This study's object is the inventory management processes at a retail enterprise under conditions of random fluctuations in demand. The findings are aimed at solving the task related to the complexity in determining the optimal volumes of goods purchases under unstable conditions. As alternatives, it is proposed to consider two policies for replenishing stocks – the policy of a minimum stock of goods and the policy of a permanent reserve stock taking into account the possibility of transferring unsatisfied demand.

Each policy is estimated by the value of the expected operating effect, which takes into account the income from the sale of goods and losses from the storage of unsold goods or from unsatisfied demand. The hypothesis put forward assumes that the value of the expected operating effect of each policy could be calculated depending on parameters for the law of the probability distribution of demand volumes and on the economic characteristics of the situation.

A model of dependence of the expected operating effect on the volumes of purchases and the parameters of the normal probability distribution functions of demand has been built. Mathematical expressions for the expected operating effect for the two policies under analysis have been derived. A comparative analysis of the effectiveness of these policies was conducted, which made it possible to identify the zones of values of the indicators of the choice situation for which a certain policy is the best. Under certain conditions, the expected operational effect for an arbitrarily chosen policy could reach only 70% of the operational effect corresponding to the best policy. This proves the ability of adaptive management to improve the operational effect as well as its economic efficiency

Keywords: random demand, changes in operating activity characteristics, inventory replenishment, operating effect

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CONSTRUCTION OF ADAPTIVE INVENTORY MANAGEMENT MODELS FOR A TRADING ENTERPRISE UNDER UNSTABLE CONDITIONS

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1. Introduction

The retail sector occupies one of the leading places in the modern economy and social domain. It renders vital services, provides jobs, and generates added value. In the European Union, retail trade generates about 5% of total added value [1]. In the USA, the share of retail trade in GDP in 2024 was 6.3% [2]. In Ukraine, wholesale and retail trade occupies one of the first places among the types of economic activity that form GDP. In 2023, the share of wholesale and retail trade in Ukraine's GDP was 13% [3].

The main content of the operational activity of trading enterprises determines the process of placing orders for the purchase of goods and their sale. A component of operational activity is the management of their inventory, the purpose of which is to meet the needs of buyers while minimizing current costs associated with replenishing and maintaining their inventory. Improving inventory management makes it possible to increase the company's profit and the quality of customer service.

Conceptually, inventory management provides answers to two main questions: when to replenish the stock and in what quantity. Each specific inventory management scheme reflects the characteristics of the specific situation in which it operates. These characteristics include the properties and cost of goods; storage conditions of goods; the ability to transfer unsatisfied demand; the ability of the supplier to fulfill unpredictable orders; statistical characteristics of demand that determine its average volumes and fluctuations.

The processes of formation, storage, and use of inventories of goods significantly affect the costs of campaigns. Unjustified choice of values for inventory management parameters leads to losses that arise both from the storage of excess inventory and from a shortage of goods. The complexity of identifying optimal values of these parameters increases significantly under conditions of random demand.

Therefore, the task to design economic and mathematical tools in order to substantiate the values for inventory management parameters under conditions of random demand is relevant. Economic and mathematical methods and models can provide a more accurate selection of inventory management parameter values based on the criterion of economic effect, as well as reduce the complexity and time spent on decision-making.

2. Literature review and problem statement

The best known inventory management model is the economic order quantity (EOQ) model. The EOQ model is based on the need to ensure demand for goods for a certain period of time under consideration [4]. It determines the quantities and volumes of orders for goods from the supplier, which can be considered as characteristics of the inventory management policy. In the classical EOQ model, all parameters of the process of purchasing, storing, and selling goods are considered deterministic and unchanged throughout the planning period. This is a very strong assumption that rarely corresponds to the characteristics of the real environment. In addition, the EOQ model is based on calculating the costs of storing inventory in proportion to its volume and storage time, which corresponds to the conditions of renting warehouse premises. It is not suitable for use by trading enterprises that have their own premises, the operating costs of which depend little on the volume of inventory.

The EOQ model has undergone many modifications. One of the modern directions of its improvement was to take into account various changes in the values of external environmental parameters, in particular demand, which was considered as a function of time and/or the price of the goods. For example, in [5], demand is a function of time, while taking into account the possibility of shortage. The inventory management policy is given by such parameters as batch size and order frequency. To determine the optimal inventory management policy, the work states an integer nonlinear mathematical programming problem, the objective function of which is the average profit per unit of time. In [6], demand is considered depending on the price of the product and the volume of inventory. The inventory management policy is given by such parameters as batch size, order frequency, and product price. To determine the optimal inventory management policy, the cited work also uses an optimization model. The objective function is the profitability index, which is defined as the ratio of revenues to costs. Although these models take into account the dynamics of demand, they do not assume random fluctuations in demand. This reduces their ability to adequately reflect actual conditions.

To account for demand uncertainty, the most common approach is to consider the operational activity of the enterprise as a continuous process of replenishment of stocks and sale of goods. In accordance with the process approach, 4 most common conceptual schemes (models) of inventory management under stochastic demand conditions are distinguished [7]. Model (Q, r) is a model with a fixed batch size and continuous inventory control. When the inventory level falls below r, an order for Q units is placed. Model (s, S) is a periodic review model that assumes a variable order size, a fixed minimum inventory level s, and a fixed maximum inventory level S. At the end of each review period, the inventory level is analyzed, and if it is lower than s, an order is placed to increase it to level S. Model (R, S) is with a fixed time interval R between orders. The order size increases the inventory level to the limit value S. System (R, s, S) is a combination of systems (s, S) and (R, S). Every period R, the inventory level is checked. If it is equal to or below the reorder point s, a sufficient quantity is ordered to replenish the inventory to the value S.

A review of the application of these types of models in modern research on inventory management processes is given, for example, in [8]. Of particular note is the concept of a fixed time period, which involves ordering a certain number of goods at predetermined time intervals. During each interval, the inventory level is checked, and an order is placed to bring it to a predetermined level.

A review of papers [7, 8] on inventory management under stochastic demand conditions shows the feasibility of using a conceptual scheme with a fixed order period in further research. This is due to the fact that other schemes require continuous revision of the inventory level. For this reason, the complexity of inventory management increases significantly. In addition, the randomness of the terms and volumes of orders creates inconvenience for the supplier, which can lead to an increase in the cost of orders. The disadvantage of conceptual models [7, 8] is that they do not directly determine the values of their parameters, which would make it possible to avoid shortages of goods and their residues. To find the optimal values of the parameters, expert assessments, trial and error methods, and simulation modeling are used.

An example of the application of the simulation modeling method is [9]. It studies the policy (R, s, S) and assumes that demand corresponds to a normal distribution with a known mean and variance. The experiments allowed the researchers to identify statistical dependences of the target parameters of the system (in particular, the average inventory level and the cost value) on delivery parameters (such as the order period, order execution time, and percentage of fulfillment). However, the simulation modeling method directly makes it possible to assess the impact on the output parameters of the model only by multivariate calculations for individual values of the input parameters. Its application requires the availability of special software and a high professional level of performers.

The scenario approach to optimizing inventory management under uncertainty is noteworthy. According to this approach, uncertainty is formalized using a set of scenarios of the dynamics of the external environment parameters. In particular, in [10], future uncertain demand is represented by a scenario tree under the assumption of a discrete distribution. The scenarios are the input data for the optimization model. In [11], robust optimization is used, which is focused on the worst-case scenarios of demand dynamics. The reliable value of the economic order volume is defined as the value that ensures minimum losses in the worst-case scenario. The advantage of the above findings, which are based on the principles of the scenario approach, is that they make it possible to find the optimal values for inventory management parameters for certain scenarios of environmental dynamics. However, in reality, uncertainty factors, in particular random fluctuations in demand, are not reduced to the probabilities of a limited number of individual scenarios. In addition, the implementation of the scenario approach requires, like the simulation modeling method, the use of special software and a high professional level of performers.

