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DEVISING A METHOD OF DISCRETE SEARCH FOR A PLANE THAT CRASHED BY USING THE BLACKWELL-BLACK-KADAN RATIO

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The object of this study is the process of searching for a plane that crashed by using search tools. The main hypothesis of the study assumes that the use of a uniformly optimal search strategy in a discrete search zone taking into account the Blackwell-Black-Kadan relation could minimize the average time for detecting a plane that crashed. An optimal Bayesian rule has been formulated, which involves determining the maximum value of the likelihood ratio in the current discrete search sector and comparing it with the threshold. A class of uniformly optimal search strategies has been introduced. A method of discrete search for a plane that crashed has been improved, according to which, unlike in the known analogs:

– the a priori probability of finding the search object in the search sector is taken into account;

– the probability that the search object will be detected when viewing the search sector is calculated;

– the Blackwell-Black-Kadan relations are determined;

– the obtained Blackwell-Black-Kadan values are ranked, and the sequence of the search sectors is examined in accordance with the obtained ranking of the Blackwell-Black-Kadan ratio values.

The average time to detect the search object was estimated. It has been established that when optimizing the search for a plane that crashed, the average search time for the search object is reduced by 12%.

The limitation of the study is a simplified representation of the search area, which is given by a regular discrete grid without taking into account complex terrain or prohibited areas. In addition, external factors such as weather conditions, wind, etc., which may affect the speed and route of the search vehicle, are not taken into account.

The disadvantage of the improved method is its application only for the case of a discrete structure search area

Keywords: Blackwell-Black-Kadan relation, discrete search, search object, search and rescue operation, uniformly optimal search strategy

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1. Introduction

The intensive development of the aviation industry contributes to the increase in the efficiency of transportation, including the reduction of the time for the delivery of cargo and passengers, as well as the integration of different types of air transport into a single logistics system [1]. At the same time, the rapid growth of air traffic creates new challenges related to ensuring flight safety and conducting air search and rescue of persons in distress. An additional significant factor in the increase in air accidents is military conflicts (wars), in which military aircraft of various types are actively involved in conducting combat operations [2, 3]. A comparative analysis of the quantitative indicators of air accidents in military aviation has shown that the losses of aircraft directly depend on the intensity of combat operations [4].

Therefore, there is a need to improve the process of air search and rescue of persons in distress. The positive outcome of a search and rescue operation (SRO) depends on how quickly and effectively the search is planned and conducted [5]. Commercial aircraft are equipped with emergency transponders of the international satellite system COSPAS-SARSAT (Search and Rescue Satellite-Aided Tracking). However, limited resources, time, and terrain (mountainous terrain, water search) significantly complicate the conduct of SROs [6, 7]. Conventional search and rescue methods involve the use of ships and aircraft (helicopters) [6, 7]. Such means are expensive, have certain limitations, and depend on the human factor.

Recently, unmanned aerial vehicles (UAVs) have been used in missile defense [8, 9]. The use of UAVs significantly increases the effectiveness of missile defense. However, search methods using UAVs need to be improved.

Thus, the area of scientific research related to the development of a method for searching for a plane that crashed is relevant.

2. Literature review and problem statement

In [10], the results of research on the detection of victims using aerial photographs of UAVs in Search and Rescue (SAR) operations are reported. Machine learning approaches are considered in detail, their comparison is carried out according to such factors as image types, data sets for training, model details, hardware for processing, and evaluation methods. Modern equipment for data processing in the design of UAVs under a real-time mode is considered. The authors of the work substantiate that the integration of UAVs and artificial intelligence (AI), in particular machine learning algorithms such as deep convolutional neural networks, could reduce the time for scanning large search areas, identifying, and determining the location of victims. The issues of the order of inspection of the search area remain unresolved. The likely reason is the failure to take into account the structure of the search area.

In [11], modern and promising methods of using UAVs in SAR-type SROs are considered. The focus is on advances in sensor integration, UAV payload, and simultaneous coordination of multiple UAVs. The paper also explores how the use of AI can improve the efficiency of these operations. It is clear from the analysis of the paper that coordinated multi-drone systems have the potential to expand the search coverage area. The paper argues that future advances in AI and autonomous technologies will allow UAVs to perform complex tasks with minimal human intervention. Improved sensor technologies will expand victim detection capabilities, including infrared imaging and biometric monitoring. However, challenges such as range, limited battery life, and UAV payload remain unaddressed.

