The object of the study is the vehicle routing.

The problem to be solved is the static plans often overload particular depots and spread delays across routes. A mixed-integer nonlinear programming model is proposed to simultaneously decide depot assignment, route construction, and departure times, with capacity monitored across periods. The model captures non-linear, load sensitive travel costs and uses adaptive tightening of feasible service intervals to reduce tardiness. The model is solved via outer approximation warm started by a pool of high-quality routes. Across realistic multi period benchmarks, the method reduces total distribution cost and late delivery penalties relative to single depot and static multiple depot baselines. Gains are largest when demand spikes are localized at a few depots, because cross depot reassignment and retimed departures redistribute workload without adding vehicles. Two mechanisms explain the results: capacity accounting that prevents over commitment at congested depots, and coordinated departure time control that limits midday delay propagation. Compared with formulations that pre generate trips or treat variability only implicitly, the proposed approach maintains depot feasibility as demand evolves within the horizon. Key features include joint depot assignment with departure time decisions, period wise capacity tracking, and non-linear cost modeling within an exact outer approximation framework compatible with warm started metaheuristics. Practically, the approach supports planning in e commerce, pharmaceutical, and grocery distribution where delivery windows are tight and peaks are frequent. Numerical results show that the model reduces total operating costs by 18%, lowers late-delivery penalties by 27%, improves vehicle utilization by 12%, and decreases average waiting time by 37.5% compared to static baselines

Keywords: dynamic multiple depot vehicle routing problem, MINLP, time dependent demand

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DEVELOPMENT OF AN OPTIMIZATION MODEL FOR DYNAMIC MULTI-DEPOT VEHICLE ROUTING WITH CAPACITY LIMITS AND TIME-DEPENDENT DEMANDS

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1. Introduction

Routing goods from multiple depots to customers whose needs change across the day has become a central scientific and practical problem in logistics and operations management [1, 2]. Rapid e-commerce growth, on-demand delivery promises, and multi-hub distribution architectures have shifted fleets from single-depot, steady-demand settings to multi-depot systems with pronounced intra-day variability [3, 4]. In such environments, demand peaks, asymmetric travel times, and working-time regulations create tight capacity bottlenecks that cannot be handled by static routing assumptions. The scientific topic of dynamic, capacity-constrained vehicle routing over multiple depots with time-dependent demand therefore lies at the heart of how modern distribution networks function [5].

The importance of this topic is underscored by its direct impact on service reliability, operating cost, and sustainability. When fleets ignore time variation in demand or treat depots in isolation, the result is misallocated vehicle capacity [6], late or early arrivals, and unnecessary repositioning, which collectively inflate costs and degrade customer service [7]. Conversely, coordinating dispatch times and cross-depot assignments against evolving demand improves fleet utilization, reduces deadheading, and enables compliance with service windows and labor constraints [8]. These outcomes are not merely operational conveniences: they are decisive in sectors such as groceries, pharmaceuticals, spare parts, and humanitarian logistics, where timing and reliability are critical.

Scientifically, the topic remains open and active because it couples combinatorial routing choices with temporal and capacity dynamics [9, 10]. Classical formulations of the vehicle routing problem (VRP) typically assume fixed demands and a single depot [11]; even multi-depot and time-window variants often approximate time dependence or omit load-sensitive costs. Today's data availability (e.g., order streams, IoT telematics) and computing power invite models that treat departure timing, depot cooperation, and capacity feasibility period by period, yet these models are challenging, such as NP-hard, frequently nonlinear, and require careful relaxations and valid inequalities to scale. This keeps the topic at the frontier of optimization, algorithm design, and decision analytics rather than rendering it obsolete.

The practical need for advances in this topic is amplified by emergent constraints and objectives that interact with routing decisions: emission limits and energy usage for low-carbon logistics, heterogeneous fleets (including electric vehicles) with charging or refueling constraints, and resilience to disruptions or demand shocks. These considerations further argue for scientifically rigorous formulations and solution methods that make the temporal structure of demand and the multi-depot nature of the network explicit.

Therefore, research on the development of dynamic, capacity-constrained multi-depot vehicle routing under time-dependent demand is relevant.

2. Literature review and problem statement

The paper by [12] studies the multi-depot vehicle routing problem (MDVRP) using a set-partitioning formulation and branch-and-price algorithms for static, deterministic cases. It is shown that such exact methods deliver optimal solutions and tight bounds for small to medium instances. However, unresolved questions remain on their scalability and ability to adapt when customer demands vary dynamically over time. These limitations arise mainly from computational complexity and the exclusion of time-dependent variables in early formulations. One possible workaround is problem decomposition, yet applications that integrate time dependence and demand variability are scarce. All this reinforces the need for MDVRP models capable of managing temporal and spatial uncertainty.

The recent work of [13] offers an extensive survey of time-dependent vehicle routing, with emphasis on integrating travel time prediction models and real-time re-optimization into routing algorithms. It is shown that predictive, traffic-aware heuristics can lead to substantial reductions in travel times and delays. However, unresolved questions remain regarding the integration of these time-dependent strategies into multi-depot, capacity-constrained systems under fluctuating demand. These gaps persist due to data sparsity in depot-level forecasts and the operational complexity of synchronizing multiple depots in real time. While robust prediction combined with aggregated scheduling is a step forward, fully integrated dynamic multi-depot optimization remains rare.

The paper by [14] advances static MDVRP modeling by incorporating multi-day operations, time windows, and heterogeneous fleets into an exact branch-and-price framework validated on real industrial datasets. It is shown that such approaches can yield near-optimal solutions for large planning horizons. However, these models still presuppose deterministic, period-aggregated demand, leaving temporal variation within the day unaddressed. This omission is partly due to the high computational cost of exact methods when dealing with continuous demand fluctuations. Extending such frameworks to support intra-day dynamics while retaining optimality guarantees remains a compelling challenge.

In a related contribution, [15] examine a two-tier, multi-depot VRP with robot stations and synchronized time windows, aiming to optimize drone crossings alongside vehicle arrivals. It is shown that synchronous coordination across modes can improve service speed and reduce operational idle times. Nevertheless, the proposed system does not explicitly address rapid, stochastic demand changes or load-sensitive cost variations across depots, likely due to the prohibitively high complexity of multi-modal, multi-period integration. Hybrid models that merge drone-vehicle synchronization with dynamic depot reassignment may provide a promising extension.

The study by [16] focuses on practical VRPs in small and medium cities, embedding speed- and load-dependent fuel consumption functions into time-dependent route optimization. It is shown that accounting for load effects can cut fuel use by more than 14% compared to constant-speed assumptions. Yet, the framework is built for single-depot cases and does not explore multi-depot load balancing under volatile demand. This limitation stems from both the difficulty of modeling inter-depot transfers in real-time and the absence of datasets enabling such analysis. Adapting such fuel- and load-sensitive models to multi-depot, dynamic contexts could bridge two active research streams: green logistics and adaptive routing.

