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IMPLEMENTATION OF NON-LINEAR DEFUZZIFICATION OF LINGUISTIC INDICATORS OF PREFERENCES IN THE SAATY'S ANALYTIC HIERARCHY PROCESS

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The object of the study is the method of hierarchy analysis, as it is more widely used for solving multicriteria decision-making problems (DM). The subject is the Saaty preference level scale and methods for establishing normalized weight coefficients (NWC). The problem is to increase the accuracy of multicriteria decisions, where the linear defuzzification of linguistic preference indicators (PI), used to form pairwise comparison matrices (PCM), does not correspond to the peculiarities of human thinking. The essence of the results is that for basic IPs (IP_9, IP_7, IP_5, IP_3, IP_1) NWCs are established by the mathematical method of prioritization (MPM), adapted for the needs of research, for intermediate IPs (IP_8, IP_6, IP_4, IP_2) – by multiplicative aggregation of NWCs of neighboring PIs. The peculiarity is that it is established that the application of the results of the II iteration of MoP with XII, which are nonlinear, form a step scale, and provide the proper accuracy of priority measurements, is acceptable. The quantitative indicator of sensitivity to the measurement of linguistic preferences increased by 4.5 times compared to the linear scale. However, the average quantitative indicator of the establishment of false IP in the nonlinear scale turned out to be 1.84 times higher than in the linear one. which places additional requirements on the competence of experts, as well as the need to control their attentiveness during the PCM formation.

The relative quantitative indicator corresponding to the increase in the NWC of the Saaty scale terms, formed using the modifier "very" by performing the fuzzy operation "concentration", has increased almost twice, "double concentration" – by a third. Which is more consistent with the quantitative-qualitative logic of the relationship between the specified terms. The practical use of the obtained results will help prevent the negative phenomenon of rank reversal in multi-criteria DM problems

Keywords: hierarchy analysis method, preference levels, linguistic indicators, normalized weight coefficients

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1. Introduction

All practical tasks of planning and decision-making (DM), conducting various examinations, etc. in many areas of research are characterized by varying degrees of complexity, hierarchy, dynamism and uncertainty of the problem situations (PS) to be solved. This makes research on improving the methodology of system analysis, DM and expert technologies used to solve such problems extremely important. The result is not only obtaining the desired result, but also improving the corresponding means of artificial intelligence (AI)/neural networks that are permanently "trained" to use real experience in solving complex DM problems (MCDMP). This involves both the development of a new methodology and new areas of application of known methods, technologies and procedures [1–3].

Multi-criteria DM (MCDMP) methods provide a structured approach to solving complex problems, which allows taking into account the influence of various factors (objective/subjective, external/internal, stochastic/non-stochastic risks) and finding optimal compromises [4]. The use of modern computer technologies significantly increases the efficiency of multi-criteria optimization, allowing to analyze large amounts of data and find optimal solutions in real time [5].

Today, Saaty's Analytic Hierarchy Process (AHP) is a more effective tool for solving MCDM [6]. It allows to structure complex problems, evaluate various criteria, indicators and alternatives in detail, which provides a more rational choice in many areas of activity: economics, management, engineering, etc. [7]. Therefore, any contribution to the development of the AHP methodology, given its popularity, will have multiple reflections in the practice of solving MCDMP.

2. Literature review and problem statement

The solution and optimization of MCDMP is provided by a wide range of methods and technologies, most of which are implemented in a special software environment [6]. Among them, the method of analysis of hierarchies [7] has been unusually popular for several decades, which is an effective tool for solving MCDMP in forensics, business, public administration, management, medicine, education, etc., based on the number of relevant publications.

The paper [8] summarizes the generally positive results of the integrated application of AHP and the analytical network method (ANM) over a twenty-year period. The difficulties of solving MCDMP in large and dynamic systems, integrating new technologies into the DM process, and taking into account the relationships between criteria are highlighted. The unsolved problem also includes the lack of uniform standards in the selection and adaptation of AHP/ANM methods, which makes it impossible to compare research results and hinders the development of their practical application. The reasons for this are the lack of interdisciplinary coordination, the complexity of integration with other methods, and the lack of a platform for sharing experience. The highlighted problems require coordination, standardization, and the development of adaptive models that combine the advantages of different approaches.

In [9], AHP was used to determine the optimal design solution in landscape design. This allowed taking into account various criteria and expert assessments and increased the validity of the choice of design solutions. However, the issues of adapting the method to different types of landscaping objects and scaling large projects remained unresolved. The reason for this may be both objective difficulties associated with the variety of criteria and the complexity of expert assessments, and the cost of organizing large-scale research. These difficulties were overcome by developing automated tools for collecting and processing expert data. However, the issues of universality and reproducibility of results remain open. Therefore, it is advisable to conduct further research to improve and scale the application of AHP in the field of organizing landscape design.

In [10], the results of research on the AHP application in the DM process in production systems are presented. It is shown that the AHP use allows to structure complex tasks, take into account multi-criteria and increase the efficiency of management decisions. But the issues related to the AHP integration into complex information systems of enterprises and the adaptation of the method to the specifics of various industries remain unresolved. This may be a consequence of both objective difficulties associated with the diversity of production tasks and the cost of implementing new approaches in practice. Their elimination should be carried out by developing flexible software solutions, especially in terms of scaling and adaptation to integrated PR support systems.

In [11], the results of research on determining the priority of land consolidation works are presented. The use of AHP allowed taking into account various criteria and expert assessments for a well-founded determination of the order of work. However, issues related to taking into account dynamic changes in land relations and adapting the model to different regional conditions remained unresolved. This is explained by objective difficulties: changes in legislation, the specifics of land resources, the costs of collecting current data. The introduction of flexible information systems for online monitoring and analysis of land resources is proposed.

