внутри области форму перевернутой «тарелочки» с плоским «дном» во внутреннем регионе области, граница которого с границей области образуют геометрический «поясок», ширина которого соответствует максимальному расстоянию по нормали к границе внутри области, на котором по показаниям данных вычислительного эксперимента еще можно решить обратную граничную задачу для исследуемого физического процесса.

Эти новые качественные особенности весовых функций ω в приближенных аналитических структурах решения краевых задач согласуются с качественными особенностями подхода к решению краевых задач в методах граничных элементов [8].

Выражаю глубокую благодарность моему аспиранту Кобринович Ю.О. за большую проделанную научно-техническую работу по созданию специализированного программного комплекса и проведению большой серии вычислительных экспериментов.

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SPECTRAL DECOMPOSITION FOR SOME TRANSPORT OPERATOR

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З допомогою перетворення Фур’є транспортний оператор представлений моделлю Фрідріхса. Використовуючи відому формулу стрибка резольвенти для оператора моделі Фрідріха, отримано рівність Парсеваля методом контурного інтегрування

Ключові слова: транспортний оператор, аналітичне продовження, модель Фрідріхса

Преобразованием Фурье транспортный оператор представлен моделью Фридрихса. Используя известную формулу скачка резольвенты для оператора модели Фридрихса, получено равенство Парсеваля методом контурного интегрирования

Ключевые слова: транспортный оператор, аналитическое продолжение, модель Фридрихса

Transport operator gives Friedrich’s model with the help of Fourier transformation. Using known formulae of jump of the resolvent for the operators of Friedrich’s model we obtain Parseval equality with the help of method of contour integrating

Keywords: transport operator, analytic extension, Friedrich’s model

Introduction

A problem about transfering of neutrons leads to the operator

$$Lf = -i\mu\frac{\partial f}{\partial x} + c(x)\int_{-1}^{1} f(x,\mu')d\mu'$$

in the space $L^2(D)$, where $D = R\times[-1,1]$. There is much literature concerning this operator and the similar ones for example works [1-3]. In the work [4] the spectrum of more general operator in the space $L^2(D)$ is studied, namely

$$Lf = -i\mu\frac{\partial f}{\partial x} + c(x)\int_{-1}^{1} b(\mu')f(x,\mu')d\mu', . \qquad (1)$$

where the function $c(x)$ as in the work [3] is exponentialy decreasing when $x \to \infty$,

$$|c(x)| \le Me^{-\varepsilon|x|}, x \in R. \qquad (2)$$

As for the function $b(\bullet)$ it is supposed the existence of analytical extension to the circle containing the interval $[-1,1]$.

Statement of the problem and denotation

The works [4-5] use the Friedrich's model to study point spectrum of transport operator. It is proposed to use known formulae of jump of the resolvent for the operators of Friedrich's model, which was obtained in the work [6], to obtain Parseval equality.

Friedrich's model

By H we denote Hilbert space of functions of two variables $\phi(\tau,\mu), (\tau,\mu) \in D = R\times[-1,1]$ with the norm

$$\|\phi\|_H^2 = \int_R\int_{-1}^{1} |\phi(\tau,\mu)|^2 \frac{1}{|\mu|}d\tau d\mu.$$

Obviously we can consider H as the space $L^2(R,H_1)$ of the functions $\phi(\tau,\bullet) \in H_1$ with values in the space

$$H_1 = L_\rho^2(-1;1),\ \rho(\mu) = \frac{1}{|\mu|},\ \|f\|_{H_1}^2 = \int_{-1}^{1}|f(\mu)|^2\frac{1}{|\mu|}d\mu.$$

Let $S: H \to H$ be the operator of multiplying by independent variable, $(S\phi)(\tau,\mu) \equiv \tau\phi(\tau,\mu)$, $\tau \in R$ with maximal domain of definition. We consider Friedrich's model $T = S + V$, where $V: H \to H$ - some integral operator. Operator V has the factorization $V = A^*B$, $A, B: H \to G$, where G - some auxiliary Hilbert space. Scalar product in the spaces H, H_1 and G we denote respectively by $(\bullet,\bullet), (\bullet,\bullet)_{H_1}$, and $(\bullet,\bullet)_G$. In work [4] it was considered Friedrich's model

$$T = S + A^*B: H \to H \qquad (3)$$

which is unitary equivalent to the operator $L: L^2(D) \to L^2(D)$ (see (1)). The factorization of the potential $c(x) = \overline{c_1(x)}c_2(x)$ such that $|c_1(x)| = |c_2(x)|, c(x) \ne 0$ and $c_1(x) = c_2(x) = 0$, if $c(x) = 0$ was used here.

