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EVALUATION OF THE EFFICIENCY OF ENERGY CHARACTERISTICS OF AN ASYNCHRONOUS MOTOR USING FREQUENCY CONVERSION WITH PULSE-WIDTH MODULATION

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The object of the study is a three-phase frequency converter (FC) with an uncontrolled rectifier and pulse-width modulation (PWM) of the output voltage, used for frequency regulation of asynchronous motors (AM). The problem to be solved is the improve the energy efficiency of the electric drive and the accuracy of speed control while reducing harmonic distortions of voltage and current, which is especially important for industrial installations operating in continuous or alternating mode. In the work in the Mathcad environment, a mathematical model of the FC was developed, including a power circuit, an algorithm for generating PWM and dependencies for calculating the power factor, efficiency and total harmonic distortion (THD). The model allows performing parametric studies of the effect of frequency and amplitude of the output voltage on the energy indicators of the drive with a high degree of detail. The simulation results showed that the optimized sinusoidal PWM algorithm with ordered switching distribution reduces THD by 18–22% and increases the power factor to 0.94–0.96 due to a more uniform harmonic spectrum distribution and reduced reactive losses. The advantage of the proposed model is the ability to adapt to various AM operating modes and load conditions without changing the structure of the power section, which expands its practical application. The obtained results can be used in the design and modernization of industrial electric drive systems, pump and fan units, as well as in automated complexes where economical, reliable and high-precision speed control is required. It further enables sensitivity checks and controller tuning to evaluate PWM strategies under realistic limit

Keywords: frequency converter, pulse-width modulation, asynchronous motor, power factor, harmonic distortion

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1. Introduction

Asynchronous electric motors (AM) occupy a leading position in the global energy and industry: according to the International Energy Agency, more than 40% of the generated electricity is consumed by electric drives, a significant share of which falls on AM. Their demand is explained by high reliability, simple design and low cost, but their efficiency directly depends on the control methods used.

Frequency regulation based on frequency converters (FC) with pulse-width modulation (PWM) has become the basic standard for modern electric drives. This technology allows changing the frequency and amplitude o the supply voltage in a wide range, providing precise speed control, smooth start and reduction of starting currents. This is especially important for pumping, compressor and ventilation units, where energy consumption can be reduced by 20-50% due to optimal speed control.

Despite the development of power electronics and control systems, the problem of increasing the energy efficiency of PWM FC remains extremely relevant. In practice, the operation of such devices is accompanied by harmonic distortions of the output voltage, increased thermal loads on the motor and a decrease in its service life. These effects cause additional energy losses and require the implementation of methods for optimizing modulation algorithms. In addition, modern trends, such as the digitalization of industry and the concept of "smart" electric drives, suggest the use of mathematical models to accurately predict the characteristics of the frequency converter at the design stage. This allows to reduce development costs, reduce implementation time and increase the reliability of the equipment. In this regard, universal mathematical models that can quickly assess the energy indicators of the system in various operating modes are of particular value.

Thus, the research aimed at developing and improving the models of frequency converters with PWM for controlling asynchronous motors have high scientific and practical significance. The results of such research are in demand in industry, energy, transport systems and automated complexes, where it is necessary to simultaneously increase the energy efficiency and reliability of electric drives.

2. Literature review and problem statement

In the work [1], a mathematical model of a variable-frequency asynchronous electric drive is presented, implemented in a simulation environment. It is shown that optimization of frequency converter control algorithms allows to increase energy efficiency and improve dynamic characteristics of the drive. However, the study did not conduct a comprehensive assessment of the relationship between the power factor, efficiency and total harmonic distortion (THD) depending on the modulation parameters, which limits the possibility of comprehensive optimization of operating modes. The reason for this is the objective difficulties associated with the need to conduct large-scale experiments and the high computational complexity of the models. A possible way to overcome these difficulties is to use universal mathematical packages with support for analytical and numerical methods. The paper [2] discusses the techniques of using Mathcad in the educational process and engineering calculations in the electrical engineering field. It is shown that Mathcad provides flexible work with analytical expressions, vector calculations and graphical visualization of data. However, the manual does not include a methodology for complex modeling of frequency converters with an assessment of energy indicators, which makes it impossible to directly use the approach for analyzing PWM frequency converters.

The paper [3] are devoted to the design of electric machines, including the calculation of magnetic and thermal characteristics. It is shown that the design parameters of the machine have a significant impact on its energy indicators. However, the paper [3] does not consider the influence of pulse width modulation algorithms on the spectral composition of the output voltage of the frequency converter and subsequent electromagnetic processes in the motor.

The paper [4] studies an AM with SVPWM control, where, based on harmonic/numerical analysis, engineering approximations are proposed for the THD of phase currents and torque ripple as functions of the switching frequency and dead-time of transistors. The authors show that increasing the switching frequency reduces the contribution of carrier harmonics to THD and decreases torque ripple, but such a gain is achieved at the cost of increasing switching losses in the power section; in addition, uncompensated dead-time introduces noticeable low-frequency distortions, which worsens the energy performance of the drive. A significant practical contribution of the paper is the derivation of closed approximations that allow one to quickly select the minimum permissible switching frequency that ensures the correct operating mode without detailed modeling of the complete electromechanical system. A limitation of the study is the lack of a direct balance of losses in the machine (iron/copper) and, accordingly, the link "quality of the current shape ? quantitative energy losses".

The paper [5] consider an inverter-fed AM and develop a methodology for estimating core losses as the sum of contributions from individual harmonics of the machine magnetic field (including slot/relaxation harmonics) and higher harmonics of the inverter voltage. A combination of analytics and field-circuit modeling (FEM/field-circuit) with experimental validation at frequencies of 10, 20, and 350 Hz is used. It is shown that at low supply frequencies, the role of harmonics generated by the inverter increases significantly, while at high frequencies, the machine's own harmonics dominate; without taking into account each node (yoke/teeth/stator-rotor), the efficiency can be overestimated. The requirement for harmonic decomposition and separate accounting of core areas is methodologically important. Limitation: PWM strategies (SPWM/SVPWM/DPWM) are not compared, the influence of dead-time and torque pulsations on losses is not analyzed.

The papers [6, 7] analyze design solutions for increasing the energy efficiency of asynchronous motors of pumping units. It is shown that optimization of the stator geometry allows reducing losses in steel and copper. However, these works do not take into account the spectral composition of the supply voltage generated by the frequency converter, which in real operating conditions can significantly affect the efficiency of the drive.

