UDC 621.317.39:531.7
DOI: 10.15587/1729-4061.2025.331740

DEVISING AN AUTOMATED METHOD FOR EXPERIMENTAL MEASUREMENT OF THE ROTOR INERTIA MOMENT IN LOW-POWER ASYNCHRONOUS GENERATORS OF RENEWABLE ENERGY SOURCES

Inna Honcharuk
Doctor of Technical Sciences, Professor
Department of Economics and Entrepreneurship*
Valerii Hraniak
Corresponding author
PhD, Associate Professor
Department of Power Engineering,
Electrical Engineering and Electromechanics*
E-mail: titanxp2000@ukr.net
Oleksii Tokarchuk
PhD, Associate Professor
Department of Engineering Mechanics
and Technological Processes at the Agricultural Complex*
Dina Tokarchuk
PhD, Associate Professor
Department of Administrative Management
and Alternative Energy Sources*
*Vinnytsia National Agrarian University
Sonyachna str., 3, Vinnytsia, Ukraine, 21008

The object of this study is the electromechanical transient process in low-power asynchronous generators, suitable for operation in conjunction with renewable energy sources, when applying and removing the supply voltage, as well as the influence of the rotor moment of inertia of a low-power asynchronous generator on the duration of the specified transient process. The study addresses the task of reducing the complexity and duration of the pre-operational testing of the specified equipment.

A new automated method for measuring the rotor moment of inertia has been proposed, which is based on the excitation of the generator under the motor mode of operation by the rated frequency voltage under the conditions of mechanical braking of the rotor. The specified measurement method is characterized by reasonable accuracy, high speed, and is suitable for use under both production and laboratory conditions.

Within the framework of the study, a structural diagram of a technical means that implements the proposed method in practice has bene designed; a corresponding mathematical model of the electromechanical system "induction motor – measuring converter" was derived.

A detailed metrological analysis revealed that the methodological error of the proposed method does not exceed 8%, which is acceptable for most applied tasks. A linear relationship between the parameters of the transient process and the characteristics of the electromechanical system was also established, which significantly simplifies the calibration of the device and increases the accuracy and repeatability of the measurement results.

The research results confirm the high efficiency and practical feasibility of using the proposed approach in the field of technical control, maintenance and modernization of electric drive systems. The method could be adapted for a wide range of models of induction motors, as well as integrated into automated equipment condition monitoring systems

Keywords: rotor moment of inertia, measurement, induction motor, mathematical model, transient process

Received 23.04.2025
Received in revised form 01.05.2025
Accepted 20.05.2025
Published 28.05.2025

***How to Cite:** Honcharuk, I., Hraniak, V., Tokarchuk, O., Tokarchuk, D. (2025). Devising an automated method for experimental measurement of the rotor inertia moment in low-power asynchronous generators of renewable energy sources. Eastern-European Journal of Enterprise Technologies, 3 (5 (135)), 49–58. https://doi.org/10.15587/1729-4061.2025.331740*

1. Introduction

The rotor moment of inertia is one of the fundamental characteristics of electrical machines in general and asynchronous generators in particular, as it determines their dynamic parameters. In technical specifications and reference materials, the value of this parameter is often given with a large error, which can reach more than 10% [1]. At the same time, when designing renewable energy systems with asynchronous generators, there is a need for a more accurate determination of the rotor moment of inertia J_{OB}. It is also worth noting that this parameter significantly affects the speed and thermal operation of the machine [2].

There are two main approaches to determining J_{OB}: computational and experimental. Computational procedures, as a rule, are complex and not always accurate due to the complex geometric configuration of the rotor and the heterogeneity of its material. In view of this, preference is mostly given to experimental methods, which provide higher reliability of the results [3].

Thus, devising express methods and means for the operational measurement of the moment of inertia of rotors of induction motors (IMs) is a relevant scientific and applied task that has significant practical importance in the field of electric drive and automation.

2. Literature review and problem statement

As shown in [4], the rotor moment of inertia is one of the key characteristics that largely determines the parameters of the control system of an induction motor. Its value, with the same settings of the regulators, largely affects the speed and magnitude of the overshoot of the control object. At the same time, in [5] it is shown that the rotor moment of inertia of IM is a complex function of a significant number of factors that affect its resulting value and must be taken into account when implementing indirect measurement methods. This causes a significant complexity of the analytical calculation of the rotor moment of inertia and significantly limits the possibility of using such an approach under real production conditions. In confirmation of this circumstance, in [6], a comprehensive analysis of the analytical and experimental calculation of the rotor moment of inertia of an asynchronous motor with a squirrel-cage rotor is described. The authors present the analytical calculation and the simplified proposed analytical approach as an alternative to the direct experimental determination of the moment of inertia. However, taking into account the simplifications adopted in deriving the mathematical model, the application of the proposed method is subject to significant limitations both in terms of the type of electrical machines to which it can be applied and in terms of the accuracy of the results.

An interesting method for measuring the moment of inertia of the rotor, from the point of view of practical implementation, is proposed in [7], in which the moment of inertia is proposed to be measured simultaneously with the braking torque of the engine. However, it is worth noting that this approach involves a rather laborious procedure for individual calculation of the optimal values of the moments of inertia of the load, which must be mechanically connected to the engine under study. And, therefore, the implementation of a universal automated means of measuring the moment of inertia based on it has significant limitations.

A similar approach to solving the problem is proposed in [8], in which the determination of the moment of inertia is combined with the determination of the engine efficiency. However, since the practical implementation of such an approach is associated with the same problem of the need to individually determine the required additional inertial mass, the possibility of its wide practical application is associated with the same limitations.

It is also worth noting that the existing problem of measurement uncertainty was studied in detail in [9], in which the complexity of ensuring the measurement of the rotor moment of inertia of low-power electric machines by indirect measurement methods with an accuracy of more than 10% was theoretically substantiated.

Therefore, taking into account the above, it can be stated that the existing approaches do not offer high-speed measuring means that would be characterized by low cost, ease of operation, and high operational reliability at an acceptable level of measurement accuracy.

There are also three direct methods for experimentally determining the moment of inertia, each of which has its own field of application [10]:

– self-braking method is used for machines with a power of more than 100 kW;

– auxiliary pendulum method is used for electric machines with a power of 10 kW to 1000 kW;

– torsional oscillation method – for engines with a power of less than 10 kW.