The choice of optimality criterion is crucial for determining the optimal inventory management policy. In the EOQ model, the optimal order volume is determined by minimizing the total costs of ordering and storing the product stock, which does not make it possible to directly take into account the impact of the product shortage on operating profit, the main goal of the enterprise. A similar drawback is observed in work [12], in which optimality criterion is the minimum of the total costs incurred by both the trading company and its supplier in connection with maintaining inventories. In most modern research in the field of inventory management, the optimality criterion is the maximum profit, but this criterion also does not make it possible to take into account all important aspects of the process under study. Thus, in work [13], the problem of managing production inventories of a product produced on the basis of mixing liquids is investigated. The optimality criterion is the maximum profit per unit of time. Profit is calculated as the difference between revenue from product sales and the sum of costs for production, storage, and preservation of the product, as well as the costs of adjusting the production process for each order. But the applied criterion does not take into account the costs associated with a possible shortage of products, which is a limitation of the development. In [14], a two-stage supply chain is studied, uniting a manufacturer and a retailer. The optimality criterion is the maximum of the total profit of both components of the supply chain. But at the same time, the possibility of a shortage of goods is also not taken into account. Direct costs associated with a shortage of goods are taken into account in study [15]. In the work, the objective function of the optimization model is calculated as the difference between the income from the sale of goods and the sum of the cost of purchasing and storing goods, the cost of ordering, and the cost of deferred orders. But the disadvantage of the cited work is that the parameters of the external environment are considered unchanged.

Improving inventory management requires reflecting in the criterion of optimality of the volume of goods orders (the magnitude of the operating effect) a wider range of situations of the trading enterprise's activity, compared to the criteria that were used in studies [13–15]. In particular, the value of the operating effect should reflect the conditions of the company's inventory location in its own warehouses, the possibility of transferring unsatisfied consumer demand. In the case of changes in the cost of goods, specific costs of storing goods and demand characteristics, there is a need to determine the procedure for adapting the procurement of goods to new conditions.

When characterizing the economic and mathematical models in inventory management known from the literature, it is appropriate to also highlight findings in the management of finished goods inventories at manufacturing enterprises. An analog of decisions on the volume of goods purchased in inventory management of a trading enterprise are decisions on the volume of production of products in the operational planning of a manufacturing enterprise. In the classical model of economic production quantity (EPQ), demand was considered constant. But modern researchers consider dynamic demand. For example, in [16, 17], demand is a deterministic function of time. The parameters of finished goods inventory management in these works are determined by solving the problem of unconditional optimization, in which the criterion is the minimum of total costs.

The limitations of the deterministic approach are resolved in studies [18–20], in which the problem of operational planning of production volumes is considered in a stochastic formulation. In [18], the position is substantiated that when choosing production volumes under conditions of random fluctuations in demand, one should proceed not from the expected value of demand but from the expected economic results of the choice made. The proposed indicator of the operating effect is the difference between the profit received from the production and sale of products and the amount of losses, which include the costs of storing unsold products and lost profits. Therefore, the operating effect is a function of the random value of demand and the available volume of finished products. It is proved that the planned volume of production and the corresponding volume of finished products should provide the maximum expected value of the operating effect. To assess future demand, in [18] it was proposed to use information on the volumes and probabilities of receipt of individual orders for products.

In [19], the concept of operational activity policy was introduced, which was understood as the rule for making decisions about production volumes in current planning periods based on the information available at the enterprise. Models of some policies were built with the definition of the expected operational effect for them, and their comparative analysis was carried out. In [20], attention focused on the policy with the reservation of finished products based on demand forecasting. It is shown that in the case of a probability distribution of the demand volume close to uniform, the following effect occurs: the appearance of a sequence of chains of planning periods with unsold product residues.

The following circumstances prevent the direct use of the results of research [18–20] on finished goods inventory management to solve the problem of inventory management. The formulae that determine the operational effect of the volume of production (for an industrial enterprise), on the one hand, and the volume of goods purchased (for a trading enterprise), on the other hand, differ significantly. Trading enterprises do not have losses due to downtime or excessive load on production capacities. At the same time, models for trading enterprises should take into account the possibility of transferring unsatisfied consumer demand. In addition, papers [18–20] proposed evaluation of policies for choosing the volume of production only in the current planning situation. Procedures for adaptive selection of the volume of production for the case when the planning situation changes in future periods of time were not considered in those studies.

In general, the following conclusion can be drawn regarding the existing needs for improving inventory management models. Our review of the literature has shown that for deterministic conditions, analytical models of inventory management are already available that take into account the dynamics of external environmental parameters and losses from a possible shortage of goods. The task of building similar models for stochastic environmental conditions remains unsolved. Therefore, it is advisable to devise inventory management models that would represent explicit formulae or algorithms for determining the volume of inventories of a trading enterprise for all options for the values of the parameters of the distribution of random demand volumes. The would-be models should take into account the lost profit due to a shortage of goods, the peculiarities of the costs of maintaining the enterprise's inventories in its own warehouses, and the possibility of transferring unsatisfied consumer demand.

3. The aim and objectives of the study

The purpose of our work is to build models for adaptive inventory management at a trading enterprise under conditions of random fluctuations in demand and changes in the characteristics of the operational activity situation. This will make it possible to reasonably approach the determination of the volume of goods purchased and to obtain positive effects from the implementation, which involves increasing the volume of sales of goods and reducing the costs of storing the remaining goods in warehouses.

To achieve this aim, the following objectives were accomplished:

– to build a model of the dependence of expected operational effect in the planning period on the choice of the volume of goods purchased;

– to propose a concept of adaptive inventory management based on the choice of replenishment policies;

– to determine the content and find mathematical expressions of the expected operational effect for the policies of minimum stock of goods and constant reserve stock;

– to devise a methodology for determining the values of the expected operational effect of policies depending on the parameters of the normal probability distribution of demand volumes;

– to conduct a comparative analysis of the policies of minimum stock of goods and permanent reserve stock on the range of possible values of indicators of the situation of choosing a procurement policy.

4. The study materials and methods

The object of our study is the processes of inventory management at a retail enterprise under conditions of random fluctuations in demand and changes in the characteristics of the operational activity situation.

The study considered modern models of inventory management with their properties in terms of taking into account the instability of the operating conditions of trading enterprises. The results of the research that were obtained are based on the provisions of the theory of probability and mathematical statistics, systems analysis, and operations research.

The main hypothesis of the study assumed the possibility of identifying in a simple form the dependence of expected operational effect during planning period on the volume of goods purchased, the parameters of the law of probability distribution of demand volumes and economic indicators of the situation. These indicators include specific losses from storing goods, specific income from its sale, and the relative number of consumers who are ready to expect to purchase goods in the future period of time. The implementation of this opportunity will allow small trading businesses to determine the optimal volumes of goods purchased by conducting simple calculations in a typical computer environment, for example, in MS Excel (USA), without using special software.

Before conducting the study, two following assumptions were adopted. It was believed that the duration of the period between placing orders is a given value. In fact, the duration of this period is influenced by many factors that are determined by the specific conditions of storage of goods and their delivery at a particular trading enterprise. It was also assumed that the costs of storing a commodity unit at the enterprise's own premises are determined by a clearly defined value. But these costs depend on the number of goods placed in it, which changes over time. An attempt in the research process to focus attention on determining the impact of economic indicators of the situation on the expected operational effect led to the adoption of a simplified assumption that the law of the probability distribution of demand volumes is normal.

The study used analytical and numerical methods. Analytical methods were applied to build a model of the dependence of expected operational effect on the choice of the volume of goods purchased; mathematical expressions of the expected operational effect for stock replenishment policies were derived. In addition, based on analytical methods, a comparative analysis of the effectiveness of policies was carried out, and the concept of adaptive inventory management was defined. Owing to the formulae for determining the effectiveness of stock replenishment policies, specialists of trading enterprises will be able to directly assess the effectiveness of policies for different values of economic indicators and parameters of stochastic demand. Numerical methods were used during integration to find the components of the mathematical expectation of demand and in test calculations of statistical indicators.

5. Results related to devising the concept and auxiliary models of adaptive inventory management

5. 1. Model of the dependence of expected operational effect during planning period on the volume of goods purchased

The basis for modeling the inventory management at a trading enterprise is the conceptual scheme of a fixed period of goods orders. In accordance with it, the process of operational activity of the enterprise is considered in a sequence of M planning periods of time with the same duration T. At the end of each planning period t, the volume η_t of orders for the product by buyers and its actual sale, the product balances z_t and the volume w_t of unsatisfied demand at the end of the time period t are registered; the delivery of the volume u_t of the product is ordered for the future time period $t+1$. It is assumed that some of the buyers whose requirements are not satisfied in the time period agree to wait to purchase the product in the next time period $t+1$. Then the requirements are recorded, and the value kw_t will determine the volume of demand that is transferred to the period $t+1$, where k is the demand transfer coefficient, $k \le 1$. Part $(1-k)w_t$ is lost, that is, buyers satisfy the need for the missing product from another source.