The works [12, 13] are devoted to the application and development of AHP in the field of knowledge management of high-tech and investment enterprises. Taking into account the multifactorial nature and complexity of modern management tasks contributed to the validity of their solutions. However, the issues of integrating AHP into complex information systems, adapting the method to the specifics of various enterprises and projects, as well as automating the DM process remained unresolved. The need to develop universal or industry-specific information systems that combine AHP with other modern methods of DM support is indicated.

In [14], AHP is used in business decisions, but the influence of subjectivity of expert assessments is pointed out. It is proposed to use AHP in combination with other methods or to automate the process of collecting and processing expert data.

In [15], a step-by-step application of AHP is implemented, which contributes to increasing the transparency and reproducibility of results. However, the issues of automating the DM process and reducing the influence of the human factor remain unresolved. The author sees the elimination of these shortcomings through the use of information systems to automate data collection and analysis.

In [16], the features of the evolution and revolution of AHP are studied. It is shown that AHP is constantly developing, integrating new approaches and technologies, which allows expanding the scope of its application. However, issues related to the use of large amounts of information, the study of dynamic systems, and the use of multidimensional criteria remain unresolved. To eliminate the listed shortcomings, the integration of AHP with modern information technologies and artificial intelligence methods is proposed.

The AHP is based on the Likert scale, which is a point-based (rating, rank) scale. However, this scale is usually interpreted as interval, which provides such processing of measurement results that are impossible in the rank scale [17]. Therefore, despite the positive results of the AHP application, they are not optimal. After all, the linear LP_{I_k} assessment scale is not always able to adequately reflect the real attitude of experts to the phenomena under study, especially in conditions of uncertainty, in particular non-stochastic nature, or strong cognitive distortions.

In [18], a study is presented that is devoted to the selection of sustainable suppliers using an approach that allows taking into account the multi-criteria and complexity of the evaluation process. The issues related to the integration of this method with other modern decision support tools in different industries remain unresolved. The reason for this may be the difficulties associated with the diversity of criteria.

Summarizing the analysis [7–19], the generally recognized advantages of AHP are formulated (without ranking):

– a systematic and structured approach to the analysis of complex problem situations (PS), which contributes to their decomposition into simpler components, which, in turn, greatly facilitates the analysis of PS and DM;

– the possibility of simultaneously applying formally incompatible numerical and linguistic criteria and indicators of solution (PS);

– the possibility of determining and controlling the consistency of the opinions of experts involved in solving PS;

– high versatility, and therefore adaptability for use in the widest range of research;

– the applicability for cases when important elements of the solution are difficult to quantify or compare, or when the effectiveness of communication of expert group members is affected by their specialization, experience, terminology or perspectives, etc.

From the analysis of sources [7–19], the following shortcomings of the studied AHP (without ranking) emerge:

– the need to obtain a significant amount of information from experts;

– the impossibility of using it in the case of group choice and identified conflicting systems of preferences (SB);

– the absence of a mechanism for prioritizing the opinions of more experienced experts;

– the phenomenon of rank reversal: adding/removing elements in the hierarchy leads to a radical change in the ranks of all elements;

– the maximum priority value is 9, which negatively affects the effectiveness of comparing alternatives;

– the applied linearization of LP estimates does not correspond to an adequate perception of their linguistic characteristics, etc.

It should be noted that the ANM under study was developed, first of all, by Saaty himself, who translated the study of hierarchies into a more advanced Analytic Network Process, ANM [7], which made it possible to solve more complex, in relation to ANM, MCDMP. And if ANM decomposes PS in the form of a hierarchy with a clearly defined goal, criteria and alternatives, then ANM structures the same PS as a network. In this case, mutual two-way dependence and connections between different components of the structure are assumed not only vertically, but also horizontally.

Common to both AHP and ANM is the use of the method of pairwise comparisons to form decision matrices, the solution of which determines local and global priority vectors. This is done in order to further rank alternatives, and therefore DM with respect to the best of them.

However, ANM does not eliminate the last two disadvantages of using AHP from those listed above.

None of the sources considered question the validity of using quantitative linear defuzzification of linguistic IPs.

Therefore, it is advisable to conduct a study devoted to improving AHP by introducing nonlinear defuzzification of linguistic IPs, which will allow more adequately taking into account the peculiarities of expert thinking and increasing the efficiency of MCDMP processing. Such an approach will help identify errors in the formation of PCM and ensure greater validity in MCDMP processing.

3. The aim and objectives of the study

The aim of the study is to optimize the process of solving multi-criteria PR problems by developing and implementing a method of nonlinear defuzzification of linguistic IPs in the Saaty AHP. This will make it possible to optimize the resulting solutions at the stage of constructing the pairwise comparison matrix.

To achieve the formulated aim, the following objectives should be solved:

– adapt MoP for research needs;

– establish nonlinear NWCs of basic (IP_9, IP_7, IP_5, IP_3, IP_1) and "intermediate (compromise)" (IP_8, IP_6, IP_4, IP_2) linguistic IPs;

– justify the MoP iteration with acceptable results.

4. Materials and methods

The object of the study is the method of analysis of hierarchies, as it is more applicable to solving multi-criteria decision-making problems. The subject of the study is the scale of preference levels and methods for establishing weighting coefficients.

Research hypothesis: since linear thinking is not inherent in humans, it is the nonlinear defuzzification of linguistic IPs in the Saaty AHP that will contribute to increasing the accuracy and validity of DM. Defuzzification should be carried out by providing the specified indicators with nonlinear NWCs.

