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DESIGNING NON-CIRCULAR WHEELS USING A FOURTH-DEGREE POLYNOMIAL

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This study's object is the process of designing closed non-circular wheels with a given center-to-center distance with external rolling, provided that it occurs without mutual slip. Non-circular wheels serve as centroids in the design of cylindrical gear transmissions with a variable gear ratio. The gear ratio is characterized by the gear function. If the gear ratio is constant, then the centroids are circles. The gear ratio in this case is the ratio of the radii of these circles. Non-circular wheels can be designed according to a given gear function, which is determined by the kinematics of the mechanism's links' actuators. In this case, non-circular wheels can be non-closed.

The study addresses another task related to designing non-circular wheels provided that they are closed. Various approaches can be used to this end. The current paper considers the use of a fourth-degree polynomial. In non-circular wheels, the radii, which are understood as the distances from the centers of rotation to the point of contact, are variable. The necessary conditions for constructing non-circular wheels are the constant value of the sum of these radii during the rotation of non-circular wheels, as well as the equality of the paths traveled, that is, the equality of the arcs that non-circular wheels pass during rotation. Pairs of wheels can have the same or different numbers of protrusions and depressions. This is explained by the use of a fourth-degree polynomial whose plot has an axis of symmetry. Accordingly, non-circular wheels or their protrusions also have an axis of symmetry.

As an example, the construction of a leading centroid with one protrusion and one depression is given, for which the maximum and minimum distances from the center are 10 and 6.1 linear units, respectively. For this centroid, a trailing centroid has been constructed, and the center-to-center distance has been found, which is 16.79 linear units

Keywords: external rolling, center-to-center distance, arc length, axis of symmetry, radius vector

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1. Introduction

The issue of designing pairs of non-circular wheels of various configurations is relevant due to their multifaceted application in modern mechanical engineering and other industries. They are indispensable in complex mechanisms with variable transmission of motion or nonlinearity in kinematics. In particular, they are used in devices for accurate measurement of fluid flow. This occurs at oil refineries and chemical plants where accurate measurement of oil or chemical reagents is required. Their use in this area makes it possible to reduce pressure fluctuations, which is important for accurate measurement. In mechanical engineering, they are applied in drives of technological machines with a variable gear ratio. With the help of non-circular wheels, the direction of rotation of gears can be changed [1]. Other examples of their use in textile, printing machines, manipulators, and step conveyors are also known. Non-circular gear transmissions are even used in the structures of exoskeleton robots [2]. All this confirms the importance of research in this area since its results are necessary for practice.

2. Literature review and problem statement

The design of centroids of non-circular wheels is based on rolling of curvilinear contours one by one without slipping. In a partial case of such rolling, one of the contours can be a straight line. Widely known examples of such rolling are rolling of a circle along a straight line when forming a cycloid and rolling of a straight line along a circle when forming an involute.

In [3], the results of research into the process of rolling a polygon along a curvilinear profile are reported. It is shown that the shape of the polygon and the features of the profile affect the dynamics of motion and contact loads. However, issues related to the optimization of such systems for complex mechanisms remain unresolved. The likely reason is difficulties associated with the instability of motion and high contact stresses. An option for overcoming the difficulties is to use optimized non-circular gears. This is the approach reported in [4], which investigated the design of non-circular gears for regulating the speed of a machine. It is shown that the use of special gears makes it possible to control the speed change.

Study [5] considers the method for generating non-circular gears with modification of the tooth head and their application in mechanisms. It is shown that the proposed method makes it possible to improve the kinematics of gears; however, its application is limited by specific loading conditions. An option for overcoming the difficulties is to introduce adaptive design methods.

In work [6], the formation of the profile of the teeth of an elliptical gear based on an involute elliptic curve is considered. It is shown that this approach makes it possible to obtain better kinematic characteristics compared to standard methods. However, there are unresolved issues related to the manufacture and quality control of such gears.

In [7], methods for computational design and optimization of non-circular gears are considered. It is proven that numerical optimization algorithms can improve the operating characteristics of gears. However, questions related to taking into account real deformations and loads in physical systems remain open. The likely reason is the limited availability of existing material models and the difficulty of their verification. In [8], the optimization of the load of non-circular gears based on asymmetric pressure angles is discussed. It is shown that the correct choice of parameters reduces contact stresses, but this complicates the production process and increases the cost. In [9], a method for measuring the pitch error of a non-circular gear based on a digital twin is proposed. This method makes it possible to increase the accuracy of error estimation. In [10], an analytical method for calculating the variable meshing stiffness of a non-circular planetary gear with a crack is reported. The authors prove that cracks significantly affect the dynamics of the system. However, there is room for further research in the direction of diagnostics and forecasting the resource of such mechanisms.

