onto a reference oscillation with an arbitrary initial phase. According to formula (16), estimates of the projections of the first signal variant $\tilde{x}_1$, $\tilde{y}_1$, $\tilde{z}_1$, are calculated, and according to formula (17), estimates for all m signal variants are calculated. Then, the obtained values are substituted into algorithm (15). This provides systematization of information packets in OFDM systems, allowing for efficient processing of multi-channel signals and ensuring high accuracy of their decoding. Such an approach is important for achieving high noise immunity under real communication conditions.

Due to this, the algorithm makes it possible to precisely adjust the initial phase of the signal, which is critically important for coherent signal processing [14]. Thus, the relationship and algorithm (15) form a comprehensive approach to coherent processing. This significantly improves noise immunity and signal decoding accuracy in complex communication channels. This approach enables high efficiency even in the presence of noise and interference.

To increase the noise immunity of the system, it is necessary to increase the equivalent energy of the signals, which involves increasing the distance between neighboring signal points. For this purpose, a signal formatting method is proposed that uses three information parameters: amplitude, phase, and time. Such a signal, built in three dimensions, is called three-dimensional or spherical OFDM, since the signal points are located in spherical space. The distance between the signal points in this case will be 1.5 times higher than for a signal formed in two-dimensional space. By forming a multi-position signal in a sphere, maximum noise immunity to white noise is achieved.

5. 2. Development of a multidimensional signal ren ception algorithm using high-order phase-difference modulation

A reception algorithm has been developed that can process signals in spherical space, taking into account amplitude, phase, and time shifts, which would ensure invariance to the initial phase and frequency shift, high accuracy, and noise immunity of reception (Fig. 4). It is necessary to ensure the adaptability of the algorithm to channel conditions based on the minimization criterion in a wide range of noise levels. The main idea is to process a signal formed using three information parameters: amplitude, phase, and time, taking into account high-order phase-difference modulation.