The AHP application consists in determining the eigenvector PCM of the studied alternatives $A=\|a_{ij}\|\Rightarrow\|\alpha_{ij}\|$, which is interpreted as a vector of priorities. The method is used to solve the problems of optimal choice from a set of alternatives.

If to denote by LP_{IP_k} the quantitative estimates of the levels of advantages (LP) defined in column 3 of Table 1 [7], then the elements a_{ij} of the matrix A are established using the ratio scale as follows

$$a_{ij}=\begin{cases} LP_{IP_k}, \text{ if } a_i \succ a_j, \\ \dfrac{1}{LP_{IP_k}}, \text{ if } a_j \succ a_i, \\ 1, \text{ if } i=j. \end{cases} \quad (1)$$

The $\|a_{ij}\|$ matrix is square and inversely symmetric, which determines the specifics of working with it.

Table 1

Saaty linguistic indicators for assessing the levels of preference of compared alternatives

IP_k	Linguistic indicators of superiority	LP_{IP_k}
IP_9	Of utmost importance	$LP_{IP_9}=9$
IP_8	Very, very strong	$LP_{IP_8}=8$
IP_7	Very strong or demonstrated importance	$LP_{IP_7}=7$
IP_6	Strong plus	$LP_{IP_6}=6$
IP_5	Strong importance	$LP_{IP_5}=5$
IP_4	Moderate plus	$LP_{IP_4}=4$
IP_3	Moderate importance	$LP_{IP_3}=3$
IP_2	Weak	$LP_{IP_2}=2$
IP_1	Equal importance	$LP_{IP_1}=1$

Note:
1) IP_k – the mark of the k-th linguistic indicator of preference;
2) LP_{IP_k} – LP quantitative assessment mark of the indicator I_k;
3) $LP_{IP_k}=1.1\div1.9$ – the alternatives being compared are very close to no advantage. Adding commas allows to show the difference;
4) $LP_{IP_8}, LP_{IP_6}, LP_{IP_4}, LP_{IP_2}$ – determine intermediate estimates (compromise solutions) between neighboring statements.

The indicators of preference (IP) IP_k, given in Table 1, form a term set (set of terms, scale) of the linguistic variable (LV) "LP"

$$T^S(LP)=\tilde{R}_{I_9}+\tilde{R}_{IP_8}+\tilde{R}_{IP_7}+\tilde{R}_{IP_6}+ \\ +\tilde{R}_{IP_5}+\tilde{R}_{IP_4}+\tilde{R}_{IP_3}+\tilde{R}_{IP_2}+\tilde{R}_{IP_1}, \quad (2)$$

where $\tilde{R}_{I_k}$ – the conditional symbol of the k-th term LV "LP"; "+" – the symbol of the logical combination of individual terms into the "LP" scale.

The results presented in column 3 of Table 1 indicate that the quantitative sensitivity of LP measurements, as the ratio of their maximum and minimum LP_{IP_k} levels, is limited and equals

$$\max\left(\frac{LP_{IP_k}}{LP_{IP_j}}\right)=\frac{\max LP_{IP_k}}{\min LP_{IP_j}}=\frac{LP_{IP_9}}{LP_{IP_1}}=\frac{9}{1}=9. \quad (3)$$

The obvious linguistic-quantitative discrepancy of other IPs of the IPk scale (2) is also shown in Table 1. For example, the linguistic description of the indicator IP_7 is formed from the previous IP_6 using the modifier "very". And this corresponds to the application of the fuzzy operation "concentration" to the term R_{IP6}. At the same time, to obtain the linguistic description of IP_8, a double "concentration" ("very, very") is required. This has the following formal description

$$\begin{cases}\mu_{IP_7}(LP)=\mu^2_{IP_6}(LP),\\ \mu_{IP_8}(LP)=\mu^2_{IP_7}(LP)=\mu^4_{IP_6}(LP).\end{cases} \quad (4)$$

However, the quantitative ratio of LP of the specified terms, measured on a linear scale (column 3 of Table 1), clearly does not correspond to the human imagination of the difference in their linguistic and informational saturation and strength in the LP measurement:

$$\frac{LP_{IP_7}}{LP_{IP_6}}=\frac{7}{6}=1.17; \quad (5)$$

$$\frac{LP_{IP_8}}{LP_{IP_7}}=\frac{8}{7}=1.14; \quad (6)$$

$$\frac{LP_{IP_8}}{LP_{IP_6}}=\frac{8}{6}=1.33. \quad (7)$$

The use of "normal concentration" in determining the linguistic content of IP IP_7 and IP_8 leads to an increase in the initial quantitative assessment of the compared LPs by a maximum of 17%, and "double" – by only 33%. Therefore, the results (5)–(7) really indicate the qualimetric illegality of using in the AHP the linear scale of correspondence of linguistic and quantitative indicators of the Saaty scale, i.e. $IP_k \Leftrightarrow LP_{IP_k}$. Moreover, without prejudice to the huge number of positive results obtained precisely with the use of linear assessments LP_{IP_k}.