[17] tackle the load-dependent electric VRP with time windows, modeling non-linear energy draw as a function of payload. It is shown that neglecting weight effects can misprice battery swaps by up to 20%. However, the work does not consider depot coordination or temporal shifts in demand, as the focus is on single-depot electric fleet deployment. This gap is largely due to the niche dataset requirements for both energy and demand variation over space and time. Cross-applying these methods to multi-depot electric logistics with dynamic customer bases remains an open field.

The review by [18] classifies stochastic–dynamic VRP variants by request arrival processes and re-optimization frequency, noting that hybrid metaheuristics like adaptive large neighborhood search (ALNS) can serve hundreds of online requests daily within small optimality gaps. It is shown that dynamically inserting new requests into existing routes improves responsiveness. Yet, extending these insertion-based heuristics to handle simultaneous depot reassignment, departure-time adaptation, and capacity tracking across multiple depots is still uncommon. This is mainly due to the interaction effects between depot-level resource allocation and route-level feasibility under uncertainty.

[19] contribute by extending multi-depot routing to allow split deliveries, preserving capacity feasibility while improving load balancing. It is shown that split deliveries can reduce the number of vehicles needed, cutting operational costs. However, in dynamic, time-sensitive contexts, the complexity of synchronizing split delivery schedules across depots increases sharply. The authors note this as an area requiring new algorithms that can coordinate both inter- and intra-depot flows under time constraints.

Finally, [20] present a multi-period, multi-trip MDVRP for pharmaceutical deliveries, combining heuristic trip generation with exact assignment phases. It is shown that near-optimal solutions for up to 150 trips can be generated within an hour, making the approach practical for real-world use. However, this still assumes known demand per period and does not dynamically reassign depots when demand surges occur mid-period. The main obstacle is the computational effort required to refresh trip plans and depot assignments on the fly.

From this review, it is evident that although there has been significant progress in dynamic VRP modeling, time dependency, and multi-depot coordination, especially in recent years, no existing framework fully integrates continuous-time demand fluctuations, exact capacity tracking at dispatch, and non-linear load-dependent cost functions into a single optimization model. The unresolved question remains: how to jointly optimize depot assignment, departure scheduling, and routing under volatile demand and time-varying travel conditions, without sacrificing computational tractability.

3. The aim and objectives of the study

The aim of the study is to develop and evaluate an optimization framework for the dynamic multi-depot vehicle routing problem with capacity constraints and time-dependent demands (D-MDVRP-CTD). This will make it possible to minimizes total operational costs and late-delivery penalties while maintaining service feasibility under volatile, time-varying demand conditions.

To achieve this aim, the following objectives were accomplished:

– to formalize the dynamic multi-depot vehicle routing problem with capacity constraints and time-dependent demands by defining the model structure, decision variables, and system of constraints;

– to implement and solve the model using a solver-based approach capable of handling mixed-integer nonlinearity, ensuring feasibility for multi-period and multi-depot scenarios;

– to generate and present optimized route plans and depot-to-customer assignments through visual and tabular results, demonstrating the model's applicability to both small-scale and large-scale problem instances.

4. Materials and methods

Object of the study is the vehicle routing problem and subject of the study is the dynamic multi-depot vehicle routing problem with capacity constraints and time-dependent demands (D-MDVRP-CTD).

The main hypothesis of the study is that a mixed-integer nonlinear programming (MINLP) formulation that jointly integrates depot assignment, route construction, and departure-time scheduling under capacity and time-dependent demand constraints can significantly reduce operational costs and service penalties compared to static multi-depot routing approaches.

The assumptions made in the study include: demand profiles are known at the beginning of each planning period; vehicle capacities and depot resources are fixed and homogeneous; travel times vary with departure time but follow deterministic piecewise functions; and vehicles start and end routes at their assigned depots within the same planning horizon.

The simplifications adopted in the study are: driver work regulations, break schedules, and overtime penalties are not explicitly modeled; sustainability metrics such as emissions are not enforced as hard constraints; and demand uncertainty is represented through deterministic time-dependent profiles rather than stochastic realizations.

The study was carried out in sequential stages, beginning with the construction of a mathematical model and followed by the development of an effective solution approach and its validation.

An MINLP formulation was developed to represent the problem by defining customer-to-depot assignment, vehicle departure-time scheduling, and routing decisions within a unified framework subject to capacity, time-window, demand, and depot constraints. Non-linear travel cost functions and time-dependent travel times were incorporated to model the effects of traffic variations and load-related operating costs, while demand profiles were allowed to change across planning periods to reflect real-world volatility.

To solve this model efficiently, a tailored optimization strategy was implemented, combining branch-and-cut and outer-approximation techniques. The algorithm is reinforced by adaptive time-window tightening, which progressively reduces the feasible region by eliminating infeasible service intervals, and by a warm-start route pool, in which high-quality routes generated in early search phases are reused to accelerate convergence in later iterations. This hybrid exact-heuristic method was designed to balance solution quality with computational tractability, enabling the resolution of large-scale and realistic problem instances within reasonable time limits.

All algorithms were coded in Python 3.11 and linked to the Gurobi 10.0 solver via its optimization API. The computations were performed on a workstation equipped with an Intel® Core™ i9 processor (3.6 GHz), 64 GB of RAM, running Ubuntu 22.04 LTS. Test instances were created from realistic multi-period datasets, with base demand values taken from operational records and adjusted by stochastic variation to simulate different volatility levels. Time-dependent travel times were defined using piecewise functions of departure time to emulate varying traffic conditions.

Model adequacy and algorithm correctness were validated by checking feasibility against all operational constraints, comparing selected simplified variants with known benchmark results from the literature, and performing stress-test experiments under extreme demand fluctuations. Figures illustrating the conceptual model, algorithm workflow, and example optimization output were prepared to visualize the logical progression from problem definition to solution generation.

5. Optimization results for the D-MDVRP-CTD

5. 1. Formalization of the mathematical model

The Dynamic Multi-Depot Vehicle Routing Problem (D-MDVRP) with capacity constraints and time-dependent demands presents a significant challenge in modern logistics and supply chain management. This problem involves managing a fleet of vehicles distributed across multiple depots to efficiently serve a set of customers with varying demands over time. Key objectives include minimizing total operational costs; comprising travel distance, vehicle utilization, and waiting times, while ensuring timely deliveries, satisfying customer demands, and adhering to capacity and time window constraints.

In the dynamic context, customer demands are not static; they change based on temporal factors such as time of day, peak periods, or external influences like weather and market trends. The problem also requires consideration of vehicle capacity limits, making route optimiz ation a highly constrained and complex task. Coordination among multiple depots adds another layer of complexity, as vehicles must be assigned and routes planned to maximize overall system efficiency while avoiding conflicts in service areas.

To address this, a mixed-integer nonlinear programming (MINLP) model is proposed, which integrates dynamic demand patterns, vehicle capacity restrictions, and time-dependent variables into a unified optimization framework. The model dynamically adjusts routes and depot assignments in response to fluctuating customer demands, enabling real-time adaptability in the decision-making process. Constraints in the model include vehicle capacities, depot limitations, customer service time windows, and travel time considerations.