The paper [8] investigated the power balance and power factors in distorted electrical systems with variable-speed asynchronous electric drives. It is shown that harmonic distortions have a significant impact on the operation of the equipment, including thermal conditions and efficiency. However, the authors do not consider the algorithmic optimization of the modulation parameters aimed at simultaneously reducing THD and increasing energy indicators.

The paper [9] consider the increase in harmonic losses in AM when powered by a PWM inverter and compare two lines of assessment: numerical calculation based on the time-stepping finite element method (time-stepping FEM) and procedures following from IEC/TS 60034 (methods for determining losses and efficiency of machines with converter power supply). The work emphasizes the importance of taking into account the switching frequency in the energy assessment of IM and justifies the joint use of FEM and IEC approaches in validation. Limitations of the study are a narrow parametric coverage (without a systematic comparison of PWM strategies and the dead-time effect), which leaves room for the integration of spectral quality metrics (THD, torque pulsation) with the quantitative balance of losses in steel/copper in applied methods.

The paper [10], a modification of the discounted cost range criterion for controlled asynchronous electric drives is proposed. It is shown that this approach allows taking into account the economic efficiency of systems when selecting control algorithms. Nevertheless, the technical aspects of spectral analysis and the influence of PWM on the energy performance of the drive remain outside the scope of the study.

Thus, the conducted analysis of the literature shows that, despite a significant number of studies, the complex problem, including simultaneous modeling of the spectral composition of the output voltage, assessment of energy performance (efficiency, cos φ, THD) and optimization of modulation parameters, remains insufficiently developed. It seems advisable to develop an adaptive mathematical model of a frequency converter with PWM, implemented in the Mathcad environment, which will provide accurate engineering calculations with minimal computational costs, will retain the versatility of application and will allow the adaptation of algorithms to various operating modes and load conditions.

3. The aim and objectives of the study

The aim of this study is to evaluate the efficiency of the energy characteristics of an asynchronous motor when powered by a frequency converter with pulse-width modulation (PWM) in various operating modes based on mathematical modeling in the Mathcad environment. The model should provide accurate analysis of power factor, efficiency and harmonic distortion, as well as optimize control parameters with minimal computational costs, while maintaining engineering practicality and versatility of application. This will improve the energy efficiency of electric drives, improve the quality of speed control and reduce operating costs when designing and upgrading industrial and automated systems.

To achieve the set aim, the following objectives are solved:

– to perform a mathematical description of the power circuit of a frequency converter with an uncontrolled rectifier and an inverter based on PWM;

– to develop a model in Mathcad with the ability to parametrically change the output parameters and calculate the efficiency, power factor and THD for various operating modes;

– to conduct a comparative analysis of the basic and optimized sinusoidal PWM with the definition of modes that ensure maximum energy efficiency at an acceptable level of harmonic distortion.

4. Materials and methods

The object of the study is a three-phase frequency converter (FC) with an unregulated rectifier and pulse width modulation (PWM) of the output voltage, used to regulate the frequency of asynchronous motors (AM).

The main hypothesis of the study is the use of frequency conversion with pulse width modulation (PWM) allows improving the energy performance of an asynchronous motor (efficiency, power factor) and reducing the level of harmonic distortion when the modulation parameters (frequency, modulation coefficient, pulse shape) are selected correctly.

Research assumptions:

1. The converter's power supply network is an ideal voltage source without internal distortions.

2. The load torque on the motor shaft is considered to be alternating current in the operating modes under study.

Simplifications of the study:

1. Neglect of switching delays of power keys and the dynamics of the switching process.

2. The model of an asynchronous motor is based on an equivalent replacement circuit, without taking into account parasitic parameters (e.g., leaks, magnetic circuit saturation).

3. Harmonic analysis is limited to a finite number of harmonics, since high-frequency components above a certain threshold do not have a significant effect on energy performance.

Modeling toolkit and data sources:

1. Software: Mathcad 15/Prime, which provides the ability to combine analytical formulas and numerical calculations with graph visualization.

2. Initial data for modeling: IM nominal parameters (voltage, frequency, power, winding resistance), IF characteristics (input voltage, modulation frequency, PWM parameters).

3. Literature sources: modern works on modeling IF with PWM, including power factor optimization, harmonic reduction and efficiency improvement.

Currently, the most common type of frequency converter (FC) for the implementation of frequency control of AC motors is a converter with an uncontrolled voltage rectifier and pulse width modulation (PWM) of the output voltage. An important advantage of such FC is the minimum number of parasitic harmonics generated in the electric motor power circuit and in the supply network. This improves the energy and operational characteristics of the electric drive, and also allows to get rid of filters and other protective equipment. Modeling of modern electric drive systems and their elements is an integral part of both the creation of new electric drives and the modernization of existing technological units and systems.

To compile a conceptual model of an FC with PWM, let's turn to the most common scheme shown in Fig. 1. Here *V*1 ... *V*6 – gate switches (usually performed on bipolar transistors with an isolated IGBT gate), $U_П$ – a constant voltage from an uncontrolled rectifier, *a*, *b*, *c* – the phases of the stator of an alternating current motor connected to a star.

As an example, let's consider a PWM in which a sawtooth reference voltage U_H with a high frequency ω_H is used as a reference voltage, with which the valves of the power groups are switched (Fig. 2).

The duration of the carrier frequency pulses varies with each period in accordance with the shape of the modulating voltage U_M. This is achieved by comparing the reference voltage with the modulating one, for which the null organ (NO) and shapers (S) are included in the FC control system, and the valves are switched at the moment of their equality (Fig. 3).