Based on the analysis of works [3, 18], it seems possible to eliminate the mentioned shortcomings of AHP and ANM by providing linguistic IPs IP_k with nonlinear normalized weight coefficients (NWC)

$$IP_k \Rightarrow \beta_{IP_k}: 0\le\beta_{IP_k}\le 1, \sum_{k=1}^{K=5}\beta_{IP_k}=1. \quad (8)$$

Expression (8) is focused on establishing NWC first of the basic IPs IP_9, IP_7, IP_5, IP_3, IP_1, and then finding their intermediate values for IP_8, IP_6, IP_4, IP_2. And if condition (8) is met, then expression (1) is transformed into the following

$$a_{ij}=\begin{cases}\beta_{I_k}, \text{ if } a_i\succ a_j,\\ \dfrac{1}{\beta_{I_k}}, \text{ if } a_j\succ a_i,\\ 1, \text{ if } i=j.\end{cases} \quad (9)$$

From Table 1 and expression (2) follows an obvious ranking of the indicators I_k and the corresponding terms LV "LP" $\tilde{R}_{I_k}$ from the standpoint of their sensitivity to the assessment of the advantages of the compared objects

$$IP_9\succ IP_8\succ IP_7\succ IP_6\succ IP_5\succ IP_4\succ IP_3\succ IP_2\succ IP_1\Leftrightarrow \\ \Leftrightarrow \tilde{R}_{IP_9}\succ\tilde{R}_{IP_8}\succ\tilde{R}_{IP_7}\succ\ldots\succ\tilde{R}_{IP_3}\succ\tilde{R}_{IP_2}\succ\tilde{R}_{IP_1}. \quad (10)$$

This opened up prospects for the application of the method of prioritization (MoP), known in systems analysis as the "leader problem", to find the NWC_{bk}. The method is effective for solving practical problems, which, taking into account the content of the works [3, 18], are formulated in the context of this publication as follows:

1) establishing a more important indicator of the advantages of IP_k – a trivial task solved in a specific case by the content of Table 1 and expression (10). After all, by definition (Table 1), such is the indicator I_9;

2) ordering IPs taking into account the degree of their qualimetric efficiency – a trivial task solved in a specific case by the content of Table 1 and expression (10);

3) determining the quantitative indicator – NWC of each IP IP_k.

From the comparative analysis of MoP and a wide range of modern methods for establishing NWC, which was conducted by the authors in the works [3, 18], the following advantages of MoP emerge:

a) practical applicability, when it is not known in advance whether there is transitivity of experts' opinions. After all, if to consider the problem of establishing NWC more broadly than in this publication, focusing first on individual SPs (ISPs)/ranking experts. Then the transitivity of ISPs does not mean at all the unambiguous transitivity of the group SP (GSP) into which they are aggregated, which is usually explained by the Condorcet paradox;

b) simplification of the task of determining the size of the group of experts involved in establishing the significance of the alternatives under study;

c) the possibility of applying several criteria for assessing the significance of alternatives;

d) the complex criterion for the significance of the alternative under study is calculated as a simple sum of the "weighted" estimates of its partial indicators.

After receiving the research results, ChatLLM was used to formulate the text in the analysis of publications and other sections of the article.

MoP should be used for defuzzification, and therefore delinearization of IPk indicators. Moreover, at each new iteration, the NWC values are refined. The use of MoP allows to take into account indirect priorities.

5. Results of the study of defuzzification of linguistic indicators of preference levels

5. 1. Adapting the prioritization method to research needs

To apply MoP, one should construct a square matrix of incidences $C=||c_{ij}||$ of linguistic IPs IP_k

$$a = \begin{Vmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{Vmatrix}, \qquad (11)$$

which elements c_{ij} – the set as the "normative" part of the total "value" (significance, importance) of the compared alternatives

$$a_{jk} = \begin{cases} 1+z - \text{advantage fact } IP_j \succ IP_k, \\ 1 - \text{fact of equivalence } IP_j \approx IP_k, \\ 1-z - \text{advantage fact } IP_j \prec IP_k, \end{cases} \qquad (12)$$

where z – a quantitative indicator of the degree of priority.

Taking into account the different significance of the compared alternatives – IP ($IP_j \neq IP_k$) and assuming $z=1$, expression (11) is transformed for the needs of research

$$a_{ij} = \begin{cases} 2, \text{ if } IP_j \succ IP_k, \\ 0, \text{ if } IP_k \succ IP_j. \end{cases} \qquad (13)$$

It is clear that all diagonal elements of the incidence matrix of the compared IPs (11) are equal to 1: $c_{ii} = 1$.

5. 2. Nonlinear defuzzification of linguistic indicators of preference in the Saaty scale

Using expression (13), a square matrix of the incidences of the studied IPs IP_k was constructed (Table 2). The essence of MoP and the technology of its application in various areas of research are considered in detail in the works [3, 18].

Table 2

Square matrix of incidences of indicators of advantage and results of its solution on the *I* iteration of the prioritization method

IP_k	IP_9	IP_7	IP_5	IP_3	IP_1	*I* iteration	
						Σ	β_k
1	2	3	4	5	6	7	8
IP_9	1	2	2	2	2	9	0.36
IP_7	0	1	2	2	2	7	0.28
IP_5	0	0	1	2	2	5	0.2
IP_3	0	0	0	1	2	3	0.12
IP_1	0	0	0	0	1	1	0.04
Σ						25	1

Therefore, the calculations for the first iteration of MoP are trivial and are presented in columns 7, 8 of Table 2. However, the obtained NWCs of the IP β_{I_k} are unacceptable, since they are linear. Table 3 presents the results of subsequent iterations of MoP.

From the contents of Table 3, at each subsequent MoP iteration, the sought NWC IP I_k are increasingly differentiated, and therefore become increasingly nonlinear. And since the calculations were performed with an accuracy of up to the fourth decimal place, the XII MoP iteration became the last in its application.