Work [11] investigates the design methodology and analysis of the characteristics of multi-segment deformed non-circular gears. A new approach is proposed that improves their operation under specific conditions, but it requires further experimental studies to confirm the theoretical results.

Our review of techniques for constructing centroid pairs shows that it is advisable to construct them from a continuous curve. In the cited works, centroids are formed from a sequence of congruent arcs of curves that intersect at a certain angle. It is advisable to construct centroids from arcs of a continuous curve that are joined together by a high order of smoothness. For this purpose, the plot of a fourth-degree polynomial could be used.

3. The aim and objectives of the study

The aim of our research is to devise a technique for constructing pairs of centroids with external rolling when rotating them around fixed centers from a continuous curve using a fourth-degree polynomial. This will make it possible to design gearing with a variable transmission function.

To achieve this aim, the following objectives were accomplished:

– to mathematically describe the contour of the driving centroid with a given number of protrusions and depressions and the corresponding contour of the trailing centroid;

– based on the results obtained, construct examples of pairs of centroids with the same and different numbers of protrusions and depressions.

4. The study materials and methods

The object of our study is the process of constructing the centroids of non-circular wheels whose contours are described by a continuous curve. It was hypothesized that a continuous curve of the contour of the leading centroid with a given number of protrusions and depressions could be obtained using a fourth-degree polynomial. The simplification of the study is that the centroids are constructed not according to a given transfer function but according to a given contour of the leading centroid.

The differential geometry of plane curves is involved in the research. Their mathematical description is considered in a polar coordinate system with a transition to a Cartesian system. At the first stage, the construction of the leading centroid is considered. First, it is advisable to consider closed non-circular wheels, in which polar radius ρ changes from the minimum to the maximum value as a function of polar angle α. Such a change is graphically represented in Fig. 1, *a*. At points A and B, polar radius ρ takes on a minimum value, and polar angle α changes from $-\pi$ to π, i.e., the polar radius rotates by 360°. Therefore, the curve will be closed and will have an axis of symmetry. A closed curve can be constructed from separate symmetric arcs (protrusions). For example, if it is required that there are n such protrusions, then at points A and B the value of the polar angle should be $-\pi/n$ and π/n, respectively. When the polar angle changes within the $-\pi/n...\pi/n$ range, a symmetric arc will be obtained, by successively rotating it by an angle of $2\pi/n$ around the pole, a closed composite non-circular wheel with symmetric protrusions can be formed.

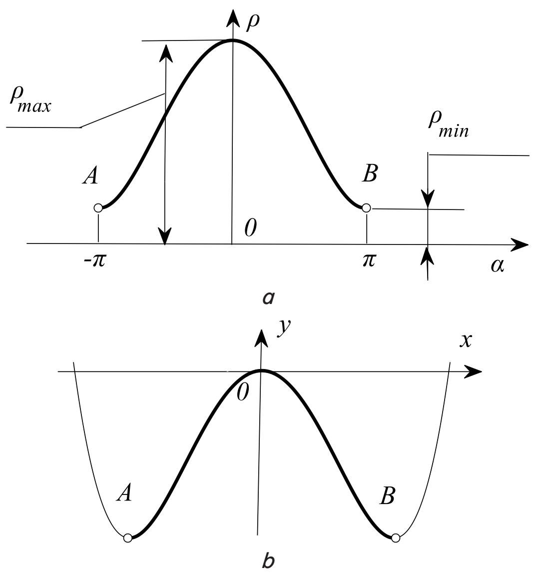

Fig. 1. Illustrating the form of dependence plot $\rho = \rho(\alpha)$: *a* — desired form of the plot with the value of argument at the extreme points; *b* — part of the plot of the fourth degree polynomial that has the desired form

The graphic dependence shown in Fig. 1, *a* corresponds to the arc of the plot for a fourth-degree polynomial in the following form

$$y = ax^4 - bx^2, \qquad (1)$$

where *a* and *b* are constants.