The main disadvantage of the considered direct methods for measuring the rotor moment of inertia is the need to dismantle individual structural elements of the electric machine, which fundamentally does not make it possible to automate the process of determining the desired parameter.

Based on our review, it can be concluded that there is no method for measuring the rotor moment of inertia in low-power induction motors suitable for solving the task of automated measurement of the rotor moment of inertia under real production conditions.

3. The aim and objectives of the study

The purpose of our research is to devise an automated method and means of experimental measurement of the rotor moment of inertia in low-power induction motors. This will make it possible to reduce the time and laboriousness of the pre-operational testing of the specified type of equipment.

To achieve the goal, the following tasks were set:

– to devise the concept of an automated method and the structure of a means for measuring the rotor moment of inertia in low-power induction motors;

– to build a mathematical model of a means for measuring the rotor moment of inertia in an electric machine;

– to investigate the metrological characteristics of the proposed method and means for measuring the rotor moment of inertia in low-power induction motors.

4. The study materials and methods

The object of the study is the electromechanical transient process in a low-power induction motor when applying and removing the supply voltage, as well as the influence of the moment of inertia of the IM rotor on its duration.

The principal hypothesis underlying this study assumes the presence of a unique functional dependence of the duration of the electromechanical transient process in an induction motor on the moment of inertia of its rotor.

When deriving the mathematical model of the object of our study, simplifications were adopted that assume instantaneous switching when applying and disconnecting the supply voltage of the stator circuit and the constancy of the stiffness value of the force sensor during its deformation.

During the study, theoretical modeling of electromechanical processes in an induction motor was applied in conjunction with a measuring transducer. To build the mathematical model, methods of classical electromechanics and the theory of transient processes were used. The model was implemented as a system of differential equations that describes the dynamics of currents in the stator and the mechanical motion of the rotor under the action of electromagnetic and mechanical moments.

Mathematical modeling and processing of the results were performed using the Mathcad environment.

5. Devising a method and means for measuring the moment of inertia of the rotor and the results of metrological research

5. 1. Devising the concept of the method and designing the structure of the measuring instrument

Considering the stated limitations in the application of the measurement methods described above, our attention focused on the method of torsional vibrations since it is the most suitable for use in conjunction with the selected object of study. The essence of the method of torsional vibrations is that the motor rotor is suspended in a vertical position on a tension and is driven into a torsional oscillatory motion. In this case, the period of small torsional vibrations is determined, which is then compared with the period of oscillations of a model body, the moment of inertia of which is known [11]

$$J_{OB} = J_M \cdot \left(\frac{T_x}{T_M} \right)^2, \tag{1}$$

where J_M – the moment of inertia of the model body;

T_M – period of oscillation of the model body;

T_x – period of oscillation of the rotor, the moment of inertia of which is being determined.

If the rotor is mounted on a prism and oscillates like a physical pendulum, then the moment of inertia of the rotor can be found from the period of oscillation T of the pendulum

$$J_{OB} = \frac{GaT^2}{4\pi^2}, \tag{2}$$

where G is the weight of the rotor;

a is the distance between the center of gravity and the axis of rotation.

The proposed express method for measuring the moment of inertia of the IM rotor is as follows. The measurement is carried out under the IM motor mode with a mechanically braked rotor (short-circuit mode). In this case, the stator windings are powered by a symmetrical voltage of rated frequency, and the rotor is rigidly connected to the measuring lever, which interacts with the force sensor. The essence of the method is that at the moment of connecting the supply voltage, the following torque is induced on the rotor of the research object [12], which acts on the force sensor (FS) through the measuring lever

$$M_k = \frac{p \cdot m \cdot R1}{2\pi f \cdot \left[(R1+R2)^2 + (X1+X2)^2 \right]} \cdot U^2, (s=1), \tag{3}$$

where p is the number of pairs of poles in IM;

m is the number of phases of the stator circuit;

f is the frequency of the supply voltage;

$R1$, $R2$ is the active resistance of the stator and rotor;

$X1$, $X2$ is the reactive resistance of the stator and rotor;

U is the effective value of the stator supply voltage;

s is the slip.

Since FS is an elastic element, this causes a transient process, the duration of which is equal to τ^1. After the end of the transient process ($t=\tau^1$), the electric machine is de-energized ($U=0$) and the torque at the output of the converter, due to the inertial properties of FS, decreases from value M_k to zero over the time interval τ^0. Since the IM rotor will then perform free damped oscillations, the duration of which is determined by the value of the rotor moment of inertia J_{OB} and stiffness C of the sensor, then, having measured the values of M_k and τ^0, and also knowing the value of C, it is possible to perform an analytical calculation of the value of the rotor moment of inertia.

The block diagram of the measuring device that implements the method described above is shown in Fig. 1.

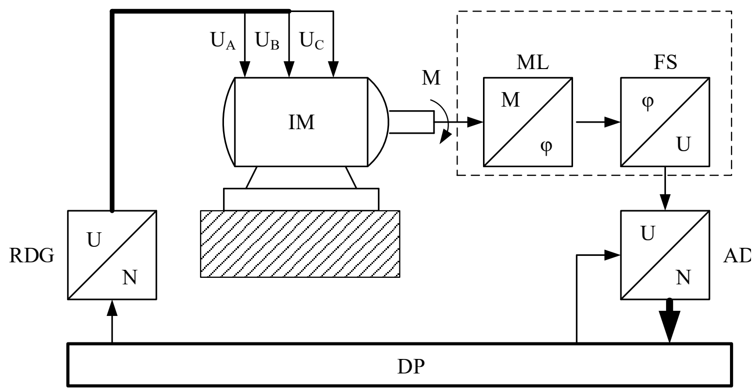


Fig. 1. Generalized structural diagram of a means for measuring the moment of inertia of the rotor in an electric machine

The basic elements of the represented generalized structural diagram (Fig. 1) are an induction motor, a coupling, a moving part of the measuring transducer, and a force sensor.

