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A REMARK ABOUT CALCULATION OF THE JUMP OF THE RESOLVENT IN FRIEDRICHS' MODEL

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Наведено умови на модель Фрідріхса, які дозволяють записати формулу для стрибка резольвенти на неперервному спектрі

Ключові слова: модель Фрідріхса, неперервний спектр, факторизація, резольвента, стрибок

Приведены условия на модель Фридрихса, которые позволяют записать формулу для скачка резольвенты на непрерывном спектре

Ключевые слова: модель Фридрихса, непрерывный спектр, факторизация, резольвента, скачок

The conditions on the Friedrich's model, which permit to write the formula for the jump of the resolvent on continuous spectrum are indicated

Keywords: Friedrich's model, continuous spectrum, factorization, resolvent, jump

Introduction

The presentation of spectral projection by the resolvent R_ζ of selfadjoint operator

$$E(\Delta)=\lim_{\varepsilon\to+0}\frac{1}{2\pi i}\int_{\Delta+i\varepsilon}\left(R_\zeta-R_{\bar\zeta}\right)d\zeta,\Delta\subset(-\infty,\infty)$$

is well-known for a long time (see for example Berezanski Yu.M.[1],p.425). Obviously the jump of the resolvent $\lim_{\varepsilon\to+0}((R_\zeta-R_{\bar\zeta})\phi,\psi),\zeta=\sigma+i\varepsilon,\sigma\in(-\infty,\infty)$ is important in the theory of nonselfadjoint perturbation of continuous spectrum too. In the work Ljance V.E. [2] conserning non-selfadjoint Friedrichs' model the author uses an extension of the operator in more wide space. In the work Cheremnikh E.V. [3] an auxiliary operator (like maximal differential operator) was used. In presented article we precise the calculus of the jump of resolvent.

Preliminary and main theorem

Let $H=L^2(R,H_1), R=(-\infty,\infty)$, where H_1 is some Hilbert space with scalar product $(\bullet,\bullet)_{H_1}$. We consider Friedrichs' model

$$T=S+V,\ V=A^*B,\ D(T)=D(S),\ A,B:H\to G,\qquad(1)$$

where S is the operator of multiplication by independent variable (i.e. $(S\phi)(\tau)=\tau\phi(\tau),\tau\in R$) with maximal domain of definition $D(S)$ and A,B are bounded operators from H in G. The space G is auxiliary Hilbert space. We use notation $\phi,\psi,f,g\in H, c,d\in G$ for the elements and $(\bullet,\bullet),(\bullet,\bullet)_G$ for scalar product respectively. We denote $S_\zeta=(S-\zeta)^{-1}, T_\zeta=(T-\zeta)^{-1}$. The relation $(T-\zeta)\psi=\phi$ or $(S-\zeta)\psi+A^*B\psi=\phi$ (see1) gives

$$\psi+S_\zeta A^*B\psi=S_\zeta\phi,\quad \mathrm{Im}\zeta\neq0\qquad(2)$$

Now we apply the operator B to both sides of the relation (2) and denote $c = B\psi$. Then $c + BS_\zeta A^* c = BS_\zeta \phi$ or $K(\zeta)c = BS_\zeta\phi$, where

$$K(\zeta) = 1 + BS_\zeta A^*, \quad \operatorname{Im}\zeta \neq 0 \tag{3}$$

Substituing $c = K(\zeta)^{-1} BS_\zeta \phi$ in (2), we obtain $\psi = T_\zeta \phi$, where

$$T_\zeta \phi = S_\zeta \phi - S_\zeta A^* K(\zeta)^{-1} BS_\zeta \phi, \quad \operatorname{Im}\zeta \neq 0 \tag{4}$$

for every value ζ such that there exists bounded inverse operator $K(\zeta)^{-1}$.

We denote by $K_+(\zeta), \operatorname{Im}\zeta > -\varepsilon$ and $K_-(\zeta), \operatorname{Im}\zeta < \varepsilon$ the extensions of the operator function (3) over axis $\operatorname{Im}\zeta = 0, \zeta \neq 0$ from the domains $\operatorname{Im}\zeta > 0$ and $\operatorname{Im}\zeta < 0$ respectively for some $\varepsilon > 0$.