The plot of polynomial (1) is shown in Fig. 1, *b*. It is proposed to use only the part of the plot between points *A* and *B*. These are the minimum points at which the value of the argument must be determined, namely $x_A = -\pi/n$ and $x_B = \pi/n$. Since at these points the derivative of the function is zero, expression (1) must be differentiated with respect to variable *x*; the variable *x* must be given the value $x = \pi/n$. After that, we can find the relationship between constants *a* and *b*

$$b = \frac{2a\pi^2}{n^2}. \qquad (2)$$

In expression (1), we should replace *y* with ρ, *x* with α and use the value of constant *b* from (2). After that, dependence $\rho = \rho(\alpha)$ takes the form

$$\rho = a\alpha^4 - \frac{2a\pi^2}{n^2}\alpha^2 + c, \qquad (3)$$

where *c* is another constant that specifies the maximum value of polar radius ρ ($\rho_{\max} = c$). The minimum $\rho_{\min}$ value depends on the value of constant *a*. Given that $\rho_{\min}$ takes its value at point *B* at $\alpha = \pi/n$ (Fig. 1, *a*), we can find the value of constant *a* that will provide the required $\rho_{\min}$ value

$$a = \frac{n^4}{\pi^4}\left(\rho_{\max} - \rho_{\min}\right). \qquad (4)$$

In the case when $\rho_{\max} = \rho_{\min}$ the constant *a* according to (4) becomes equal to zero and dependence (3) is transformed into a constant value corresponding to the circle.

From polar equation (3) it is necessary to proceed to the parametric equations of the curve:

$$x = \left(a\alpha^4 - \frac{2a\pi^2}{n^2}\alpha^2 + c\right)\cos\alpha,$$

$$y = \left(a\alpha^4 - \frac{2a\pi^2}{n^2}\alpha^2 + c\right)\sin\alpha. \qquad (5)$$

When constructing the arc of the curve according to equations (5), angle α varies within the limits $\alpha = -\pi/n...\pi/n$.

As an example, the construction of non-circular wheels for $\rho_{\max} = 10$ and for different values of number *n* can be considered. Let the values of the constants be: $c = 10$, $n = 1$, $a = 0.04$. When changing angle α within the limits $\alpha = -\pi...\pi$ according to equations (5), it is possible to construct a curve, which is shown in Fig. 2, *a*. According to formula (3), at $\alpha = \pi$, it is possible to find the minimum value of radius ρ: $\rho_{\min} = 6.1$. At $\rho_{\min} = 2$ according to formula (4), it is possible to determine: $a = 0.082$. This curve is shown in Fig. 2, *b*.

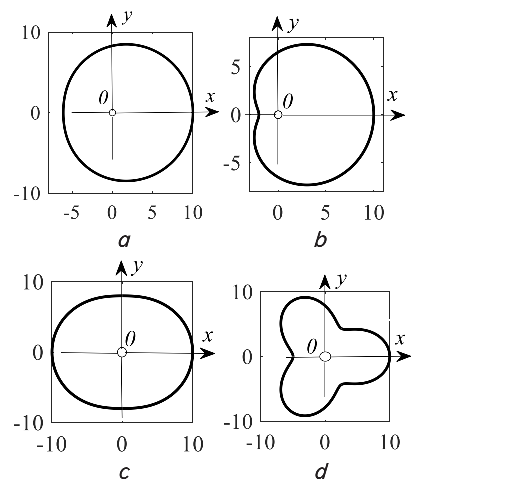

Fig. 2. Closed curves constructed for a given value of $\rho_{\max} = 10$: *a* — $n = 1$, $\rho_{\min} = 6.1$; *b* — $n = 1$, $\rho_{\min} = 2$; *c* — $n = 2$, $\rho_{\min} = 8$; *d* — $n = 3$, $\rho_{\min} = 5$

It is also possible to construct composite curves. In Fig. 2, *a*, a curve is constructed that consists of two arcs, i.e., for the value $n = 2$. For it, $\rho_{\min} = 8$. For $n = 3$ and $\rho_{\min} = 5$, the curve in Fig. 2, *d* is constructed.