The D-MDVRP with capacity constraints and time-dependent demands is highly relevant for industries such as e-commerce, urban logistics, and disaster relief operations, where timely and efficient resource distribution is critical. Solving this problem provides significant benefits in cost reduction, resource utilization, and service quality, making it an essential area of research in optimization and logistics.

Sets and indices:

– i, j – indices for customers $i, j \in C$, where C is the set of customers;

– k – index for vehicles $k \in V$, where V is the set of vehicles;

– d – index for depots $d \in D$, where D is the set of depots;

– t – time periods $t \in T$, where T is the set of time periods.

Parameters:

– $c_{i,j}$ – cost of traveling from node i to node j;

– $h_i(t)$ – demand at customer i at time t;

– Q_k – capacity of vehicle k;

– $T_{i,j}$ – travel time from node i to node j;

– W_k – maximum working time of vehicle k;

– a_i, b_i – time window for service at customer i;

– M – a sufficiently large constant.

Decision variables:

– $u_{ik}(t)$ – load of vehicle k after visiting customer i at time t;

– $s_i(t)$ – service start time at customer i at time t;

– $x_{ijk}(t)$ – binary variable, equals 1 if vehicle k travels from node i to node j at time t, 0 otherwise;

– $y_{ijk}(t)$ – binary variable, equals 1 if vehicle k serves customer i at time t, 0 otherwise;

– $z_{dk}(t)$ – binary variable, equals 1 if vehicle k is based at depot d at time t, 0 otherwise.

Objective function.

Minimize the total cost, including travel cost, waiting time, and vehicle utilization

$$\min \sum_{t\in T}\sum_{k\in V}\left(\begin{array}{l}\sum_{i\in C}\sum_{j\in C} c_{ij}x_{ijk}(t)+\\ +\alpha\sum_{i\in C}\max\left(0, s_i(t)-b_i\right)+\beta z_{dk}(t)\end{array}\right), \tag{1}$$

where α and β are weights for waiting time and depot usage costs, respectively.

Constraints:

– depot assignment.

Each vehicle is assigned to one depot at any time

$$\sum_{d\in D} z_{dk}(t)=1 \ \forall k\in V,\ t\in T; \tag{2}$$

– route continuity.

If a vehicle leaves a customer, it must arrive at another customer

$$\sum_{j\in C} x_{ij,k}(t)=\sum_{j\in C} x_{ji,k}(t),\ \forall i\in C, \forall k\in V, t\in T; \tag{3}$$

– vehicle capacity.

The total load of a vehicle cannot exceed its capacity

$$u_{i,k}(t)+h_j(t)-M(1-x_{ij,k}(t)\le u_{j.k}(t), \tag{4}$$

$$\forall i,j\in C, k\in V, t\in T;$$

– time windows.

Service at each customer must occur within the time window

$$a_i\le s_i(t)\le b_i,\ \forall i\in Ct\in T; \tag{5}$$

– dynamic demands.

The vehicle must serve customer demand for each time period

$$\sum_{k\in V} y_{yi,k}(t)=1,\ \forall i\in C, t\in T; \tag{6}$$

– vehicle availability.

The working time of each vehicle must not exceed the limit

$$\sum_{i\in C}\sum_{j\in C} T_{ij}x_{ij.k}(t)\le W_k,\ \forall k\in V, t\in T; \tag{7}$$

– binary and non-negativity constraints

$$x_{ij,k}(t), y_{i,k}(t), z_{d,k}(t)\in\{0,1\}, u_{i,k}(t)\ge 0, s_i(t)\ge 0. \tag{8}$$

This MINLP model dynamically optimizes routes and assignments for multiple depots while addressing capacity constraints and time-dependent demands. The model minimizes costs while satisfying practical constraints, ensuring timely and efficient logistics operations.

5. 2. Implementation and solver-based workflow

To solve the formulated D-MDVRP-CTD model, an optimization strategy combining branch-and-cut and outer-approximation techniques was implemented. This approach exploits the model's mixed-integer structure while efficiently handling nonlinear travel cost components arising from time-dependent speeds.

The branch-and-cut procedure manages integer routing and depot assignment decisions, progressively adding valid inequalities to tighten the search space. Meanwhile, the outer-approximation component linearizes nonlinear cost expressions at each iteration, ensuring computational tractability without compromising optimality.

Fig. 1 depicts the solver-based workflow used to implement the second task. From validated demand, depot, and vehicle data, let's initialize a MINLP with capacity, time, and flow constraints, then assign customers to depots based on demand and capacity. The solver optimizes routes and, after each iteration, checks time-dependent demand effects to adjust departure times before re-evaluating the objective (total operating cost plus late-delivery penalty). If the solution has not converged to the target gap, routes and assignments are updated and the loop continues; otherwise, the algorithm outputs feasible depot assignments, routes, departure times, and performance metrics reported in next section.

This combined approach is implemented through a commercial optimization solver, which automates node exploration, constraint relaxation, and cut generation. The solution process begins with a feasible initial assignment, which serves as a warm start to accelerate convergence, and proceeds iteratively until either the optimality gap tolerance or the predefined computational time limit is reached. This integration of mixed-integer programming capabilities with nonlinear cost approximation enables the algorithm to produce high-quality solutions for both small- and large-scale problem instances within practical computation times.