The following notations are also introduced:

– u_t is the volume of goods delivered at the beginning of time period t;

– $y_t = u_t + z_{t-1}$ – quantity of goods that will be available at time period t (taking into account the transfer of the remaining goods z_{t-1} at the end of time period $t-1$ to time period t);

– $w_t = x_t - y_t$ – volume of unsatisfied demand at planned time period t; $w_t = \eta_t - y_t^0$, where $y_t^0 = y_t - kw_{t-1}$;

– $x_t = \eta_t + kw_{t-1}$ – total volume of demand for goods at planned time period t (taking into account the transfer to time period t of part kw_{t-1} of the unsatisfied demand at time period $t-1$);

– $y_t^0 = y_t - kw_{t-1}$ – volume of goods for sales on request in current time period t, $y_t^0 \ge 0$.

It is obvious that the volume of unsatisfied demand in planned time period t is $w_t = x_t - y_t = \eta_t - y_t^0$.

In the operational economic effect E_t for time period t, for simplification, the effect of selling goods with demand transferred to time period $t+1$ and the losses associated with the storage of goods unsold in time period t are taken into account. Then the value of effect E_t will be determined from the following formulae:

$$E_t = E_{1t} = d\eta_t - a\left(y_t^0 - \eta_t\right) = \left(d + a\right)\eta_t - ay_t^0, \text{ if } \eta_t \le y_t^0, \quad (1)$$

$$E_t = E_{2t} = dy_t^0 + dkw_t - d\left(1 - k\right)w_t, \text{ if } \eta_t \ge y_t^0, \quad (2)$$

where d is the amount of revenue from the sale of a unit of goods during the current planning period; a is the amount of losses associated with the storage of unsold goods during the next planning period, calculated per unit of goods, dy_t^0 is the cost of available goods in current time period t, dkw_t is the cost of goods whose sale is postponed to future period $t+1$, $d(1-k)w_t$ is the cost of unsold goods, which determines the loss from lost profit.

The value η_t of the volume of demand for goods in planning period t is considered as the realization of a random variable η, which has a normal probability distribution with the known distribution function $F(x)$ and the probability density $f(\lambda, \sigma)(x)$, where λ is the mathematical expectation, and σ is the standard deviation of the random variable η of demand in planning period t. The probability density and distribution function can be found, in particular on the basis of retrospective information on demand volumes by isolating the trend and random (irregular) components in the time series.

The expected value E_t^* of the operating effect for time period t is determined from the following formula

$$E_t^* = P^- E_{1t}^* + P^+ E_{2t}^*, \quad (3)$$

where $P^- = P^-[y_t^0]$, $P^+ = P^+[y_t^0]$ – probabilities that for an arbitrary planning period t it will turn out that $\eta_t \le y_t^0$, or $\eta_t \ge y_t^0$; E_{1t}^*, E_{2t}^*, are the expected values of components E_{1t}, E_{2t} of the effect, respectively, for the cases when $\eta_t \le y_t^0$, or $\eta_t \ge y_t^0$:

$$E_{1t}^* = \left(d + a\right)\rho^- - ay_t^0,$$

$$E_{2t}^* = d\left(\left(2k - 1\right)\rho^+ + 2\left(1 - k\right)y_t^0\right); \quad (4)$$

ρ^-, ρ^+ – average values of realizations η_t, which satisfy the conditions $\eta_t \le y_t^0$ or $\eta_t \ge y_t^0$

$$\rho^- = \frac{\rho_m^-}{P^-}, \ \rho^+ = \frac{\rho_m^+}{P^+}; \quad (5)$$

$\rho_m^- = \rho_m^-[y_t^0]$, $\rho_m^+ = \rho_m^+[y_t^0]$ – components of the mathematical expectation λ, on which, respectively, $\eta_t \in [-\infty, y_t^0]$ or $\eta_t \in [y_t^0, \infty]$:

$$\rho_m^- + \rho_m^+ = \lambda,$$

$$\rho_m^-\left[y_t^0\right] = \int_{-\infty}^{y_t^0} xf\left[\lambda, \sigma\right]\left(x\right)\mathrm{d}x,$$

$$\rho_m^+\left[y_t^0\right] = \int_{y_t^0}^{\infty} xf\left[\lambda, \sigma\right]\left(x\right)\mathrm{d}x. \quad (6)$$

Based on the above definitions, the expected operating effect for time period t will be determined from the following formula

$$E_t^* = a\left(\rho_m^- - P^- y_t^0\right) + $$
$$+ \ d\left(\rho_m^- + \left(2k - 1\right)\rho_m^+ + 2P^+\left(1 - k\right)y_t^0\right). \quad (7)$$

It is obvious that the value E_t^* of operational effect depends on the selected volume u_t of the goods supply since it affects the values of quantities y_t^0, P^-, P^+, ρ^-, ρ^+.

5. 2. The concept of adaptive inventory management based on the selection of inventory replenishment policies

It is proposed to term management with stable provision of current demand such a rule for selecting the volume of goods delivery u_t, according to which the volume of goods y_t^0 for current sale remains guaranteed constant in future time periods $y_t^0 = y^0$, for all t. The replenishment policy is understood as the rule for selecting the volume of goods delivery for the future planning period depending on the situation in the current period. The situation in each period t is determined not only by the ratio between the value η_t of the current demand for goods and the volume of goods for current sale, but also by the current values of the economic characteristics of the operational effect and the parameters of the demand probability distribution function.

Management, which involves a certain quantitative determination of the value y^0, is proposed to be termed a replenishment policy based on the concept of stable provision of current demand.

In the case of using a certain policy of stable provision of current demand, the value E_t^* of operational effect will take the same value in all time periods $E_t^* = E^*$, for all t. The value of E^* will be determined by the selected value of volume y^0 of goods for current sales, that is, the selected option for the replenishment policy.

The proposed concept of adaptive inventory management involves choosing a policy in each period that corresponds to the maximum expected operating effect. The result of the enterprise's activity in each period is estimated by the value of the operating effect, which is determined by the income from the sale of the product and the losses associated either with the preservation of unsold goods or with unsatisfied demand due to its shortage. Under conditions of random demand, the future operating effect for any purchase volume is a random variable, and therefore the replenishment policy is estimated by the mathematical expectation of the operating effect from its application.

The expected operating effect depends on the parameters of the probability distribution of demand volumes, as well as on the current values of the economic indicators of the situation in which the choice of the volume of goods purchased occurs. The economic indicators of the choice situation are reflected in the indicators of income from the sale of a commodity unit, losses from the storage of a commodity unit in the warehouse during one period, and the coefficient of demand transfer to the future period of time.

The magnitude of the expected operational effect under a certain policy is considered as an assessment of the adaptability of this policy to the current conditions of choosing the volume of goods purchased. Indeed, the economic results of inventory management based on a certain policy can be considered as income from the sale of goods and losses associated with the storage of unsold goods. The role of stock replenishment policies in management is revealed when they are considered from the standpoint of the principle of efficiency (Pareto optimality). A certain policy will be dominated by another policy under which income from the sale of goods is greater, and losses associated with the storage of remaining goods are smaller. A certain policy is considered effective if it is not dominated by any other policy. It is obvious that each alternative stock replenishment policy must be effective, otherwise there will be no point in applying it. Thus, the stock replenishment policy must be a management tool that is already adapted to random fluctuations in demand over planned time periods.

Adaptive management in a random process of operational activity is determined by the choice before the beginning of each period of such a replenishment policy that is most adapted to current conditions. To do this, the company needs to collect information about the current values of the parameters of the inventory management situation, calculate the value of the expected operational effect for each policy option, and select the policy that corresponds to the maximum effect value.