5. Results related to devising a technique for constructing pairs of centroids with external rolling when rotating them around fixed centers from a continuous curve

5. 1. Establishing the relationship between the polar angles of pairs of centroids for constructing a trailing centroid

The origin of coordinates (pole) of one curve, given by radius vector $\rho = \rho(\alpha)$ is point *O*, and the second, given by radius vector $\rho_1 = \rho_1(\varphi)$, is point O_1. At zero polar angles α and φ, the curves touch each other at point *T* (Fig. 3). If functions ρ and ρ_1 are even, as in the case under consideration (Fig. 1), then at negative values of polar angles the polar radii will have the same values as at positive angles and the curve will be symmetric (in Fig. 3, the symmetric branch is depicted by a dashed line). Under the condition of external rolling of the curves one by one without slipping, the lengths of arcs TT_1 and TT_2 must be equal. At $\alpha=\varphi=0$, we can write: $\rho+\rho_1=r$, where r denotes the center-to-center distance OO_1. Let the initial condition be the fulfillment of this equality for all corresponding points of both curves. Then we can write: $\rho_1=r-\rho$. The parametric equations of the curves take the following form:

$$x=\rho\cos\alpha,$$

$$y=\rho\sin\alpha,$$

$$x_1=\left(r-\rho\right)\cos\varphi+r,$$

$$y_1=\left(r-\rho\right)\sin\varphi. \tag{6}$$

For specific values of angles α and φ according to equations (1) points on the curves, for example, T_1 and T_2 (Fig. 3), will be obtained.

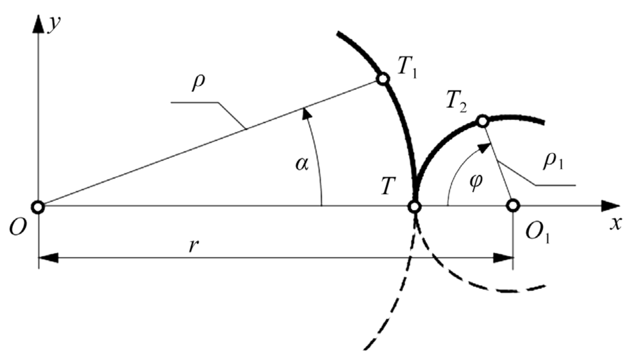

Fig. 3. Illustrating the modeling of curves with a common point of contact at their external contact

Angle φ depends on angle α, i.e., $\varphi=\varphi(\alpha)$. Dependence $\varphi=\varphi(\alpha)$ can be found on the basis of the equality of arcs TT_1 and TT_2 for the current value of angle α. The derivative of the arc length for both curves is determined from the following formula

$$\frac{ds}{d\beta}=\sqrt{x'^2+y'^2}. \tag{7}$$

To determine the derivatives of arcs using formula (7), the derivatives of the curves (6) must be found:

$$x'=\rho'\cos\alpha-\rho\sin\alpha,$$

$$y'=\rho'\sin\alpha+\rho\cos\alpha,$$

$$x_1'=\rho'\cos\varphi-\varphi'\left(\rho-r\right)\sin\varphi,$$

$$y_1'=\rho'\sin\varphi+\varphi'\left(\rho-r\right)\cos\varphi. \tag{8}$$

After substituting the derivatives (8) into formula (7), the following expressions for the derived arcs of the curves will be obtained:

$$\frac{ds}{d\alpha}=\sqrt{\rho^2+\rho'^2},$$

$$\frac{ds_1}{d\beta}=\sqrt{p'^2+\varphi'^2\left(\rho-r\right)}. \tag{9}$$

Expressions (9) must be equated with each other and solved with respect to dependence $\varphi=\varphi(\alpha)$

$$\varphi=\int\frac{\rho}{r-\rho}\mathrm{d}\alpha. \tag{10}$$

Dependence (10) in its final form cannot be obtained for all possible functions $\rho=\rho(\alpha)$. In the case under consideration, this can be done, although the result has a somewhat cumbersome appearance. After substituting (3) into (10) and integrating, we can obtain

$$\varphi=\frac{rn^2}{2\sqrt{a^2\pi^2\left(c-r\right)-an^4\left(c-r\right)^2}}\times$$
$$\times\left(B\,\mathrm{Arctanh}\frac{\sqrt{a}}{A}\alpha-A\,\mathrm{Arctanh}\frac{\sqrt{a}}{B}\alpha\right)-\alpha, \tag{11}$$

where

$$A=\sqrt{\frac{a\pi^2}{n^2}+\sqrt{a\left(\frac{a\pi^4}{n^4}+r-c\right)}};\ B=\sqrt{\frac{a\pi^2}{n^2}-\sqrt{a\left(\frac{a\pi^4}{n^4}+r-c\right)}}.$$

From dependence (11), we can obtain rotation angle φ of the trailing centroid, which corresponds to rotation angle α of the leading centroid.