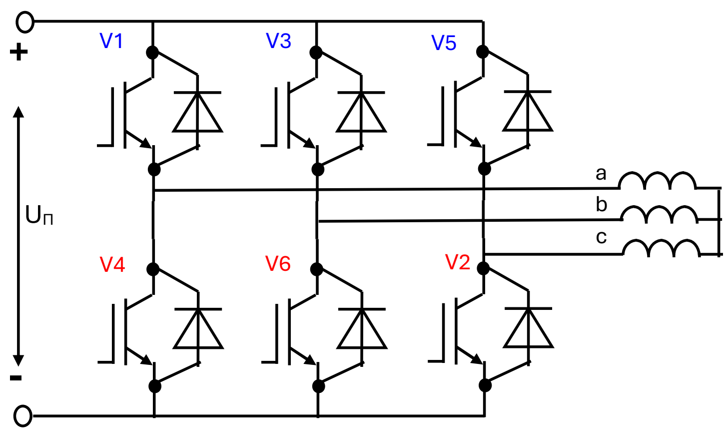

Fig. 1. Power circuit of a three-phase voltage inverter

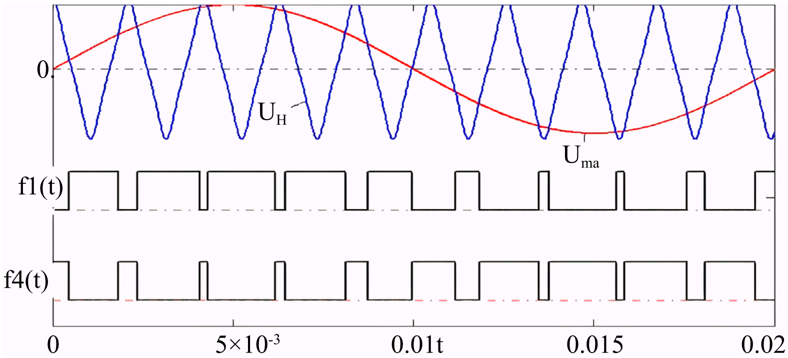

Fig. 2. Generation of control pulses with Formation of control pulses with pulse-width modulation

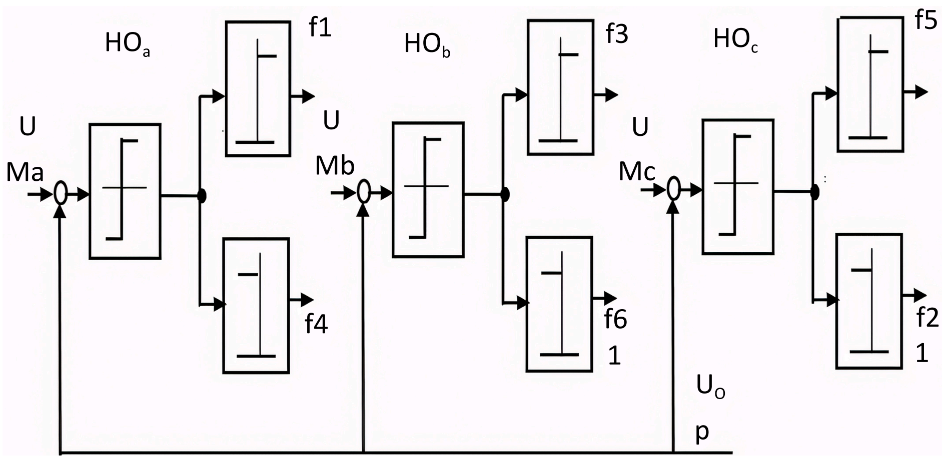

Fig. 3. Pulse width modulation system in three-phase frequency converter

If $U_M > U_H$ the signal at the output of the null organ is positive and there is a positive signal f_n at the output of the Φ_n shaper that unlocks the corresponding transistor.

The three-phase FC used in the electric drive has its own characteristics. In its control algorithm, it is necessary to take into account the switching circuits of the stator windings at different states of the inverter keys and the values of phase voltages. This is necessary to form a system of three-phase voltages shifted relative to each other by 120 electrical degrees.

Using numerical modeling in Mathcad 15, it is possible to obtain visualization of the results in the form of graphs of the dependence of the engine parameters on the frequency and amplitude of the output voltage of the frequency converter.

Numerical calculation methods. It is necessary to represent the function $U_a(t)$ with a period T – the output voltage of the phase of the frequency converter obtained in the mathematical model of the frequency converter – in the form of successive readings (vector) Y_i with a given number of elements. In the example, 28 = 256 elements are accepted, which is sufficient for high convergence of $U_a(t)$ and Y_i (Fig. 4).

The sequence of calculating the Fourier coefficients is shown in Fig. 4, 5.

Fig. 4 shows the results of discretization and vector representation of the phase voltage $U_a(t)$, generated by the frequency converter using the pulse-width modulation (PWM) algorithm. The original signal $U_a(t)$ has the form of a high-frequency pulse sequence, the amplitude of which changes in accordance with the modulating sinusoidal law.

The first graph shows the original time distribution of the $U_a(t)$ values in continuous form, and the second – the result of its transformation into a discrete vector Y_i using the Mathcad 15 (USA) function, which implements signal sampling with a given discretization step. This approach ensures the formation of a data array suitable for further numerical analysis, including spectral decomposition using the fast Fourier transform (FFT), calculation of harmonic components, total harmonic distortion (THD) and assessment of the energy contribution of individual frequency components.

Vector representation of the output voltage is a key stage of digital data processing, allowing to move from the time domain to the frequency domain while preserving full information about the time structure and phase relationships. The use of discrete vectors in the analysis of FC signals facilitates the automation of calculations, ensures compatibility with mathematical packages for modeling electric drives (MATLAB/Simulink, PSCAD, ANSYS Simplorer, etc.) and increases the accuracy of the analysis of spectral characteristics. Fig. 5 shows the results of the spectral (harmonic) analysis of the output voltage of a frequency converter (FC) operating in pulse-width modulation (PWM) mode. The analysis was performed in the Mathcad 15 environment based on the previously obtained discrete vector of the output voltage.

For spectral analysis of the FC voltage, it is possible to use the classical representation of the periodic non-sinusoidal function in the form of a trigonometric Fourier series

$$Ua(t)=\sum_{k=0}^{N}\left(A_k\cos\left(\frac{2k\pi t}{T}\right)+B_k\sin\left(\frac{2k\pi t}{T}\right)\right), \qquad (1)$$

where k – ordinal number of harmonics;

T – the oscillation period of the first harmonic;

A_k, B_k – amplitudes of sine and cosine components.

In addition to the form (1), it is more convenient to represent the Fourier series in the form

$$Ua(t)=\sum_{n=1}^{N}M_n\cos\left(2\pi nft+\psi_n\right), \qquad (2)$$

where is the amplitude of the n-th harmonic

$$M_n=\sqrt{A_n^2+B_n^2}, \qquad (3)$$

and the phase of the n-th harmonic

$$\psi_n=-\mathrm{arctg}\left(\frac{B_n}{A_n}\right). \qquad (4)$$

The block numerically calculates the amplitudes of the sine and cosine components of the harmonics A_k and B_k of the FC output voltage, represented as (1), the amplitude M_k and the phase ψ_k of the harmonics in (2).