Let $G = L^2(R)$. Then in the presentation (3) we have:

$$A^*c(s,\mu) = \frac{1}{2\pi}\int_R \overline{c_1(x)}c(x)e^{-ix\frac{s}{\mu}}dx,\ B\phi(x) = \\ = c_2(x)\int_R e^{ix\tau}\left(\int_{-1}^{1} b(\mu')\phi(\tau\mu',\mu')d\mu'\right)d\tau, \qquad (4)$$

where operators $A, B: H \to G$ are bounded. We recall for reader's convenience the formulae of the resolvent of the operator T. Let's denote $S_\zeta = (S-\zeta)^{-1}, T_\zeta = (T-\zeta)^{-1}$ and

$K(\zeta) = 1 + BS_\zeta A^*, \zeta \notin R$. If the operator $K(\zeta): G \to G, \zeta \notin R$ has the inverse operator $K(\zeta)^{-1}$, then

$$T_\zeta = S_\zeta - S_\zeta A^*K(\zeta)^{-1}BS_\zeta.$$

If the operator $K(\zeta)^{-1}$ is bounded then T_ζ is the resolvent of the operator T.

We denote by $\Phi \subset H$ the subspace of elements $\phi(\tau,\mu)$, which admit holomorph extension $\phi(z,\mu), |\mathrm{Im} z| < \varepsilon$ in the domain $\Omega = \{z \in C: |\mathrm{Im} z| < \varepsilon\}$, where ε is defined in (2). We need the extension of the operator functions $K_+(\zeta), \mathrm{Im}\zeta > -\varepsilon$ and $K_-(\zeta), \mathrm{Im}\zeta < \varepsilon$ over axis $\mathrm{Im}\zeta = 0$ in the domain Ω.

Denote

$$(A(\zeta)\psi)(\tau) = \psi(\tau) - B^*K(\bar{\zeta})^{-1*}AS_\zeta\psi, \\ (B(\zeta)\psi)(\tau) = \psi(\tau) - A^*K(\zeta)^{-1}BS_\zeta\psi. \qquad (5)$$

In the work [4] it was proved that

C_1) there exist the extension $K_\pm(\zeta), \zeta \in \Omega$ and $\lim_{\zeta\to\infty}\|K_\pm(\zeta) - 1\|_G = 0$ uniformly in the domain Ω,

C_2) the operators $K_\pm(\zeta)^{-1}$ exist and are holomorph in Ω except may be finite set of poles.

Let $A_-(\sigma), B_+(\sigma): H \to H$ are following operator function

$$A_-(\sigma)\psi = \psi - B^*K_-(\sigma)^{-1*}(AS_\sigma\psi)_+, \\ B_+(\sigma)\phi = \phi - A^*K_+(\sigma)^{-1}(BS_\sigma\phi)_+, \sigma \in R. \qquad (6)$$

If $\sigma = \mathrm{const}, \sigma \ne 0$, then the operators

$\psi \to (A_-(\sigma)\psi)(\sigma) \in H_1$ $\phi \to (B_+(\sigma)\phi)(\sigma) \in H_1$ represent a family of eigenfunctionals corresponding to σ of the operators T and T^* respectively. Namely, if $\phi, \psi \in D(T) \cap \Phi$ then in the space H_1 we have the equalities

$$\left(A_+(\sigma)(T^* - \sigma)\psi\right)(\sigma) = 0, \left(B_+(\sigma)(T - \sigma)\phi\right)(\sigma) = 0. \qquad (7)$$

For arbitrary orthonormal base $\{e_k\}$ in H_1 we have infinite system of linearly independent eigenfunctionals

$$(\phi, b_{\sigma,k}) = \left((B_+(\sigma)\phi)(\sigma), e_k\right)_{H_1}, (a_{\sigma,k}, \psi) = \left(e_k, \left(A_+(\sigma)\psi\right)(\sigma)\right)_{H_1},$$

$k = 1, 2, \ldots$

respectively of the operators T and T^* corresponding to σ.