5. 2. Examples for constructing pairs of centroids with the same and different numbers of protrusions and depressions

It is advisable to start with non-circular wheels, for which the number of protrusions n is equal. Let $n=1$ (Fig. 2, *a*, *b*). In this case, in expression (11), angle α must vary within the limits $\alpha=-\pi...\pi$, that is, the curve (leading centroid) must be closed. Angle φ must vary within the same limits so that the second curve (following centroid) is also closed. When $\alpha=0$, from formula (11) we can obtain: $\varphi=0$. With a linear change in angle α, the change in angle φ will be nonlinear, but the final values must be equal to π. For arbitrary constants a, r, c, this will not happen, so they must be selected accordingly. Since $c=\rho_{max}$, and the ρ_{min} value depends on constant a, it is necessary to select the center-to-center distance r. It is not possible to solve equality (11) with respect to r; therefore, it is necessary to find the required value of r by numerical methods. For example, for the curve (Fig. 2, *a*) by substituting in (11) $a=0.04$, $n=1$, $c=10$, $\alpha=\pi$, $\varphi=\pi$ and by numerical methods it is possible to find: $r=16.79$. The trailing centroids x_1, y_1 were constructed according to equations (6) by substituting into them expressions $\rho=\rho(\alpha)$ from (3) and $\varphi=\varphi(\alpha)$ from (11). Similarly, the trailing centroid can be constructed for the curve shown in Fig. 2, *b*. The result of constructing the leading and trailing centroids with a common contact is shown in Fig. 4.

When $n>1$, the leading centroid will consist of n identical arcs. The trailing centroid will consist of the same number of arcs. When angle $\alpha=-\pi/n...\pi/n$ changes, angle φ must also change within these limits. In this case, the arc of the leading centroid will correspond to the arc of the trailing centroid, as shown in Fig. 5, *b* for $n=3$. The arcs of both curves are depicted on the centroids with thin lines for clarity. To obtain closed curves, these arcs must be sequentially rotated around their centers O and O_1 by angle $2\pi/n$, i.e., by 120°. The center-to-center distance r is found similarly to the previous case for $n=1$. In this case, the values of a, c, n and $\alpha=\pi/n$, $\varphi=\pi/n$ must be substituted into equation (11). For example, for $n=3$,

$c = \rho_{\max} = 10$, $a = 4.158$, which corresponds to $\rho_{\min} = 5$ according to formula (4), $\alpha = \varphi = 60°$, we can obtain: $r = 16.05$. In Fig. 5, pairs of non-circular centroids were constructed for different values of n with the specified $\rho_{\max}$, $\rho_{\min}$ and r values.

Of interest are pairs of wheels with different numbers of protrusions. The number of protrusions on the trailing centroid is denoted by letter m. Fig. 6 shows pairs of centroids for $n = 1$ and different values of m.

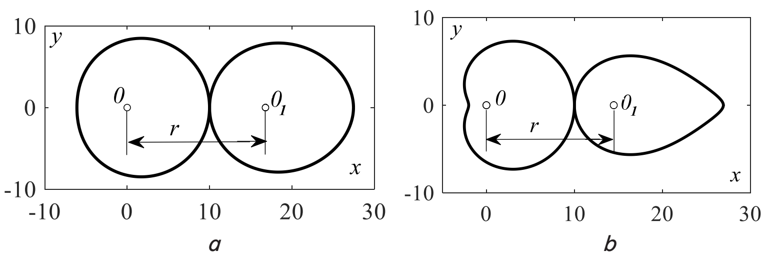

Fig. 4. Leading and trailing centroids constructed at $\rho_{\max} = 10$, $n = 1$: a — $\rho_{\min} = 6.1$, $r = 16.79$; b — $\rho_{\min} = 2$, $r = 14.47$