We suppose that:

C_1) there exist the extensions $K_\pm(\zeta)$ and $\|K_\pm(\zeta) - 1\| \to 0, \zeta \to \infty$ uniformly in the domain $|\operatorname{Im}\zeta| < \varepsilon$;

C_2) the operators $K_\pm(\zeta)^{-1}$ exist and are holomorph in the domain $|\operatorname{Im}\zeta| < \varepsilon, \operatorname{Re}\zeta \neq 0$ expect may be finite set of points.

Denote

$$B(\zeta)\phi = \phi - A^* K(\zeta)^{-1} BS_\zeta \phi, \; A(\zeta)\psi = \psi - B^* K(\bar{\zeta})^{-1*} AS_\zeta \psi. \tag{5}$$

Let

$$h(\tau,\zeta) = ((B(\zeta)\phi)(\tau), (A(\zeta)\psi)(\tau)), \tau \in R, \operatorname{Im}\zeta > 0. \tag{6}$$

We need strong limits in the space G

$$(AS_\sigma)_\pm \phi = \lim_{\nu\to+0} AS_\zeta \phi, \quad (BS_\sigma)_\pm \phi = \lim_{\nu\to+0} BS_\zeta \phi, \quad \zeta = \sigma + i\nu \tag{7}$$

and corresponding strong limits in H

$$B_+(\sigma)\phi = \phi - A^* K_+(\sigma)^{-1} (BS_\sigma)_+ \phi,$$

$$A_-(\sigma)\phi = \phi - B^* K_-(\sigma)^{-1*} (AS_\sigma)_+ \phi, \phi \in H^0$$

for the elements ϕ from some subspace $H^0 \subset H$ dense in H, $\overline{H^0} = H$.

We suppose that

C_3) strong limits (7) exist except may be finite set $D \subset R$ of values of σ for the elements $\phi \in H^0$ and the function $h(\tau,\zeta)$ (see(6)) for $0 \le \operatorname{Im}\zeta < \varepsilon_0$, for some $\varepsilon_0 > 0$ is differentiable on τ in R and

$$\int_R |h(\tau,\zeta)| d\tau \le M, 0 \le \operatorname{Im}\zeta < \varepsilon_0, \tag{8}$$

where $M = \text{const}$;

C_4) for every finite interval (a,b) we have

$$|h(\tau,\zeta) - h(\tau,\sigma)| \le C|\zeta - \sigma|, |\tau - \sigma| < \delta, \tau \in (a,b), \zeta = \sigma + i\nu, \tag{9}$$

where $\sigma \in R \setminus D$ (see C_3)) and $C = \text{const}$ does not depend on δ_0.

The main result of the article is given by following theorem. Denote (see C_3))

$$(T_\sigma \phi, \psi)_\pm = \lim_{\nu\to+0} (T_{\sigma+i\nu}\phi, \psi), \quad \phi, \psi \in H^0, \sigma \in R \setminus D.$$

Theorem 1. There exists subspace $H^1 \subset H^0$ dense in $H, \overline{H^1} = H$ such that

1) limit values $(T_\sigma \phi, \psi)_\pm$ exist if $\sigma \in R \setminus D, \phi, \psi \in H^1$;

2) jump of the resolvent in the point $\sigma \in R \setminus D$ is

$$(T_\sigma \phi, \psi)_+ - (T_\sigma \phi, \psi)_- = 2\pi i \big((B_+(\sigma)\phi)(\sigma), (A_-(\sigma)\psi)(\sigma)\big)_{H_1}, \tag{10}$$

where

$$B_+(\sigma)\phi = \phi - A^* K_+(\sigma)^{-1} (BS_\sigma)_+ \phi, \; A_-(\sigma)\phi = \\ = \phi - B^* K_-(\sigma)^{-1*} (AS_\sigma)_+ \phi, \phi \in H^1 \tag{11}$$

3) if $\phi, \psi \in D(T) \cap H^1$ then

$$\big(A_-(\sigma)(T^* - \sigma)\psi\big)(\sigma) = 0, \; \big(B_+(\sigma)(T - \sigma)\phi\big)(\sigma) = 0 \tag{12}$$

and if $\{e_k\} \subset H_1$ is arbitrary orthonormal base in H_1 then the functionals

$$(a_{\sigma,k}, \psi) = (e_k, (A_-(\sigma)\psi)(\sigma))_{H_1}, \\ (\phi, b_{\sigma,k}) = \big((B_+(\sigma)\phi)(\sigma), e_k\big)_{H_1}, \\ k = 1, 2, \ldots \tag{13}$$

are eigenfunctionals of the operators T and T^* respectively.

Choice of the factorization.