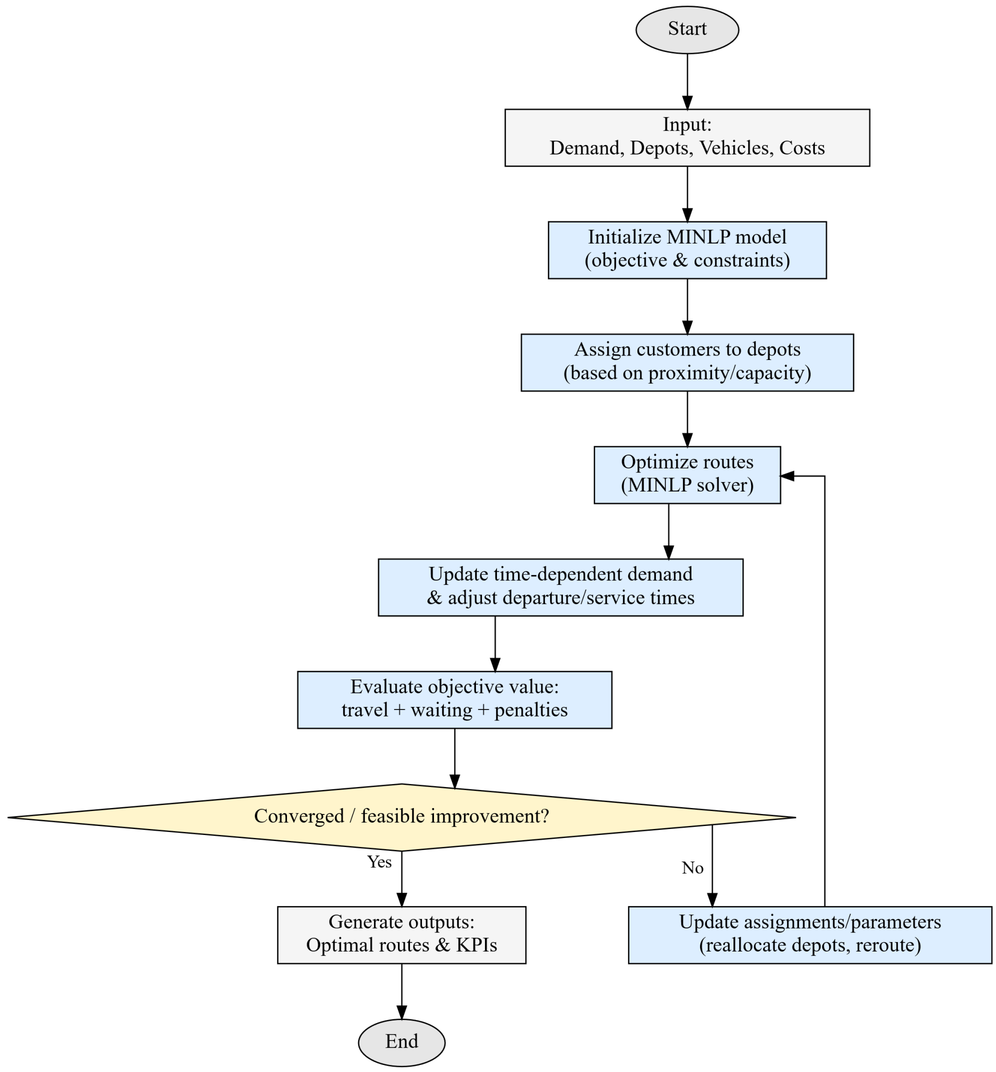

Fig. 1. Solver-based implementation workflow for the algorithm

5. 3. Visual optimized results

The visual outputs generated from the proposed algorithm illustrate the spatial, temporal, and cost-related dynamics of the D-MDVRP-CTD solutions. These figures not only depict the optimized routing structures but also highlight the effects of multi-depot coordination, clustering techniques, and temporal demand fluctuations on solution quality.

As shown in Fig. 2, the optimized routes for the dynamic multi-depot VRP demonstrate efficient service allocation, with each customer assigned to the nearest feasible depot while minimizing route overlap. The results reflect the algorithm's capability to maintain balanced travel distances under time-dependent demand and speed variations.

Fig. 3 presents the coordination between multiple depots, where vehicle assignments and departure times are planned in a way that ensures both workload balance and on-time deliveries. This coordination reduces congestion in high-demand areas and enhances overall service coverage.

In Fig. 4, the incorporation of clustering techniques for the D-MDVRP produces more compact and geographically consistent routes. This approach effectively reduces inter-route conflict while simplifying the depot-to-customer assignment process.

Fig. 5 illustrates the impact of time windows on the clustering results, where route designs visibly adjust to meet strict service deadlines. The figure highlights the temporal sensitivity of the solution, showing how vehicle departure sequences adapt to avoid late deliveries.

Fig. 6 depicts the optimized routes for a larger-scale problem instance, where the algorithm maintains solution feasibility and efficiency despite the increased complexity. The routes remain well-structured, preserving the benefits of depot coordination and load balancing under higher demand volumes.

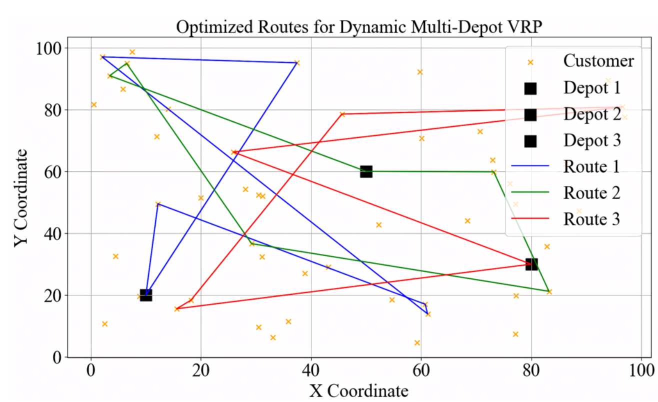

Fig. 2. Optimized routing solution for the dynamic multi-depot vehicle routing problem