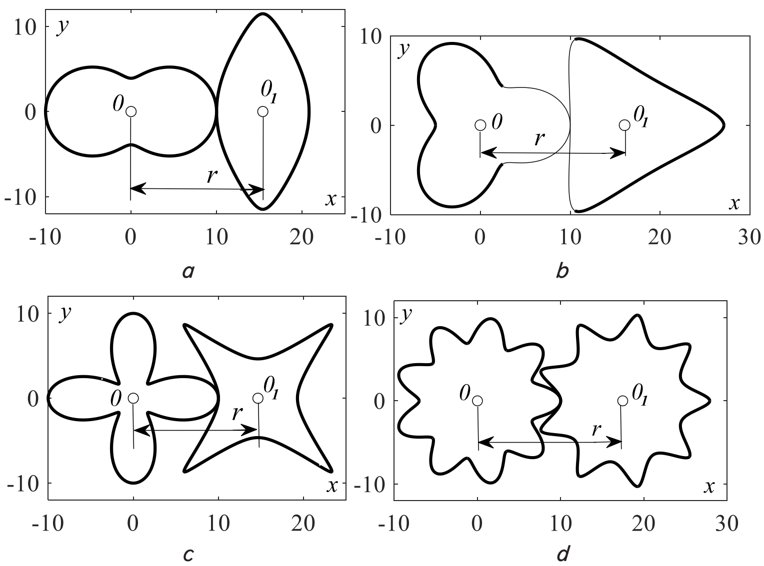

Fig. 5. Pairs of non-circular wheels constructed for $\rho_{\max} = 10$ and different values of r: a — $\rho_{\min} = 4$, $r = 15.41$; b — $\rho_{\min} = 5$, $r = 16.05$; c — $\rho_{\min} = 2.4$, $r = 14.64$; d — $\rho_{\min} = 7$, $r = 17.45$

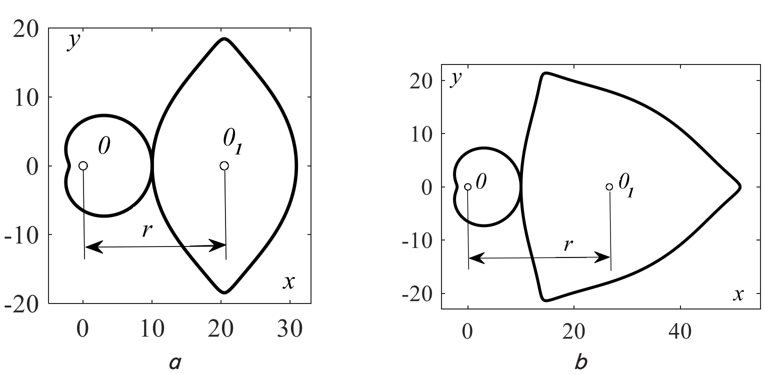

Fig. 6. Leading and trailing centroids constructed at $\rho_{\max} = 10$, $\rho_{\min} = 2$, $n = 1$: a — $m = 2$, $r = 20.45$; b — $m = 3$, $r = 26.61$

So, for the leading centroid angle $\alpha = \pi$, and angle $\varphi = \pi/m$. From the values of these angles, the center-to-center distance r can be found numerically using formula (11). At $m = 2$ (Fig. 6, a) an open curve of the trailing centroid will be obtained, which must be rotated around center O_1 by 180°, and at $m = 3$ – sequentially rotated twice by 120° (Fig. 6, b). With a full rotation of the leading centroid, the trailing one will make half a rotation in the first case and a third of a rotation in the second. The construction of centroid pairs can be generalized for different numbers n and m. In Fig. 7, centroid pairs were constructed for $n = 2$ and different values of m, and in Fig. 8 – for $n = 3$ and different values of m.

In Fig. 7, b, a pair of centroids was constructed for $n = 2$ and $m = 3$. Arcs of the same length, which roll over each other when rotating around their centers, are highlighted with a thin line. The arc of the leading centroid is constructed when angle α changes within $\alpha = -\pi/2 \ldots \pi/2$. The arc of the trailing centroid is constructed when angle φ changes within $\varphi = -\pi/3 \ldots \pi/3$. Thus, when determining the center-to-center distance using formula (11), the value of angle φ must be determined from expression $\varphi = \pi/m$.

For clarity, in Fig. 7, b, as well as in Fig. 8, a, arcs of equal length are depicted with thin lines.