5. 2. Construction of a mathematical model of a means for measuring the moment of inertia of the rotor in an electric machine

To build a generalized mathematical model of an induction motor, the model of a generalized electric machine [13] was chosen as the initial one, for which the following are valid:

– voltage equations

$$\begin{bmatrix} u_{s\alpha} \\ u_{s\beta} \\ u_{r\beta} \\ u_{r\alpha} \end{bmatrix} = \begin{bmatrix} r_{s\alpha} + \frac{d}{dt} L_{s\alpha} & \frac{d}{dt} M & 0 & 0 \\ 0 & 0 & \frac{d}{dt} M & r_{s\beta} + \frac{d}{dt} L_{s\beta} \\ -M\omega_r & -L_{r\alpha} & r_{r\beta} + \frac{d}{dt} L_{r\beta} & \frac{d}{dt} M \\ \frac{d}{dt} M & r_{r\alpha} + \frac{d}{dt} L_{r\alpha} & L_{r\beta}\omega_r & M\omega_r \end{bmatrix} \times \begin{bmatrix} i_{s\alpha} \\ i_{r\alpha} \\ i_{r\beta} \\ i_{s\chi} \end{bmatrix}; \tag{4}$$

– electromagnetic moment equation

$$M_{em} = M \cdot \left(i_{s\beta} i_{r\alpha} - i_{s\alpha} i_{r\beta} \right); \tag{5}$$

– equation of motion

$$J \cdot \frac{d\omega_r}{dt} \pm M_o = M_{em}. \tag{6}$$

Therefore, the system of equations for electromechanical energy conversion of an induction motor consists of four Kirchhoff equations (4), as well as equations (5), (6).

The next step was to obtain the transformation function of the moving part of the measuring device. The structural diagram of the measuring instrument is shown in Fig. 1. In this measuring instrument (MI), the following sequence of measuring transformations is performed. The starting torque from the output shaft of IM through the coupling and the measuring lever with a length l acts on the force sensor.

Using the principles of the theory of electromechanical measuring transformation [14], a differential equation was derived that describes the process of motion of the moving part of the proposed measuring instrument. During the rotation of a solid body (the moving part of MI) around its axis, the product of the moment of inertia and the angular acceleration is equal to the sum of the moments of forces acting on the body relative to the same axis, i.e.

$$J\cdot\frac{d^2\varphi(t)}{dt^2}=\sum_{i=1}^{n}M_i, \tag{7}$$

where J is the moment of inertia of the moving part of the transducer;

φ is the angle of rotation of the moving part of the transducer;

M is the moments acting on the moving part of the transducer.

The following moments act on the moving part of the measuring instrument during its movement:

1. The torque, which is generally $M(t)$, which is caused by the action of the input quantity [$M_p=f(\alpha)$, $M_k=f(U_k)$, J_r, M_E].

2. The counter-torque, which is caused by the elastic properties of the force sensor

$$M_{\bullet p}=-C\cdot\varphi(t), \tag{8}$$

where C is the stiffness of the force sensor.

3. The moment of stabilization, which is equal to the product of the stabilization coefficient η and the angular velocity of rotation

$$M_{,}=-\eta\cdot\frac{d\varphi(t)}{dt}. \tag{9}$$

When substituting the values of the obtained moments in (7), the following was obtained

$$J\cdot\frac{d\varphi^2(t)}{dt^2}+\eta\cdot\frac{d\varphi(t)}{dt}+C\cdot\varphi(t)=M(t). \tag{10}$$

Converting (10) to the commonly used form, we have

$$\frac{d^2\varphi(t)}{dt^2}+2\cdot\varepsilon\cdot\omega\frac{d\varphi(t)}{dt}+\omega^2\varphi(t)=\frac{M(t)}{J}, \tag{11}$$

where $\omega=\sqrt{C/J}$ is the natural frequency of free (undamped) oscillations of the converter;

$\varepsilon=\eta/\left(2\cdot\sqrt{JC}\right)$ – degree of damping of free oscillations.

Having carried out transformation (11), we obtain

$$J\cdot\frac{d\varphi^2(t)}{dt^2}+\eta\cdot\frac{d\varphi(t)}{dt}+C\cdot\varphi(t)=K\cdot U^2+A. \tag{12}$$

After converting (12) to the commonly used form, we shall have

$$\frac{d^2\varphi(t)}{dt^2}+2\cdot\varepsilon\cdot\omega\frac{d\varphi(t)}{dt}+\omega^2\varphi(t)=\frac{1}{J}\cdot\left(KU^2+A\right), \tag{13}$$

where $\omega=\sqrt{C/J}$ – natural frequency of free (undamped) oscillations of the converter;

$\varepsilon=\eta/\left(2\cdot\sqrt{JC}\right)$ – degree of damping of free oscillations.

Expression (13) allows us to simulate the measuring transformation of the input quantity into the output quantity only for a constant torque (if $M_k=\text{const}$ then $U=\text{const}$). Therefore, this model can be used to analyze measuring converters that operate under a semi-automated mode of operation and have a rather low speed (one measured value of the starting torque is obtained in approximately 10 s). Therefore, the task of building a mathematical model for automated measuring devices that must have a much higher speed (hundreds or more measured values of M_k during the time interval when the winding temperature changes from room to the calculated operating one) arises.

Taking this into account, the following mathematical model of the input signal for the converter was proposed

$$M_k=K\left(U+k\cdot t\right)^2+A, \tag{14}$$

where $k=h/\tau$; h – quantization step of the supply voltage, which is formed using a variator;τ – duration of the transient process of the measuring converter.

Taking into account (14), equation (13) can be represented as

$$\frac{d^2\varphi(t)}{dt^2}+2\cdot\varepsilon\cdot\omega\frac{d\varphi(t)}{dt}+\omega^2\varphi(t)= =\frac{1}{J}\cdot\left[K\cdot\left(U+kt\right)^2+A\right]. \tag{15}$$

Using the above results, a differential equation was derived that describes the physical processes in the converter during power supply and de-energizing of the stator windings with a braked rotor

$$J\frac{d^2\varphi(t)}{dt^2}+P\frac{d\varphi(t)}{dt}+C\varphi(t)=\begin{cases}M_{ps},0\le t\le\tau^1,\\0,t>\tau^1,\end{cases} \tag{16}$$

where $J=J_{OB}+J_{BB}$ – moments of inertia of the motor rotor and the measuring lever, respectively.