$i := 0..255$ element number in the created vector

$t := 0, 0.0001..0.02$ range and step of change of the ti me variable

$Y_i := Ua1\left(\frac{i \cdot 0.02}{255}\right)$

resulting vector (transposed):

$Y^T =$

	0	1	2	3	4	5	6	7
0	0	-300	-300	300	300	0	300	...

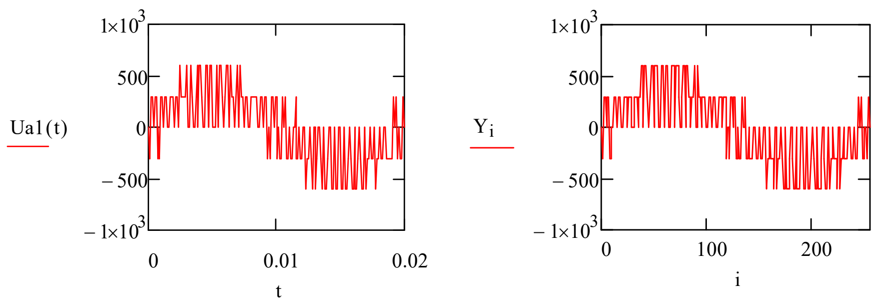

Fig. 4. Converting $U_a(t)$ to Y_i vector ("Mathcad 15")

$N := \text{length}(Y)$ $N = 256$ the number of elements in the vector Y

$f1 := 50$ first harmonic frequency

$M := 128$ number of harmonics (no more than N /2)

$dt := \frac{1}{f1 \cdot N}$ $p := 2 \cdot \pi \cdot f1 \cdot dt$

Calculation of Fourier coefficients (numerical integration):

$k := 0..M$ $i := 0..N-1$ $j := \sqrt{-1}$

$A_k := \sum_i \left(Y_i \cdot \cos(p \cdot k \cdot i)\right)$ $B_k := \sum_i \left(Y_i \cdot \sin(p \cdot k \cdot i)\right)$

$M_k := \left|A_k + j \cdot B_k\right| \cdot \frac{2}{N}$ amplitude of k-th harmonic

$\psi_k := -\arg\left(A_k + j \cdot B_k\right)$ phase of the k-th harmonic

tabular calculation results

$M^T =$

	0	1	2	3	4	5	6
0	0.07	6.305	0.148	0.02	0.095	0.019	...

$\psi^T =$

	0	1	2	3
0	0	-1.048	-1.401	...

graphical calculation results:

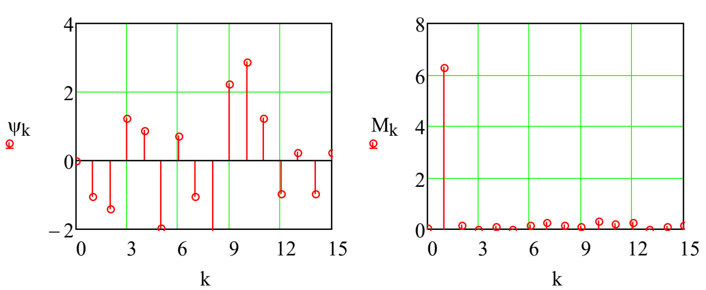

Fig. 5. Harmonic analysis of the output voltage of a PWM frequency converter (Mathcad 15)

The first graph illustrates the distribution of the amplitudes of the harmonic components in relative units in the vicinity of the fundamental frequency. The fundamental harmonic ($h = 1$) corresponding to the frequency of the IF output voltage is clearly visible, as well as the side harmonics caused by the PWM modulation process and located around the carrier frequency and its multiples. The characteristic alternation of positive and negative amplitudes in the diagram is associated with phase shifts of the harmonic components relative to the fundamental sinusoid.

The second graph shows the total spectrum of the signal in a wider frequency range, allowing to visually assess the energy distribution and the contribution of individual harmonics to the overall shape of the output voltage. It is evident that the main energy is concentrated in the fundamental harmonic, while the amplitudes of higher harmonics decrease with increasing frequency, which corresponds to the theoretical PWM model at a high switching frequency.

The analysis performed allows to quantify the total harmonic distortion (THD) and identify the dominant frequency components that affect the quality of the output voltage and the level of electromagnetic interference. The results confirm that the use of a high carrier frequency ensures effective spectral separation of the main harmonic and switching components, which facilitates filtering and improves the quality of power supply to loads in electric drive systems.

For an in-depth interpretation of the results, a comparative analysis of the spectral characteristics of the output voltage was performed. This approach made it possible to compare the obtained indicators with existing control methods and determine the optimal PWM parameters that minimize energy losses and harmonic distortions.

5. Results of the study of the mathematical model of a three-phase converter with an uncontrolled rectifier

5. 1. Mathematical description of the frequency converter power circuit

To create a model, it is possible to use the possibilities of implementing numerical calculation methods and mathematical modeling of the "Mathcad" mathematical application. As initial data, the frequency of the carrier voltage ω_H, the frequency of the modulating voltage $\omega_H = \omega_{out}$ and the modulation depth (relative amplitude) of the output voltage are entered.