In [12], the automatic detection of victims and objects in images and videos obtained from UAVs during missile defense was investigated. The reliability of modern detectors, such as Faster Region-based Convolutional Neural Network, You Only Look Once, version 4 (YOLOv4), Retina Network, and Cascade Region-based Convolutional Neural Network, was considered. The results of automatic detection of victims were compared. Due to its high speed and accuracy, as well as a small number of false positives, the YOLOv4 detector was selected for further research. However, the issues of determining the strategy for searching for victims and the procedure for inspecting the search area remained unresolved.

In [13], the process of planning the route of a swarm of UAVs involved in the search for people in distress is considered. The use of the original Dynamic Butterfly Optimization (DBO) algorithm leads to an imbalance between global research and local operation. Based on the well-known DBO algorithm, an improved version of it, Logistic Opposition-Based Dynamic Butterfly Optimization (LODBO), was developed in the study. The main goal was to introduce landmark operators into the movement behavior of dung beetles to expand the search range. At the same time, the adapted factor for dung beetles, which balances the exploration and exploitation strategies, is modified to improve the local search capabilities of the affected individuals. Then, the algorithm is applied to the missile defense environment and UAV swarms. Experimental results showed that the optimal route length of the proposed LODBO algorithm is 10.47% less than that of the known DBO algorithm, and the calculation time is reduced by 16.99%. However, in [13], the structure of the search area and the availability of a priori information about the coordinates of the affected individuals are not taken into account.

In [14], the case of maritime search and rescue is considered. In this case, UAVs are used. A mathematical programming model was built, taking into account the size of the search area and the total time allocated for the search. The total time is represented as an objective function for the maritime search and rescue route planning problem. A bee algorithm based on the Turn-Based Butterfly Algorithm (TBBA) was also proposed, which takes into account the complexity of the terrain and the limitations of the missile defense. TBBA uses a strategy to control the search of bees and improves the individual structure to reduce the probability of conducting an incorrect search. Also, the proposed algorithm uses an individual replacement strategy, which randomly generates a new search area to replace the worst one when the search results are unsatisfactory. The effectiveness of the proposed algorithm was confirmed by various experiments. TBBA allows one to obtain a more optimal solution for missile defense compared to the search algorithm on neighboring search areas. But in [14] the case of a continuous search area is considered, the search time is not specified, and the features of determining the current search area are not taken into account.

In order to improve the efficiency of using UAVs in missile defense, in [15] a method is presented that uses a modern object detection network. The method involves detecting victims in real time on board the UAV using standard Red, Green, Blue (RGB) cameras, with minimal operator intervention.

This provides increased autonomy of the UAV and increases the search range. In addition, UAVs transmit images of detected persons and coordinates of places containing potential victims, which are determined by the on-board detector. It is proposed to organize parallel operation of several UAVs with transmission of information to one operator. The YOLOv4 detection network was selected, pre-trained on the Common Objects in Context dataset and adapted for detecting victims at sea. For this purpose, special datasets were recorded and annotated that simulate the presence of victims at sea. The proposed approach was tested by the authors of the work on an independent test dataset, demonstrating high capabilities for detecting the search object. However, in [15], the search time limit was not taken into account. This could lead to a search for a significant amount of time, which is unacceptable in the case of missile defense.

In [16], an ant colony algorithm was developed for designing a search and rescue route at sea. To solve the problem that the ant colony algorithm easily falls into local optimal solutions during the search, a strategy for updating the pheromone concentration in the original ant colony algorithm is proposed. According to the real conditions of search and rescue at sea, the route weight based on the time of falling into the water is introduced into the algorithm, which allows obtaining the optimal route. The simulation results show that the improved algorithm can be effectively used for route design and obtaining the optimal path suitable for sea-based missile defense. However, in [16], the search area restriction, search time, and search strategy are not considered.