We denote limit values of resolvent by

$$\left(T_\sigma\phi, \psi\right)_\pm = \lim_{\nu\to+0}\left(T_{\sigma\pm i\nu}\phi, \psi\right), \phi, \psi \in \Phi, \sigma \in R.$$

If the scalar function scalar function

$$h(\tau,\zeta) = ((B(\zeta)\phi)(\tau), (A(\zeta)\psi)(\tau))_{H_1}$$

satisfies following conditions

$C_3)$ function $h(\tau,\zeta)$ is differentialble on τ in R and

$$\int_R |h(\tau,\zeta)| d\tau \le M, 0 \le \mathrm{Im}\zeta < \varepsilon_0,$$

where $M = \mathrm{const}$.

$C_4)$ for every finite interval (a,b)

$$|h(\tau,\zeta) - h(\tau,\sigma)| d\tau \le C|\zeta - \sigma|, \zeta = \sigma + i\nu, \tau \in (a,b),$$

where $C = \mathrm{const}$, then (see (6)) jump of the resolvent on axis $\mathrm{Im}\zeta = 0$ is

$$(T_\sigma \phi, \psi)_+ - (T_\sigma \phi, \psi)_- = 2\pi i\big((B_+(\sigma)\phi)(\sigma), (A_-(\sigma)\psi)(\sigma)\big)_{H_1}$$

Estimates of eigenfunctions

Conserning the factorization $c(x) = c_1(x)\overline{c_2(x)}$, we suppose that there exist the derivatives $c'_{1,2}(x), c''_{1,2}(x)$, that $c'_{1,2}(x), c''_{1,2}(x) \to 0$, exponentialy when $|x| \to \infty$ and $c', c'' \in G$.

Lemma 1. Let $c \in G = L^2(R)$, then

1) function $\mu \to A^* c(s,\mu)$ belongs to H_1 everywhere for $s \in R$,

2) if $c' \in G$ then the function $\mu \to A^* c(s,\mu)$ belongs to H_1 for every $s \in R, s \ne 0$ and

$$\|A^* c(s,\bullet)\|_{H_1} \le \frac{C}{|s|}\left(\|c\|_G + \|c'\|_G\right), \tag{8}$$

if $c', c'' \in G$ then

$$\left\|\frac{d}{ds} A^* c(s,\bullet)\right\|_{H_1} \le \frac{C}{s^2}\left(\|c\|_G + \|c'\|_G + \|c''\|_G\right), \tag{9}$$

where $C = \mathrm{const}$.

Analogous statement for the operator B^* holds too.

The proof is based directly on the presentation (4).

Denote by $D\psi$ or $\frac{d}{d\tau}\psi(\tau)$ strong derivate $\frac{\partial}{\partial\tau}\psi(\tau,\mu)$ of vector- function $\tau \to \psi(\tau,\bullet) \in H_1$.

Denote

$$p(s,\nu) = 1 + \left|\frac{s}{\nu}\right| + \frac{s^2}{\nu^2}, \|\psi\|_p = \|p\psi\| + \|pD\psi\|,$$

where $s,\nu \in R, \nu \ne 0$ and introduce linear subspace

$$H_D = \left\{\psi = \psi(s,\mu) : \psi'_s(s,\mu) \in H, \|\psi\|_p < \infty\right\} \tag{10}$$

Lemma 2. If $\psi \in H_D, \zeta = \mathrm{const}$, then vector functions $\tau \to (A(\zeta)\psi)(\tau)$, $(B(\zeta)\psi)(\tau) \in H_1$ admit strong derivates

$$\frac{d}{d\tau}(A(\zeta)\psi)(\tau), \frac{d}{d\tau}(B(\zeta)\psi)(\tau) \in H_1, \tau \ne 0.$$