In accordance with the operation principle of the FC with PWM, the sawtooth reference voltage is set as

$$U_H(t)=k\cdot\text{tg}\left(\cos\frac{\omega_H t}{2}\right), \tag{5}$$

and the modulating curve of phase "a" as

$$U_{Ma}(t)=\mu\sin\omega_{out}t. \tag{6}$$

Accordingly, for phases "b" and "c", the curves of the modulating voltage are shifted by 120 degrees:

$$U_{Mb}(t)=\mu\sin\left(\omega_{out}t-\frac{2\pi}{3}\right), \tag{7}$$

$$U_{Mc}(t)=\mu\sin\left(\omega_{out}t-\frac{4\pi}{3}\right). \tag{8}$$

The condition for switching on the valves $V1...V6$ with pulse width modulation is recorded in "Mathcad" using the logical function if:

$$f1(t):=if\left(U_H(t)<U_{Ma}(t),1,0\right);$$

$$f3(t):=if\left(U_H(t)<U_{Mb}(t),1,0\right);$$

$$f5(t):=if\left(U_H(t)<U_{Mc}(t),1,0\right);$$

$$f4(t):=if\left(U_H(t)>U_{Ma}(t),1,0\right);$$

$$f2(t):=if\left(U_H(t)>U_{Mc}(t),1,0\right);$$

$$f6(t):=if\left(U_H(t)>U_{Mb}(t),1,0\right). \tag{9}$$

The phase voltage is generated in accordance with the logic of switching valves in a three-phase system (Fig. 1). The logical "AND" of the "Boolean" group of the "Mathcad" toolbar is used for implementation. For phase "a":

$$Ua1(t):=if\left(f1(t)\cdot f5(t)\cdot f6(t)=1,E,0\right);$$

$$Ua5(t):=if\left(f3(t)\cdot f4(t)\cdot f5(t)=1,-2E,0\right);$$

$$Ua2(t):=if\left(f1(t)\cdot f2(t)\cdot f6(t)=1,2E,0\right);$$

$$Ua6(t):=if\left(f4(t)\cdot f5(t)\cdot f6(t)=1,-E,0\right);$$

$$Ua3(t):=if\left(f1(t)\cdot f2(t)\cdot f3(t)=1,E,0\right);$$

$$Ua7(t):=if\left(f1(t)\cdot f3(t)\cdot f5(t)=1,0,0\right);$$

$$Ua4(t):=if\left(f2(t)\cdot f3(t)\cdot f4(t)=1,-E,0\right);$$

$$Ua8(t)=if\left(f2(t)\cdot f4(t)\cdot f6(t)=1,0,0\right). \tag{10}$$

As a result, the output voltage of phase "a"

$$Ua(t)\cdot\sum_{i=1}^{8}U_{ai}(t). \tag{11}$$

For phases "b" and "c", similar (9) and (10) logical expressions are composed. The line voltage is defined as $Uab(t)=Ua(t)-Ub(t)$.

5. 2. Implementation of the model in the Mathcad environment

Implementation of the model in the Mathcad environment according to the developed mathematical model, the following tasks must be completed:

– plotting graphs using Mathcad;

– obtaining the spectral composition of the output voltage of the frequency converter with PWM in the form of tables;

– plotting a time diagram of the formation of output phase and line voltages of the frequency converter (FC);

– obtaining the spectral composition of the output voltage of the frequency converter with PWM at f_{OUT}=75 Hz, f_H=20000 Hz, μ=1 in the form of tables.

Identification of dependencies between the operating modes of the IM and changes in its energy characteristics. The developed approach allows to evaluate the efficiency of the FC with PWM in various operating conditions, optimize control algorithms, and formulate recommendations for practical application in industry and energy.

Fig. 6 shows the timing diagram of the output voltage generation of the frequency converter (FC) implementing the pulse-width modulation (PWM) algorithm.

The upper part of the graph illustrates the three-phase modulating sinusoidal signals $U_{ma}(t)$, $U_{mb}(t)$ and $U_{mc}(t)$, as well as the high-frequency reference signal of the carrier frequency f_H = 6000 Hz. The output voltage frequency is f_{OUT} = 50 Hz, the amplitude modulation coefficient is taken to be equal to μ = 1, which corresponds to the maximum voltage utilization mode. The subsequent timing diagrams show the pulse signals of the control functions $f_1(t)$–$f_6(t)$, determining the switching of the inverter power keys in accordance with the algorithm for comparing the instantaneous values of the modulating sinusoids with a sawtooth (triangular) carrier. It is clearly seen that the switching moments of the keys correspond to the intersection points of the signals $U_m(t)$ and $U_H(t)$. The middle part of the figure shows the obtained phase voltages $U_a(t)$ and $U_b(t)$, which have the nature of multi-level pulse sequences, the amplitude of which is determined by the DC link voltage. The presence of a multi-level pulse structure indicates that the circuit operates in PWM mode with a high carrier frequency, which ensures effective suppression of low-frequency harmonics and improved output voltage quality after filtering.

The lower part of the graph illustrates the line voltage $U_{ab}(t)$, formed as the difference between the phase voltages $U_a(t)$ and $U_b(t)$. It is evident that the voltage shape is an alternating pulse sequence with a constant amplitude level determined by the modulation parameters.

This method of generating output voltage based on PWM ensures high accuracy of reproduction of the required sinusoidal shape with minimal distortion, and also allows to adjust the frequency and amplitude of the output signals in a wide range. This method is used in modern frequency converters for electric drives that require smooth speed control and high-quality power supply for asynchronous and synchronous motors.

2. Obtaining the spectral composition of the output voltage of a frequency converter with PWM in the form of tables, which allows for a quantitative analysis of the distribution of harmonics, assessing the contribution of individual components to the total level of distortion and using the results when calculating the power factor, efficiency and THD.

Table 1 shows the spectral composition of the primary frequency output voltage with pulse width modulation.

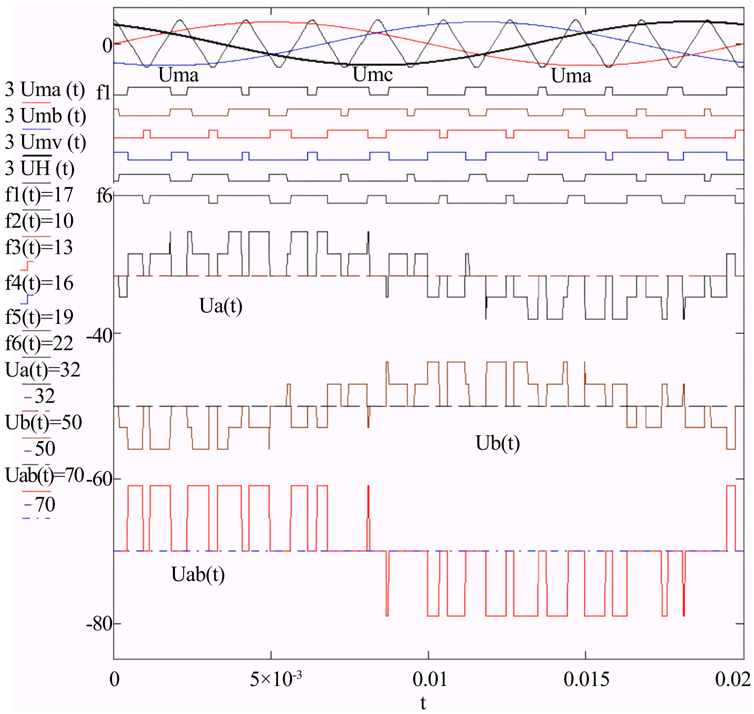

Fig. 6. Formation of output voltage of frequency converter with pulse-width modulation $fout$ = 50 Hz, f_H = 6000 Hz, μ = 1