Having transformed (16) into the commonly used form, a linear inhomogeneous differential equation with constant coefficients was derived

$$\varphi''(t)+2\varepsilon\omega\varphi'(t)+\omega^2\varphi(t)=\begin{cases}\lambda,0\le t\le t_*^1,\\0,t>t_*^1,\end{cases} \tag{17}$$

where $\lambda=M_k/J$ is the torque constant.

To find a partial solution to this equation, the following initial conditions were adopted, corresponding to the operating mode of the machine

$$\varphi(0)=\varphi'(0)=0. \tag{18}$$

The differential equation was solved in the operator form. For this purpose, the right-hand side of equation $f(t)$ was represented by the Heaviside unitary function $H(t)$, in which [15]

$$H(t)=\begin{cases}1,t\ge0,\\0,t<0,\end{cases} \tag{19}$$

then

$$f(t)=\lambda\cdot\left[H(t)-H\left(t-\tau^{1}\right)\right]. \quad (20)$$

Using the delay theorem [16], on the basis of (20), we obtained

$$f(t)=L^{-1}\left(\frac{\lambda}{s}\cdot\left(1-e^{-s\tau^{1}}\right)\right). \quad (21)$$

Since the initial conditions are zero, then by accepting $\varphi(t)=L^{-1}(\Phi(s))$, we can derive the following operator equation

$$s^{2}\Phi(s)+2\varepsilon\omega s\Phi(s)+\omega^{2}\Phi(s)=\frac{\lambda}{s}\cdot\left(1-e^{-s\tau^{1}}\right), \quad (22)$$

from which it follows that

$$\Phi(s)=\frac{\lambda}{s\left(s^{2}+2\varepsilon\omega s+\omega^{2}\right)}\cdot\left(1-e^{-s\tau^{1}}\right). \quad (23)$$

To find the original φ(*t*), the decomposition theorem [17] was applied

$$\frac{\lambda}{s\left(s^{2}+2\varepsilon\omega s+\omega^{2}\right)}=\frac{B_{1}}{s}+\frac{B_{2}s+B_{3}}{(s+\varepsilon\omega)^{2}+\omega^{2}\left(1-\varepsilon^{2}\right)}, \quad (24)$$

where B_1, B_2, B_3 are the integration constants, which are found by the method of undetermined coefficients.

As a result, we obtain

$$\begin{cases} B_{1}=\dfrac{1}{\omega^{2}};\\ B_{2}=-\dfrac{1}{\omega^{2}};\\ B_{3}=-\dfrac{2\varepsilon}{\omega}. \end{cases} \quad (25)$$

Using the table of images and the properties of Laplace transformations, in particular the theorem of delay [18], we obtained

$$\varphi(t)= \\ =\begin{cases} \dfrac{\lambda}{\omega^{2}}\cdot\left[\begin{array}{l}1-e^{-\varepsilon\omega t}\times\\ \times\left(\begin{array}{l}\cos\left(\sqrt{1-\varepsilon^{2}}\omega t\right)+\\ +\dfrac{\varepsilon}{\sqrt{1-\varepsilon^{2}}}\cdot\sin\left(\sqrt{1-\varepsilon^{2}}\omega t\right)\end{array}\right)\end{array}\right], t<\tau^{1},\\ \dfrac{\lambda}{\omega^{2}}\cdot\left\{\begin{array}{l}e^{-\varepsilon\omega\left(t-\tau^{1}\right)}\times\\ \times\left[\begin{array}{l}\cos\left(\sqrt{1-\varepsilon^{2}}\omega\left(t-\tau^{1}\right)\right)+\\ +\dfrac{\varepsilon}{\sqrt{1-\varepsilon^{2}}}\times\\ \times\sin\left(\sqrt{1-\varepsilon^{2}}\omega\left(t-\tau^{1}\right)\right)-\\ -e^{-\varepsilon\omega t}\left[\begin{array}{l}\cos\left(\sqrt{1-\varepsilon^{2}}\omega t\right)+\\ +\dfrac{\varepsilon}{\sqrt{1-\varepsilon^{2}}}\times\\ \times\sin\left(\sqrt{1-\varepsilon^{2}}\omega t\right)\end{array}\right]\end{array}\right]\end{array}\right\}, t>\tau^{1}. \end{cases} \quad (26)$$

The first equation in (26) allows us to determine the duration of the transient process in the converter without taking into account the transient process that occurs in the object of study when the supply voltage is applied to the stator windings. Since the IM is under the “short circuit” mode, the duration t_{OB} of the transient process of the engine with a braked rotor can significantly exceed the duration τ^1. Therefore, the first equation (26) was supplemented with a system of differential equations of the engine under study

$$\varphi\begin{cases} \dfrac{dI_{r\alpha}(t)}{dt}=k\left(L_{m}\left(-U_{s\alpha}(t)+R_{s}I_{s\alpha}(t)\right)\right)-\\ -L_{s}\left(R_{r}I_{r\alpha}(t)+n_{r}(t)\left(L_{r}I_{r\beta}(t)+L_{m}I_{s\beta}(t)\right)\right);\\ \dfrac{dI_{s\alpha}(t)}{dt}=k\left(L_{r}\left(U_{s\alpha}(t)-R_{s}I_{s\alpha}(t)\right)\right)+\\ +L_{m}\left(R_{r}I_{r\alpha}(t)+n_{r}(t)\left(L_{r}I_{r\beta}(t)+L_{m}I_{s\beta}(t)\right)\right);\\ \dfrac{dI_{r\beta}(t)}{dt}=k\left(L_{m}\left(-U_{s\beta}(t)+R_{s}I_{s\beta}(t)\right)\right)-\\ -L_{s}\left(R_{r}I_{r\beta}(t)-n_{r}(t)\left(L_{r}I_{r\alpha}(t)+L_{m}I_{s\alpha}(t)\right)\right);\\ \dfrac{dI_{s\beta}(t)}{dt}=k\left(L_{r}\left(U_{s\beta}(t)-R_{s}I_{s\beta}(t)\right)\right)+\\ +L_{m}\left(R_{r}I_{r\beta}(t)-n_{r}(t)\left(L_{r}I_{r\alpha}(t)+L_{m}I_{s\alpha}(t)\right)\right);\\ \dfrac{dn_{r}(t)}{dt}=\dfrac{p}{J}\cdot\left(M_{e}(t)-M_{0}\right);\\ k=\dfrac{1}{L_{r}L_{s}-L_{m}^{2}};\\ J=J_{me}+J_{ee};\\ M_{0}=K\cdot U^{2};\\ K=\dfrac{pmR1}{2\pi f\left[(R1+R2)^{2}+(X1+X2)^{2}\right]};\\ M_{D}(t)=\left[M_{e}(t)-M_{0}\right];\\ \rho=\dfrac{l\cdot\xi\cdot h^{3}}{e\cdot g\cdot r^{4}};\\ M_{c}(t)=g\cdot\rho\cdot\dfrac{M_{D}(t)}{J\cdot\omega^{2}}\times\\ \times\left[1--e^{-\varepsilon\omega t}\left(\begin{array}{l}\cos\left(\sqrt{1-\varepsilon^{2}}\omega t\right)+\\ +\dfrac{\varepsilon}{\sqrt{1-\varepsilon^{2}}}\cdot\sin\left(\sqrt{1-\varepsilon^{2}}\omega t\right)\end{array}\right)\right], \end{cases} \quad (27)$$