Let $\nu_0 \in (0,1)$, we introduce linear subspace $H_{\nu_0} \subset H$ as

$$H_{\nu_0} = \left\{\psi \in H : \psi(s,\nu) \equiv 0, |\nu| < \nu_0, s \in R\right\}. \tag{11}$$

The proof uses the property of the operator-function $K(\zeta)$. For example the equation $K(\bar{\zeta})^* c = d$ signifies

$$c(x) + \frac{1}{2\pi} c_1(x) \int_R \overline{c_2(y)}\, \overline{I(y - x, \zeta)} c(y) dy = d(x),$$ where $I(u,\zeta)$ is defined by the function $b(\mu)$.

Lemma 3. Let (a,b) be arbitrary finite interval such that $0 \notin (a,b)$ and $\sigma \in R$. Then for $\psi \in H_{\nu_0}$

$$\|(A(\zeta)\psi)(\tau) - (A(\sigma)\psi)(\tau)\|_{H_1} \le C|\zeta - \sigma|, \tau \in (a,b), \tag{12}$$

where $C = \mathrm{const}$ and $|\zeta - \sigma| < \varepsilon_1$ for some $\varepsilon_1 > 0$.

To prove the Lemma we prove that the expression $B^* K_-(\sigma)^{-1*}(AS_\sigma \psi)_+$

Admits analytic extension. Here the estimate (2) is important.

Theorem 1. If $\phi, \psi \in H_D \cap H_{\mu_0}, \mu_0 \in (0,1)$ then the jump of the resolvent is

$$(T_\sigma \phi, \psi)_+ - (T_\sigma \phi, \psi) = 2\pi i\big((B_+(\sigma)\phi)(\sigma), (A_-(\sigma)\psi)(\sigma)\big)_{H_1},$$

$\sigma \ne 0$

Let $\psi = \psi(s,\mu) \in H$, denote

$$\|\psi\|_1^2 = \int_R \int_{-1}^{1} (1 + s^2)\left[|\psi(s,\mu)|^2\right] \frac{1}{|\mu|^{\frac{7}{2}}} d\mu ds. \tag{13}$$

Lemma 4. Let $\delta > 0$ then

$$\int_{|\sigma|>\delta} \|(A_-(\sigma)\psi)(\sigma)\|_{H_1}^2 d\sigma, \int_{|\sigma|>\delta} \|(B_+(\sigma)\psi)(\sigma)\|_{H_1}^2 d\sigma \le \frac{M}{\delta^2}\|\psi\|_1, \tag{14}$$

where $M = \mathrm{const}$.

Method of contour integration

Recall that continuous spectrum of the operator T coinsides with real axis R. We suppose that the set of eigen values is finite and there are not spectral singularities.

We will use the method of contour integration i.e. if $\phi \in D(T)$, then

$$(\phi, \psi) = \left(T_\zeta (T - \zeta)\phi, \psi\right) = \left(T_\zeta T\phi, \psi\right) - \zeta(T_\zeta \phi, \psi),$$

or

$$\frac{1}{\zeta}(\phi, \psi) = \frac{1}{\zeta}(T_\zeta T\phi, \psi) - (T_\zeta \phi, \psi).$$

Integrating through the contour $|\zeta| = R$ for some $R > 0$, we obtain

$$2\pi i(\phi, \psi) = \int_{|\zeta|=R} \frac{1}{\zeta}\left(T_\zeta T\phi, \psi\right) d\zeta - \int_{|\zeta|=R} \left(T_\zeta \phi, \psi\right) d\zeta. \tag{15}$$

Let $\{\zeta_k\}, k = 1,2,\dots k_0$ be set of eigenvalues and $P_{\zeta_k} = -\mathrm{Res}_{\zeta=\zeta_k} T_\zeta$.