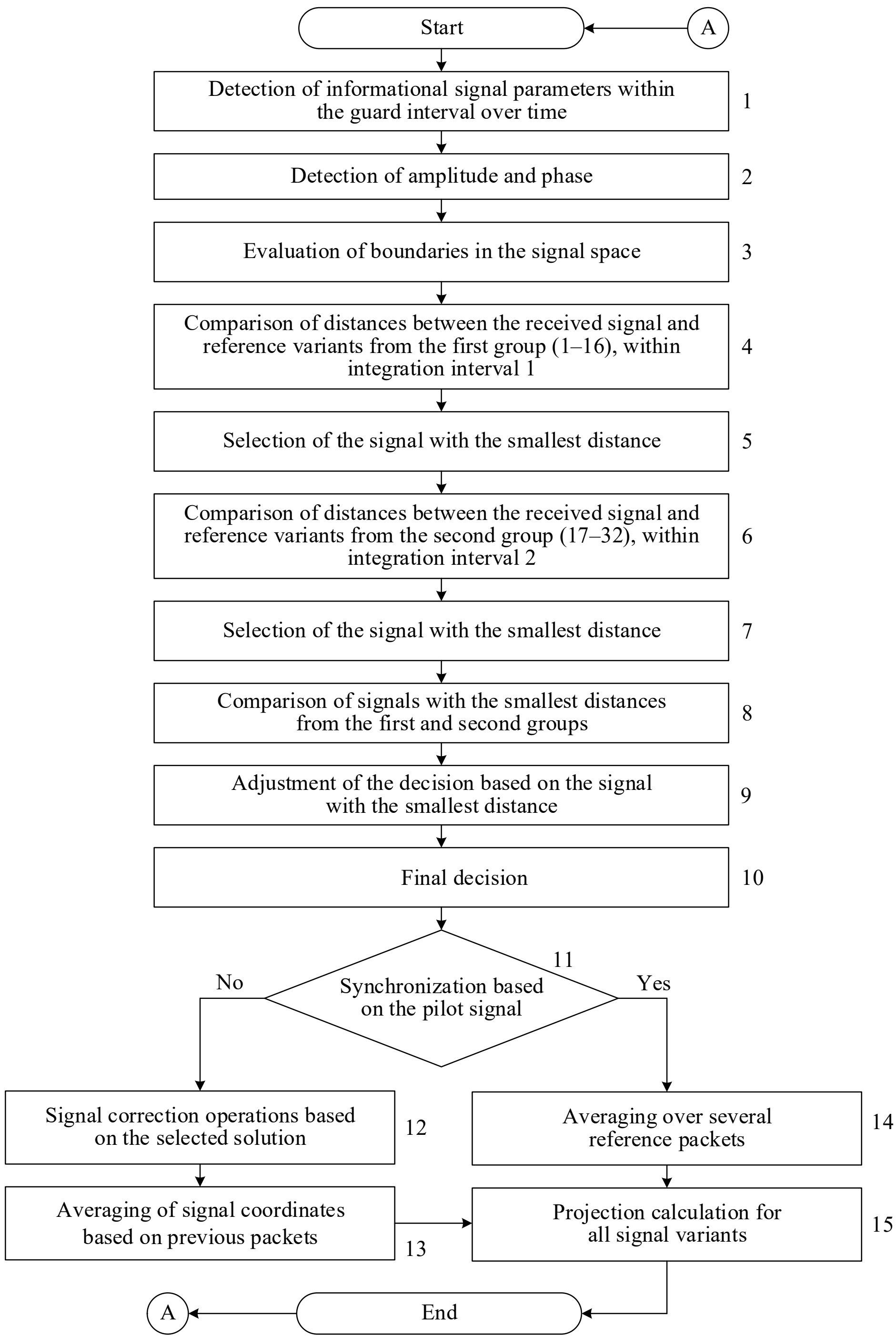

Fig. 4. Block diagram of the algorithm for receiving a three-dimensional multiposition signal

It is assumed that the proposed algorithm should be invariant to the initial phase and frequency shift, while ensuring the maximum possible noise immunity and adaptive matching with the channel parameters. The development of such an algorithm is a key step in creating reliable reception schemes for 5G and 6G mobile networks.

A step-by-step description of the algorithm is given:

1–3: Initial measurements. At the initial stage, key parameters of the received signal are measured. First, the time guard interval boundary is determined, which corresponds to the coordinate z_j in the three-dimensional signal space. Then, the amplitude and phase of the signal are measured, which specify the coordinates x_j and y_j, respectively. The final stage of this part is to determine the boundaries in the signal space necessary for further comparison with the reference variants.

4–7: Comparison with signal variants. The received signal is compared with known reference signals, which are divided into two separate groups. Each of the groups (the first – signals 1–16, the second – signals 17–32) is processed using a separate integration interval. This makes it possible to adapt the reception to different transmission conditions. Within each group, the signal variant that has the smallest distance to the received one is determined, taking into account the Euclidean or scalar similarity measure.

8–10: Decision making. After one closest signal has been determined from each group, these two signals are compared with each other. The option that has a smaller distance to the received signal is chosen. This signal is recognized as the most probable, that is, the system makes the final decision about which signal was transmitted.

11–15: Synchronization and refinement. Further processing depends on whether synchronization with the pilot signal is possible. If such synchronization is available, averaging is performed over several packets to increase accuracy, after which projections of all possible reference signals are calculated according to established formulas. If synchronization is not possible, a decision is made in favor of the closest signal, and this signal is used as a reference for further reception. In particular, for correction and averaging (of signal parameters over several packets to reduce the influence of noise and obtain effective estimates of three-dimensional coordinates) in subsequent packets.

Based on the described algorithm (Fig. 4), a simulation of the operation of a three-dimensional multi-position signal reception system with phase-difference modulation was carried out. The purpose of the simulation is to assess the efficiency of the reception algorithm under noise conditions, check the noise immunity depending on the averaging parameters, and also compare it with theoretically achievable characteristics. Below is a simulation modeling of the efficiency of 3D AFM-32 in comparison with other methods.