Table 3

Results of successive iterations of the prioritization method and establishment of normalized weight coefficients of preference indicators

IP_k	Normalized weight coefficients of preference indicators obtained in different iterations of the prioritization method											
	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
IP_9	0.36	0.4824	0.5733	0.6407	0.6914	0.7303	0.761	0.7856	0.8011	0.8144	0.8293	0.844
IP_7	0.28	0.2941	0.28	0.2575	0.2345	0.2136	0.1952	0.1793	0.1647	0.1587	0.1487	0.1387
IP_5	0.2	0.1529	0.1111	0.0818	0.0619	0.0482	0.0384	0.0312	0.0308	0.0248	0.0203	0.017
IP_3	0.12	0.0588	0.0311	0.018	0.0112	0.0074	0.0051	0.0037	0.0027	0.002	0.0016	0.0012
IP_1	0.04	0.0118	0.0044	0.002	0.0010	0.0006	0.0003	0.0002	0.0001	0.0001	0.0001	0
Σ	1	1	1	1	1	1	1	1	1	1	1	1

5. 3. Justification of the iteration of the prioritization method with acceptable results

Fig. 1 illustrates the corresponding dynamics of NWC. It shows the more interesting results of the I, II, V, IX iterations of MoP with solid lines (columns 1, 3, 6, 10 of Table 1). Thus, the I iteration provided clearly unacceptable linear results. The results of the II, V, IX iterations are nonlinear and meet the following possible requirements for the degree of acceptability of the accuracy of calculations: the results of the II iteration give an idea of the accuracy – to the second decimal place, V – to the third, IX – to the fourth.

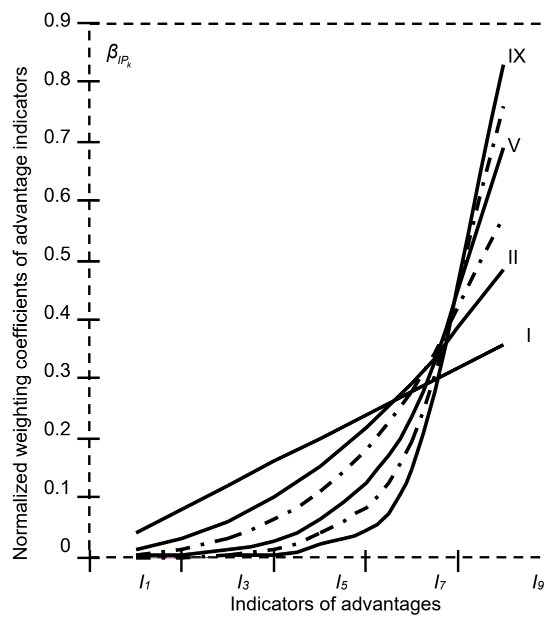

Fig. 1. The influence of the number of iterations of the prioritization method on the nonlinearity of the normalized weight coefficients of the preference indicators

The sought NWCs should be set with an accuracy of up to two decimal places. Then, from the analysis of Table 2, it follows that the results of the second iteration of MoP (column 3) are more acceptable for research purposes, since:

1. The requirement for the accepted accuracy of calculations to the second decimal place is satisfied.
2. The obtained NWCs β_{I_k} are nonlinear.
3. Based on the recommendations for the formation of linguistic scales, they correspond to a greater extent to the logic

of the quantitative-linguistic correlation of the studied IP_k indicators than in the case of using linear estimates LP_{IP_k}.

Thus, the above justification for choosing the results of the second iteration of MoP for further application in AHP is correct.

Below is an analytical description of the corresponding curve

$$\beta_{I_k} = 0.0119 \cdot x\left(IP_k\right)^{2.312}. \quad (14)$$

From expression (14), the quantitative assessment of the IP is transferred from a linear scale (column 3 of table 1) to a nonlinear step scale, which is more flexible for determining the attitude of experts to the phenomena under study.

To find the NWC of the intermediate "compromise" terms (IP) IP_8, IP_6, IP_4, IP_2, a multiplicative aggregation of the NWC of the neighboring terms of the scale (1) was performed

$$\beta_{IP_{2k}} = \sqrt{\beta_{IP_{2k+1}} \cdot \beta_{IP_{2k-1}}}. \quad (15)$$

Substituting into expression (15) the data of nonlinear NWCs of basic IPs of the Saaty scale, established on the 2nd iteration of MoP (Table 3), the required NWCs for "compromise" IPs were obtained. Table 4 summarizes the NWCs of all studied IPs.

Table 4

Comparison of quantitative values of indicators of preference levels in the Saaty scale

Value	Indicators of preference								
	IP_9	IP_8	IP_7	IP_6	IP_5	IP_4	IP_3	IP_2	IP_1
Acting value LP_{IP_k}	9	8	7	6	5	4	3	2	1
Recommended values β_{IP_k}	0.48	0.38	0.29	0.21	0.15	0.09	0.05	0.03	0.01

Similarly, expressions (3), (5)–(7) were applied to estimate the overall change in sensitivity in measures of linguistic preference ratings:

$$\max\left(\frac{\beta_{IP_k}^{II}}{\beta_{IP_j}^{II}}\right) = \frac{\max \beta_{IP_k}^{II}}{\min \beta_{IP_k}^{II}} = \frac{\beta_{IP_9}^{II}}{\beta_{IP_1}^{II}} = \frac{0.4824}{0.0118} = 40.88; \quad (16)$$

$$\frac{\beta_{IP_7}^{II}}{\beta_{IP_6}^{II}} = \frac{0.2941}{0.2121} = 1.39; \quad (17)$$

$$\frac{\beta_{IP_8}}{\beta_{IP_7}} = \frac{0.3767}{0.2941} = 1.28; \quad (18)$$

$$\frac{\beta_{IP_8}}{\beta_{IP_6}} = \frac{0.3767}{0.2121} = 1.78. \quad (19)$$

From expression (16), due to the nonlinearity of the obtained NWC β_{IP_k}, the quantitative measure of sensitivity LP reached a value of 40 and increased in relation to the result (3) obtained for linear indicators LP_{IP_k} by 4.5 times. The use of fuzzy "ordinary concentration" and "double concentration" in the linguistic definition of the content of the IP IP_7 and IP_8 doubled the quantitative assessment of the corresponding priority in relation to linear assessments LP_{IP_k}. This indicates the effectiveness of the proposed nonlinear scale for assessing IP_k of the type (14).