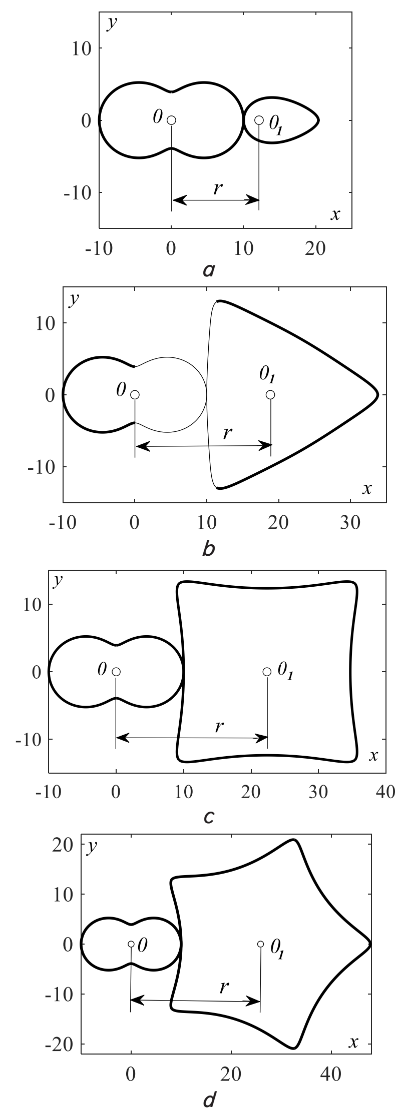

Fig. 7. Pairs of non-circular wheels constructed for $\rho_{\max} = 10$, $\rho_{\min} = 4$, $n = 2$: a — $m = 1$, $r = 12.15$; b — $m = 3$, $r = 18.85$; c — $m = 4$, $r = 22.35$; d — $m = 5$, $r = 25.88$

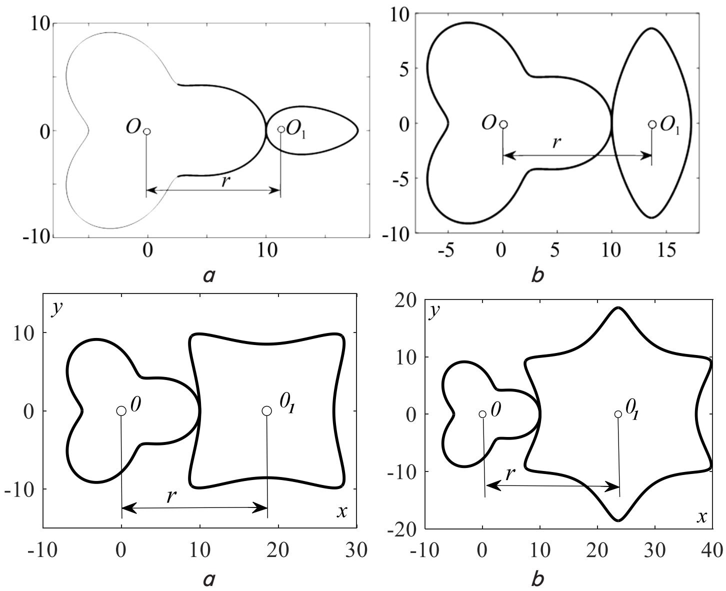

Fig. 8. Pairs of non-circular wheels constructed for $\rho_{\max} = 10$, $\rho_{\min} = 5$, $n = 3$: a – $m = 1$, $r = 11.38$; b – $m = 2$, $r = 13.63$; c – $m = 4$, $r = 18.54$; d – $m = 6$, $r = 23.57$

6. Discussion of results based on the mathematical description and construction of centroids using a fourth-degree polynomial

To construct pairs of centroids with external rolling, a fourth-degree polynomial plot was used. Due to the fact that it has an axis of symmetry and the conjugation of individual centroid arcs occurs at symmetric points, the centroid can be equated to a continuous curve. For example, at points A and B (Fig. 1), the numerical values of any derivative curve (3) will be equal. Obviously, this also applies to curve (5), in which the corresponding arcs of the curves after the transition from the Cartesian coordinate system (3) to polar (5) with the deformation of the curve retain these properties. In this case, the points of adjacent arcs of the centroid curves are common conjugation points with the same differential characteristics. This ensures the absolute smoothness of the closed leading centroid.