Signal processing algorithms using amplitude-phase and amplitude-differential-phase modulation have demonstrated high efficiency in multi-channel systems that use orthogonal channel signals. The process of separating such signals is based on calculating the received signal into a pair of mutually perpendicular reference harmonics with an arbitrary initial phase. This approach allows for accurate signal separation, which significantly improves noise immunity and transmission efficiency in multi-channel communication.

In OFDM systems, the reference oscillations of all channels are usually synthesized from the same frequency, while the initial phases of the signals may have unsynchronized shifts or low correlation [15]. This makes the controlled phase adjustment methods ineffective [16].

The proposed algorithm for coherent reception in three-dimensional signal space (Fig. 4) avoids the need to accurately fix the absolute phase of the signal, which is characteristic of traditional phase modulation. Instead, phase difference relations are used that are invariant to the initial phase and frequency shift.

5. 3. Assessing the noise immunity of a multidimeni sional signal signal reception method

To confirm the reliability of the proposed method, it is necessary to determine the probability of an error during information transmission. Based on simulation modeling, it is necessary to calculate the noise immunity of the system. That is, calculate the dependence of the error probability on the ratio of signal energy to the spectral density of the interference power at different signal parameters. The results should make it possible to determine the reception parameters for practical implementation in the new generation mobile networks.

In order to evaluate the algorithm proposed in the work, calculations of projections of the multi-position 3D AFM-32 signal for different averaging intervals ($M = 1$, 5, 10, 20, 100) were performed. The results are represented on the plot of the dependence of error probability on the signal-to-noise ratio (Fig. 5). The first curve shows the maximum possible noise immunity of the system under conditions of strict coherent reception. Curve 2 ($M = 100$) demonstrates the closest to theoretical noise immunity, while smaller averaging intervals ($M < 20$) show significantly lower signal reception accuracy.

The simulation results showed that to achieve maximum noise immunity, it is necessary to use an averaging interval of at least $M = 20$. This value provides coherent reception efficiency close to the ideal case.

The results of simulation experiments for an OFDM modem with incoherent reception are also presented. The data (Fig. 5, curve 7) show the real noise immunity of the system under conditions close to practical ones.

Separately, an assessment of the required sample size was carried out to achieve the specified error probability intervals (formula ≥), which made it possible to determine the parameters for further research and practical application.

When assessing the required sample size, the results given should be taken into account. For each value of Q and a specified confidence level probability β, it is possible to deterł mine the region where the error probability (P_{error}) and its empirical value $\left(P^*_{erorr}\right)$. are located. The plot in Fig. 5 shows the curves limiting these regions for different values of Q for a specified confidence level $\beta = 0.9$. For example, at $P^*_{erorr} = 0.1$ to ensure the interval [0.09–0.11], the required sample size is 200. For $P^*_{erorr} = 0.07$ and the interval [0.065–0.075], a sample size is needed. In general, the sample size should be determined from formula $n \geq \frac{20}{P^*_{erorr}}$.

The result in plots (Fig. 5) confirms the energy advantage of coherent methods of receiving multi-position signals, which is achieved due to the improved signal system and the developed algorithm.

Comparison of curves 2 and 6 (Fig. 5) shows that the energy gain is 3 dB. To achieve real noise immunity, which is as close as possible to the potential one, it is necessary to ensure a minimum averaging interval $M = 20$. This will ensure a high level of noise immunity during coherent signal processing.