Charging the contour $|\zeta| = R$ to the segment of real axis $(-R,R)$ in the second integral of (15) we obtain

$$2\pi i(\phi,\psi)=\int\limits_{|\zeta|=R}\frac{1}{\zeta}\left(T_\zeta T\phi,\psi\right)d\zeta+$$
$$+\int\limits_{-R}^{R}\left(\left(T_\sigma\phi,\psi\right)_+-\left(T_\sigma\phi,\psi\right)_-\right)d\sigma+\sum_{k=1}^{k_0}(P_{\zeta_k}\phi,\psi). \quad (16)$$

As there are no spectral singularities then the jump of the resolvent is integrable on the interval $(-R,R)$. For putting $R\to\infty$ we need the next Lemma.

Lemma 5. If the elements $\phi,\psi\in H$ satisfy the conditions $\phi,\psi\in H_2\subset H$, where

$$H_2=\left\{\phi\in H:\|\phi\|_2^2\equiv\int\limits_{-1}^{1}\int\limits_{R}\frac{1+t^2}{\mu^2}\left[\frac{1}{\mu^2}|\phi(t,\mu)|^2+\left|\phi_t^{'}(t,\mu)\right|^2\right]dtd\mu<\infty\right\} \quad (17)$$

then

$$\lim_{R\to\infty}\oint\limits_{|\zeta|=R}\frac{1}{\zeta}\left(T_\zeta\phi,\psi\right)d\zeta=0. \quad (18)$$

Theorem 2. If $\phi,\psi\in H_D\cap H_{v_0}\cap H_2$ then following Parseval equality of the operator T holds

$$(\phi,\psi)=\int\limits_{-\infty}^{\infty}\left((B_+(\sigma)\phi)(\sigma),(A_-(\sigma)\psi)(\sigma)\right)_{H_1}d\sigma+\sum_{k=1}^{k_0}\left(P_{\zeta_k}\phi,\psi\right), \quad (19)$$

where P_{ζ_k} are finite dimension operators.

Proof results from Theorem 1, Lemma 5 and equality (16). The operator is compact, therefore $\dim P_{\zeta_k}<\infty$.

The functionals $\phi\to\left(B_+(\sigma)\phi\right)(\sigma,\mu)$, $\psi\to\left(A_-^{*}(\sigma)\psi\right)(\sigma,\mu)$ for every $\mu\in(-1,1)$ are eigen functionals for the operators T and T^*, namely (see(7))

$$\left(B_+(\sigma)T\phi\right)(\sigma,\mu)=\sigma\left(B_+(\sigma)\phi\right)(\sigma,\mu)\quad\left(A_+(\sigma)T^*\psi\right)(\sigma,\mu)=\sigma\left(A_+(\sigma)\psi\right)(\sigma,\mu),\ \sigma\in R \quad (20)$$

corresponding to the point σ to continuous spectrum of T (see(10), (11) and (16)).

Theorem 3. 1) Parseval equality

$$(\phi,\psi)=\int\limits_{-\infty}^{\infty}\left((B_+(\sigma)\phi)(\sigma),(A_-(\sigma)\psi)(\sigma)\right)_{H_1}d\sigma+\sum_{k=1}^{k_0}\left(P_{\zeta_k}\phi,\psi\right)$$

holds for the elements $\phi,\psi\in H$ such that $\|\phi\|_1,\|\psi\|_1<\infty$ (see(13)).

2) If $h(\tau)$ is bounded function on R, holomorph in neighborhood of points ζ_k, then the function of operator $h(T)$ is defined by the equality

where $\|\phi\|_1,\|\psi\|_1<\infty$.

$$(h(T)\phi,\psi)=\int\limits_{-\infty}^{\infty}h(\sigma)\left((B_+(\sigma)\phi)(\tau),(A_-(\sigma)\psi)(\sigma)\right)_{H_1}d\sigma+\sum_{k=1}^{k_0}\left(P_{\zeta_k}h(T)\phi,\psi\right), \quad (21)$$

As conclusion note that formulae (21) generalize partial case, considered in the work [7], where it was considered the function $h(\tau)=e^{i\tau t},-\infty<\tau<\infty$.

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