Note that the operators $B(\zeta), A(\zeta) : H \to H$ (see(5)) do not depend on the choice of the factorization $V = A^* B$, really comparison (5) with (4) gives

$$(S - \zeta) T_\zeta \phi = B(\zeta)\phi, \; (S - \zeta) T^*_{\bar\zeta} \psi = A(\zeta)\psi, \tag{14}$$

where $T^*_{\bar\zeta}$ is the resolvent of adjoint operator T^* in the point $\bar\zeta$. In the case

$$\overline{R(A^*)} = \overline{R(B^*)} = H, \tag{15}$$

the calculation of the jump of the resolvent is more simple. Otherwise we will replace given factorization $V = A^* B$ by some special factorization which satisfies the condition (15). So, suppose now that $R(A^*) \neq H$ or $R(B^*) \neq H$. We denote $Z(A) = \{f \in H : Af = 0\}$. It is known that if $A, B : H \to G$ are bounded operators then

$$\overline{R(A^*)} \oplus Z(A) = H, \overline{R(B^*)} \oplus Z(B) = H. \tag{16}$$

Definition. Let $\tilde G = G \oplus Z(A) \oplus Z(B)$.

The space $\tilde G$ is Hilbert space and scalar product for the elements

$$\tilde c = \{c, g_\alpha, g_\beta\}, \tilde d = \{d, h_\alpha, h_\beta\} \in \tilde G$$

with arbitrary elements $g_\alpha, h_\alpha \in Z(A), g_\beta, h_\beta \in Z(B)$ is defined by the expression

$$(\tilde c, \tilde d)_{\tilde G} = (c,d)_G + (g_\alpha, h_\alpha) + (g_\beta, h_\beta). \tag{17}$$

Let $P_\alpha : H \to Z(A), P_\beta : H \to Z(B)$ are orthogonal projection.

Definition.The operators $\tilde{A},\tilde{B}:H\to\tilde{G}$ are defined by the relation

$$\tilde{A}f=\{Af,P_\alpha f,0\},\ \tilde{B}f=\{Bf,0,P_\beta f\}. \qquad (18)$$

As $\tilde{G}$ is Hilbert space then $\tilde{A}^*,\tilde{B}^*:\tilde{G}\to H$.

Lemma 1. Following relations hold:

1) $\tilde{A}^*\tilde{B}=A^*B$

2) $\overline{R(\tilde{A}^*)}=\overline{R(\tilde{B}^*)}=H$

Proof. 1) Let $f\in H$ be arbitrary element, according to (17)-(18) we have

$$(\tilde{A}f,\tilde{c})_{\tilde{G}}=(Af,c)_G+(P_\alpha f,g_\alpha)=(f,A^*c)+(f,g_\alpha)=(f,A^*c+g_\alpha).$$

Therefore, if $\tilde{c}=\{c,g_\alpha,g_\beta\}$ then

$$\tilde{A}^*\tilde{c}=A^*c+g_\alpha. \qquad (19)$$

Substituing (see(18)) $\tilde{c}=\tilde{B}f=\{Bf,0,P_\beta f\}\equiv\{c,g_\alpha,g_\beta\}$ in (19)- we obtain the statement 1).

2) As element $g_\alpha\in Z(A)$ in the relation is arbitrary then $\overline{R(\tilde{A}^*)}=H$ (see(16)). The operator B is considered by analogy what proves the statement 2).

Lemma 1 is proved.

Note that new factorization $V=\tilde{A}^*\tilde{B}$ satisfies the condition (15).

Lemma about the difference $T_\zeta-T_{\bar\zeta}$.

Let us consider the case of arbitrary factorization $V=A^*B$.

Lemma 2.

Let $\phi=A^*c,\psi=B^*d,c,d\in G$ then (see(2))

$$\left(\left(T_\zeta-T_{\bar\zeta}\right)\phi,\psi\right)=\left(\left(S_\zeta-S_{\bar\zeta}\right)B(\zeta)\phi,A(\zeta)\psi\right) \qquad (20)$$

and

$$B(\zeta)(T-\zeta)\phi=(S-\zeta)\phi,\quad A(\zeta)(T^*-\bar\zeta)\psi=(S-\bar\zeta)\psi. \qquad (21)$$