In [17], a two-phase method for solving the problem of planning a coverage route for many UAVs in SAR-type maritime missile defense systems was devised. In the first stage, a grid-based domain decomposition method was developed that minimizes the divided search area by transforming it into a graph consisting of vertices and edges. In the second stage, a mixed integer linear programming model is formed to obtain the optimal coverage route of the search area that minimizes the missile defense completion time. To solve the model in large-scale cases, a heuristic randomized search algorithm is developed. Experiments are conducted to verify the effectiveness of the algorithm. The experimental results show that this algorithm provides a better solution with an optimality gap of approximately 0.7% and significantly faster computation than known algorithms. The work also demonstrates the results of real field experiments in a marine environment using the proposed algorithm. However, in [17], the discrete structure of the search area is not taken into account, and the search strategy is not determined. This imposes certain restrictions on the use of the results reported in [17].

In [18], a method was devised that combines color analysis and frequency pattern detection using an inexpensive optoelectronic camera. It was implemented through an adaptive algorithm capable of operating under conditions of dynamically changing background. The proposed method was successfully tested in various environments and showed high efficiency. However, the method from [18] does not provide for optimization of the search trajectory of the search tool, does not take into account the limited search time.

In [19], the possibilities, performance indicators, and limitations of using UAVs in humanitarian logistics are studied. The features of conducting searches in logistics tasks are taken into account. Search methods based on the traveling salesman problem are proposed. However, in [19], the structure of the search area, the limited search time, etc. are not determined. The introduction of such restrictions requires a significant revision of the stated optimization problem.

In [20], UAV systems and the organization of communication on them are considered, as well as the methods used for connecting and transmitting data to a ground station. The concepts of UAV systems functioning are investigated, as well as the details of the development of communication systems and a ground control station. An overview and analysis of security protocols for UAV control and types of attacks that can be carried out to disrupt UAV system communication are presented. The results from [20] cannot be used to optimize the search trajectory of a search tool. They are aimed only at building a data transmission system regarding the current situation in the search area.

Thus, our review of the literature [10–20] led to the conclusion that the use of UAVs is proposed as a search tool for missile defense. One of the main conditions for the successful use of UAVs is planning the movement route in a given search area, especially when several UAVs are involved in missile defense. In the considered works, the authors proposed methods for searching for victims at sea, but they do not take into account the peculiarities of conducting an aviation search for planes that crashed on land. Therefore, it is relevant to devise a method for discrete search for a plane that crashed, taking into account the peculiarities of conducting such a search.

3. The aim and objectives of the study

The aim of our study is to reduce the average time to detect the search object by applying the devised discrete search method using the Blackwell-Black-Kadan relation. This will make it possible to find the aircraft faster and, if necessary, redistribute the remaining search resource.

To achieve this goal, the following research tasks were formulated:

– to list the main stages in the discrete search method for a plane that crashed;

– to estimate the average time to detect the search object.

4. The study materials and methods

The object of our study is the search process for a plane that crashed.

The main hypothesis of the study assumes that the use of a uniformly optimal strategy (UOS) of search in a discrete search area taking into account the Blackwell-Black-Kadan relation could minimize the average time for detecting a plane that crashed.

Assumptions and limitations that we accepted in this work;

– the search area is discretized into a limited number of sectors of a fixed area;

– the plane that crashed is stationary throughout the search process;

– the search for the aircraft is carried out on the earth's surface (on land);

– there is a priori information about the location of the plane that crashed;

– the search tool is not specified, an aircraft-type UAV can be used as a search tool;

– the search tool (UAV) has a limited flight range and flight altitude;

– the planes of the search sectors in the search area are constant and do not change in time;

– external factors (wind, fog, air currents, weather conditions, etc.) do not affect the search process.

The following equipment was used to conduct the study:

– hardware: Dell laptop Intel® Core™ i7-8650U CPU@ 1.90 GHz;

– software: interpreted object-oriented programming lanaguage Python 3.11.

To study and improve the method of discrete search for a plane that crashed, the following research methods were used:

– when devising the main stages of the method of discrete search for a plane that crashed: theoretical methods of analysis and synthesis, methods of probability theory, methods of mathematical statistics, methods of combinatorics, mathematical apparatus of matrix theory, methods of differential calculus, methods of optimization theory, metaheuristic optimization methods, methods of optimal search theory;

– when estimating the average time to detect the search object: theoretical methods of analysis and synthesis, methods of mathematical modeling, methods of optimization theory, methods of optimal search theory, analytical and empirical methods of comparative research.