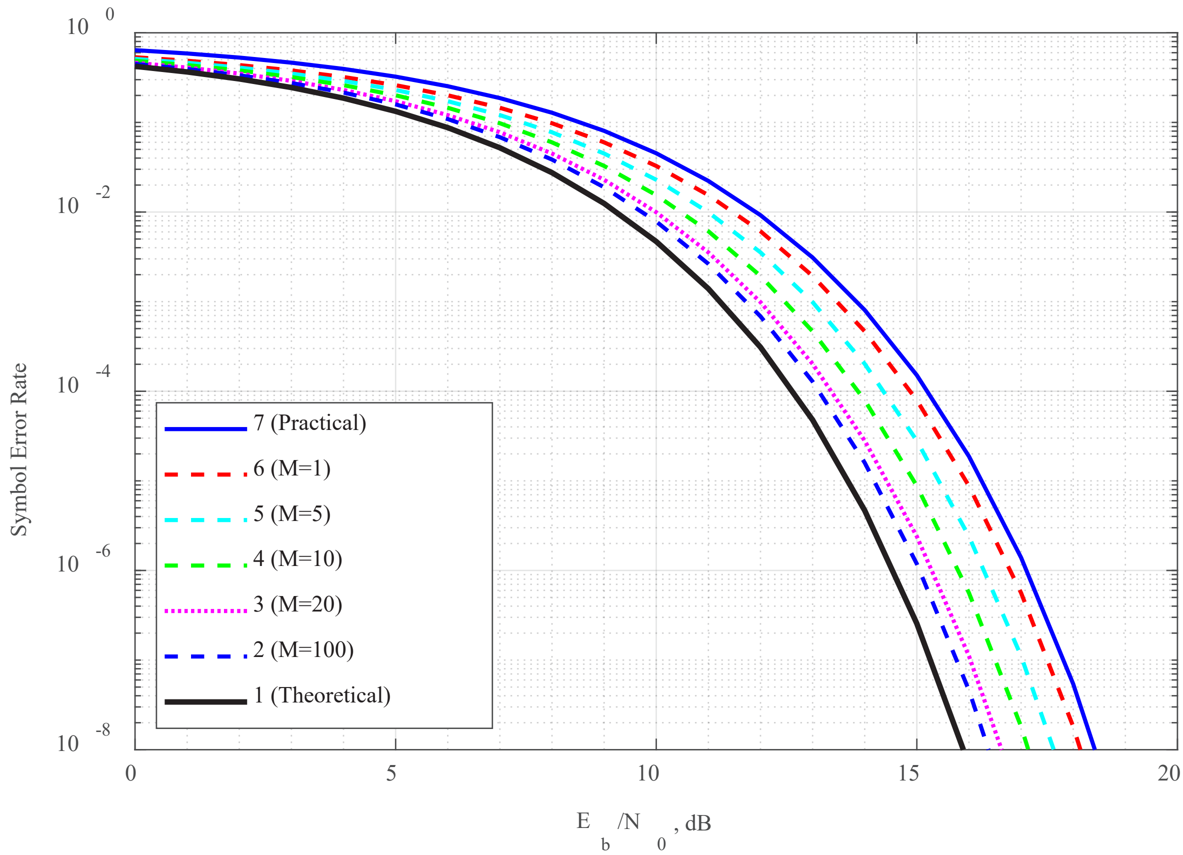

Fig. 5. Dependence of error probability on signal-to-noise ratio in the 3D AFM-32 multi-position signal system

In order to fully analyze the effectiveness of the proposed system with a multidimensional 3D AFM-32 signal, it is advisable to compare it with typical modulation methods used in modern mobile networks. For this purpose, Fig. 6 shows the results of modeling noise immunity for systems with coherent reception of QAM-32 signals depending on the averaging interval. The obtained curves make it possible not only to illustrate the nature of the influence of parameter M on the reception accuracy but also compare the overall level of noise immunity of QAM-32 with the corresponding characteristics of the proposed 3D AFM-32. This makes it possible to establish that only under conditions of a sufficient averaging interval ($M \geq 20$) both systems demonstrate an approach to the potential theoretical limit; however, 3D AFM-32 provides increased noise immunity due to three-dimensional signal formatting. Thus, the inclusion of Fig. 6 makes it possible to objectively confirm the advantages of the developed system over existing modulation schemes and justify its feasibility for use in mobile networks of the next generations. The designations of the curves correspond to the previous figure.