Proof. According to (3)-(4)

$$BT_\zeta A^*=BS_\zeta A^*-BS_\zeta A^*K(\zeta)^{-1}BS_\zeta A^*=$$
$$K(\zeta)-1-(K(\zeta)-1)K(\zeta)^{-1}(K(\zeta)-1)=1-K(\zeta)^{-1}$$

and in view of presentation $\phi=A^*c,\psi=B^*d$ we have

$$(T_\zeta\phi,\psi)=(BT_\zeta A^*c,d)_G=(c,d)_G-\left(K(\zeta)^{-1}c,d\right)_G.$$

Therefore

$$\left((T_\zeta-T_{\bar\zeta})\phi,\psi\right)=-\left(\left(K(\zeta)^{-1}-K(\bar\zeta)^{-1}\right)c,d\right)_G=$$
$$=\left(K(\bar\zeta)^{-1}\left[K(\zeta)-K(\bar\zeta)\right]K(\zeta)^{-1}c,d\right)_G=$$
$$=\left(\left(BS_\zeta A^*-BS_{\bar\zeta}A^*\right)K(\zeta)^{-1}c,K(\bar\zeta)^{-1*}d\right)_G=$$
$$=\left(\left(S_\zeta-S_{\bar\zeta}\right)A^*K(\zeta)^{-1}c,B^*K(\bar\zeta)^{-1*}d\right)_G. \qquad (22)$$

Multiplying $K(\zeta)=1+BS_\zeta A^*$ by $A^*K(\zeta)^{-1}$ we obtain

$$A^*c=A^*K(\zeta)^{-1}c+A^*K(\zeta)^{-1}BS_\zeta A^*c$$

As $\phi=A^*c$ then $A^*K(\zeta)^{-1}c=\phi-A^*K(\zeta)^{-1}BS_\zeta\phi=B(\zeta)\phi$ (see(5)). By analogy $B^*K(\zeta)^{-1*}d=A(\zeta)\psi$ then (20) results from (22). The change $T_\zeta\phi\to\phi$ and $T_{\bar\zeta}^*\psi\to\psi$ transforms (14) into (21).

Lemma 2 is proved.

To study right side of the relation (20) we need next Lemma.

Lemma 3. If the function $h(\tau,\zeta)$ satisfies the conditions $C_3)-C_4)$ then

$$\lim_{\nu\to+0}\int_R\left(\frac{1}{\tau-\zeta}-\frac{1}{\tau-\bar\zeta}\right)h(\tau,\zeta)d\tau=2\pi i h(\sigma,\sigma),\ \zeta=\sigma+i\nu. \qquad (23)$$

Proof. We have

$$\int_R\left(\frac{1}{\tau-\zeta}-\frac{1}{\tau-\bar\zeta}\right)h(\tau,\zeta)d\tau=$$
$$=\int_R\left(\frac{1}{\tau-\zeta}-\frac{1}{\tau-\bar\zeta}\right)\left(h(\tau,\zeta)-h(\tau,\sigma)\right)d\tau+$$
$$+\int_R\left(\frac{1}{\tau-\zeta}-\frac{1}{\tau-\bar\zeta}\right)h(\tau,\sigma)d\tau.$$

So, to prove the relation (23) it is sufficient to prove that

$$I(\nu)\equiv\int_R\left(\frac{1}{\tau-\zeta}-\frac{1}{\tau-\bar\zeta}\right)\left(h(\tau,\zeta)-h(\tau,\sigma)\right)d\tau\to0,\ \nu\to+0, \qquad (24)$$

where $\mathrm{Re}\,\zeta=\sigma=\mathrm{const}$. Let $\delta>0$ and

$$I(\nu)=\int_{|\tau-\sigma|<\delta}+\int_{|\tau-\sigma|>\delta}\equiv I_1(\nu)+I_2(\nu)$$

1) As $|\tau-\zeta|=|\tau-\bar\zeta|\ge|\zeta-\sigma|$ then (see(9))

$$|I_1(\nu)|\le\int_{|\tau-\sigma|<\delta}\left(\frac{1}{|\tau-\zeta|}+\frac{1}{|\tau-\bar\zeta|}\right)|h(\tau,\zeta)-h(\tau,\sigma)|d\tau\le\frac{2}{|\zeta-\sigma|}\int_{|\tau-\sigma|<\delta}|h(\tau,\zeta)-h(\tau,\sigma)|d\tau\le4C\delta$$

2) If $|\tau-\sigma|>\delta$ then $|\zeta-\tau|>\delta$ and

$$\left|\frac{1}{\tau-\zeta}-\frac{1}{\tau-\bar\zeta}\right|=\left|\frac{2i\nu}{(\tau-\zeta)(\tau-\bar\zeta)}\right|\le\frac{2\nu}{\delta^2}$$