The structure of the methodological justification for the development of a method for discrete search for a distressed aircraft involves the integration of scientific methods, research objectives, and expected research results into a single, consistent system.

5. Results of investigating the discrete search method for a plane that crashed

5. 1. Main stages in the discrete search method for a plane that crashed

The search task arises when it is necessary to determine the position of the search object, which is located in a given search area (region) Ω of physical space using search tools. The search area Ω has a discrete structure.

Let us consider the task of optimizing search routes using a UAV as a search tool. The a priori probabilities of finding the search object in the search area are considered known.

In discrete search tasks, the area to be surveyed is represented as a set of sectors (subregions) but the kinematic aspects of the search are not considered. The search process is represented as a sequential or parallel survey of selected sectors by available search tools. Detection of the search object is possible only during the survey of the search sector.

Discrete search takes into account:

– the last location of the aircraft;

– the possible route of the aircraft;

– meteorological conditions;

– features of the terrain in the search area;

– available search facilities.

Let Ω be the search area, which is divided into n discrete search sectors Ω_1, Ω_2,...Ω_n.

The prior probability $P(\Omega_i)$ of finding the aircraft in all search sectors Ω_1, Ω_2,...Ω_n must meet condition (1)

$$\sum_{i=1}^{n} P(\Omega_i) = 1. \quad (1)$$

The a priori probabilities of finding the search object in all sectors are calculated before the search is conducted, in accordance with the available information about the event, and do not depend on the search results (expression (2))

$$P(\Omega_i) = 1 - \exp\left(\frac{-M_o}{t_c}\right), \quad (2)$$

where M_0 is the mathematical expectation of detecting the search object in one cycle;

t_c is the time allocated for one search cycle.

The current search sector of the plane that crashed is denoted by $\Omega(t_i) = \sum_{j(t_i)} \Omega_j$, where j is the sequence number of the search sector and the detection of the search object at time t_i. To conduct the search, it is necessary to solve the problem of finding a Bayesian decision rule in a discrete sector $\Omega(t_i)$. Such a problem requires optimization of the search process, namely: the size and location of the discrete search sector $\Omega(t_i)$ in the search area Ω.

Thus, the prerequisites are created for determining the optimal search strategy and detection of the search object in a discrete sector, which minimizes the average risk according to the Bayesian criterion [21].

The average risk value in the current search sector $\Omega(t_i)$ is determined from expression (3) [21]

$$R(t_i) = \sum_{\Omega(t_i)} R_j = R_0 - \begin{pmatrix} (I_{10} - I_{11}) \sum_{\Omega(t_i)} P_{1j}(\gamma_1, t_i) - \\ -(I_{01} - I_{00}) \sum_{\Omega(t_i)} P_{0j}(\gamma_1, t_i) \end{pmatrix}, \quad (3)$$

where $P_{1j}(\gamma_1,t_i)$ is the probability of correct detection of the search object in the jth search sector at time t;

$P_{0j}(\gamma_1,t_i)$ is the current probability of false detection of the search object in the jth search sector at time t.

It was assumed that R_0 is a positive fixed value for a given search sector $\Omega(t_i)$ at the current time t. The Bayesian criterion for evaluating the simple hypothesis H_0 against the simple alternative H_1 in the current discrete search sector $\Omega(t_i)$ of the search area Ω can be written as expression (4) [21]

$$\frac{\sum_{\Omega(t_i)} P_{1j}(\gamma_1, t_i)}{\sum_{\Omega(t_i)} P_{0j}(\gamma_1, t_i)} \mathop{\gtrless}_{\gamma_0}^{\gamma_1} \frac{I_{01} - I_{00}}{I_{10} - I_{11}}. \quad (4)$$

The next step is to move to the unconditional relation of plausibility (expression (5))

$$l(t_i) = \frac{\sum_{\Omega(t_i)} P_{1j}(\gamma_1, t_i)}{\sum_{\Omega(t_i)} P_{0j}(\gamma_1, t_i)}. \quad (5)$$

Expression (4) can be written in the form of (6)

$$l(t_i) \mathop{\gtrless}_{\gamma_0}^{\gamma_1} \frac{I_{01} - I_{00}}{I_{10} - I_{11}}. \quad (6)$$

According to expressions (4) and (5), the optimal Bayesian rule (6) involves determining the maximum value of the dimensionless likelihood ratio (expression (5)) in the current discrete search sector $\Omega(t_i)$ and comparing it with the threshold criterion (7)

$$K_b = \frac{I_{01} - I_{00}}{I_{10} - I_{11}}. \quad (7)$$

At the same time, taking into account:

– if $l(t) \geq K_b$, the solution γ_1 is chosen, the hypothesis H_0 is rejected;

– if $l(t) < K_b$, the solution γ_0 is chosen, the hypothesis H_0 is accepted.