where $M_e(t)$ is the electromagnetic moment of the induction motor;

M_0 – starting moment of the induction motor;

J_{OB} – moment of inertia of the motor rotor;

J_{BB} – moment of inertia of the measuring lever;

$M_D(t)$ – dynamic moment on the motor rotor shaft;

ρ – constant of the strain-resistive transducer;

$M_c(t)$ – moment acting on the force sensor.

Next, the duration τ^0 of the transient process that occurs in the measuring transducer during de-energizing of the induction motor was found.

When expressing the function of the moment acting on the force sensor from system (27), the following was obtained:

$$M_c(t)=g\rho\frac{\lambda}{\omega^2}\times\left\{\begin{array}{l}e^{-\varepsilon\omega t_*^0}\times\\ \times\left[\begin{array}{l}\cos\left(\sqrt{1-\varepsilon^2}\omega\tau^0\right)+\dfrac{\varepsilon}{\sqrt{1-\varepsilon^2}}\cdot\sin\left(\sqrt{1-\varepsilon^2}\omega\tau^0\right)-\\ -e^{-\varepsilon\omega t}\cos\left(\begin{array}{l}\left(\sqrt{1-\varepsilon^2}\omega t\right)+\\ +\dfrac{\varepsilon}{\sqrt{1-\varepsilon^2}}\cdot\sin\left(\sqrt{1-\varepsilon^2}\omega t\right)\end{array}\right)\end{array}\right]\end{array}\right\}, \quad (28)$$

where $\tau^0=t-\tau_\Sigma^1$ is the duration of the transient process that occurs in the converter during the machine de-energizing.

Since the second component in equation (28) characterizes the transient process in the converter during IM energizing, at time point $t>\tau_\Sigma^1$ it is an infinitely small value compared to the first component. Therefore, (28) was written in the form

$$M_c(t)\cong g\rho\cdot\frac{\lambda}{\omega^2}\times\left\{e^{-\varepsilon\omega\tau^0}\cdot\left[\begin{array}{l}\cos\left(\sqrt{1-\varepsilon^2}\omega\tau^0\right)+\\ +\dfrac{\varepsilon}{\sqrt{1-\varepsilon^2}}\cdot\sin\left(\sqrt{1-\varepsilon^2}\omega\tau^0\right)\end{array}\right]\right\}. \quad (29)$$

For simplification, the following substitutions have been introduced

$$\begin{cases}1=a\cdot\sin\gamma;\\ \dfrac{\varepsilon}{\sqrt{1-\varepsilon^2}}=a\cdot\cos\gamma,\end{cases} \quad (30)$$

where a and γ are constants defined as

$$\begin{cases}a=\dfrac{1}{\sqrt{1-\varepsilon^2}};\\ \gamma=\mathrm{arctg}\dfrac{1}{\sqrt{1-\varepsilon^2}}.\end{cases} \quad (31)$$

Taking into account the entered variables, expression (29) can be represented in the following form

$$M_{\tilde{n}}=g\rho\cdot\frac{\lambda}{\omega^2}\cdot\left[e^{-\varepsilon\omega\tau^0}\cdot a\cdot\sin\left(\omega\sqrt{1-\varepsilon^2}\tau^0+\gamma\right)\right]. \quad (32)$$

Consequently, oscillations occur with a damping amplitude

$$A=g\lambda\cdot\frac{a\cdot\lambda}{\omega^2}\cdot e^{-\varepsilon\omega\tau^0}, \quad (33)$$

frequency $\omega_1=\omega\cdot\sqrt{1-\varepsilon^2}$ and period $T=\dfrac{2\pi}{\omega\cdot\sqrt{1-\varepsilon^2}}$.

Next, the time points at which the oscillation speed was zero were calculated

$$\left(M_c(t)\right))=g\rho\cdot\frac{a\cdot\lambda}{\omega}\cdot e^{-\varepsilon\omega\tau^0}\times\left[\begin{array}{l}-\varepsilon\cdot\sin\left(\omega\sqrt{1-\varepsilon^2}\tau^0+\gamma\right)+\\ +\sqrt{1-\varepsilon^2}\cdot\cos\left(\omega\sqrt{1-\varepsilon^2}\tau^0+\gamma\right)\end{array}\right]. \quad (34)$$

Equating (34) to zero, we obtained

$$\left(M_c(t)\right)'=0\Rightarrow \mathrm{tg}\left[\omega\sqrt{1-\varepsilon^2}t_k+\gamma\right]=\frac{\sqrt{1-\varepsilon^2}}{\varepsilon}. \quad (35)$$