The result of the two plots (Fig. 5, 6) shows that the 3D AFM 32 system provides a lower probability of error over the entire range of signal-to-noise values at the same averaging intervals. In particular, at $M = 20$, which is a practically achievable interval for coherent reception, the difference between the curves is about 2–3 dB in favor of 3D AFM-32. This indicates better energy efficiency and higher noise immunity compared to the traditional QAM-32 scheme. Such an assessment is important for determining the efficiency of the system under real conditions when disturbances or errors may occur in the process of signal transmission.

In both cases, the actual noise immunity approaches the potential one only if the number of averaged packets is not less than 20. This emphasizes the importance of choosing the correct averaging interval to achieve maximum system efficiency, as well as to reduce errors in signal reception, which is critically important under conditions of a noisy environment and interference. The specified choice of interval M determines the balance between the accuracy of estimates and the speed of the system's response to changing communication conditions.

To clearly illustrate the effectiveness of the proposed three-dimensional phase-difference modulation of high orders (3D AFM-32), a simulation of comparison with the AFM and QAM modulation schemes is performed in the work. The inclusion of these figures provides an illustration of the effectiveness of the developed algorithm (Fig. 4), which is important for substantiating its feasibility under the conditions of modern and promising mobile communications.

Fig. 7 illustrates the dependence of symbol error probability on the signal-to-noise ratio for three types of phase-difference modulation: AFM-16, AFM-32, and the proposed 3D AFM-32. At the SER $\approx 10^{-8}$ level, it is seen that the system with 3D AFM-32 ensures the achievement of the required noise immunity at $E_b/N_0 \approx 17.4$ dB, which is ≈ 1.8 dB more effective than in the case of AFM-16 (≈ 19.2 dB), and at the same time significantly outperforms AFM-32, which does not reach the specified SER level even at $E_b/N_0 > 21$ dB.

Fig. 8 shows a graphical comparison of the noise immunity of 3D AFM-32 with traditional quadrature amplitude modulations QAM-16 and QAM-32. At the SER $\approx 10^{-8}$ level, the system with three-dimensional modulation achieves the required reception accuracy at $E_b/N_0 \approx 17.4$ dB, which provides an energy gain of about 1 dB compared to QAM-16 and more than 3.5 dB compared to QAM-32.

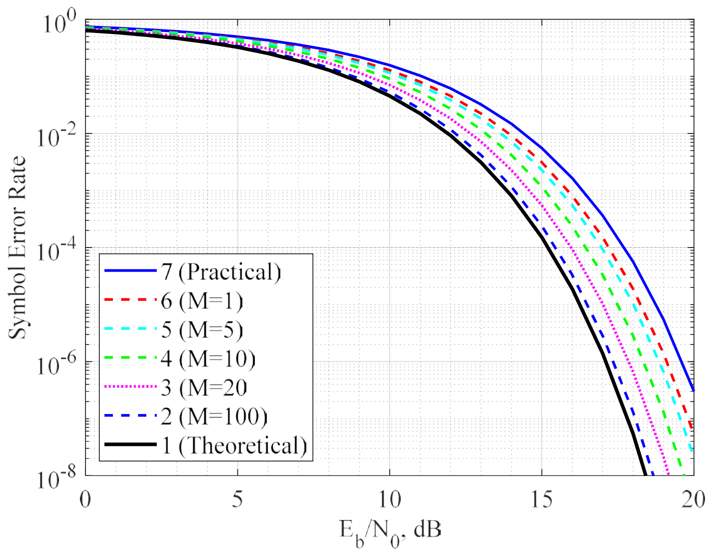

Fig. 6. Dependence of error probability on signal-to-noise ratio in the QAM-32 multi-position signal system