According to expression (9), the optimization is performed according to:

– the parameters of the conditional probability of correct detection $P(\gamma_1/H_1,x)$ of the search object in sectors $\Omega(t_i)$;

– the parameters of the current discrete search sector $\Omega(t_i)$.

For further investigation, let us consider an important special case. Similarly to the Neumann-Pearson criterion [22, 23], let us assume that the unconditional probability of false detection of the search object in sector Ω_j remains fixed at a constant level at time t, which is given by expression $t–P_{0j}(\gamma_1,t_i)$.

Thus, according to expression (4), determining the largest value of $l(t_i)$ reduces to determining the largest value of $P_{1j}(\gamma_1,t_i)$. In order to determine the Bayesian decision rule in the current discrete sector $\Omega(t_i)$ of the search area Ω, the problem of finding the minimum risk value $\lambda(\Omega_j,t_i)$ arises. This strategy $\lambda(\Omega_j,t_i)$ has the essence that at any time step t_λ it determines in which sector Ω_j of the search area the search should be carried out and by what search means.

Let us introduce some restrictions for the search strategy. We shall require that the search strategy $\lambda(\Omega_j,t_i)$ be truncated, namely: $\lambda(\Omega_j,t_i)=0$ for $t_i > T$ and $x \in \Omega$. That is, the mandatory condition must be met that the entire search area Ω will be explored during the search time T.

Also, the search strategy $\lambda(\Omega_j,t_i)$ must satisfy conditions (8) and (9):

$$\lambda(\Omega_j,t_i) > 0 \text{ for } \Omega_j \in \Omega(t_i), \tag{8}$$

$$\lambda\left(\widetilde{\Omega},t_i\right) > 0 \text{ for } \widetilde{\Omega} \in \Omega / \Omega\left(t_i\right). \tag{9}$$

It is reasonable to assume that the search strategy will be the same for all search sectors in the search area that are surveyed at time t_i. Taking into account the above properties for $\lambda(\Omega_j,t_i)$, it is necessary that it satisfies the optimality condition (expression (10))

$$P\left(\lambda_{opt}\left(\Omega_j,t_i\right)\right) = \sup P\left(\lambda\left(\Omega_j,t_i\right)\right), \tag{10}$$

where $P(\lambda(\Omega_j,t_i))$ is the probability of correctly detecting the search object at time t_i when using the strategy $\lambda(\Omega_j,t_i)$.

The search strategy must be optimal in any time period T, until the time when the search sector is completed. That is, at any time when the search is interrupted, it must be optimal until that moment. Conditions (8) to (10) are satisfied by the class of uniformly optimal search strategies (UOS).

The search strategy $\lambda(\Omega_j,t_i)$ will be UOS if any of its T-truncated $\lambda(\Omega_j,t_i)$ has an optimal value (expression (11))

$$P\left(\lambda\left(\Omega_j,t_i\right)\right) = P\left(\lambda_{opt}\left(\Omega_j,t_i\right)\right), \quad \forall t_i, \leq T. \tag{11}$$

Therefore, for the task of determining the minimum value of the average risk and detecting the search object using the Bayesian criterion, the optimal one is UOS $\lambda(\Omega_j,t_i)$. According to UOS rules, the size of the search sector $\Omega(t_i)$ in the search area Ω must be determined. According to expression (4), with a fixed value of false detection of the search object, the optimization problem is stated in expressions (12) to (16):

$$P_1\left(\lambda_1,t_i\right) \to \max; \tag{12}$$

$$\lambda\left(\Omega_j,t_i\right) \geq 0, \quad t_i > 0; \tag{13}$$

$$\sum_{\Omega(t_i)} \lambda\left(\Omega_j,t_i\right) = L_0, \quad t_i > 0; \tag{14}$$

$$\sum_j \lambda\left(\Omega_j,t_i\right) = \varphi\left(t_i\right); \tag{15}$$

$$\sum_{\Omega(t_i)} \varphi\left(t_i\right) = L_0 t_i, \tag{16}$$

where $P_1(\gamma_1,t_i)$ is the probability of detecting the search object at time t_i in the search sector $\Omega(t_i)$;

L_0 is the power of the search system;

$\varphi(t_i)$ is the search resources (search effort) in the search area Ω at time t_i.