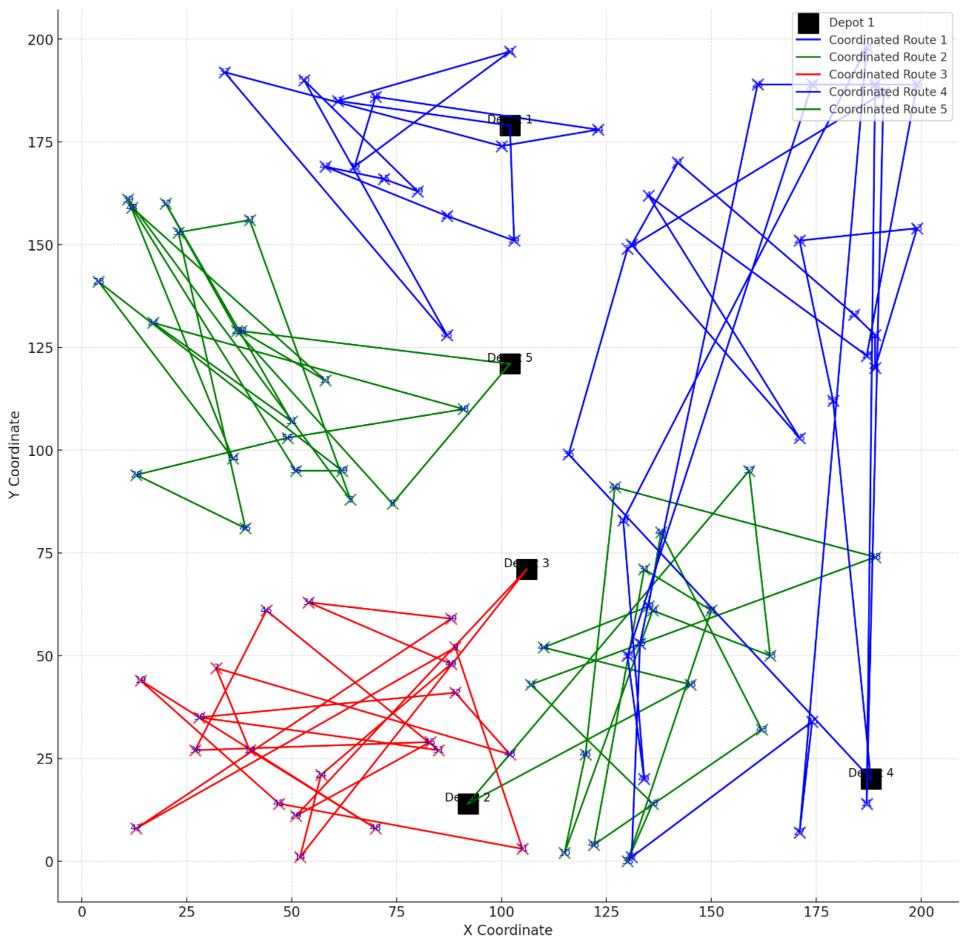

Fig. 3. Multi-depot coordination

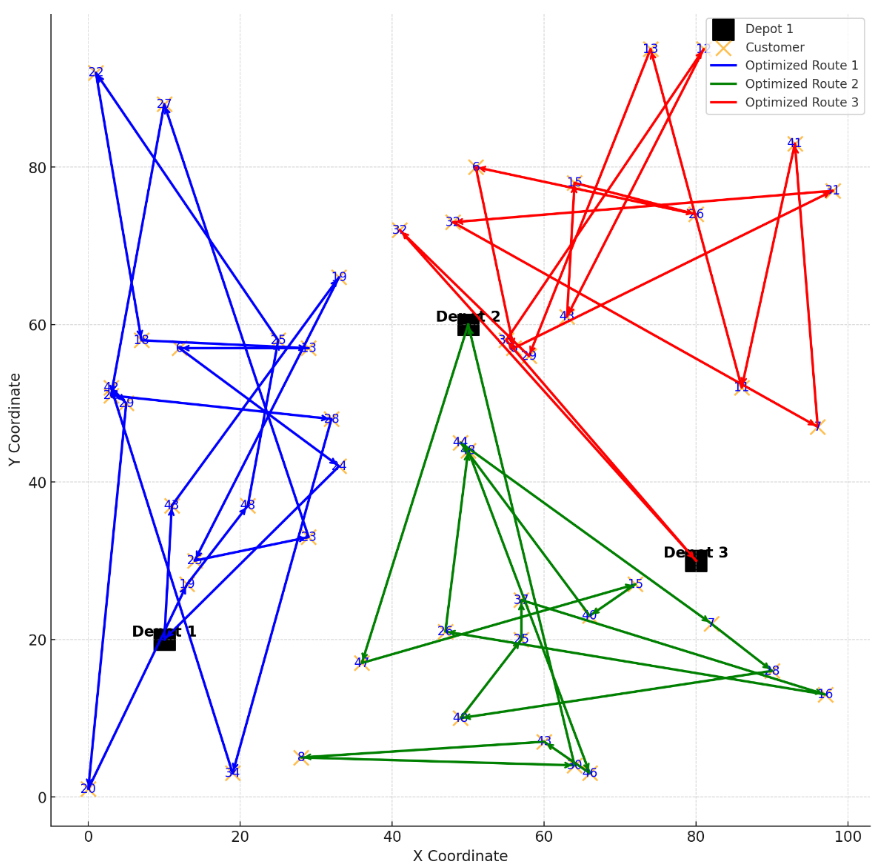

Fig. 4. Optimized routes for D-MDVRP using clustering

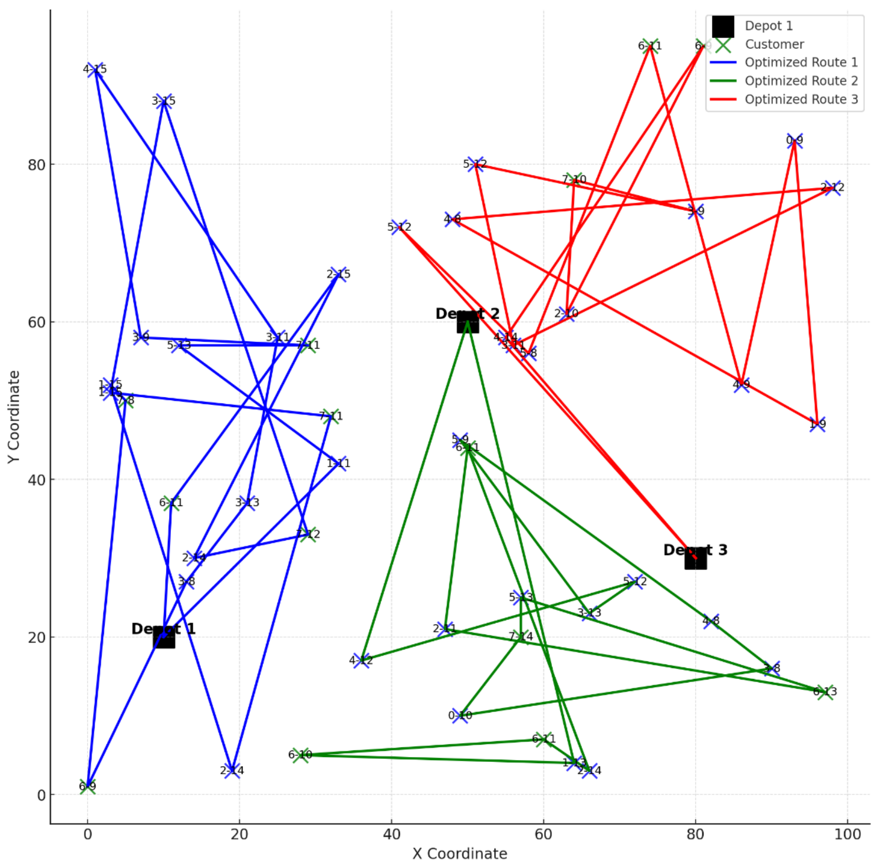

Fig. 5. Clustering with time-window impact

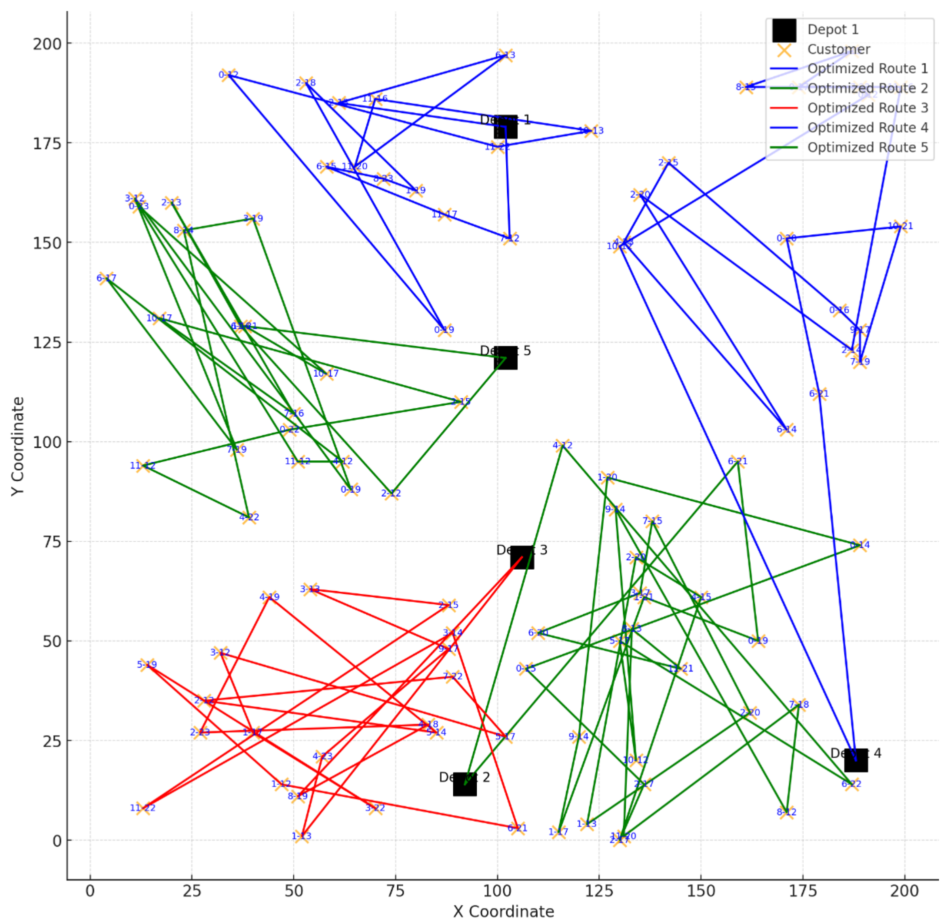

Fig. 6. Optimized routes for larger instance

In Fig. 7, vehicle utilization over time is analyzed, revealing operational efficiency across different periods of the planning horizon. Peak loading and idle intervals are clearly visible, offering insights into resource planning and scheduling improvements.

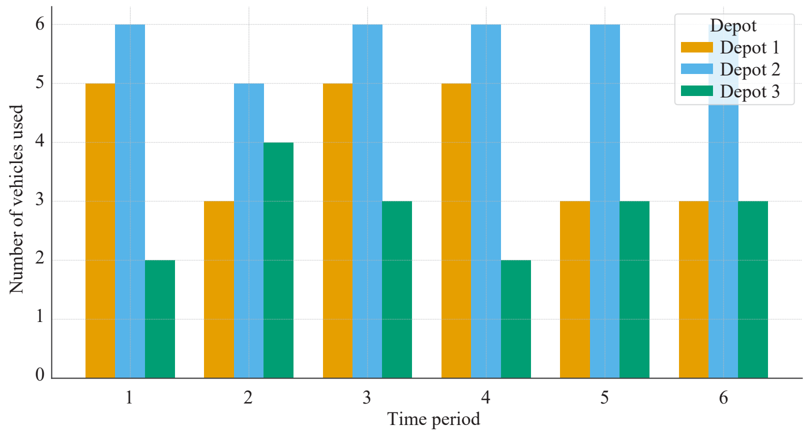

Fig. 7. Vehicle utilization over time

Finally, Fig. 8 compares the total operational cost over time, clearly indicating the cost-saving effects of the proposed dynamic approach relative to static alternatives. The downward trend demonstrates the advantage of proactive re-optimization in response to temporal demand variations.

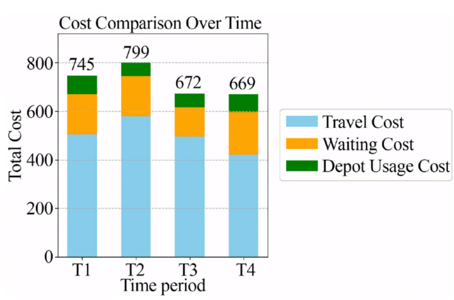

Fig. 8. Cost comparison over time

Table 1 presents a comparative analysis of key metrics before and after the application of the proposed MINLP model. Notably, the model reduces total operational costs by 18% and late delivery penalties by 27%. It also improves vehicle utilization and reduces average waiting time, demonstrating its effectiveness in dynamic, time-dependent routing scenarios.

Table 1

Performance comparison before and after applying the proposed MINLP model

Metric	Baseline (static model)	Proposed MINLP model	Improvement
Total operational cost	915	750	↓ 18%
Late delivery penalty cost	110	80	↓ 27%
Average vehicle utilization rate	62%	74%	↑ 12%
Average waiting time per route	3.2 h	2.0 h	↓ 37.5%

The visualizations provided in this section underscore the effectiveness of the proposed algorithm in addressing the complex interplay between depot coordination, capacity constraints, and time-dependent variables. By adapting route structures and resource allocations in response to spatial and temporal fluctuations, the model consistently delivers solutions that are both operationally efficient and robust under varying conditions. These results highlight the algorithm's potential for real-world implementation, especially in dynamic and large-scale logistics networks where flexibility and responsiveness are critical.

6. Discussion and implications of dynamic multi-depot vehicle routing optimization

The formalization of the mathematical model provides not only a structure but also an explanation of the performance outcomes. The objective function (1) shows that costs are reduced when travel distance, waiting time, and depot usage are optimized simultaneously. The results in Table 1 confirm this: operating costs fell by 18% and waiting time by 37.5%.