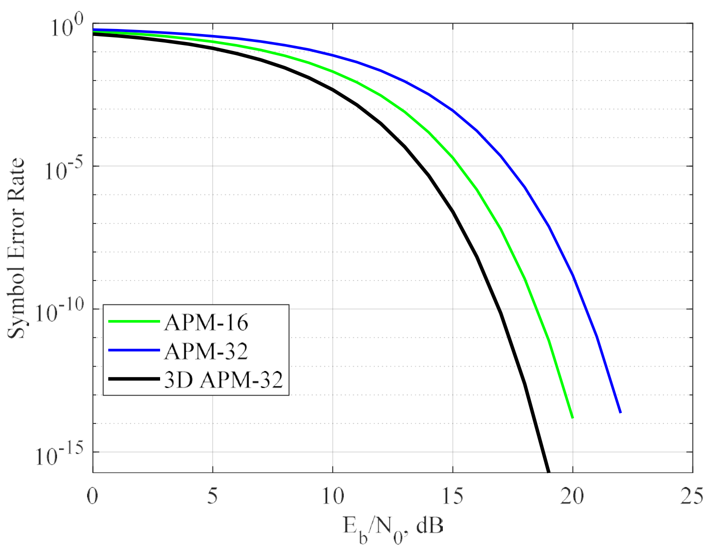

Fig. 7. Comparison of the efficiency of 3D AFM-32 with AFM-16 and AFM-32

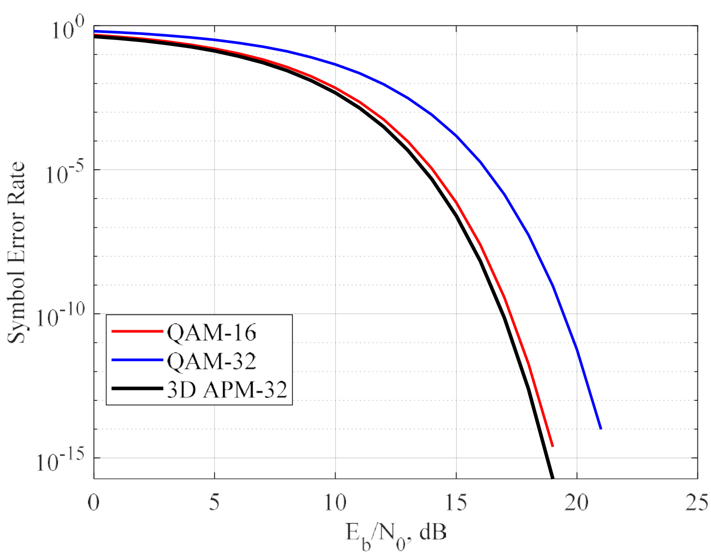

Fig. 8. Comparison of the efficiency of 3D AFM-32 with QAM-16 and QAM-32

This confirms the advantage of the proposed three-dimensional OFDM method, which allows for maximum information transmission speed, high reliability, and invariance to random signal disturbances.

6. Discussion of results based on studying the effectiveness of the multidimensional signal reception algorithm

A three-dimensional signal with thirty-two-position phase-difference modulation of high orders (3D AFM-32) has been synthesized, which simultaneously uses three independent information parameters: amplitude, phase, and time. The peculiarity of this signal is that the signal points are not located in a plane, as in classical QAM or two-dimensional AFM systems. They are located on the surface of a sphere, which makes it possible to increase the distance between points with an unchanged signal energy level.

To improve noise immunity, it is necessary to increase the equivalent energy, which involves increasing the distance between neighboring signal points. For this purpose, a signal formatting method is proposed that uses three information parameters: amplitude, phase, and time. Such a signal, built in three dimensions, will be called three-dimensional or spherical OFDM, since the signal points are located in spherical space.

The distance between signal points in this case will be 50% higher than for a signal generated in two-dimensional space.

Having formed a multi-position signal in the sphere, we achieve maximum noise immunity to white noise.