The solution to the optimization problem (expression (12)) for conducting a discrete search is quite difficult to obtain. It is necessary to reformulate the optimization problem (expression (12)).

The main variable is the value $0 \leq P_{1j} \leq 1$ – the probability that the object is found when viewing the search sector with number k. At the same time, the mandatory condition must be met that the search object is exactly in the search sector Ω_j and has not been detected before. Another important point is the fact that the search sectors with numbers 1,2,....,k–1 were inspected earlier.

Then, the notation $q_{kj} = 1 - P_{kj}$ is introduced. We shall assume that the search sector Ω_j is scanned n_j times. Therefore, the probability that the search object in sector Ω_j will be detected during scan number k will be as follows (17)

$$P_{ij}\prod_{i=1}^{k-1} q_{ij}, \tag{17}$$

In the case where all sectors Ω_j, $j=\overline{1,N}$, are subject to inspection, and sector Ω_j is inspected n_j times, the probability of detecting the search object during these inspections is calculated from expression (18)

$$\sum_{j=1}^{N} P_j \sum_{k=1}^{n_j} P_{ij} \prod_{i=1}^{k-1} q_{ij}, \tag{18}$$

where P_j is the prior probability of the distribution of the coordinates of the search object in the search sector Ω_j.

In the next step, we introduce condition (19)

$$\sum_{j=1}^{N} n_j S_j \leq U, \tag{19}$$

where U is the search potential used for time T;

S_j is the area of the search sector in the search domain Ω.

The following optimization problem is stated (expressions (20) to (22)):

$$\sum_{j=1}^{N} P_j \sum_{k=1}^{n_j} P_{ij} \prod_{i=1}^{k-1} q_{ij} \to \max; \tag{20}$$

$$\sum_{j=1}^{N} n_j S_j \leq U; \tag{21}$$

$$n \in Z, \quad j = \overline{1,N}. \tag{22}$$

In expressions (20) to (22) there are two types of unknown quantities:

– search efforts $n_1,n_2,...,n_i$;

– search sectors $\Omega_1,\Omega_2,...,\Omega_N$.

Expressions (20) to (22) are reformulated into a dynamic programming problem (expressions (23), (24))

$$f_N(Z) = \max_{n_j,\, j=\overline{1,N}} \left\{ \sum_{j=1}^{N} P_j \sum_{k=1}^{n_j} P_{ij} \prod_{i=1}^{k-1} q_{ij} \right\}, \tag{23}$$

where

$$\sum_{j=1}^{N} n_j, S_j \le Z. \qquad (24)$$

In the case of solving the dynamic programming problem (expressions (23) to (24)) using the Bellman optimization theorem [24], we shall obtain the optimal number of inspections of each search sector. After the optimal number of inspections of each search sector is known, it is necessary to determine the sequence of inspection of the search sectors. The main condition in this case will be the detection of the search object in the minimum possible search time. For this purpose, the Blackwell-Black-Kadan relations (expression (25)) are used

$$\frac{P_j P_{ij} \prod_{k=1}^{i-1} \overline{P_{ij}}}{S_j}, \qquad (25)$$

where $P_{ij} = 1 - \overline{P_{ij}}$;

i is the number of surveys of the j-th sector, $i = \overline{1, u_{i-1}^{opt}}$;

$j = \overline{1,m}$, m – total number of sectors in the search area Ω.

Having arranged relation (25) in descending order, the search and detection of the search object should be performed in the same sequence.

Thus, the main stages in the method of discrete search for a plane that crashed using the Blackwell-Black-Kadan relation are (Fig. 1).

1. Input data input:

– search area Ω;

– search sectors Ω_j;

– a priori probabilities P_i of finding the search object in the search sectors Ω_j;

– area of the search sectors S_j.