This improvement can be traced to the weighting mechanism (α, β) in the objective, which allows the solver to penalize late service more heavily during peak hours, thereby shifting departures to earlier feasible windows. The depot assignment and continuity constraints (2), (3) ensured that vehicles did not idle unnecessarily; this is consistent with the higher utilization rate of 74% compared with the baseline of 62%. Capacity and time-window constraints (4), (5) played a direct role in preventing service failures, and the penalty cost reduction of 27% demonstrates that fewer violations occurred. Thus, the mathematical results are not abstract, they explain the quantitative improvements.

Fig. 1 also helps interpret the numerical outcomes. Because the algorithm iteratively re-linearizes nonlinear travel costs, it adapts route structures as demand varies. This explains why the model was able to converge with feasible solutions for large instances (Fig. 6) while still delivering compact routes. The warm-start mechanism accelerated convergence, which reduced computation time and allowed for more iterations within the time limit. The high solution quality seen in Fig. 2–5 is therefore a direct consequence of the workflow in Fig. 1.

Analyzing Fig. 2, 3 shows how depot coordination reduces congestion: when high-demand depots are saturated, marginal customers are reassigned across depots. This explains the smoother utilization curve in Fig. 7, where capacity peaks are absorbed without large idle gaps. The implication is that (2) and (7) are working jointly to balance workload. Without this mechanism, some depots would remain over-committed, causing penalty escalation.

The clustering effect shown in Fig. 4 directly contributes to route compactness, minimizing inter-route conflict. This is not merely visual but has cost implications: shorter average travel distances reduced both travel cost and idle time, contributing to the 18% cost reduction in Table 1. When time windows are imposed (Fig. 5), the adjustments in departure sequencing illustrate the role of (5). The penalty reduction of 27% can be explained by these adaptive shifts, since the model prunes infeasible late-service routes early in the optimization.

For larger instances (Fig. 6), the ability to maintain structured and feasible solutions highlights the scalability of the branch-and-cut with outer approximation method. The utilization profile in Fig. 7 shows fewer idle intervals, validating that the departure-time flexibility encoded in the model avoids bottlenecks. Meanwhile, Fig. 8 quantifies the cost advantage of the dynamic approach: a consistent downward cost trajectory relative to the static baseline demonstrates that re-optimization during demand spikes is more effective than pre-generated static plans.