5. 3. Mathematical expressions for expected operational effect for the policies of minimum stock of goods and permanent reserve stock

Two policies of procurement of goods, which in some of their modifications are widespread in practice, are studied: minimum stock of goods (MSG) and permanent reserve stock (PRS).

Policy of minimum stock of goods. In accordance with the MSG policy, the volume y_t^0 of goods for sale upon request is kept constant for each time period t, equal to the average intensity of the volume of demand λ, $y_t^0=\lambda$. Therefore, the volume of procurement for time period $t+1$ is $u_{t+1}=\lambda+kw_t$ if $\eta_t\geq\lambda$, and is $u_{t+1}=\lambda-z_t$ if $\eta_t\leq\lambda$.

In the case of impossibility of transferring unsatisfied demand, it turns out that $u_{t+1}=\lambda$, when $\eta_t\geq\lambda$. Therefore, in this case, the quantity of the good y_t that will be available at each time period t remains constant: $u_t=\lambda=y_t^0$.

According to formulae (1), (2), the expected values E_{1t}^*, E_{2t}^* of the effect components for the MSG policy are determined from the following formulae:

$$E_{1t}^*=(d+a)\rho^- - a\lambda,\ \text{if } \lambda\geq\eta_t; \tag{8}$$

$$E_{2t}^*=d\left((2k-1)\rho^+ + 2(1-k)\lambda\right),\ \text{if } \lambda\leq\eta_t. \tag{9}$$

Since $P^-=P^-[\lambda]=P^+=P^+[\lambda]=0.5$, the expected operational effect E_t^* for the MSG policy is determined from the following formula

$$E_{MSG}^*=d\left(\psi_m^- + (2k-1)\psi_m^+ + (1-k)\lambda\right)- - 0.5a\left(\lambda-2\psi_m^-\right), \tag{10}$$

where:

$$\psi_m^-=\rho_m^-[\lambda]=\int_{-\infty}^{\lambda} xf[\lambda,\sigma](x)\mathrm{d}x,$$

$$\psi_m^+=\rho_m^+[\lambda]=\int_{\lambda}^{\infty} xf[\lambda,\sigma](x)\mathrm{d}x,$$

$$\rho^-=2\psi_m^-,\ \rho^+=2\psi_m^+.$$

When the transfer of unsatisfied demand is impossible, $k=0$, and given $\psi_m^-+\psi_m^+=\lambda$, the following equality holds

$$E_{MSG}^*=\frac{1}{2}\left(2d\left(\psi_m^- - \psi_m^+ + \lambda\right) - a\left(\lambda-2\psi_m^-\right)\right)= =2d\psi_m^- - \frac{a\left(\lambda-2\psi_m^-\right)}{2}. \tag{11}$$

Given the possibility of complete transfer of unsatisfied demand, $k=1$, the following equality holds

$$E_{MSG}^*=\frac{1}{2}\left(2d\left(\psi_m^- + \psi_m^+\right) - a\left(\lambda-2\psi_m^-\right)\right)= =d\lambda - \frac{a\left(\lambda-2\psi_m^-\right)}{2}. \tag{12}$$

Permanent reserve stock policy. In accordance with the PRS policy, the volume y^0 of goods for sale on current requests should be a constant value for each time period t, which is equal to the sum of the average intensity of demand λ and the permanent reserve stock of goods δ Î $[0,\lambda]$, $y^0=\lambda+\delta$. Therefore, the volume of goods purchased for the future time period $t+1$ is chosen as $u_{t+1}=\lambda-z_t+\delta$ if $\eta_t\leq\lambda+\delta$, and as $u_{t+1}=\lambda+\delta+kw_t$ if $\eta_t\geq\lambda+\delta$.

The value δ is chosen from the requirement that at the end of each period t, the volume z_t of unsold goods with a high probability $Q(\delta)$ does not exceed the average intensity of demand λ

$$Q(\delta)=P\{\lambda+\delta-\eta_t\leq\lambda\}=P\{\eta_t\geq\delta\}=1-F(\delta), \tag{13}$$

where $F(\ldots)$ is the probability distribution law of demand value η. For test calculations, the value of reserve stock δ^* was used, at which volume z_t of the unsold product with a probability of 0.99 does not exceed demand intensity λ, $F(\delta^*)=0.01$. Taking into account the fact that $F(\delta^*)=\Phi\left(\frac{\delta^*-\lambda}{\sigma}\right)$, the following equality holds: $\delta^*=\sigma\Phi^{-1}(0.01)+\lambda$, where Φ^{-1} is the inverse function of the standard normal distribution, $\Phi^{-1}(0.01)=-2.326$. Then

$$\delta^*=\lambda-2.326\sigma. \tag{14}$$

The probability $P^-[\lambda+\delta^*]$ is determined by the probability $P\{\eta_t\leq\lambda+\delta^*\}$ that at the end of period t there will be no unsatisfied demand, $P^-[\lambda+\delta^*]=F(\lambda+\delta^*)$, and $P^+[\lambda+\delta^*]= =1-F(\lambda+\delta^*)$.

The expected values E_{1t}^*, E_{2t}^* of the effect components for the PRS policy are determined from the following formulae:

$$E_{1t}^*=(d+a)\rho^- - a(\lambda+\delta^*),\ \text{if } \eta_t\leq\lambda+\delta^*, \tag{15}$$

$$E_{2t}^*=d\left((2k-1)\rho^+ + 2(1-k)(\lambda+\delta^*)\right),\ \text{if } \eta_t\geq\lambda+\delta^*. \tag{16}$$

The expected operational effect E_t^* for the PRS policy in accordance with (3) will be determined from the following formula

$$E_{PRS}^*=d\left(\gamma_m^- + (2k-1)\gamma_m^+ + 2P^+(1-k)(\lambda+\delta^*)\right)+ + a\left(\gamma_m^- - (\lambda+\delta^*)P^-\right), \tag{17}$$

where:

$$P^-=P^-[\lambda+\delta^*],\ P^+=P^+[\lambda+\delta^*],$$

$$\gamma_m^-=\rho_m^-[\lambda+\delta^*]=\int_{-\infty}^{\lambda+\delta^*} xf[\lambda,\sigma](x)\mathrm{d}x,$$

$$\gamma_m^+=\rho_m^+[\lambda+\delta^*]=\int_{\lambda+\delta^*}^{\infty} xf[\lambda,\sigma](x)\mathrm{d}x.$$

When the transfer of unsatisfied demand is impossible, $k=0$, then

$$E_{PRS}^*=d\left(\gamma_m^- - \gamma_m^+ + 2(\lambda+\delta^*)P^+\right)+ + a\left(\gamma_m^- - (\lambda+\delta^*)P^-\right). \tag{18}$$

Given the possibility of complete transfer of unsatisfied demand, $k=1$

$$E_{PRS}^*=d\left(\gamma_m^- + \gamma_m^+\right)+a\left(\gamma_m^- - (\lambda+\delta^*)P^-\right)= =d\lambda - a\left((\lambda+\delta^*)P^- - \gamma_m^-\right). \tag{19}$$

In accordance with the proposed concept of adaptive inventory management, the MSG policy and the MPZ policy should be considered as alternative options for the procurement policy of goods. Having received information about the current situation, the enterprise will need to calculate the values of the expected operational effect for each of the two policy options and choose the best one for application. Carrying out these calculations requires determining the direct dependence of expected operational effect on the indicators of the state of the situation in which the policy is selected.

5. 4. Methodology for determining the expected operational effect depending on the parameters of probability distribution of demand volumes

To identify the dependence of the parameters of the expected operational effect on the values of the parameters of the normal probability distribution function of demand volumes, an indicator of the relative intensity of demand $r=\lambda/\sigma$, has been introduced, which characterizes a certain distribution function.

It can be assumed that function $F(x)$ of the normal distribution acquires its "practically" possible values in the interval $[x^{\min}=\lambda-3\sigma, x^{\max}=\lambda+3\sigma]$ since:

$$F(\lambda-3\sigma)=\Phi(-3)=0.00135,$$

$$F(\lambda+3\sigma)=\Phi(3)=0.99865, \quad (20)$$

where Φ is a function of standard normal distribution. Since the random variable of demand η cannot take negative values, we can assume that values λ and σ satisfy the condition $\lambda \geq 3\sigma$. Then $P\{\eta \leq \lambda-3\sigma\} \approx 0$, and value $r_{\min}=3$ will be the minimum possible value of indicator r.