It is natural that an expert can make mistakes, like any other person. The influence of the human factor on the MCDMP decision can be significant. Thus, based on the obtained nonlinear indicators NWC β_{IP_k} and linear indicators LP_{IP_k}, the potential expected error of choosing the IP assessment of experts for the compared alternatives was determined. At the same time LP_{IP_k}, the mentioned error of the estimates LP_{IP_k} or β_{IP_k} should not exceed the nearest IP value.

A quantitative error indicator is introduced, which concerns the choice of an inadequate linear estimate of the value of the compared alternatives, which arises when constructing a matrix of pairwise comparisons

$$\varepsilon_{IP_k}^{L} = \frac{\left|LP_{IP_k} - LP_{IP_j}\right|}{LP_{IP_k}},\ j = \begin{cases}\left(k+1\right) \le 9, \\ \left(k-1\right) \ge 1,\end{cases} \quad (20)$$

where LP_{IP_k} selected from the last column of Table 1.

It is possible to suppose that the expert mistakenly chose the inadequate IP IP_8 instead of the true IP IP_9 during a pairwise comparison of alternatives for the corresponding matrix composition. Then, according to expression (23), the error estimate for such a comparison can be calculated as

$$\varepsilon_{IP_9}^{L} = \frac{\left|LP_{IP_9} - LP_{IP_8}\right|}{LP_{IP_9}} = \frac{\left|9-8\right|}{9} = 0.11. \quad (21)$$

Or the expert can set intermediate (compromise) IPs, such as IP_8 or IP_6 instead of IP_7. This leads to the following quantitative assessment

$$\varepsilon_{IP_7}^{L} = \frac{\left|LP_{IP_7} - LP_{IP_8}\right|}{LP_{IP_7}} = \frac{\left|LP_{IP_7} - LP_{IP_6}\right|}{LP_{IP_7}} = = \frac{\left|7-8\right|}{7} = \frac{\left|7-6\right|}{7} = 0.14. \quad (22)$$

Similarly, other quantitative estimates of other potential errors in estimating the IP of the true comparable alternatives are calculated and presented in column 3 of Table 5.

Table 5

Predicted quantitative estimates of errors in identifying indicators of real preferences

Level of preference		Error, ε	
Adequate IP_k	With an error $IP_k = IP_{k\pm1}$	Linear, LP_{IP_k}	Nonlinear, β_{IP_k}
IP_9	IP_8	0.11	0.22
IP_8	IP_9	0.13	0.28
	IP_7		0.22
IP_7	IP_8	0.14	0.28
	IP_6		0.28
IP_6	IP_7	0.17	0.39
	IP_5		0.28
IP_5	IP_6	0.20	0.39
	IP_4		0.38
IP_4	IP_5	0.25	0.61
	IP_3		0.38
IP_3	IP_4	0.33	0.61
	IP_2		0.55
IP_2	IP_3	0.50	1.24
	IP_1		0.55
IP_1	IP_2	1	1.23
–		$\bar{\varepsilon}_{IP_k}^{L} = 0.27$	$\bar{\varepsilon}_{IP_k}^{N-L} = 0.49$

Predicted estimates of potential errors in correct IP identification increase with decreasing qualimetric power (although this is linguistic), as shown in Table 1. Errors are also called a larger spread of linguistic assessment in the "lower" part of the IP scale (taking into account intermediate estimates IP_8, IP_6, IP_4, IP_2) compared to the "higher" part.

In the example of using nonlinear NWC β_{IP_k} to determine the predicted error of inadequate expert identification of certain IPs of compared alternatives, formula (23) is transformed into the following form

$$\varepsilon_{IP_k}^{N-L} = \frac{\left|\beta_{IP_k} - \beta_{IP_j}\right|}{\beta_{IP_k}}, \; j = \begin{cases} (k+1) \le 9, \\ (k-1) \ge 1. \end{cases} \quad (23)$$

The calculations are based on column 3 of Table 3. It is possible to suppose that the expert, when comparing two alternatives, used the incorrect IP_8 instead of the correct IP9. Then formula (23) is implemented as follows

$$\varepsilon_{IP_9}^{N-L} = \frac{\left|\beta_{IP_9} - \beta_{IP_8}\right|}{\beta_{IP_9}} = \frac{\left|0.4824 - 0.3767\right|}{0.4824} = 0.22. \quad (24)$$

Thus, due to the use of nonlinear NWC, the error in identifying the true IP has doubled (last row of Table 5).