2. Formation of value P_i when reviewing each search sector.

3. Determining the value of $\overline{P_{ij}}$.

4. Determining the Blackwell-Black-Kadan ratio.

$$\frac{P_j P_{ij} \prod_{k=1}^{i-1} \overline{P_{ij}}}{S_j};$$

5. Ranking of the obtained Blackwell-Black-Kadan values.

6. Searching in the search sectors Ω_j in descending order of the obtained Blackwell-Black-Kadan values.

8. Detection of the search object in the search sector Ω_j.

9. Evaluation of the search results.

Thus, the method of discrete search for a plane that crashed has been improved, in which, unlike in the known analogs:

– the a priori probability of finding the search object in the search sector is taken into account;

– the probability that the search object will be detected when viewing the search sector is calculated;

– the Blackwell-Black-Kadan ratios are determined;

– the obtained Blackwell-Black-Kadan values are ranked; the sequence of surveying the search sectors is carried out in accordance with the obtained ranking of the values of the Blackwell-Black-Kadan ratios.

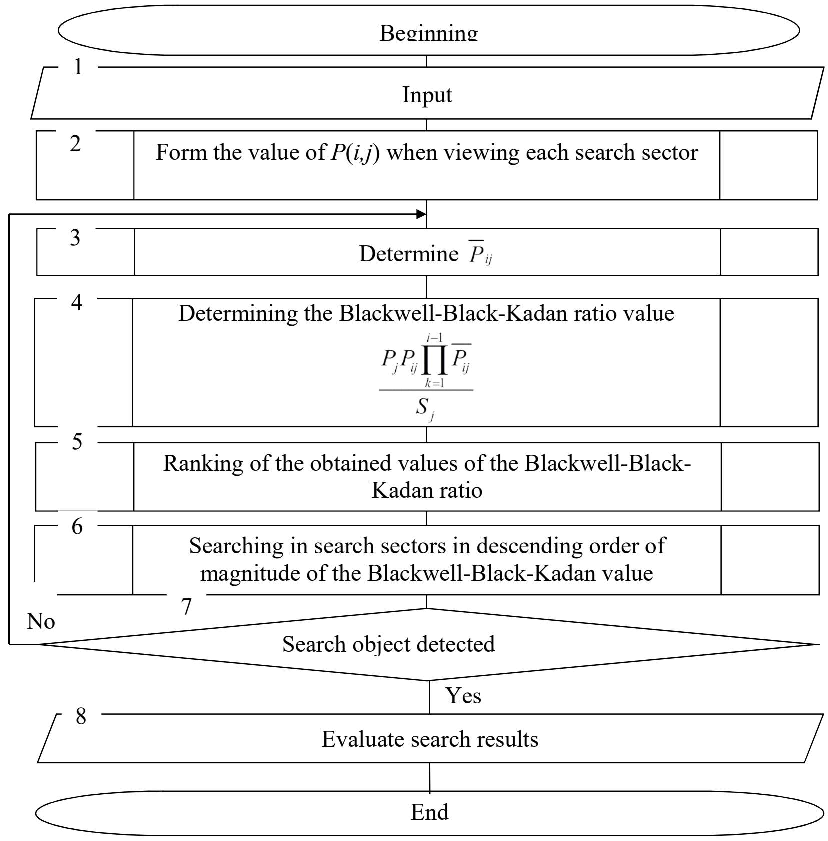

Fig. 1. Block diagram of the algorithm for a discrete method to search for a plane that crashed using the Blackwell-Black-Kadan relation

5. 2. Estimating the average time to detect the search object

The average time to detect the search object is calculated using expression (26)

$$T_{ave} = \int_0^T Q(t)\,\mathrm{d}t, \qquad (26)$$

where $Q(t)$ is the probability of not detecting the search object;

Taking into account that $Q(t) = 1 - P_{ij}(t)$, expression (26) can be written in the following form (27)

$$T_{ave} = \sum_j \left(1 - P_j P_{ij}(t)\right)\Delta t_j, \qquad (27)$$

where $P_1(t_j)$ is the probability of detection of the search object in the search sector Ω_j;

Δt_j – specified time for surveying the search sector Ω_j.

It is obvious that $\sum \Delta t_j = T$, and for quantitative estimates it should be assumed that $P_j = P_{ij}(t)$, $\Delta t_j = \Delta t$.