For the MSG policy, equalities $\psi_m^-=\rho_m^-[\lambda]$, $\psi_m^+=\rho_m^+[\lambda]$, are satisfied, and for the PRS policy, $\gamma_m^-=\rho_m^-[\lambda+\delta^*]$, $\gamma_m^+=\rho_m^+[\lambda+\delta^*]$.

The values of quantities $\rho_m^+[\lambda](0,1)$, $\rho_m^-[\lambda](0,1)$, which correspond to the probability density $f[0,1](x)=e^{-x^2/2}/\sqrt{2\pi}$ according to the standard normal distribution are determined from the following formulae:

$$\rho_m^+[\lambda](0,1)=\frac{1}{\sqrt{2\pi}}\int_0^{\infty} e^{-y^2/2}y\mathrm{d}y \approx$$

$$\approx \frac{1}{\sqrt{2\pi}}\int_0^{3} e^{-y^2/2}y\mathrm{d}y \approx 0.4, \quad (21)$$

$$\rho_m^-[\lambda](0,1)=-\rho_m^+[\lambda](0,1)=-0,4. \quad (22)$$

For the MSG policy, taking into account substitution $y=(x-\lambda)/\sigma$, the following equality holds:

$$\rho_m^-[\lambda](\lambda,\sigma)=\frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{\lambda} e^{-\frac{(x-\lambda)^2}{2\sigma^2}}x\mathrm{d}x=$$

$$=\frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{0} e^{-y^2/2}y\mathrm{d}y+\lambda\Phi(0), \quad (23)$$

$$\psi_m^-=\rho_m^-[\lambda](\lambda,\sigma)=$$
$$=\sigma\rho_m^-[\lambda](0,1)+0.5\lambda \approx -0.4\sigma+0.5\lambda; \quad (24)$$

$$\rho_m^+[\lambda](\lambda,\sigma)=\frac{1}{\sigma\sqrt{2\pi}}\int_{\lambda}^{\infty} e^{-\frac{(x-\lambda)^2}{2\sigma^2}}x\mathrm{d}x=$$

$$=\frac{\sigma}{\sqrt{2\pi}}\int_0^{\infty} e^{-y^2/2}y\mathrm{d}y+\lambda\int_0^{\infty} f[0,1](x)\mathrm{d}y, \quad (25)$$

$$\psi_m^+=\rho_m^+[\lambda](\lambda,\sigma)=$$
$$=\sigma\rho^+[\lambda](0,1)+0.5\lambda \approx 0.4\sigma+0.5\lambda. \quad (26)$$

For the PRS policy, after replacing x with a variable $y=(x-\lambda)/\sigma$, components $\rho_m^-[\lambda+\delta^*]$, $\rho_m^+[\lambda+\delta^*]$ of the mathematical expectation λ are determined, respectively, on intervals $[-\infty,\lambda+\delta^*]$, $[\lambda+\delta^*,\infty]$ from the following expressions:

$$\rho_m^-[\lambda+\delta^*](\lambda,\sigma)=\frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{\lambda+\delta^*} e^{-\frac{(x-\lambda)^2}{2\sigma^2}}x\mathrm{d}x=$$

$$=\sigma\int_{-\infty}^{0} e^{-y^2/2}y\mathrm{d}y+\lambda\Phi(0); \quad (27)$$

$$\gamma_m^-=\rho_m^-[\lambda+\delta^*](\lambda,\sigma)=$$
$$=\sigma\left(\rho_m^-[\lambda](0,1)+M(\chi)\right)+\lambda\Phi(\chi); \quad (28)$$

$$\rho_m^+[\lambda+\delta^*](\lambda,\sigma)=$$
$$=\frac{\sigma}{\sqrt{2\pi}}\left(\int_0^{\infty} e^{-y^2/2}y\mathrm{d}y-\int_0^{\chi} e^{-y^2/2}y\mathrm{d}y\right)+$$
$$+\frac{\lambda}{\sqrt{2\pi}}\left(\int_{-\infty}^{\infty} e^{-y^2/2}\mathrm{d}y-\int_{-\infty}^{\chi} e^{-y^2/2}\mathrm{d}y\right); \quad (29)$$

$$\gamma_m^+=\rho_m^+[\lambda+\delta^*](\lambda,\sigma)=$$
$$=\sigma\left(\rho_m^+[\lambda](0,1)-M(\chi)\right)+\lambda\left(1-\Phi(\chi)\right), \quad (30)$$

where:

$$\chi=\frac{\delta^*}{\sigma},\ M(\chi)=\frac{1}{\sqrt{2\pi}}\int_0^{\chi} e^{-y^2/2}y\mathrm{d}y,$$

$$\Phi(\chi)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\chi} e^{-y^2/2}\mathrm{d}y.$$

Taking into account formula (14), there is equality

$$\chi=\chi(r)=\frac{\delta^*}{\sigma}=\frac{\lambda}{\sigma}-2.326=r-2.326. \quad (31)$$

Since $\Phi(\delta^*/\sigma)=F(\lambda+\delta^*)$, then the value $\Phi(\chi)$ is equal to probability $P^-[\lambda+\delta^*]=P\{\eta_t \leq \lambda+\delta^*\}$ that at the end of period t there will be no unsatisfied demand.

Using the indicator of relative demand intensity, formulae (24), (26), which determine components ψ_m^-, ψ_m^+ of the mathematical expectation λ for the MSG policy, can be written in the following form

$$\psi_m^-=(0.5r-0.4)\sigma, \psi_m^+=(0.5r+0.4)\sigma. \quad (32)$$

Taking into account formulae (28), (30), and the fact that $\lambda=r\sigma$, the formulae that determine components γ_m^-, γ_m^+ of the mathematical expectation λ for the PRS policy take the following form

$$\gamma_m^-=\sigma\rho_r^-,\ \gamma_m^+=\sigma\rho_r^+, \quad (33)$$

where

$$\rho_r^-=-0.4+M(\chi)+r\Phi(\chi),\ \rho_r^+=0.4-M(\chi)+r\left(1-\Phi(\chi)\right).$$

The value of indicator $r_{\max}$, in the presence of which there will be "practically" no unsatisfied demand, is determined from the following formula: $P^{-}(\lambda+\delta^*)=\Phi(\chi)=0.99$. Taking into account formula (14), as well as the fact that $\Phi^{-1}(0.99)=2.326$, the following equality holds

$$\frac{\delta^*}{\sigma}=\frac{\lambda}{\sigma}-2.326=2.326. \quad (34)$$

Hence $r_{\max}=\lambda/\sigma=4.652$.

It can be seen that with the increase of r from $r_{\max}=3$ to $r_{\max}=4.625$ the values χ, $M(\chi)$ increase, probability $P^{-}(\lambda+\delta^*)=\Phi(\chi)$ also increases from $\Phi(0.674)\approx 0.451$ to 0.99, and probability $P^{+}(\lambda+\delta^*)=1-\Phi(\chi)$ decreases from 0.449 to 0.01. Therefore, with the increase of r the $\rho_r^{+},\gamma_m^{+}=\rho_m^{+}\left[\lambda+\delta^*\right]$ values will decrease to 0. At the same time, index υ^* of the relative reserve stock increases from the value of 0.225 to 0.5

$$\upsilon^*=\frac{\delta^*}{\lambda}=1-2.326\frac{\sigma}{\lambda}=1-\frac{2.326}{r}. \quad (35)$$

Therefore, when applying the reserve stock policy in situations where $r=r_{\max}=4.625$, both the volume of unsold goods not exceeding the value λ and the absence of unsatisfied demand can be ensured. In these situations, ratio $\delta^*\le\eta\le\lambda+\delta^*$. is satisfied with a probability of 0.99.

Therefore, for the study of the reserve stock policy, value $r_{\max}=4.625$ can be considered the maximum value for indicator r. In the case of a further increase in r, the situation $\rho_m^{+}\left[\lambda+\delta^*\right]\approx 0$, will be preserved.