As is known, the use of phase-difference modulation allows the information transmission system to be insensitive to phase jumps. It is shown that the use of high-multiplicity phase-difference modulation ensures the system's insensitivity to frequency jumps.

The choice of three information parameters (amplitude, phase, and time), according to expressions (14), (15), provided an increase in the distance between signal points by 50% compared to traditional two-dimensional phase-difference modulation (Fig. 1). Due to such a geometric configuration (Fig. 3), the distance between signal points increases on average by 50% compared to two-dimensional AFM, which is confirmed by both analytical calculations and graphical visualization. This, in turn, provides a significant increase in the equivalent signal energy and a decrease in the probability of error. Fig. 2 confirms that it is the use of such an approach that ensures invariance to the initial phase and frequency shifts of the signal. This becomes possible due to the use of first- and second-order phase differences (formulas (2), (4)), which ensure invariance to the initial phase and frequency shifts (Table 1).

An algorithm for coherent reception of a three-dimensional multi-position signal based on high-order phase-difference modulation has been developed. The algorithm takes into account three coordinate components, amplitude, phase, and time shift (which reflects the duration of the guard interval), which allows for accurate localization of the signal point in the spherical signal space (Fig. 4).

The decision-making is based on the estimation of the scalar distance between the received signal and all possible reference signals (formulas (15) to (17)). The proposed approach ensures invariance to the initial phase and frequency perturbations. This is achieved by calculating the first and second order phase differences (formulas (2), (4), (8), (10)), which do not change under the influence of phase instabilities.

The key feature of the algorithm is its adaptability. It makes it possible to compare signals in different groups of variants (signals 1–16 and 17–32) and averaging the estimates using the parameter M (as shown in Fig. 5). The algorithm also provides the possibility of synchronization using pilot signals. As a result, high-precision reception is ensured even in the presence of noise and intersymbol interference.

The proposed algorithm for receiving a three-dimensional signal $S_i(t)$ enables the selection of the correct signal variant if condition (12) is satisfied for all other variants. The calculations of the first and second order phase differences, the results of which are given in formulas (2), (4), made it possible to determine the patterns and characteristics of changes in phase differences during the signal transmission process. This makes it possible to determine the received signal with high accuracy under conditions of noise and interference.

A quantitative assessment of the noise immunity of the transmission system based on a three-dimensional multi-position signal with high-order phase-difference modulation (3D AFM-32) has been performed. For this purpose, simulation modeling methods were used, the results of which are shown in Fig. 5–8.

The simulation results showed that to achieve maximum noise immunity, it is necessary to use an averaging interval of not less than $M = 20$. This value provides the efficiency of coherent reception, close to the ideal case.

The result in plots (Fig. 5) confirms the energy advantage of coherent methods of receiving multi-position signals, which is achieved owing to the improved signal system and the developed algorithm.

A comparison of curves 2 and 6 (Fig. 5) shows that the energy gain is 3 dB. To achieve real noise immunity, which is as close as possible to the potential one, it is necessary to provide a minimum averaging interval of $M = 20$. This will provide a high level of noise immunity during coherent signal processing.

It is shown that the use of a reception algorithm with an averaging interval of $M \geq 20$ provides a reduction in the error probability to the level of 10^{-8} at $E_b/N_0 \approx 17.4$ dB. This exceeds the efficiency of traditional modulation methods such as QAM-32 (≈20.9 dB) and even QAM-16 (≈ 18.4 dB), as illustrated in Fig. 8. According to Fig. 7, 3D AFM-32 also significantly outperforms classical two-dimensional phase difference modulation schemes (AFM-16, AFM-32). Thus, the efficiency of 3D AFM-32 approaches the efficiency of QAM-16.

In addition, it has been found that increasing the averaging interval makes it possible to bring the real system characteristics closer to the theoretically achievable limit. The use of three information parameters (amplitude, phase, time) provides an expansion of the signal space without increasing the spectrum width.