The construction of the trailing centroid is based on meeting the following two requirements:

1) the sum of distances from the point of contact of the centroids to their centers of rotation is constant and equal to the center-to-center distance r (Fig. 3);

2) the rolling of the centroids one by one during their rotation around fixed centers O and O_1 (Fig. 3) occurs without mutual sliding. This means the equality of the lengths of the centroid arcs, that is, the path they have traveled in a certain time. This equality is ensured by integrating expression (10), which for the considered polynomial has a finite form.

The leading centroid can have a different number of protrusions and depressions. The trailing centroid can have the same or a different number of protrusions and depressions. The simplest case, when the number of protrusions and depressions is equal to one, is shown in Fig. 4 for different center-to-center distances. Centroids with the same number of protrusions and depressions are constructed in Fig. 5. The developed algorithm allows centroids with a different number of protrusions and depressions of the leading and trailing centroids to be constructed. Examples for these cases are shown in Fig. 7, 8. Their number can be greater on the trailing centroid and vice versa – on the leading one.

There are different approaches to designing non-circular wheels. In them, the type of curve from which the driving wheel is formed is given, and the trailing wheel is located under the condition of a constant center-to-center distance and equality of the wheel arcs. In work [12], an equilateral polygon is considered as the driving wheel, that is, its profile is formed by straight line segments. The trailing wheel has the same number of curved sides that intersect at the same angle as the sides of the polygon. If the polygon is a square, then there are right angles on both wheels. The minimum number of them is four, there cannot be a triangular wheel since the angle of intersection of wheel elements cannot be acute. As the number of sides of the polygon increases, non-circular wheels approach round ones. In work [13], the profile of the driving wheel is formed using the hyperbolic cosine. This dependence describes the change in the radius vector in the polar coordinate system, and the curve itself, which is described by the end of the radius vector, is a component of the wheel arc. The arcs can intersect at a given angle, which cannot be acute. In this design, the protrusions are convex curves, and the depressions are the vertices of the angles or vice versa. The number of angles on both wheels is equal to the number of such elements.

The design of a driving wheel from the arcs of a logarithmic spiral is considered in [14]. Its profile is similar to the profile of a gear wheel. The protrusions and depressions are the vertices of the angles of intersection of adjacent arcs that form the wheel. Thus, the number of angles is twice the number of wheel elements. A common property of the described techniques for constructing pairs of non-circular wheels is the presence of angles of intersection of adjacent arcs in them. If non-circular wheels are used as centroids as a basis for manufacturing gear wheels, then their profile must be smooth, described by a smooth continuous curve. The use of a fourth-degree polynomial has made it possible to achieve this.

There are limitations to the proposed approach to designing non-circular wheels. The shape of the curve, similar to the curve in Fig. 1, can be described by other dependences. However, in this case, an expression can be obtained that cannot be integrated according to formula (10).

The disadvantage of our study is that the center-to-center distance is not specified but depends on the structural parameters of non-circular wheels. This study in the future will address the design of pairs of non-circular wheels with a given center-to-center distance.

7. Conclusions

1. The contour of the driving non-circular wheel is given by the dependence of the radius vector on the angle of its rotation in the polar coordinate system. A fourth-degree polynomial was chosen as this dependence, which made it possible to describe the wheel contour by a smooth curve with a given number of protrusions and depressions. The minimum value of the radius, which corresponds to the depressions, and the maximum value, which corresponds to the protrusions, can be specified through the values of the constant polynomials. The contour of the trailing wheel is determined on the basis of a constant value of the center-to-center distance and the equality of the lengths of the wheel contours. This enables their mutual rolling without slipping during simultaneous rotation around fixed centers.

2. Based on our results, pairs of non-circular wheels of different configurations have been constructed. The number of protrusions and depressions on the driving and trailing wheels can be the same or different. Depending on the number of protrusions n in the polar system, a corresponding arc of the driving wheel is constructed, which sequentially rotates around the origin by an angle of $2\pi/n$. The trailing wheel is constructed in a similar manner, except that its corresponding arc is found from the correspondence between the angles of rotation of the wheels and the equality of the arcs. The center-to-center distance depends on the number of protrusions of both wheels and is found by solving the equation numerically. Such wheels can serve as centroids in the design of gearing with non-circular wheels.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study, as well as the results reported in this paper.

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Data availability

All data are available, either in numerical or graphical form, in the main text of the manuscript.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

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