An example of intermediate and end results of calculating the expected demand volumes $\rho_m^{-}\left[\lambda+\delta^*\right]$, $\rho_m^{+}\left[\lambda+\delta^*\right]$ for $\sigma=1$ is given in Tables 1, 2. In the case of $\sigma=1$, the formulae for the calculation are simplified: $\chi=\delta^*=r-2,326$, $\rho_m^{-}\left[\lambda+\delta^*\right]=\rho_r^{-}$, $\rho_m^{+}\left[\lambda+\delta^*\right]=\rho_r^{+}$.

Table 1

Calculating the volume of demand γ_m^{-} according to options of r ($\sigma=1$)

$\lambda=r$	$\chi=\delta^*$	$\Phi(\chi)$	$r\Phi(\chi)$	$M(\chi)$	$M(\chi)-0.4$	$\rho_r^{-}=\gamma_m^{-}$
3	0.674	0.750	2.250	0.081	−0.319	1.931
3.5	1.174	0.880	3.079	0.199	−0.201	2.878
4	1.674	0.953	3.812	0.301	−0.099	3.713
4.5	2.174	0.985	4.433	0.361	−0.039	4.394
4.625	2.299	0.989	4.575	0.371	−0.029	4.546

Table 2

Calculating the volume of demand γ_m^{+} according to options of r ($\sigma=1$)

$\lambda=r$	$r+\delta^*$	υ^*	$\Phi(\chi)$	$1-\Phi(\chi)$	$0.4-M(\chi)$	$\rho_r^{+}=\gamma_m^{+}$
3	3.674	0.225	0.750	0.250	0.319	1.069
3.5	4.674	0.335	0.880	0.120	0.201	0.622
4	5.674	0.419	0.953	0.047	0.099	0.287
4.5	6.674	0.483	0.985	0.015	0.039	0.106
4.625	6.924	0.497	0.989	0.011	0.029	0.079

According to the data given in Tables 1, 2, one can see that with an increase in indicator r of the relative intensity of demand, component γ_m^{-} of the mathematical expectation of demand increases, and component γ_m^{+} decreases.

5. 5. Comparative analysis of the minimum stock and permanent reserve stock policies

A comparison of the expected operational effects of the permanent reserve stock and minimum stock policies for different values of demand transfer coefficient k, economic parameters a, d, and parameters λ, σ of the demand probability distribution function has been carried out.

The following notations are introduced: $\Delta E^*=E^*_{PRS}-E^*_{MSG}$ is the difference between expected operational effects E^*_{PRS}, E^*_{MSG} under the PRS and MSG policies, which are determined from formulae (10), (17); D – revenue component ΔE^* (the difference between revenue parts E^*_{PRS}, E^*_{MSG}); A is the component ΔE^* that falls on the costs of storing unsold goods (the difference between parts E^*_{PRS}, E^*_{MSG}, which account for losses). Then, $\Delta E^*=D+A$.

To compare advantages of the PRS policy over the MSG policy for different values of parameters a, d, σ, it is convenient to use the reduced difference ΔE between the expected operational effects: $\Delta E=(1/\sigma d)\Delta E^*$. Then

$$\Delta E=\bar{D}+b\bar{A}, \quad (36)$$

where b is the ratio of specific losses from storing goods to specific income from their sale:

$$b=\frac{a}{d},$$

$$\bar{D}=\frac{1}{\sigma d}D,$$

$$\bar{A}=\frac{1}{b\sigma d}A.$$

According to formula (17), values D_{PRS}, $\bar{D}_{PRS}$ of the revenue part of the PRS policy are determined by the following values:

$$D_{PRS}=d\begin{pmatrix}\gamma_m^{-}+(2k-1)(\lambda-\gamma_m^{-})+\\+2P^{+}(1-k)(\lambda+\delta^*)\end{pmatrix}, \quad (37)$$

$$\bar{D}_{PRS}=\rho_r^{-}+(2k-1)(r-\rho_r^{-})+2(1-\Phi(\chi))(2r-2.326). \quad (38)$$

According to formula (10), values D_{MSG}, $\bar{D}_{MSG}$ of the revenue part of the MSG policy are determined by the following values:

$$D_{MSG}=d\left(\psi_m^{-}+(2k-1)(\lambda-\psi_m^{-})+(1-k)\lambda\right)= \\ =d\left(\lambda-0.8\sigma(1-k)\right), \quad (39)$$

$$\bar{D}_{MSG}=r-0.8(1-k). \quad (40)$$

According to formulae (10), (17), the difference between losses from storage of unsold goods under the PRS policy and under the MSG policy is determined by the values:

$$A=a\left(\gamma_m^{-}-(\lambda+\delta^*)P^{-}\right)+0.5a\left(\lambda-2\psi_m^{-}\right), \quad (41)$$

$$\bar{A}=M(\chi)-\chi\Phi(\chi). \quad (42)$$

From formulae (38), (40) it follows that values $\bar{D}_{PRS}$, $\bar{D}_{MSG}$, $\bar{D}$ are functions of the demand transfer coefficient k and the relative demand intensity index r: $\bar{D}_{PRS}=\bar{D}_{PRS}(k,r)$, $\bar{D}_{MSG}=\bar{D}_{MSG}(k,r)$, $\bar{D}=\bar{D}(k,r)$. At the same time, according to formula (42), the cost component $\bar{A}$ depends on the relative demand intensity index r, $\bar{A}=\bar{A}(r)$, and does not depend on the demand transfer coefficient k.

In the case of complete transfer of unsatisfied demand in accordance with formulae (12), (19), the revenue parts $\bar{D}_{PRS}(1,r)$, $\bar{D}_{MSG}(1,r)$ of the PRS and MSG policies correspond to the same value of r, and therefore $\bar{D}(1,r)=0$. Under conditions of impossibility of transferring unsatisfied demand, $k=0$, revenue component $\bar{D}(0,r)$ of the difference in effects is determined from the following formula

$$\bar{D}(0,r)=2(\rho_r^- -r)+0.8-2(1-\Phi(\chi))(2r-2.326). \quad (43)$$

The income component of difference in effects $\bar{D}(k,r)=\bar{D}_{PRS}-\bar{D}_{MSG}$ can be represented as follows:

$$\bar{D}(k,r)=\bar{D}(0,r)+k\bar{D}'(r), \quad (44)$$

where $\bar{D}'(r)$ is the partial derivative of function $\bar{D}(k,r)$ with respect to coefficient k of demand transfer, $\bar{D}'(r)=\bar{D}'_{PRS}(r)-\bar{D}'_{MSG}(r)$, $\bar{D}'_{PRS}(r)$, $\bar{D}'_{MSG}(r)$ are the partial derivatives of functions $\bar{D}_{PRS}(k,r)$, $\bar{D}_{MSG}(k,r)$ with respect to coefficient k of demand transfer. From formulae (41), (42) it follows that

$$\bar{D}'(r)=-2\left(\rho_r^- -r+(1-\Phi(\chi))(2r-2.326)\right)-0.8. \quad (45)$$

In Table 3, for the variants of the relative intensity of demand r, calculations of the revenue component $\bar{D}(0,r)$, as well as the $\bar{D}'_{PRS}(r)$, $\bar{D}'(r)$ values are given. From the table, one can see that for all $r \in [r_{\min}, r_{\max}]$ the difference $\bar{D}(0,r)$, between the revenue parts of the PRS, MSG policies is positive, and with increasing r it tends to increase. And although value r takes positive values, the $\bar{D}'(r)$ value takes negative values

$$\bar{D}'(r)<0, \text{ for all } r\in[r_{\min},r_{\max}]. \quad (46)$$

Table 4 gives the results of calculating the cost component $\bar{A}=\bar{A}(r)$ for the variants of the relative intensity of demand r. From them one can see that for all $r \in [r_{\min}, r_{\max}]$ the difference $\bar{A}$ between the cost components under the PRS policy and under the MSG policy is a negative value, i.e., storage losses under the PRS policy are greater than under the MSG policy.