Similarly, the expert can set intermediate (compromised) neighboring IP values IP_8 or IP_6 instead of the true one. This leads to the following quantitative assessment

$$\varepsilon_{IP_7}^{N-L} = \begin{cases} \frac{\left|\beta_{IP_7} - \beta_{IP_8}\right|}{\beta_{IP_7}} = \frac{\left|0.2941 - 0.3767\right|}{0.2941} = \\ = 0.2809 = 0.28, \\ \frac{\left|\beta_{IP_7} - \beta_{IP_6}\right|}{\beta_{IP_7}} = \frac{\left|0.2941 - 0.2121\right|}{0.2941} = \\ = 0.2788 = 0.28. \end{cases} \quad (25)$$

Similarly, quantitative estimates of other possible errors of the compared alternatives related to the identification of the true IP are presented in column II of Table 5. The same trend is observed in the case of using nonlinear NWC to identify quantitative estimates of IP misidentification in the case of pairwise comparison of the studied alternatives during the compilation of PCM. The linguistic significance of the IP and the quantitative estimate of the potential error caused by misidentification demonstrate an inverse relationship. For nonlinear estimates, the summed error estimate is 2 times larger than the corresponding summed measure for linear rating estimates. It is also obvious that a large number of MoP iterations provides greater nonlinearity, which leads to a higher "price" of the predicted IP identification error, as can be seen from Table 6.

Table 6

The impact of the nonlinearity of normalized weight coefficients on the predicted estimate of the identification error of preference indices (fragment)

Identification error IP	Normalized weight coefficients obtained at different MoP stages					
	I	II	…	V	…	IX
$\sum_{k=1}^{8} \varepsilon_{IP_k}^{N-L}$	4.53	7.89	…	15.11	…	40.39
$\bar{\varepsilon}_{IP_k}^{N-L}$	0.28	0.49	…	0.94	…	2.52

Considering the "price" of a potential error, all of the above points to the need to involve more mature experts in the MCDMP as a key requirement.

6. Discussion of the results of the defuzzification study of linguistic indicators of preference levels

All known methods for establishing NWCs involve conducting special expert studies. At the same time, if the alternatives for which NWCs are determined are ordered, then there is no need for such studies, and the desired NWCs are established by applying MoP, known in systems analysis as the "leader problem". This is the advantage of MoP. The ranking of the Saaty scale IP is obvious, therefore the establishment of nonlinear NWCs is carried out using MoP.

Twelve iterations of MoP were carried out (Tables 3, 4 and Fig. 1). The peculiarity of using MoP is that each new iteration automatically provides greater delinearization of the NWC of the basic IPs (IP_9, IP_7, IP_5, IP_3, IP_1), and therefore greater accuracy of their calculation. The value of the NWC of the "intermediate (compromise)" IPs (IP_8, IP_6, IP_4, IP_2) in accordance with expression (15) is set as the geometric mean of the coefficients of neighboring terms (Table 4).

The results of the second iteration of MoP, which, on the one hand, are nonlinear and, on the other hand, satisfy the established accuracy of NWC calculations, are considered more acceptable for research. These results are mathematically described by the power function (14).

The results presented indicate the following advantages over the linear LP score scale:

– from expression (16) it follows that due to nonlinear NWCs, the sensitivity of the scale as the ratio of the maximum and minimum IP score increased to an indicator of 41 (40.88), which is 4.5 times greater than the corresponding indicator (3) obtained for linear scores;

– quantitative indicators of the predictive assessment of the error of expert identification of the true IP (see expressions (20) and (23)) during the construction of PCM were introduced. It was found that the average estimate of the IP identification error increased in relation to linear measurements by 1.84 times (Tables 5, 6). This requires experts to be more competent and responsible during the construction of PCM;

– the relative quantitative indicator corresponding to the increase in the NWC of the Saaty scale IP, formed using the modifier "very" by performing the fuzzy operation "concentration" on the previous term (see expression (4)), increased almost twice (the ratio of the results (17) and (5), (18) and (6)), "double concentration" – by a third (the ratio of the results (19) and (7)). This is more consistent with the quantitative-qualitative logic of the ratio of the corresponding terms.

The scope of application of the obtained results covers all tasks of multi-criteria DM, where AHP is used, and therefore PCM is built. The practical significance of the work lies in the possibility of increasing the validity and accuracy of decisions by taking into account the nonlinearity of human perception when forming PCM. The obtained results can also be used as a methodological content of the created intelligent DM support systems.

The conditions for the effective application of the developed method are tasks where expert assessments are of a linguistic nature, and requirements are put forward for the accuracy of identification and quantitative assessment of IP.

The disadvantages of the study are the increased demands on preventing the influence of the human factor on the adequacy of IP identification.

The Saaty scale should be considered as holistic, however, "following its lead", the corresponding IPs were artificially divided into "basic" and "compromise", which could not but affect the obtained results of the application of MoP.

Further development of the obtained results consists in re-solving known MCDMPs, which will allow to realistically assess their effectiveness. After all, the NWC application concerns only the PCM formation and does not affect the general technology of determining local and global priority vectors of the solved MCDMPs.

7. Conclusions

1. It is adapted for the needs of MoP research. The acceptability of the results of the second iteration of MoP, which are nonlinear and satisfy the established accuracy of calculations, is justified. The implementation of MoP brought nonlinear NWCs into line with the basic linguistic IRs of the Saaty scale (IP_9, IP_7, IP_5, IP_3, IP_1). The NWC values for the "intermediate (compromise)" linguistic IPs (IP_8, IP_6, IP_4, IP_2) are found by multiplicative aggregation of the NWCs of neighboring IRs.

2. The nonlinear NWC scale of the IP is mathematically described by a power function, which facilitates analytical research. The quantitative relative indicator corresponding to the terms formed by the fuzzy "ordinary concentration" and "double concentration" in the linguistic IP scale has doubled. The quantitative indicator of the sensitivity of the Saaty IP scale to the distinction of priorities increases in the nonlinear scale by 4.5 times.

3. It is found that the average price of false identification of IP increased in the nonlinear scale by 1.84 times, which places demands on the competence of experts and ensuring their attentiveness in the process of building PCM. From the above, a generalized conclusion can be drawn regarding a more accurate reflection of expert preferences in the process of forming PCM, which is important for increasing the validity of decisions in MCDMP.