Table 1

Spectral composition of the output voltage of a frequency converter with pulse width modulation

No. Harmonic *h*	Frequency *fh*, Hz	Amplitude, V	Relative contribution, %	Note
1	50	220	100.0	Fundamental harmonic
119	5950	15.8	7.18	Carrier harmonic
121	6050	15.8	7.18	Carrier harmonic
239	11950	9.2	4.18	2nd sideband from carrier
241	12050	9.2	4.18	2nd sideband from carrier
359	17950	6.4	2.91	3rd sideband from carrier
361	18050	6.4	2.91	3rd sideband from carrier
...	...	...	...	...
THD	–	–	≈ 10.2	Harmonic distortion coefficient

The Table 1 reflects the results of the spectral analysis of the output voltage of the frequency converter with PWM at f_{OUT} = 50 Hz, f_H = 6000 Hz and the amplitude modulation factor μ = 1. The fundamental harmonic with a frequency of 50 Hz has an amplitude of 220 V and is taken as 100% as a base level for assessing the relative contribution of higher harmonics. The greatest energy contribution among higher-order harmonics is made by components located near the carrier frequency and its sidebands – 5950 and 6050 Hz (119th and 121st harmonics), each of which is about 7.18% of the amplitude of the fundamental harmonic. Further spectral components are concentrated in the vicinity of multiple carrier frequencies (e.g. 11950/12050 Hz and 17950/18050 Hz), their amplitudes gradually decrease, which corresponds to what is expected for PWM shaping with a high switching frequency.

THD (Total Harmonic Distortion) is calculated using the formula (12)

$$\mathrm{THD}=\frac{\sqrt{\sum_{h=2}^{\infty}U_h^2}}{U_1}\times 100\%. \quad (12)$$

The relatively low value of the harmonic distortion coefficient THD ≈ 10.2% indicates the high quality of the sinusoidal component and the possibility of further harmonic reduction using L or LC-type output filters.

The spectral distribution confirms that the selected carrier frequency provides sufficient spectral separation of the fundamental harmonic and higher harmonics, which facilitates the design of filtering devices and minimizes electromagnetic interference in the electric drive system.

3. Construction of a time diagram of the formation of output phase and line voltages of the frequency converter (FC), which allows to clearly trace the modulation process and evaluate the compliance of the signal shape with the specified parameters.

Fig. 7 shows the timing diagram of the formation of output phase and line voltages of the frequency converter (FC) operating in the pulse-width modulation (PWM) mode with a high carrier frequency. The output voltage frequency is f_{OUT} = 75 Hz, which corresponds to the increased rotation speed of the electric drive, and the carrier frequency f_H = 20 kHz is selected to shift the spectrum of switching harmonics to the high-frequency region, minimizing low-frequency distortions and simplifying filtering. The amplitude modulation coefficient is taken to be equal to μ = 1, which ensures maximum use of the DC link voltage.

The upper part of the graph illustrates the shape of the phase voltage $U_a(t)$, the middle part – the phase voltage $U_b(t)$, and the lower part – the line voltage $U_{ab}(t) = U_a(t) – U_b(t)$. All voltages have the form of high-frequency pulse sequences with an amplitude determined by the DC link voltage. It is clearly seen that the change in the polarity of the output voltage occurs at the moments of switching the power keys, which is a consequence of the PWM algorithm based on the comparison of modulating sinusoidal signals with a high-frequency triangular carrier.

The characteristic multi-level structure of the pulses reflects the operation of the circuit in the high-frequency PWM mode, in which the switching harmonics are grouped around the carrier frequency and its multiples, leaving the band around the fundamental harmonic (75 Hz) practically free. This ensures high quality of the sinusoidal component of the output voltage, low total harmonic distortion (THD) and efficient operation of the output LC filters. This mode is preferred for precision electric drive systems that require high quality motor winding power, minimization of acoustic noise and electromagnetic interference, and an extended speed control range.

4. Obtaining the spectral composition of the output voltage of a frequency converter with PWM at f_{OUT} = 75 Hz, f_H = 20000 Hz, μ = 1 in the form of tables, which provides the possibility of quantitative comparison of harmonic components in a specific operating mode.

Table 2 shows the spectral composition of the primary frequency output voltage with pulse width modulation.

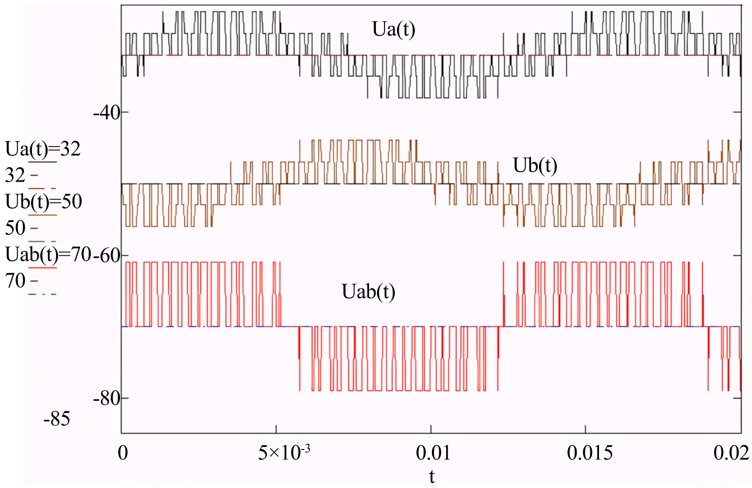

Fig. 7. Output voltage of frequency converter with pulse width modulation f_{out} = 75Hz, f_H = 20000Hz, μ = 1

Table 2

Spectral composition of the output voltage of the frequency converter with pulse-width modulation at f_{OUT} = 75 Hz, f_H = 20000 Hz, μ = 1