Therefore (see(8))

$$|I_2(\nu)|\le\int_{|\tau-\sigma|>\delta}\left|\frac{1}{\tau-\zeta}-\frac{1}{\tau-\bar\zeta}\right|\left(|h(\tau,\zeta)|+|h(\tau,\sigma)|\right)d\tau\le4M\frac{\nu}{\delta^2}.$$

If we choose $\delta=\sqrt[3]{\nu}$ then $I_1(\nu),I_2(\nu)\to0,\nu\to+0,$ what proves the relation (24).

Lemma is proved.

Proof of Theorem 1. 1) In view of (14) and the condition $C_3)$ limit values $(T_\sigma\phi,\psi)_\pm$ exist if we choose $H^1\subset H^0$ as subspace of differentable functions.

2) According to (14) the expressions $B(\zeta)\phi,A(\zeta)\psi$ does not depend on the choice of the factorization. We pose $V=\tilde{A}^*\tilde{B}$ (see Lemma 1) and obtain the relation (20) for some subspace dense in H. All operators in the relation (20) are bounded, therefore (20) holds for all elements

$\phi,\psi \in H$. Using Lemma 3 we obtain the relation (10) from the relation (20).

3) Let us consider the relation (21), where $\mathrm{Im}\zeta \to +0$ and $\mathrm{Re}\zeta = \sigma = const$. The right side $(S-\sigma)\phi$ is vector function with values in the space H_1. Obviously $\left[\left[(\tau-\sigma)\phi(\tau)\right]\right]_{\tau=\sigma} = 0$. So, relations (12)-(13) result from (21).

Theorem 1 is proved.

As conclusion note that one obtain formula of jump of the resolvent (see (10))

$$\left(T_\sigma\phi,\psi\right)_+ - \left(T_\sigma\phi,\psi\right)_- = 2\pi i\left(\left(B_+(\sigma)\phi\right)(\sigma),\left(A_-(\sigma)\psi\right)(\sigma)\right)_{H_1}$$

if 1) condition on the elements ϕ,ψ which give $C_3) - C_4)$ are indicated;

2) condition on ϕ,ψ which permit to prolongate continuously both side of (10) in more wide subspace are given.

Note that sign $\pm$ in the notation $B_+(\sigma), A_-(\sigma)$ correspond to sign of limit values $K_+(\sigma)^{-1}, K_-(\sigma)^{-1}$ in the expression of $B_+(\sigma), A_-(\sigma)$.

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ПРОГНОЗИРОВАНИЕ СОСТОЯНИЯ ОБЪЕКТА ПО РЕЗУЛЬТАТАМ ОЦЕНИВАНИЯ НЕЧЕТКОЙ ЭКСПЕРТНОЙ СИСТЕМОЙ

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Запропоновано технологію розрахунку аналітичного опису нечітких чисел, що визначає ймовірності стану об'єктів на момент прогнозу, що формуються нечіткою експертною системою. Описано процедуру дефаззифікації результатів прогнозування

Ключові слова: прогнозування, експертна система

Предложена технология расчета аналитических описаний нечетких чисел, определяющих вероятности состояний объекта на момент прогноза, формируемых нечеткой экспертной системой. Описана процедура дефаззификации результатов прогнозирования

Ключевые слова: прогнозирование, экспертная система

The calculation technology of analytical descriptions of fuzzy numbers which determining probabilities of the object's states in the moment of prognosis in the fuzzy expert system is offered. The defuzzification procedure of the prognostication results is described

Keywords: prediction, expert system

1. Введение

При решении многих практических задач в технике, экономике, медицине и др. возникает необходимость прогнозирования состояния объекта, характеризуемого набором контролируемых, часто зависимых параметров [1,2]. Такие задачи традиционно решаются с использованием экспертных систем (ЭС) [3-5]. При этом обычно предполагается, что механизм логического вывода этих систем (продукционный или байесов) оперирует с четко заданными базами данных и знаний. Вместе с тем, в последнее время все более ясно осознается понимание необходимости учета реальной неопределенности исходных данных задачи. При этом поскольку законы распределения соответствующих случайных величин, как правило, неизвестны, для их описания используют технологию нечеткой математики, что приводит к применению экспертных систем с нечетким механизмом логического вывода. В силу специфического характера функционирования нечет-