Unlike works [1, 2], which focus mainly on the characteristics of communication channels and signal propagation without analyzing multidimensional modulation schemes, our study proposes a method that takes into account high-order phase-difference modulation and multidimensionality of the signal (formulas (2), (4), (8), Fig. 1, 3). This provided a significant increase in noise immunity and signal reception accuracy.

Compared with studies [3–5], which consider the issues of multiplexing and interchannel interference in traditional OFDM and QAM schemes without taking into account high-order phase shifts, the proposed method has shown significant advantages in the stability of signal reception due to the use of first- and second-order phase differences ((6), (7), (9), Fig. 2).

In [6], only two-dimensional signals with second-order phase-difference modulation were considered in detail. Our method provides an additional increase in the distance between signal points due to three-dimensional signal formatting (formulas (14), (15), Fig. 3).

Unlike the index modulation methods reported in [7, 10], the proposed method has ensured invariance to phase and frequency shifts due to the use of phase difference relations and adaptive reception algorithms ((16), (17), Fig. 4), which is important for practical application under the conditions of next-generation mobile networks.

In [8, 9], attention was paid to modulation schemes with high spectral efficiency and digital systems with significant phase noise, but the issue of multidimensionality of the signal and phase-difference modulation of high orders was not resolved. The proposed method successfully resolved these issues and demonstrated significant energy efficiency and reduced error probability (Fig. 5–8). Thus, unlike existing approaches, our method makes it possible to take into account phase shifts of the first and second orders. It also provides invariance to frequency deformations and adaptive signal processing under conditions of white noise and interference.

The results of tour study prove that the stated scientific problem of the lack of an effective method for forming a multidimensional signal of n-th multiplicity and receiving based on phase-difference modulation of high orders has been solved. The proposed method makes it possible to fill the existing scientific niche and increase the reliability of data transmission in 5G/6G mobile networks without increasing the signal spectrum width.

However, some limitations of the study should be noted, including the idealized simulation conditions. Real mobile networks may be characterized by additional factors, such as rapidly changing signal propagation conditions, nonlinear distortions, and interference, which may affect the accuracy of the estimates of our results.

The main drawback is the lack of practical implementation of the prototype in a hardware environment and verification of the power consumption of the developed algorithm. In addition, there is no complete optimization of the time parameters under the dynamic mode of switching between signal groups.

Further research should be directed to experimental verification of the proposed method under actual operating conditions of mobile networks, as well as to the development of adaptive algorithms that could allow for even more precise adjustment of signal parameters taking into account dynamic changes in the communication channel.

7. Conclusions

1. A multidimensional signal has been synthesized by using three information parameters – amplitude, phase, and time of thirty-two-position phase-difference modulation (3D FM-32). This provided an increase in the distance

between signal points by 50% and, accordingly, a decrease in the probability of signal transmission error without expanding the spectral width of the signal.

2. An algorithm for coherent reception of a three-dimensional signal has been developed, which adaptively responds to changes in the parameters of the communication channel, which was confirmed by the simulation results. Our study showed that under reception conditions (averaging interval of at least 20) the proposed method provides an energy gain of up to 3.5 dB compared to traditional modulation methods, such as QAM-32, with an error probability level of 10^{-8}.

3. The conducted noise immunity assessment confirmed that the proposed transmission system with 3D AFM-32 demonstrates a significant energy gain – up to 3.5 dB – compared to QAM-32 at an error probability level of 10^{-8}. A feature of our result is that noise immunity is ensured without increasing the spectral width of the signal, but due to three-dimensional formatting, which increases the distance between signal points by 50%. Unlike known methods, achieving maximum efficiency requires a minimum averaging interval ($M \geq 20$), which ensures proximity to the potential theoretical limit. This is explained by the use of high-order phase-difference modulation in combination with a coherent reception algorithm capable of adapting to changes in the communication channel.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study, as well as the results reported in this paper.

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Data availability

All data are available, either in numerical or graphical form, in the main text of the manuscript.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

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