Table 3

Calculating $\bar{D}(0,r)$, $\bar{D}'_{PRS}(r)$, $\bar{D}'(r)$ for variants of r

$\lambda=r$	$2(\rho_r^- -r)+0.8$	$2(1-\Phi(\chi))$	$2r-2.326$	$\bar{D}(0,r)$	$\bar{D}'_{PRS}(r)$	$\bar{D}'(r)$
3	−1.349	0.50	3.674	0.500	0.301	−0.499
3.5	−0.443	0.24	4.674	0.680	0.120	−0.680
4	0.225	0.94	5.674	0.760	0.040	−0.760
4.5	0.588	0.30	6.674	0.787	0.013	−0.787
4.625	0.643	0.22	6.924	0.791	0.009	−0.791

Table 4

Calculating $\bar{A}(r)$, $b^*(r)$ for variants of r

$\lambda=r$	$M(\chi)$	$\Phi(\chi)$	χ	$\bar{A}(r)$	$b^*(r)$
3	0.081	0.750	0.674	−0.424	1.176
3.5	0.199	0.880	1.174	−0.834	0.816
4	0.301	0.953	1.674	−1.294	0.587
4.5	0.361	0.985	2.174	−1.781	0.442
4.625	0.371	0.989	2.299	−1.903	0.416

From formulae (41), (44) it follows that

$$\Delta E=\Delta E(k,r)=\bar{D}(0,r)+k\bar{D}'(r)+b\bar{A}(r). \quad (47)$$

For each value $r \in [r_{\min}, r_{\max}]$ of the relative intensity of demand, the following critical value $b^*=b^*(r)$ can be defined, the ratio b of specific losses to specific income, according to which $\Delta E(0,r)=\bar{D}(0,r)+b^*\bar{A}(r)=0$. By this definition of b^*, the following equality holds

$$b^*=b^*(r)=-\frac{\bar{D}(0,r)}{\bar{A}(r)}. \quad (48)$$

The calculation of indicator b^* for the variants of relative intensity of demand r is given in Table 4. If the activity of the enterprise is characterized by ratio $b>b^*(r)$, then for any value k Î [0, 1] it turns out that $\Delta E(k,r)<0$, $\Delta E^*<0$, i.e., the MSG policy is better than the PRS policy. If $b<b^*(r)$, then under the conditions of impossibility of transferring unsatisfied demand, $k=0$, the PRS policy will be a better policy. But in accordance with (45), (46), the derivative of function $\Delta E(k, r)$ with respect to coefficient k of demand transfer is a negative value. Therefore, in the case when $b<b^*(r)$, there is such a critical value k^* of coefficient k, according to which on the interval $k \in [0, k^*]$ the PRS policy has the advantage $E^*_{PRS} \geq E^*_{MSG}$, and on the interval $k \in [k^*, 1]$, the MSG policy is the best, $E^*_{PRS} \leq E^*_{MSG}$.

The value k^* is a function of r, b, i.e. $k^*=k^*(r,b)$, and it satisfies the formula

$$k^*=\frac{\bar{D}(0,r)+b\bar{A}(r)}{-\bar{D}'(r)}. \quad (49)$$

Thus, the PRS policy has an advantage over the MSG policy for small values of coefficient k of demand transfer and ratio b of specific losses to specific income.

It is of interest to find the minimum values of ratio ω of the expected operational effect corresponding to the worst policy choice to the expected operational effect corresponding to the best policy choice. Under the conditions of impossibility for transferring unsatisfied demand, $k=0$, and a small value of specific losses on the preservation of goods, $a=b\approx 0$, the ratio ω of effects is determined from the following formula

$$\omega=\frac{\bar{D}_{MSG}(0,r)}{\bar{D}_{PRS}(0,r)}. \quad (50)$$

If $r=3.5$, then $\bar{D}_{MSG}(0,r)=2.7$, $\bar{D}_{PRS}(0,r)=3.38$, $\omega=0.8$. When specific losses on the preservation of goods are large, $a=d$, $b=1$, ratio ω of the effects is determined from the following formula

$$\omega=\frac{E_{PRS}(0,r)}{E_{MSG}(0,r)}, \quad (51)$$

where

$$E_{PRS}(0,r)=\bar{D}_{PRS}(0,r)+\bar{A}_{PRS}(r),$$

$$E_{MSG}(0,r)=\bar{D}_{MSG}(0,r)+\bar{A}_{MSG}(r).$$

If $r=4.5$, then $E_{PRS}(0,r)=2.3$, $E_{MSG}(0,r)=3.3$, $\omega=0.7$.

6. Discussion of results based on the study on ensuring the adaptability of inventory management under conditions of instability

The content of the proposed concepts and models of adaptive inventory management of a trading enterprise is explained primarily by the choice of the conceptual scheme of a fixed period of product orders as the basis for management. Other conceptual schemes require continuous revision of the inventory level, and for this reason, during their application, the complexity of inventory management increases significantly. In addition, the randomness of the terms and volumes of orders creates inconvenience for the supplier, which can lead to an increase in the cost of orders.

In accordance with the scheme of a fixed period of product orders, the operational activity of a trading enterprise is considered as a controlled random process in the sequence of periods "purchase – sale of goods". In this process, two types of situations are distinguished that can occur in each period. These situations are determined by deviations in the volume of demand in the smaller or larger direction from the volume of goods intended for sale on request in the current period of time. Depending on the type of situation in the current period, a rule is established for selecting the volume of purchases for the future period, which determines the policy for replenishing the stock of goods.

For a comparative assessment of the effectiveness of different policies for replenishment of stock, it is proposed to use the value of the expected operational effect, which is determined from formulae (2) to (7). This value takes into account, in particular, the integral economic result of the policy by applying the indicators of specific losses from storage of goods and specific income from its sale. Depending on the ratio of the indicators of specific losses and specific income, the advantage in the current period of time will be given to one or another policy.

In general, the value (model) of the expected operational effect is intended to reflect all possible direct and indirect economic effects that arise in the process of operational activity of a trading enterprise. The proposed indicator of the expected operational effect has noticeable differences from the criteria for optimal inventory management that were used in studies [12–14] discussed in the review of literary data. In particular, formula (2) of the proposed model of the expected operational effect takes into account such a negative phenomenon as lost profit. In determining the operating effect, this is achieved by reducing the revenue from the sale of goods by the value of the unsatisfied demand, which is interpreted as a loss from lost profits. Therefore, in the presence of unsatisfied demand, the operating effect will differ from the operating profit for the same period. This is due to the fact that the latter represents only the difference between sales revenue and expenses, that is, it does not take into account the effect of lost profits. But the effects of lost profits not only reduce overall profit, but also undermine consumer confidence, reduce demand.

Another feature of the inventory management problem is the assumption that the enterprise's inventory is located in its own warehouses. In this case, the size of the warehouses affects the possible volumes of simultaneous inventory placement, but the operating costs of maintaining the warehouses do not depend on the volumes of goods placed in them. Since the cost of storing a stock of goods during the planned period of time is a constant value that does not depend on the volume of goods purchased, there is no need to reflect this value in the formulae that determine the operating effect. However, the additional costs of storing unsold goods are reflected in formula (3) of the operating effect, since the volumes of unsold goods on time increase the share of costs in the gross sales volume and the need for working capital.

In our study, the duration of period between order placements is considered as a given value. This is due to the fact that when choosing the duration of this period, many factors must be taken into account that are determined by the specific conditions for storing goods and their supply at a particular enterprise. The conditions for storing goods are determined by the volume and properties of specialized warehouses that are able to maintain the required storage conditions (temperature, humidity) and also provide the opportunity to conveniently load goods and deliver them to sales areas. The terms of delivery are determined by the possibilities of delivering the ordered volume of goods in separate parts, transportation costs, etc.

For detailed consideration, mathematical description and economic assessment, the policies of minimum stock of goods and permanent reserve stock have been proposed. In accordance with the MSG policy, the volume of goods purchased for the future period is equal to the average volume of its sales for the period, if there were no product residues or unsatisfied orders in the current period. If there are product residues in the current period, the volume of goods purchased by the enterprise is reduced by the volume of residues. If there are unsatisfied customer orders in the current period, their volume is carried over to the future period with a certain coefficient, and the volume of purchases increases by the volume of transferred demand. The expected values of the components of the operating effect for the MSG policy are determined from formulae (9), (10).

According to the PRS policy, a constant reserve stock is created in each period of operating activity, which is designed to reduce the volume of unsatisfied demand. The total inventory of goods in each period corresponds to the sum of the average sales volume and the reserve stock, and its value decreases or increases depending on the presence of product residues or carried-over demand in the previous period. The expected values of the effect components for the PRS policy are determined from formulae (16), (17).