Then

$$\omega\sqrt{1-\varepsilon^2}t_k+\gamma=\mathrm{arctg}\frac{\sqrt{1-\varepsilon^2}}{\varepsilon}+\pi k, \quad (36)$$

on the basis of which we obtained

$$t_k=\frac{\pi}{\omega\cdot\sqrt{1-\varepsilon^2}}\cdot k. \quad (37)$$

Amplitude values are found taking into account expression (37)

$$\left(M_c\right)_k=g\rho\cdot\frac{\lambda}{\omega^2}\cdot e^{-\frac{\varepsilon}{\sqrt{1-\varepsilon^2}}\pi k}\cdot a\cdot\sin\left[\pi k+\gamma\right]=$$
$$=g\rho\cdot\frac{\lambda}{\omega^2}e^{-\frac{\varepsilon}{\sqrt{1-\varepsilon^2}}\pi k}\cdot a\cdot(-1)^k\sin\gamma. \quad (38)$$

The value of τ^0 was found from the condition

$$\left(M_c\right)_k\le\delta\Rightarrow g\rho\cdot\frac{\lambda}{\omega^2}\cdot e^{-\frac{\pi\varepsilon}{\sqrt{1-\varepsilon^2}}k}\cdot a\cdot\sin\gamma\le\delta. \quad (39)$$

Number of periods of damping oscillations

$$N\ge\frac{\sqrt{1-\varepsilon^2}}{\pi\varepsilon}\cdot\ln\left(g\rho\cdot\frac{a\lambda\cdot\sin\gamma}{\omega^2\cdot\delta}\right). \quad (40)$$

Then the duration τ^0 of the transient process that occurs in the converter during de-energizing of the object of study is equal to

$$\tau^0\ge\frac{\pi}{\omega\sqrt{1-\varepsilon^2}}\cdot N\ge\frac{1}{\varepsilon\omega}\cdot\ln\left(\frac{g\cdot K1\cdot\lambda}{\delta\cdot\omega^2}\right). \quad (41)$$

Taking into account the notations taken in (17), equation (41) will take the final form

$$\tau^0=J\cdot\frac{2}{\eta}\cdot\ln\left(M_k\cdot\frac{g\cdot\rho}{\delta\cdot\tilde{N}}\right). \quad (42)$$

Dependence (42) establishes a unique relationship between the moment of inertia of the IM rotor and the duration of transient process when removing voltage from the stator winding and takes into account the design parameters of the measuring device. Therefore, when designing a suitable measuring instrument, it can be used as its transformation equation.

5. 3. Studying the metrological characteristics of the proposed method and measuring device

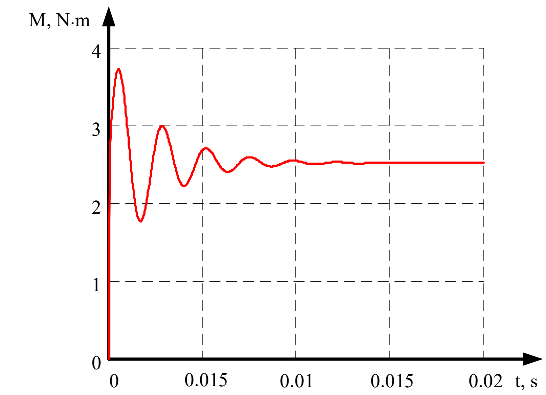
The modeling results obtained based on the solution to the system of nonlinear equations (27) for the parameters of the induction motor model I4BI-03T-71 are shown in Fig. 2.

In this case, the dynamic error that occurs in the converter can be calculated as follows [19]

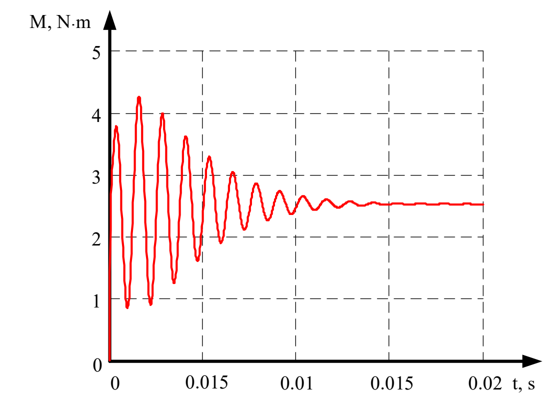
$$\delta_M(t)=\frac{M_c(t)-M_a}{M_a}\cdot 100\%, \quad (43)$$

where M_a is the actual value of the moment.

The graphical representation is shown in Fig. 3.



a



b

Fig. 2. The form of the transient process at the converter output at the moment of switching on the control object: *a* — without taking into account the transient process of the induction motor; *b* — taking into account the transient process of the induction motor

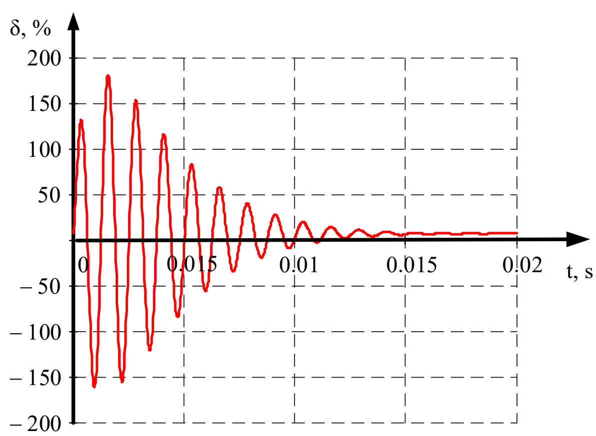


Fig. 3. Graphical representation of dynamic error

Modeling of dependence (29) and transformation equation (42), which describe the motion process of the measuring transducer of the moment of inertia, was carried out for IM model I4BI-03T-71 [20] and the measuring transducer with technical characteristics given in Table 1.

A graphical representation of the dependence of change in torque over time ($M_c = f(t)$) when the supply voltage is turned off is shown in Fig. 4.

Fig. 5 shows changes in torque over time at the converter output during energizing (upper dependence $M_{c1} = f(t)$) and after de-energizing the stator windings (lower dependence $M_{c0} = f(t)$).

To obtain a static characteristic of the means for measuring the moment of inertia of the rotor, the following restrictions were introduced

$$\tau^0 = f(J)\begin{cases} M_{ps} = \text{const}; \\ \delta = \text{const}; \\ C = \text{const}. \end{cases} \tag{44}$$

The general view of the static characteristic of the measuring transducer is shown in Fig. 6.