No. Harmonic *h*	Frequency *fh*, Hz	Amplitude, V	Relative contribution, %	Note
1	75	220.0	100.0	Fundamental harmonic
267	20025	7.8	3.55	Carrier harmonic
269	20175	7.8	3.55	Carrier harmonic
533	39975	4.2	1.91	2nd sideband from carrier
535	40125	4.2	1.91	2nd sideband from carrier
799	59925	2.9	1.32	3rd sideband from carrier
801	60075	2.9	1.32	3rd sideband from carrier
…	…	…	…	…
THD	–	–	≈ 5.4	Harmonic distortion coefficient

Table 2 shows the spectral composition of the output voltage of the frequency converter with PWM at f_{OUT} = 75 Hz, f_H = 20000 Hz and amplitude modulation factor μ = 1. The fundamental harmonic with a frequency of 75 Hz and an amplitude of 220 V is taken as the base level (100%), relative to which the contribution of higher harmonics is estimated. The largest contribution among higher-order harmonics is made by components located near the carrier frequency and its sidebands – 20025 Hz and 20175 Hz (267th and 269th harmonics), each of which is about 3.55% of the amplitude of the fundamental harmonic. Further spectral components are grouped in the vicinity of multiple carrier frequencies (for example, 39975/40125 Hz and 59925/60075 Hz), and their amplitudes successively decrease, which corresponds to the nature of energy distribution in PWM with a high switching frequency.

The obtained value of the harmonic distortion coefficient THD ≈ 5.4% indicates a high quality of the sinusoidal component and confirms the efficiency of the selected PWM parameters. The high carrier frequency ensures significant spectral separation of the fundamental harmonic and switching components, which simplifies the design of output filters and helps to reduce acoustic noise and electromagnetic interference in the electric drive system.

5. 3. Comparative analysis of the basic and optimized sinusoidal PWM

To verify the correctness of the spectral analysis, a time curve of the output voltage $U_a(t)$ was synthesized in accordance with (1), using the calculated amplitudes and phases of the harmonic components (Fig. 8). The same figure shows the time dependence of the output voltage $U_{aa}(t)$ of the frequency converter, reconstructed based on the results of harmonic analysis. To confirm the correctness of the spectral decomposition, an additional reverse reconstruction of the signal was carried out based on expression (2), where the amplitudes and phase shifts of the harmonic components calculated in the Mathcad 15 environment were used as the initial data.

The synthesized curve reproduces the output voltage shape typical for the operation of the frequency converter in the pulse-width modulation (PWM) mode with a high carrier frequency. The time diagram clearly shows high-frequency oscillations caused by switching processes superimposed on a slowly changing envelope corresponding to the fundamental harmonic of the output voltage.

Comparison of the obtained synthesized curve with the original signal $U_a(t)$ demonstrates a high degree of coincidence, which confirms the correctness of the calculation of the amplitudes and phases of the harmonic components, as well as the reliability of the performed spectral analysis. This approach allows not only to estimate the quantitative parameters of the spectrum (THD, the contribution of individual harmonics), but also to carry out numerical modeling of time processes in electric drives, providing a connection between the frequency and time domains of analysis.

Below, for comparison, are the output voltage curves obtained as a result of modeling a frequency converter with PWM ($U_{a1}(t)$, Fig. 9) and the calculated data of the Y_i vector with a given number of elements (Y_i, Fig. 10).

Fig. 9, 10 show the time dependences of the phase output voltage of a frequency converter (FC) with pulse-width modulation (PWM), obtained by two methods: numerical modeling in a simulation design environment and calculated restoration by the Y_i vector with a given number of discrete elements.

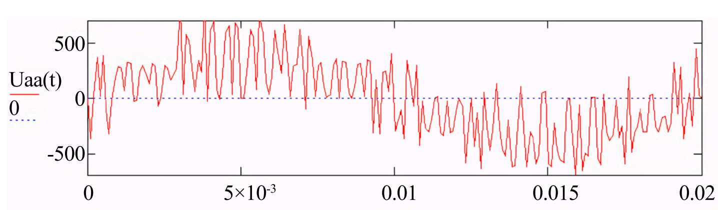

Fig. 8. Output voltage curve of a pulse-width modulated frequency converter, synthesized by harmonic composition

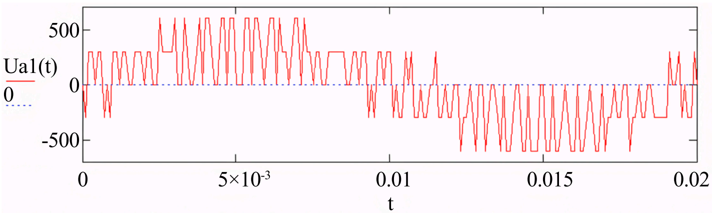

Fig. 9. Output voltage of frequency converters with pulse-width modulation obtained as a result of simulation

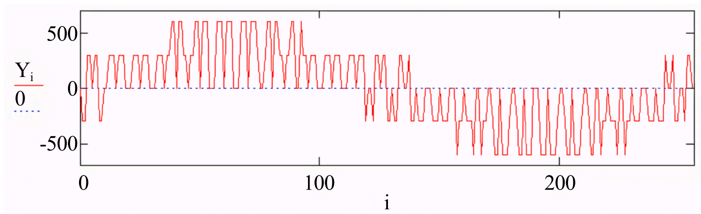

Fig. 10. Output voltage of frequency converters with pulse width modulation (calculated data of vector Y_i)

The $U_{a1}(t)$ curve obtained as a result of modeling reflects the signal type typical for the PWM mode: the presence of high-frequency switching pulses superimposed on the fundamental harmonic component, the frequency of which corresponds to the specified output value of the IF. The amplitude of the levels is determined by the DC link voltage, and the switching frequency is determined by the parameters of the carrier wave.

The calculated dependence Y_i, constructed on the basis of predetermined amplitudes and phases of the harmonic components, almost identically reproduces the shape of the $U_{a1}(t)$ signal. This confirms the correctness of the harmonic analysis and the adequacy of the chosen method for synthesizing a time signal based on its spectral characteristics.

Comparison of the two curves demonstrates a high degree of coincidence of the shape and amplitude levels, which indicates minimal discrepancies caused mainly by numerical approximation and data sampling. This result proves that the proposed algorithm for calculating and restoring the time dependence of the FC output voltage can be used to predict the electromagnetic characteristics of the system, estimate the harmonic distortion coefficient (THD), and optimize the PWM parameters in order to improve the quality of the load power supply.

