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DEVELOPMENT OF A POLYMODEL RESOURCE MANAGEMENT COMPLEX FOR INTELLIGENT DECISION SUPPORT SYSTEMS

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The object of the study is intelligent decision support systems. The problem addressed in the research is to improve the accuracy of modeling the functioning process of intelligent decision support systems.

A polymodel complex for resource management of intelligent decision support systems has been developed. The originality of the study lies in:

– the comprehensive description of the functioning processes of intelligent decision support systems;

– the capability to model both an individual process occurring in intelligent decision support systems and to perform comprehensive modeling of the processes taking place within them;

– the establishment of conceptual dependencies in the functioning process of intelligent decision support systems. This makes it possible to describe the interaction of individual models at all stages of solving computational tasks;

– the description of coordination processes in hybrid intelligent decision support systems, which ensures an increase in the reliability of managerial decision-making;

– the modeling of processes for solving complex computational tasks in intelligent decision support systems through the conceptual description of the specified process;

– the coordination of computational processes in intelligent decision support systems, which leads to a reduction in the number of computational resources of the systems;

– the comprehensive resolution of conflicts through a set of corresponding mathematical models.

The proposed polymodel complex is advisable for use in solving the problem of managing intelligent decision support systems characterized by a high degree of complexity

Keywords: efficiency, reliability, decision-making, coordination, interaction, computational tasks, artificial intelligence

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1. Introduction

Intelligent decision support systems (IDSS) are an integral component of all spheres of human social activity and are applied to solve a wide range of tasks, from entertainment to highly specific ones [1, 2].

The main tasks addressed by IDSS are [3, 4]:

– solving various computational tasks in the interests of a wide range of users, regardless of the field of their application;

– storing the results of computations as well as intermediate results for user needs [5];

– supporting the decision-making process for decision-makers;

– providing the prerequisites for intelligent decision-making.

The trends in the development of modern IDSS are aimed at solving the following tasks [6]:

– increasing the efficiency of processing heterogeneous data and ensuring their reliability [7];

– improving the accuracy of modeling their functioning process [8];

– maintaining a balance between efficiency and reliability in the process of solving computational tasks, among others.

At the same time, existing scientific approaches to the synthesis and functioning of IDSS exhibit insufficient accuracy and convergence. This is due to the following reasons [1–9]:

– the significant role of the human factor in the initial configuration process of IDSS;

– the large number of heterogeneous information sources that must be analyzed and further processed during the operation of IDSS;

– the operation of IDSS under conditions of uncertainty, which causes delays in data processing;

– the presence of numerous destabilizing factors affecting the functioning of IDSS, and others.

This stimulates the implementation of various strategies to improve the efficiency of IDSS operation when solving computational tasks. One such approach is the enhancement of existing (or the development of new) mathematical models of the functioning of intelligent decision support systems.

2. Literature review and problem statement

In [6], it is proposed to use Bayesian hierarchical networks to determine the quantitative assessment of cybersecurity risk levels in special-purpose information systems. However, this approach is limited by the statistical distribution that can be used, as well as by the extensibility of the model structure. This imposes restrictions on the architecture of the information system and does not consider qualitative factors that affect the cybersecurity of the information system.

In [7], a security certification methodology is proposed for information systems to provide various stakeholders with the ability to automatically assess security decisions for large-scale deployments of information systems. The methodology ensures transparency regarding the security level of information systems for consumers, as it provides labeling as one of the main outcomes of the certification process. The drawbacks of the proposed approach include the inability to train knowledge bases for new threats, the difficulty of generalizing and analyzing heterogeneous data circulating within the network.

In [8], a model is proposed that combines fault tree analysis, decision theory, and fuzzy theory to determine the current causes of failures to prevent cyberattacks. The model was applied to assess cybersecurity risks associated with attacks on e-commerce websites and enterprise resource planning systems, as well as to evaluate the possible consequences of such attacks. Although this model has a flexible architecture, its drawbacks include the accumulation of estimation errors during fuzzification and defuzzification procedures.

In [9], a model of resource allocation in an automated control system for special purposes under conditions of insufficient information about the development of the operational environment is proposed. The model introduces mechanisms for resource allocation in automated control systems considering the impact of cyberattacks. This enables the representation of the solution to a vector optimization problem in binary relations of conflict, assistance, and indifference. It also considers the operational situation and allows forecasting of the system state under external influences, constructing utility and guaranteed gain functions, as well as a numerical optimization scheme over this set. However, this model does not allow for the processing of multidimensional indicators.

In [10], a hierarchical concept for implementing a management model based on e-government is proposed. The study examines major threats to critical cyber-physical systems as a foundation for the functioning mechanisms of e-governance. The hierarchical system is based on the use of symmetric and asymmetric cryptosystems, which makes it unsuitable for the task of identifying cyber impacts on the system.

In [11], a model is proposed for selecting the optimal set of cybersecurity insurance policies for a firm, considering the limited number of policies offered by one or several insurance companies. The model enables a systematic evaluation of various insurance policies as a function of the probability that a cybersecurity breach will occur during the policy term and the premiums associated with the policies. The proposed model provides a risk allocation approach that assists in mean-square selections of cybersecurity insurance policies, thereby contributing to an efficient cybersecurity insurance market. However, the drawbacks of this approach include the inability to add new risks to the knowledge base during operation and the limited number of assumptions, which makes its real-time operation impossible.