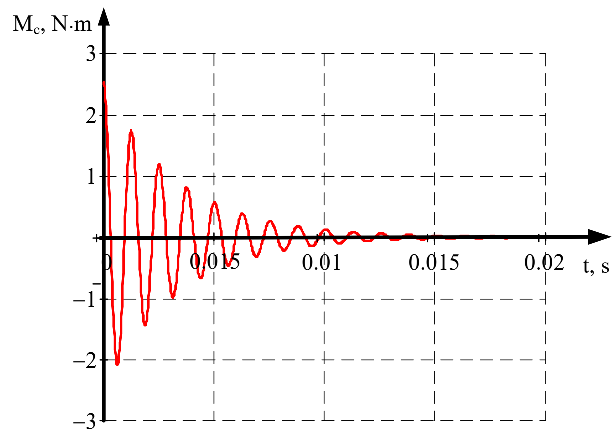


Fig. 4. Change of moment in time at the output of the measuring transducer

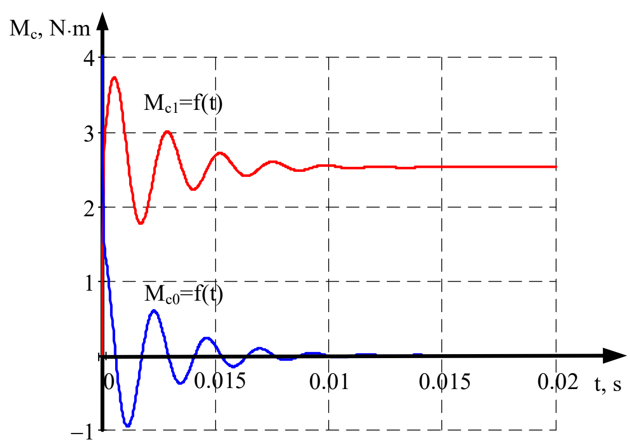


Fig. 5. Forms of torque dependences at the output of the measuring transducer

Table 1

IM and measuring transducer parameters used for simulation

IM characteristics		MI characteristics	
Designation	Value	Designation	Value
$R1$, Ohm	8.1	$C,\ \frac{\text{N}\cdot\text{m}}{\text{rad}}$	$3\cdot10^{5}$
$R2$, Ohm	6.3	$\eta,\ \frac{\text{N}\cdot\text{m}\cdot\text{s}}{\text{rad}}$	30
$X1$, Ohm	204.83	δ, N	10^{-4}
$X2$, Ohm	368.69	λ, N	712.8
J, N m^2	0.0078	τ_{Σ}^{1}, s	0.5

The influence of stiffness C, moment of inertia J, and error δ on the duration of the transient process τ^0 was evaluated. To evaluate this influence, surfaces were modeled that characterize the dependences $\tau^0 = f(J, C)$ and $\tau^0 = f(J, \delta)$. The modeling results are shown in Fig. 7.

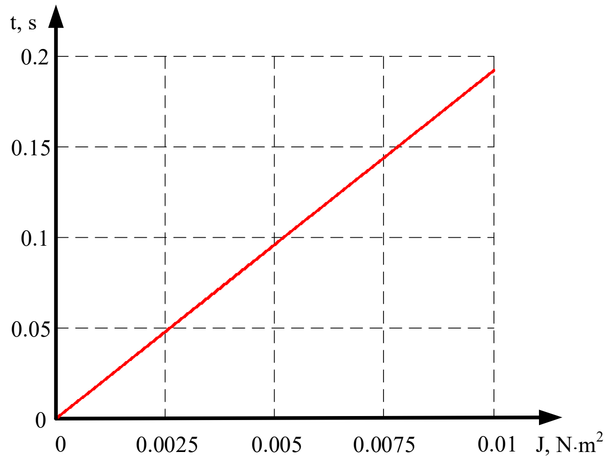


Fig. 6. Static characteristic of the measuring transducer of moment of inertia

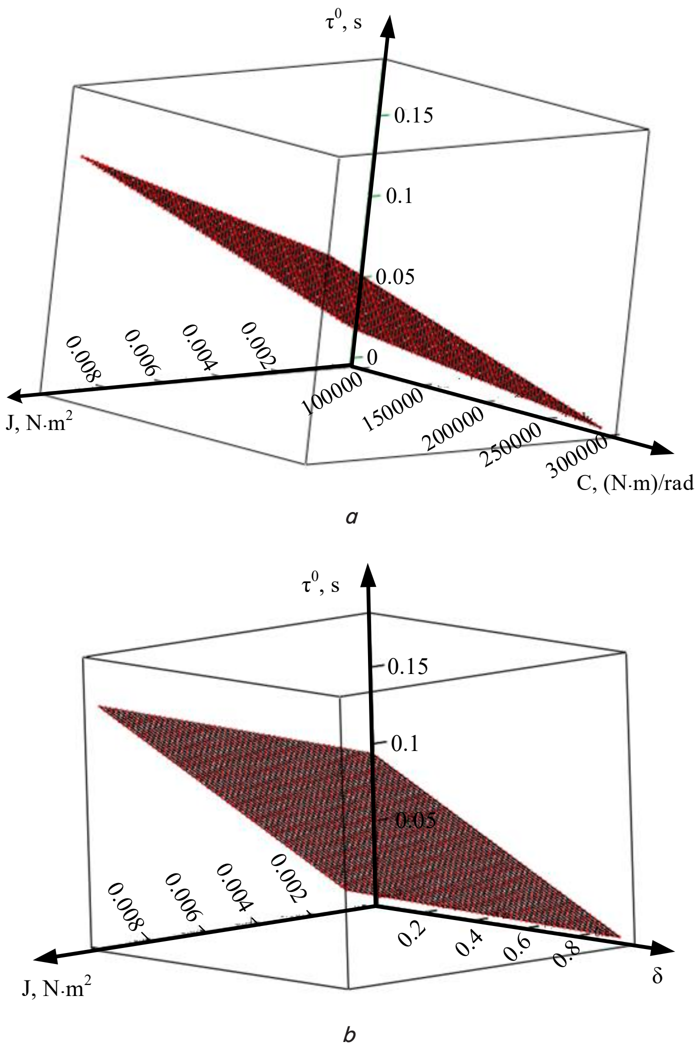


Fig. 7. Results of modeling the dependence of duration of the transient process on the parameters of IM and measuring transducer: *a* – dependence $\tau^0 = f(J, C)$; *b* – dependence $\tau^0 = f(J, \delta)$

The studied metrological characteristics make it possible to fully assess the specificity and practical possibility of applying the proposed method and measuring instrument.