6. Discussion of the results of the study of the mathematical model of a three-phase converter with an uncontrolled rectifier

The PWM proposed model correctly reproduces the operation of a three-phase inverter: the reference sawtooth voltage $U_H(t)$ according to (5) is compared with the sinusoidal modulating signals of phases U_{Ma}, U_{Mb}, U_{Mc} with 120° shifts according to (6)–(8), forming control functions $f_1 \dots f_6$ through the if operator (9); further, Boolean combinations of these functions set eight key states ($U_{a1} \dots U_{a8}$) and corresponding phase voltage levels ($\pm E_0$ and $\pm 2E_0$) (10), after which the phase voltage $U_a(t)$ is obtained by summation (11), and the linear voltage is obtained as $U_{ab}(t) = U_a(t) - U_b(t)$. Such an explicit mapping "comparison → state → voltage level" ensures reproducibility of results, simplifies parametric studies of the influence of ma, ωout, and ω_H on the output voltage spectrum, and serves as a basis for calculating THD, cosφ, and efficiency coefficient.

The implementation of the mathematical model in the Mathcad environment made it possible to obtain time diagrams of the formation of phase and line voltages of the frequency converter (Fig. 6, 7), as well as to perform a spectral analysis of the output voltage in various operating modes (Table 1, 2).

The analysis of the results (Table 1) showed that at a carrier frequency of 6 kHz, the harmonic distortion coefficient is about 10.2%, which indicates an acceptable quality of the output signal, providing the possibility of reducing harmonics when using output filters. At the same time, increasing the carrier frequency to 20 kHz (Table 2) makes it possible to significantly reduce the distortion level: the THD value decreases to 5.4%, which confirms the effectiveness of the selected PWM parameters.

The time diagrams of the formation of output voltages (Fig. 6, 7) clearly demonstrate the nature of modulation and the presence of a multi-level pulse structure of the signal, which ensures the spectral transfer of switching components to the high-frequency region. This facilitates filtering, reduces the level of electromagnetic interference and improves the quality of the power supply of the asynchronous motor. Thus, the conducted study confirms that the correct choice of pulse-width modulation parameters (carrier frequency, modulation coefficient) directly affects the energy performance of the "converter-asynchronous motor" system and ensures a reduction in harmonic distortion, increased efficiency and quality of operation of electric drives.

The analysis performed showed that the synthesis of the output voltage time dependence based on the spectral decomposition results (Fig. 8) provides almost complete coincidence with the original signal obtained during the simulation of the frequency converter operation in the PWM mode. This confirms the correctness of the calculation of the amplitudes and phases of the harmonic components and the reliability of the spectral analysis performed.

Comparison of the time curves obtained by different methods – numerical simulation (Fig. 9) and signal restoration by harmonic components using the Y_i vector (Fig. 10) – revealed a high degree of coincidence in both shape and amplitude levels. Minor discrepancies are due only to the approximation and discretization of data.

Thus, the adequacy of the proposed algorithm for restoring time dependences based on spectral characteristics is confirmed. This approach allows reliable linking of frequency and time domains of analysis, numerical modeling of electromagnetic processes in electric drives, evaluation of harmonic distortion coefficient (THD) and optimization of pulse-width modulation parameters to improve the quality of power supply of an asynchronous motor.

The disadvantages of the study include the lack of consideration of thermal modes of power switches, which limits the possibilities of predicting their reliability. To eliminate this disadvantage, it is proposed to integrate the thermal model into the calculation algorithm. Another aspect that requires revision is the lack of modeling of output voltage filters, which can be compensated for by adding an LC filter optimization module, which reduces the level of high-frequency harmonics.

Prospects for further development of the model are associated with taking into account the parameters of a real motor, including experimental data, integrating filters and analyzing resonance phenomena, as well as adapting the method to multichannel control systems. In this case, difficulties of various natures may arise: mathematical - associated with the complication of the system of equations when taking into account parasitic elements; methodological – caused by the need to align the algorithm with industry standards; and experimental – caused by the need to ensure high accuracy of measurements of harmonic components in conditions of electromagnetic interference and noise.

7. Conclusions

1. A mathematical description of a frequency converter with an uncontrolled rectifier and a PWM inverter has been performed. Adequacy has been confirmed: the model correctly generates a three-phase system of output voltages at a given frequency and modulation coefficient, stably reproducing modes at different carrier frequencies. The procedures of signal sampling, spectral decomposition and synthesis of the time form by amplitudes/phases of harmonics gave a high coincidence with the simulation results; the detected minimal discrepancies are due to the features of the numerical approximation.

2. A parametric model has been developed in Mathcad with the ability to vary the output frequency/amplitude, modulation coefficient and switching frequency; calculations of the efficiency, power factor and THD for various modes have been implemented. Harmonic analysis showed that increasing the switching frequency to 20 kHz transfers switching harmonics to the high-frequency region and reduces THD from approximately 10.2% to 5.4%, which improves the quality of the asynchronous motor power supply and increases the energy performance of the drive.

3. The analysis showed that the optimal range for achieving maximum energy efficiency with an acceptable level of harmonics is the modulation factor $ma = 0.85 - 0.95$ and the carrier frequency $f_H = 18 – 20$ kHz. Under these conditions, a low THD level is maintained (5-6%), the power factor reaches 0.94–0.96, and the efficiency of the electric drive increases by 3–4% compared to modes with a lower carrier frequency (6–10 kHz). When the modulation frequency decreases below 15 kHz, the amplitude of higher harmonics increases by 20–25%, which leads to an increase in losses and a decrease in efficiency. When the frequency exceeds 22 kHz, on the contrary, the share of switching losses increases, which limits further growth of energy efficiency and requires additional measures for cooling the power keys. Thus, the optimal operating modes of the frequency converter are determined by a compromise between minimizing harmonic distortion and ensuring high energy indicators, which is especially important when designing industrial electric drive systems and automated complexes.

Conflict of interest

The authors declare that they have no conflict of interest in relation to this study, whether financial, personal, authorship or otherwise, that could affect the study and its results presented in this paper.

Financing

The study was performed without financial support.

Data availability

Manuscript has associated data in a data repository.

Use of artificial intelligence

The authors have used artificial intelligence technologies within acceptable limits to provide their own verified data, which is described in the research methodology section.

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