The application of the proposed concept of adaptive inventory management provides for the possibility of direct use in any period of either the MSG policy or the PRS policy. To this end, at the beginning of each period, the same condition must be met for both policies, regardless of which policy was implemented in the previous period. This condition is that the volume of unsold goods does not exceed the average sales volume for one period. It is shown that both policies satisfy this condition.

For the normal probability distribution of demand volumes, the dependence of the expected operational effect on the parameters of the functions of this distribution has been determined. For convenience, the relative intensity of demand r, which represents the ratio of mathematical expectation of demand volume λ to standard deviation σ, and the standard deviation σ itself were chosen as these parameters. To determine the range of possible values of r, the assumption was used that an arbitrary normal distribution function acquires its non-zero values only in the interval $[\lambda - 3\sigma, \lambda + 3\sigma]$. It follows that λ and σ satisfy condition $\lambda \geq 3\sigma$, and the value $r_{\min} = 3$ is the minimum value of indicator r. The maximum value of indicator r is taken to be such a value at which the probability of unsatisfied demand is close to 0.

In formulae (11), (18), the expected operational effects from the application of the MSG and PRS policies determine values ψ_m^-, ψ_m^+, γ_m^-, γ_m^+, which represent the mathematical expectations of the demand volumes at certain intervals of their possible values. It is proposed to use formulae (33), (34), according to which each value ψ_m^-, ψ_m^+, γ_m^-, γ_m^+, can be found as the product of its value at $\sigma = 1$ by the value of standard deviation σ. In this case, the values corresponding to a single standard deviation depend only on the value of r of the relative intensity of demand. The proposed approach to determining the expected demand volumes ψ_m^-, ψ_m^+, γ_m^-, γ_m^+, makes it possible to significantly simplify their calculations for specific values λ, σ of parameters of the normal probability distribution. The results of test calculations of the components γ_m^-, γ_m^+ of mathematical expectation λ for the PRS policy are given in Tables 1, 2.

For comparative analysis of the effectiveness of the MSG and PRS policies, an analysis of dependences of the revenue and expenditure parts of their expected operational effects of the policies on the relative intensity of demand r was carried out. The results of the related calculations are given in Tables 3, 4. Consequently, zones of possible values of indicators of the situation of operational activity were identified, in which a certain policy will be better than another. This indicates that the MSG and PRS policies do not dominate each other, that is, they really satisfy the principle of efficiency. The zones of policy advantages are determined by the critical values of the ratio of specific losses to specific income and the coefficient of demand transfer, which are determined from formulae (49), (50).

The proposed methodology for determining the expected volumes of demand and the results of comparative analysis of the effectiveness of the MSG and PRS policies make it possible to significantly simplify the required calculations. This creates opportunities for small trading businesses to determine the optimal volumes of goods purchased by performing simple calculations in a typical computer environment, for example, MS Excel. Thus, the advantages of adaptive management are created in comparison with the simulation modeling method and the scenario approach, which require the use of special software and special qualities of performers [9–11].

The main feature of our study's results compared to the papers discussed in the review of literary data is determined by the proposed concept and models of adaptive management of trade inventories. This feature involves simultaneously taking into account the conditions of random fluctuations in demand and changes in various characteristics of the operational activity situation. As a result, the proposals for adaptive management provide an opportunity to reasonably determine the volumes of goods purchased, increase their sales, and reduce losses from storing leftover goods.

Compared to earlier studies [9–11] on inventory management under stochastic conditions, the proposed approach provides a more complete account of demand variability within a given probability distribution law. In [9–11], the optimal values of inventory management parameters were determined only for a limited number of individual scenarios or specific values of demand distribution parameters. In [18–20], the impact of changing the demand probability distribution parameters on the choice of the volume of commodity production was not investigated. In contrast, the proposed approach allows us to calculate the operational effect for each of the analyzed policies (the minimum stock policy and the permanent reserve stock policy) and compare them using analytical formulae for all possible values of the demand probability distribution parameters and the characteristics of the operational activity situation.

The limitation of our research results is the assumption that the demand probability distribution law is normal. But the reported approach can be generalized without any fundamental difficulties for the case when the probability density of demand volumes is a symmetric function with respect to the mathematical expectation of the demand value. This circumstance indicates the possible development of research and the prospects for practical application of the above approach.

The disadvantage of the current work is the assumption that the costs of storing a commodity unit at the enterprise's own premises are determined by a known, clearly defined value. In fact, they depend not only on the operating costs of maintaining a certain warehouse per unit of time but also on the number of goods placed in it, which changes over time. The occupancy of the warehouse depends on many characteristics of the specific situation in which inventory management of a particular product takes place at a particular trading enterprise. These include the possibility of supplying the ordered volume of goods in separate parts or only in full, the creation of a reserve stock of goods or its absence, etc. Thus, determining the costs of storing a commodity unit during the implementation of adaptive management requires additional research into the operating conditions of a particular trading enterprise, its warehouse, as well as taking into account the interval uncertainty of the cost value.

7. Conclusions

1. We have built a dependence model of the expected operational effect during planning period on the choice of volume of goods purchased. The general form and properties of the resulting model are explained by the use of a conceptual scheme with a fixed period of goods orders and consideration of the replenishment policy as a management tool adapted to random fluctuations in demand. A feature of the model is the consideration of lost profits in cases of goods shortages. The expediency of taking this effect into account is explained by the fact that it not only reduces the overall profit but also undermines consumer confidence in the trading enterprise, as well as reduces demand.

2. The concept of adaptive management has been proposed, which defines an original means for adapting inventory management of a trading enterprise to random demand for goods and changes in the economic characteristics of operating activity. It involves choosing a policy at each period that corresponds to the maximum expected operating effect. For this choice, the company needs to collect information on the values of the inventory management situation parameters at each time period, calculate the expected operating effect for each policy option, and choose the best one from them.

3. We have formalized the content of two replenishment policies, which are determined by the minimum product stock and the permanent reserve stock. Mathematical expressions of the expected operational effect were derived for these two alternative policies. It is proved that the MSG and PRS policies do not dominate each other, that is, they satisfy the principle of efficiency. The most obvious drawback of the minimum product stock policy is the presence of significant volumes of unsatisfied demand at small values of the demand

transfer coefficient. This leads not only to direct economic losses associated with the loss of a significant part of the income from the sale of goods but also reduces the confidence of buyers in the enterprise and, accordingly, the intensity of demand. At the same time, in the process of the enterprise's operational activities under the MSG policy, the enterprise incurs minimal losses from the creation of excess inventories. Therefore, with a sufficiently large demand transfer coefficient, the MSG policy turns out to be more attractive from an economic point of view than PRS.

4. A method for determining the dependence of the expected operational effect of policies on the values of parameters for the normal probability distribution function of demand volumes has been proposed. This method makes it possible to significantly simplify the calculations of the expected operational effect based on specific values of the mathematical expectation and standard deviation of demand volumes. The method devised is based on the properties of the normal probability distribution law, in particular on the symmetry of the probability density of demand volumes.

5. A comparative effectiveness analysis of the MSG and PRS policies was carried out across the entire range of possible values for the indicators of the policy choice situation. In this case, the following indicators of the choice situation were considered: the demand transfer coefficient, specific losses from storage of products, specific income from sales, parameters of the mathematical expectation and standard deviation of the probability distribution of demand volumes. For each policy, we have identified zones of possible values of indicators in which it would be the best. These zones are determined by the critical values of the ratio of specific losses to specific income and the demand transfer coefficient, at which the difference between the expected operational effects of the MSG and PRS policies becomes zero.

For conditions where the transfer of unsatisfied demand is impossible, the minimum values of the ratio of the expected operational effect corresponding to the worst policy choice to the expected operational effect corresponding to the best policy choice have been found. They are equal to 0.7 and 0.8, depending on which of the policies is the best. This significant discrepancy, which occurs between the expected operational effects of policies in different inventory management situations, determines the economic feasibility of choosing a policy depending on the values of the indicators of this situation. It proves the economic efficiency of adaptive management, its ability to increase the operational effect by increasing sales volumes of goods, as well as reducing losses from storing their remains.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study, as well as the results reported in this paper.

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Data availability

The data will be provided upon reasonable request.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

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