6. Discussion of results based on studying the automated method and means for experimental measurement of the moment of inertia

Analysis of the results shown in Fig. 2 reveals that the transient process in the measuring converter taking into account the induction motor is much longer in time and is $\tau^1_\Sigma = \tau_{me} + \tau^1$. Therefore, it is necessary to de-energize the control object at time τ^1_Σ.

From the results shown in Fig. 3, it follows that the proposed model (27) reproduces the physical processes of measuring the rotor moment of inertia with high accuracy. After the completion of the transient process, the error does not exceed 8%, which is a completely acceptable result.

As can be seen from Fig. 4, after the power supply is turned off, the torque at the output of the converter performs free damped oscillations and changes from the value of M_k to zero over time τ^0 (for IM of type I4BI-03T-71, $\tau^0 = 0.15$ s).

The curves shown in Fig. 5 confirm the above assumption that the forms of the torque dependences at the output of the measuring transducer for a real induction motor both when applying and removing the stator voltage have the character of a damped oscillation.

Fig. 6 demonstrates that the static characteristic of the means for measuring the moment of inertia $J = f(\tau^0)$ is linear in the range of change of the informative parameter for low-power electric machines.

From the results of analysis of the dependences shown in Fig. 7, it is obvious that the surfaces that characterize the influence of the selected parameters $\tau^0 = f(J, C)$ and $\tau^0 = f(J, \delta)$, are also linear, which potentially simplifies the hardware and software implementation of the measuring device [21].

Our results are attributed to considering the physical and mathematical features of the process of interaction of the electromagnetic field in IM with the moving part of the measuring transducer, as well as the features of the rotor dynamics during transient processes.

The proposed method is based on the analysis of free damped oscillation of the rotor after its short-term excitation. Applying voltage to the stator windings during mechanical locking of the rotor generates electromagnetic forces that excite the rotating part of the system. After removing the voltage, the rotor, being locked by an elastic measuring transducer, performs free oscillations, and their duration determines the moment of inertia.

Our mathematical model takes into account the electromechanical characteristics of the induction motor and the interaction with the measuring transducer. It makes it possible to quantitatively describe the relationship between the duration of damped oscillations and the moment of inertia of the rotor. This approach ensures the uniqueness and reproducibility of the results. An error of up to 8% indicates that the model adequately reflects the real process and can be used in practical measurements without significant loss of accuracy.

The linear relationship between the duration of the transient process and the moment of inertia is very important because it simplifies both the hardware and software implementation of the measuring instrument. This means that complex nonlinear transformations, which usually complicate accurate measurements, are absent, and simple algorithms can be used to evaluate the results. Such simplification opens up opportunities for designing inexpensive and reliable instruments for mass use in the industry [22].

The specified method and means of measurement are characterized by limitations associated with the limiting mechanical moment, and therefore the power of IM, since with the increase of the latter it is necessary to increase the dimensions of the measuring transducer, which would inevitably lead to an increase in the instrumental component of the error. Therefore, it is advisable to limit the scope of application of this approach to low-power electrical machines.

Further development of this research may address several areas, covering both the improvement of the measurement method itself and the expansion of its application. In particular, the search for ways to adapt the method to higher-power electrical machines, as well as to other types of machines, such as synchronous or collector, is promising. This could significantly expand the scope of using our technological advancement but, at the same time, it creates new challenges associated with the complication of the dynamics of objects and the need to take into account additional energy and thermal factors.

Another direction may be to increase the measurement accuracy by improving the mathematical model, in particular by taking into account friction losses, nonlinearities of the magnetic circuit or temperature changes in parameters. At the same time, such detailing inevitably leads to the complication of calculations and the need for more accurate identification of the object parameters.

7. Conclusions

1. The concept of an automated method and the structure of a means for measuring the rotor moment of inertia in low-power induction motors have been devised. The implementation of the proposed method involves short-term operation of the electric machine under the short-circuit mode. The proposed concept implements indirect measurement of the rotor moment of inertia in IM, as a parameter that is functionally related to the duration of the electromagnetic transient process of the machine when removing voltage from the stator winding. In this case, the measurement experiment is performed under an automatic mode for a period of time significantly shorter than the heating time of the machine windings from a cold state to the rated temperature (as follows from the modeling results, the total duration of the measurement experiment is at the order of 0.3 s). The advantage of the proposed concept of the specified method in comparison with known analogs is the possibility of automating the measurement experiment, the reduced labor intensity and duration of the experiment.

The structure of the designed measuring instrument, which implements the specified method, is characterized by relative simplicity of implementation, low cost, and compatibility with IMs of various designs. At the same time, taking into account the absence of moving electrical contacts and mechanical connections, as well as a small number of structural elements, we can conclude that it has high operational reliability.

2. A mathematical model of IM has been built in combination with a measuring transducer, which provides the ability to uniquely relate the duration of the transient process of rotor settling after removing the supply voltage under the condition of mechanical blocking of the latter with its moment of inertia. Based on the specified mathematical model, the equation of the transformation of the measuring instrument for the moment of inertia of the rotor, which implements the proposed measurement method, has been derived. It is shown that the error of the resulting mathematical model does not exceed 8%.

3. The metrological characteristics of the proposed method for measuring the moment of inertia of the rotor have been analyzed. As a result of our analysis, it was found that the dependences of duration of the transient process on the parameters of IM and measuring transducer are linear. This feature potentially simplifies the hardware and software implementation of the measuring instrument.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study, as well as the results reported in this paper.

Funding

The research was conducted with funding from the state budget of Ukraine within the framework of the applied scientific work "The latest concept of the development of the Ukrainian agro-industrial complex on the principles of a "green" economy" (State registration number 0124U000470).

Data availability

The data will be provided upon reasonable request.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

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