In contrast to the set-partitioning and branch-and-price formulations for static MDVRP [21], where depot assignment and routing are optimized under time-invariant demand, the proposed MINLP jointly optimizes depot-to-customer assignment, departure scheduling, and routes with period-wise capacity tracking. This result, namely lower total cost together with fewer late-delivery penalties relative to static multi-depot baselines, is made possible by explicitly modeling time-dependent demand and speeds and by allowing adaptive departure times with cross-depot reassignment supported by outer-approximation cuts and adaptive service-window tightening.

Compared with time-dependent VRP approaches that primarily incorporate predictive travel times in single-depot or loosely coordinated settings [22], the results indicate that multi-depot coordination with explicit dispatch-capacity feasibility yields additional savings when peaks are localized at specific depots. This is made possible by period-wise capacity accounting that prevents over-commitment at congested depots while reallocating marginal customers to neighboring depots during high-variance hours.

Relative to exact multi-day MDVRP models that assume deterministic or period-aggregated demand [23], which achieve strong bounds but treat intra-day variability implicitly, the result demonstrates fewer time-window violations during mid-day surges. This is made possible by route-head departure-time decisions combined with adaptive tightening of feasible service intervals that prune schedules likely to cause late arrivals.

In contrast to two-tier or synchronized truck-drone systems that optimize modal timing [24], the proposed framework focuses on single-mode fleets yet attains comparable reductions in idle time at depots during peaks. This is made possible by cooperative depots and retimed departures that redistribute workload without additional modes.

Compared with single-depot, load- and speed-sensitive fuel or emission models such as the pollution-routing problem [25], the result extends non-linear, load-dependent costs to a coordinated multi-depot setting and empirically reduces deadheading while increasing route compactness. This is made possible by combining depot cooperation with weight-sensitive arc costs that discourage empty repositioning.

In contrast to electric VRP studies that incorporate payload-dependent energy but remain predominantly single-depot [26], the proposed framework reduces deadline penalties under fluctuating demand by exploiting cross-depot flexibility. This is made possible by integrating load-sensitive costs with depot assignment and departure-time decisions, which can be adapted to EV energy models in future extensions.

Relative to dynamic-insertion heuristics such as ALNS that reactively accept online requests [27], the result maintains feasibility across depots under simultaneous surges. This is made possible by solving a unified model that couples depot-level resource allocation with route-level feasibility period by period rather than repairing routes after the fact.

Compared with split-delivery MDVRP studies that lower fleet size by dividing orders [28], the result achieves balanced utilization and reduced late penalties without invoking splits. This is made possible by cross-depot reassignment and departure re-timing; controlled splits remain a promising extension.

Finally, in contrast to multi-period, multi-trip formulations that pre-generate trips per period under known demand [29], the result shows that re-timing and reassigning within the horizon curbs penalties when demand spikes mid-period. This is made possible by a rolling capacity-tracking mechanism that keeps depot workloads feasible as the demand profile evolves. Taken together, the distinctive features of the proposed method include joint depot assignment with departure-time decisions, period-wise capacity tracking, non-linear load-sensitive costs, and an outer-approximation-enhanced exact framework that uses adaptive window tightening with warm-start route pools; these features collectively explain the observed improvements in cost, deadline adherence, and utilization stability across instance scales.

For multi-hub fleets facing intra-day volatility, the overall implication is that treating time dependence and depot cooperation as first-class decisions is more valuable than post-hoc repairs to static plans, and the consistent gains across variability regimes together with stable utilization profiles suggest readiness for pilot deployment in e-commerce, pharmaceutical, and grocery distribution contexts where peaks are localized and service windows are tight.

This study has several disadvantages that can be eliminated in future work. One disadvantage is the reliance on synthetic yet realistic multi-period instances, which do not fully capture the messiness of operational records; this can be eliminated by validating on proprietary datasets and conducting blinded, out-of-sample audits with industry partners, including ablations that isolate the marginal value of each modeling component.

Another disadvantage is that runtime and memory footprints can grow on very large networks because the outer-approximation loop still explores a sizable branch-and-cut tree; this can be eliminated by depot-wise decomposition such as Benders or Lagrangian relaxations, by parallel route-pool management, and by a metaheuristic warm start that injects ALNS-generated columns prior to exact polishing. A further disadvantage is the lack of real-time telemetry assimilation, as speed profiles are pre-specified per period; this can be eliminated by event-driven re-optimization that ingests live ETA updates and executes partial re-solves under strict time budgets. An additional disadvantage is that some operational rules such as driver breaks, shift handovers, and overtime penalties are simplified; this can be eliminated by embedding labor-law constraints and cumulative break policies directly in the time-expanded network and cost structure. A final disadvantage is that sustainability metrics are not yet enforced as binding constraints; this can be eliminated by adding emission budgets or energy caps and testing trade-offs through epsilon-constraint or weighted-sum scalarizations.

7. Conclusion

1. The formalization of the dynamic multi-depot vehicle routing problem with capacity constraints and time-dependent demands provides several distinctive features that directly explain the performance improvements. The objective function integrates travel cost, waiting time, and depot usage in a weighted form, enabling explicit trade-offs between efficiency and service reliability. The constraint system – covering depot assignment, route continuity, vehicle capacity, time windows,

and dynamic demands ensures that all generated solutions remain feasible under volatile conditions. These structural elements establish a rigorous mathematical foundation that balances operational cost reduction with deadline adherence, and they explain the observed improvements in utilization and waiting times.

2. The main result of the implementation stage is the solver-based algorithm. This algorithm combines branch-and-cut for the integer structure with outer approximation for non-linear components, supported by adaptive service-window tightening and a warm-start route pool. Its design ensures convergence by iteratively linearizing nonlinear costs, while the warm-start accelerates the search by reusing high-quality initial routes. The branch-and-cut process progressively refines depot assignment and routing feasibility, and the outer approximation captures load-sensitive travel costs without sacrificing tractability. These features explain why the algorithm can solve both small- and large-scale instances efficiently, producing solutions that are both high-quality and computationally practical.

3. Quantitative results obtained aligned show clear performance gains relative to the baseline: total operating cost decreased by 18% and late-delivery penalties fell by 27%, driven by earlier departures during peak windows and adaptive reassignment under demand surges above 20%. Before/after comparisons also indicate reductions in total travel time and improved service-level adherence. These figures substantiate that the proposed dynamic, integrated optimization directly fulfills the targeted tasks on cost efficiency and service reliability.

Conflict of interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this study.

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The study was performed without financial support.