Conflict of interest

The authors declare that they have no conflict of interest regarding this study, including financial, personal, authorship, or other, that could influence the study and its results presented in this article.

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Data availability

The article has no associated data.

Using artificial intelligence tools

The authors used artificial intelligence technologies within acceptable limits to provide their own verified data, which is described in the research methodology section.

References

1. Rotshtein, O. P., Shtovba, S. D., Kozachko, O. M. (2007). Modeliuvannia ta optymizatsiya nadiynosti bahatovymirnykh alhorytmichnykh protsesiv. Vinnytsia: UNIVERSUM-Vinnytsia, 211.
2. Kovalchuk, V. M. (2010) Osoblyvosti rozv'yazannia bahatokryterialnykh zadach pryiniattia rishen u nechitkomu seredovyshchi. Finansova systema Ukrainy. Naukovi zapysky. Seriya "Ekonomika", 14, 447–456. Available at: https://eprints.oa.edu.ua/id/eprint/6803/
3. Reva, O. M., Kamyshyn, V. V., Borsuk, S. P., Yarotskyi, S. V., Sahanovska, L. A. (2023). Metodolohiya systemno-informatsiynoi kvalimetriyi investytsiynoi pryvablyvosti obiektiv ekspertyzy. Kyiv: UkrINTEI, 150.
4. Taherdoost, H., Madanchian, M. (2023). Multi-Criteria Decision Making (MCDM) Methods and Concepts. Encyclopedia, 3 (1), 77–87. https://doi.org/10.3390/encyclopedia3010006
5. Metody vyrishennia zadach bahatokryterialnoi optymizatsiyi (2024). Odesa: Odes. nats. un-t im. I.I. Mech-nykova, 50. Available at: https://dspace.onu.edu.ua/items/0a0bbbbe-81da-45e2-9703-fed7261b3a11
6. Saaty, T. L. (2009). Mathematical Principles of Decision Making (Principia Mathematica Decernendi). Pittsburgh: RWS.
7. Millar, L. A., McCallum, J., Burston, L. M. (2010). Use of the conjoint value hierarchy approach to measure the value of the national continence management strategy. The Australian and New Zealand Continence Journal, 16 (3), 81–86.
8. Khan, A. U., Ali, Y. (2020). Analytical hierarchy process (AHP) and analytic network process methods and their applications: a twenty year review from 2000-2019. International Journal of the Analytic Hierarchy Process, 12 (3). https://doi.org/10.13033/ijahp.v12i3.822
9. Ivanova, N., Ganzha, O., Podkovyrov, I. (2020). The use of the Analytic Hierarchy Process in Determination of the Design Solution for Vertical Gardening. Journal of Physics: Conference Series, 1614 (1), 012035. https://doi.org/10.1088/1742-6596/1614/1/012035
10. Krenicky, T., Hrebenyk, L., Chernobrovchenko, V. (2022). Application of Concepts of the Analytic Hierarchy Process in Decision-Making. Management Systems in Production Engineering, 30 (4), 304–310. https://doi.org/10.2478/mspe-2022-0039
11. Oleniacz, G., Skrzypczak, I., Leń, P. (2019). Decision-making models using the Analytical Hierarchy Process in the urgency of land consolidation works. Journal of Water and Land Development, 43 (1), 144–150. https://doi.org/10.2478/jwld-2019-0072
12. Vartanyan, V. M., Shteinbrekher, D. A. (2019). Application of the hierarchy analysis technique for the design of high-technological projects knowledge management strategy. Radioelectronic and Computer Systems, 2 (90), 118–126. https://doi.org/10.32620/reks.2019.2.11

13. Nisfoian, S., Sysolina, N., Savelenko, H. (2020). Improving the Efficiency of Investment Project Selection Mechanisms at the Enterprise. Central Ukrainian Scientific Bulletin. Economic Sciences, 5 (38), 228–237. https://doi.org/10.32515/2663-1636.2020.5(38).228-237
14. Alshaibi, A., Kahraman, G. D., Qasim, A. (2016). Analytic Hierarchy Process (AHP) as criteria in business decision making and their implementation in practice. International Journal of Management and Business Studies, 6 (2), 209–220. Available at: https://www.internationalscholarsjournals.com/abstract/analytic-hierarchy-process-ahp-as-criteria-in-business-decision-making-and-their-implementation-in-practice-61841.html
15. Taherdoost, H. (2017). Decision Making Using the Analytic Hierarchy Process (AHP); A Step by Step Approach. International Journal of Economics and Management System, 2, 244–246. Available at: https://hal.science/hal-02557320
16. Tavana, M., Soltanifar, M., Santos-Arteaga, F. J. (2021). Analytical hierarchy process: revolution and evolution. Annals of Operations Research, 326 (2), 879–907. https://doi.org/10.1007/s10479-021-04432-2
17. Koo, M., Yang, S.-W. (2025). Likert-Type Scale. Encyclopedia, 5 (1), 18. https://doi.org/10.3390/encyclopedia5010018
18. Mahmoudi, A., Deng, X., Javed, S. A., Zhang, N. (2020). Sustainable Supplier Selection in Megaprojects: Grey Ordinal Priority Approach. Business Strategy and the Environment, 30 (1), 318–339. https://doi.org/10.1002/bse.2623
19. Reva, O., Kamyshyn, V., Borsuk, S., Nevynitsyn, A. (2022). α-Method of Air Traffic Controllers' Preference System Specification Over Typical Mistakes Severities. ICT with Intelligent Applications, 679–685. https://doi.org/10.1007/978-981-16-4177-0_68