For example, consider two partial cases.

The elements of the initial data of the study are as follows:

– the search area consists of two search sectors (j=2);

– the planes of the search sectors are assumed to be the same;

– the a priori probabilities of detection of search objects are assumed to be equal (with the classical search method) $P_1=P_2=0.5$;

– with the improved search method, we assume that the distribution of the aircraft position is not uniform: P_1=0.8; P_2=0.2;

– the total search time is equal to T;

– probability of detecting the search object in the search sector Ω_j: $P_1(t_j = P_1$ for each of the search sectors;

– the specified time for the inspection of each of the search sectors is taken equal and is equal to Δt=0.3 T.

For the refined search method taking into account the same planes of the search sectors, the Blackwell-Black-Kadan ratio in accordance with expression (25) is:

– for the first inspection of the first sector, the Blackwell-Black-Kadan ratio is 0.87;

– for the second inspection of the first sector, the Blackwell-Black-Kadan ratio is 0.74;

– for the third inspection of the first sector, the Blackwell-Black-Kadan ratio is 0.61;

– for the first inspection of the second sector, the Blackwell-Black-Kadan ratio is 0.55;

– for the fourth inspection of the first sector, the Blackwell-Black-Kadan ratio is 0.47;

– for the second inspection of the second sector, the Blackwell-Black-Kadan ratio is 0.36.

It is in this order that the aircraft search is carried out in two search sectors.

At the same time, in accordance with expression (27): $T_{ave} \approx 0.396\ T$.

For a uniform (known search), the aircraft search is carried out alternately in each of the two sectors.

In this case, in accordance with expression (27): $T_{ave} \approx 0.45\ T$.

Thus, based on our results, it can be concluded that when optimizing the search for a plane that crashed, the average search time for the search object is reduced by 12%.

6. Discussion of results related to devising a method for discrete search for a plane that crashed

Unlike known works (for example, [10, 13, 14]), the search area is represented in a discrete form. A formalization of the discrete search for a plane that crashed has been carried out. The a priori probability of finding an aircraft in all search sectors must meet condition (1). An optimal Bayesian rule (expression (6)) is formulated, which involves determining the maximum value of the likelihood ratio (expression (5)) in the current discrete search sector and comparing it with the threshold criterion (expression (7)). Unlike known works (for example, [12, 15, 16])), a class of uniformly optimal search strategies (expression (11)) has been introduced.

The method for discrete search for a plane that crashed has been improved, in which, unlike in the known analogs [10–20]:

– the a priori probability of finding the search object in the search sector is taken into account;

– the probability that the search object will be detected when viewing the search sector is calculated;

– the Blackwell-Black-Kadan relations are determined;

– the obtained Blackwell-Black-Kadan values are ranked, and the sequence of inspection of the search sectors is carried out in accordance with the obtained ranking of the values of the Blackwell-Black-Kadan relations.

The average time to detect the search object has been estimated (expressions (26), (27)). It was established that when optimizing the search for a plane that crashed, the average search time for the search object is reduced by 12%.

This became possible due to the use of the search UOS in a discrete search zone taking into account the Blackwell-Black-Kadan relation.

The limitation of our study is the simplified representation of the search area, which is represented by a regular discrete grid without taking into account complex terrain or prohibited areas. In addition, external factors such as weather conditions, wind, etc., which can affect the speed and route of the search vehicle, are not taken into account.

The disadvantage of the improved method is its application only for the case of a discrete structure search area.

Further development of the study should be focused on:

– integration of the search method with real geographic information systems;

– adaptation of the search method to changing environmental conditions in real time.

7. Conclusions

1. The main stages of the discrete search method for a plane that crashed are:

– the a priori probability of finding the search object in the search sector is taken into account;

– the probability that the search object will be detected when scanning the search sector is calculated;

– the Blackwell-Black-Kadan ratios are determined;

– the obtained Blackwell-Black-Kadan values are ranked, and the sequence of surveying the search sectors is carried out in accordance with the obtained ranking of the values of the Blackwell-Black-Kadan ratios.

2. The average time to detect the search object has been estimated. It was found that when optimizing the search for a plane that crashed, the average search time for the search object is reduced by 12%.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study, as well as the results reported in this paper.

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Data availability

The data will be provided upon reasonable request.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

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