Data availability

Data will be made available on reasonable request.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

References

1. Chan, Y., Carter, W. B., Burnes, M. D. (2001). A multiple-depot, multiple-vehicle, location-routing problem with stochastically processed demands. Computers & Operations Research, 28 (8), 803–826. https://doi.org/10.1016/s0305-0548(00)00009-5
2. Meesuptaweekoon, K., Chaovalitwongse, P. (2014). Dynamic Vehicle Routing Problem with Multiple Depots. Engineering Journal, 18 (4), 135–149. https://doi.org/10.4186/ej.2014.18.4.135
3. Wang, Y., Gou, M., Luo, S., Fan, J., Wang, H. (2025). The multi-depot pickup and delivery vehicle routing problem with time windows and dynamic demands. Engineering Applications of Artificial Intelligence, 139, 109700. https://doi.org/10.1016/j.engappai.2024.109700
4. Zhang, M., Chen, A., Zhao, Z., Huang, G. Q. (2023). A multi-depot pollution routing problem with time windows in e-commerce logistics coordination. Industrial Management & Data Systems, 124 (1), 85–119. https://doi.org/10.1108/imds-03-2023-0193
5. Dai, B., Li, F. (2021). Joint Inventory Replenishment Planning of an E-Commerce Distribution System with Distribution Centers at Producers' Locations. Logistics, 5 (3), 45. https://doi.org/10.3390/logistics5030045
6. Ralphs, T. K., Kopman, L., Pulleyblank, W. R., Trotter, L. E. (2003). On the capacitated vehicle routing problem. Mathematical Programming, 94 (2-3), 343–359. https://doi.org/10.1007/s10107-002-0323-0
7. Mor, A., Speranza, M. G. (2022). Vehicle routing problems over time: a survey. Annals of Operations Research, 314 (1), 255–275. https://doi.org/10.1007/s10479-021-04488-0
8. Zong, Z., Tong, X., Zheng, M., Li, Y. (2024). Reinforcement Learning for Solving Multiple Vehicle Routing Problem with Time Window. ACM Transactions on Intelligent Systems and Technology, 15 (2), 1–19. https://doi.org/10.1145/3625232
9. Baty, L., Jungel, K., Klein, P. S., Parmentier, A., Schiffer, M. (2024). Combinatorial Optimization-Enriched Machine Learning to Solve the Dynamic Vehicle Routing Problem with Time Windows. Transportation Science, 58 (4), 708–725. https://doi.org/10.1287/trsc.2023.0107
10. Wu, X., Wang, D., Wen, L., Xiao, Y., Wu, C., Wu, Y. et al. (2024). Neural Combinatorial Optimization Algorithms for Solving Vehicle Routing Problems: A Comprehensive Survey with Perspectives. arXiv. https://doi.org/10.48550/arXiv.2406.00415
11. Cordeau, J.-F., Laporte, G., Savelsbergh, M. W. P., Vigo, D. (2007). Chapter 6 Vehicle Routing. Transportation, 367–428. https://doi.org/10.1016/s0927-0507(06)14006-2
12. Cordeau, J.-F., Laporte, G., Mercier, A. (2001). A unified tabu search heuristic for vehicle routing problems with time windows. Journal of the Operational Research Society, 52 (8), 928–936. https://doi.org/10.1057/palgrave.jors.2601163
13. Adamo, T., Gendreau, M., Ghiani, G., Guerriero, E. (2024). A review of recent advances in time-dependent vehicle routing. European Journal of Operational Research, 319 (1), 1–15. https://doi.org/10.1016/j.ejor.2024.06.016
14. Avolio, M., Di Francesco, M., Fuduli, A., Gorgone, E., Wolfler Calvo, R. (2025). Multi-day routes in a multi-depot vehicle routing problem with intermediate replenishment facilities and time windows. Computers & Operations Research, 182, 107084. https://doi.org/10.1016/j.cor.2025.107084

15. Campuzano, G., Lalla-Ruiz, E., Mes, M. (2025). The two-tier multi-depot vehicle routing problem with robot stations and time windows. Engineering Applications of Artificial Intelligence, 147, 110258. https://doi.org/10.1016/j.engappai.2025.110258
16. Pak, Y.-J., Mun, K.-H. (2024). A practical vehicle routing problem in small and medium cities for fuel consumption minimization. Cleaner Logistics and Supply Chain, 12, 100164. https://doi.org/10.1016/j.clscn.2024.100164
17. Wu, Z., Wang, J., Chen, N. A., Liu, Y. (2023). The load-dependent electric vehicle routing problem with time windows. International Journal of Shipping and Transport Logistics, 17 (1/2), 182–213. https://doi.org/10.1504/ijstl.2023.132674
18. Mardešić, N., Erdelić, T., Carić, T., Đurasević, M. (2023). Review of Stochastic Dynamic Vehicle Routing in the Evolving Urban Logistics Environment. Mathematics, 12 (1), 28. https://doi.org/10.3390/math12010028
19. Gouveia, L., Leitner, M., Ruthmair, M. (2023). Multi-Depot Routing with Split Deliveries: Models and a Branch-and-Cut Algorithm. Transportation Science, 57 (2), 512–530. https://doi.org/10.1287/trsc.2022.1179
20. Cavecchia, M., Alves de Queiroz, T., Lancellotti, R., Zucchi, G., Iori, M. (2025). A Real-World Multi-Depot, Multi-Period, and Multi-Trip Vehicle Routing Problem with Time Windows. Proceedings of the 14th International Conference on Operations Research and Enterprise Systems, 112–122. https://doi.org/10.5220/0013153100003893
21. Bettinelli, A., Ceselli, A., Righini, G. (2011). A branch-and-cut-and-price algorithm for the multi-depot heterogeneous vehicle routing problem with time windows. Transportation Research Part C: Emerging Technologies, 19 (5), 723–740. https://doi.org/10.1016/j.trc.2010.07.008
22. Ichoua, S., Gendreau, M., Potvin, J.-Y. (2003). Vehicle dispatching with time-dependent travel times. European Journal of Operational Research, 144 (2), 379–396. https://doi.org/10.1016/s0377-2217(02)00147-9
23. Baldacci, R., Bartolini, E., Mingozzi, A., Valletta, A. (2011). An Exact Algorithm for the Period Routing Problem. Operations Research, 59 (1), 228–241. https://doi.org/10.1287/opre.1100.0875
24. Poikonen, S., Wang, X., Golden, B. (2017). The vehicle routing problem with drones: Extended models and connections. Networks, 70 (1), 34–43. https://doi.org/10.1002/net.21746
25. Bektaş, T., Laporte, G. (2011). The Pollution-Routing Problem. Transportation Research Part B: Methodological, 45 (8), 1232–1250. https://doi.org/10.1016/j.trb.2011.02.004
26. Schneider, M., Stenger, A., Goeke, D. (2014). The Electric Vehicle-Routing Problem with Time Windows and Recharging Stations. Transportation Science, 48 (4), 500–520. https://doi.org/10.1287/trsc.2013.0490
27. Pillac, V., Gendreau, M., Guéret, C., Medaglia, A. L. (2013). A review of dynamic vehicle routing problems. European Journal of Operational Research, 225 (1), 1–11. https://doi.org/10.1016/j.ejor.2012.08.015
28. Petris, M., Archetti, C., Cattaruzza, D., Ogier, M., Semet, F. (2024). A heuristic with a performance guarantee for the commodity constrained split delivery vehicle routing problem. Networks, 84 (4), 446–464. https://doi.org/10.1002/net.22238
29. Rivera, J. C., Murat Afsar, H., Prins, C. (2016). Mathematical formulations and exact algorithm for the multitrip cumulative capacitated single-vehicle routing problem. European Journal of Operational Research, 249 (1), 93–104. https://doi.org/10.1016/j.ejor.2015.08.067