In [12], the importance of including vulnerability analysis in cybersecurity is discussed not only as part of process hazard analysis but also from the perspective of protecting process control networks and implementing adequate safeguards against cyber threats in general. The protection level analysis is adapted to assess potential weaknesses and ensure the security of critical applications against cyberattacks. The integration of cybersecurity with hazard and risk analysis, as well as other elements of process safety management, is demonstrated through examples, making a plant more resilient to both traditional and cyber threats. However, the proposed approach is adapted only for fixed architectures and is not designed for reconfiguration during operation.

In [13], a risk management process is proposed for identifying, analyzing, evaluating, responding to cyber threats, and monitoring risks at each stage of the cybersecurity chain. This approach can be used by organizations planning to implement security mechanisms to align with current requirements or reduce cyber risks to an acceptable level. The risk assessment method is based on a continuous Markov chain. However, the shortcomings of the proposed method include the inability to simultaneously consider both quantitative and qualitative indicators and the inability to adapt to new threats within the system.

In [14], a theoretical and analytical approach is proposed for analyzing the impact of information transmission delays in traffic control systems caused by cyber influences. The assessment is performed using the method of successive averages. However, this approach is limited to use in traffic control systems and is not adaptable to other systems.

In [15], it is proposed to consider the cybersecurity of an object in the form of a graph of transient processes. This approach allows for describing threats that affect the object and determining their degree of influence on cybersecurity. The drawbacks of the proposed approach include the ability to operate only with one-dimensional quantities and the inability to add new threats during operation.

In [16], a method for constructing and solving a game theory model for cybersecurity issues is presented, particularly for advanced manufacturing systems with highly integrated computer-based integration. This method introduces a unique approach to defining the content of the game payoff matrix, incorporating defensive strategies, production losses, and recovery from attacks as part of the cost function. However, the drawbacks of this method include high computational complexity and the ability to work only with one-dimensional quantities.

Thus, summarizing the above, the common disadvantage of all these approaches is the inability to process multidimensional data in real time. Let's consider the known studies that allow this drawback to be addressed. Several different solutions have been proposed to eliminate this shortcoming.

In [17], an approach is presented for evaluating input data for decision support systems. The essence of the proposed approach lies in the clustering of a basic set of input data, their analysis, followed by system training based on the analysis results. The disadvantages of this approach include the gradual accumulation of estimation and training errors due to the lack of an adequacy assessment of the decisions made.

In [18], an approach is presented for processing data from various information sources. This approach enables data processing from multiple sources. However, the drawbacks of this approach include the low accuracy of the obtained estimates and the inability to verify their reliability.

In [19], a comparative analysis of existing decision support technologies is conducted, namely: the analytic hierarchy process, neural networks, fuzzy set theory, genetic algorithms, and neuro-fuzzy modeling. The advantages and disadvantages of these approaches are indicated, and their areas of application are defined. It is shown that the analytic hierarchy process works well under conditions of complete initial information but, due to the need for experts to compare alternatives and select evaluation criteria, it involves a high degree of subjectivity. For prediction tasks under conditions of risk and uncertainty, the use of fuzzy set theory and neural networks is justified.

In [20], the use of combined strategies of various metaheuristic algorithms is discussed. The drawbacks of this approach include insufficient efficiency in processing heterogeneous data when several metaheuristic algorithms are used jointly.

The analysis of works [9–20] has shown that the common shortcomings of the aforementioned studies are as follows:

– modeling within each approach is carried out only at a separate level of IDSS functioning;

– in a comprehensive approach, usually only two components of IDSS functioning are considered. This does not allow for a full assessment of the impact of control decisions on their subsequent operation;

– the above models, which are constituent parts of the mentioned approaches, provide weak integration with one another, preventing their unification for joint functioning;

– the aforementioned models employ different mathematical apparatuses, which require appropriate mathematical transformations, thereby increasing computational complexity and reducing modeling accuracy, among other drawbacks.

3. The aim and objectives of the study

The aim of the study is the development of a polymodel complex for resource management of intelligent decision support systems. This will make it possible to model the functioning of intelligent decision support systems at different levels of their operation for the formulation of subsequent managerial decisions. It will also allow for the development (or improvement) of software for modern and advanced intelligent decision support systems through the integration of these models.

To achieve this aim, the following objectives were accomplished:

– to develop a conceptual model of intelligent decision support systems;

– to develop coordination models in hybrid intelligent decision support systems;

– to develop a mathematical model of consistency in intelligent decision support systems;

– to develop a fuzzy system for resolving model conflicts in an intelligent decision support system.

4. Materials and methods of research

The object of the study is intelligent decision support systems. The problem addressed in the study is the improvement of the accuracy of modeling the functioning process of intelligent decision support systems. The subject of the study is the process of functioning of intelligent decision support systems using a set of mathematical models of their operation. The research hypothesis is the possibility of increasing the efficiency and accuracy of the functioning of intelligent decision support systems through the development of a set of models of their operation.

To solve the specified partial tasks, the following research methods were used:

– artificial neural networks with evolving structure – to obtain a generalized assessment of the state of processes occurring in IDSS. This is due to the ability of artificial neural networks to flexibly adjust their structure to the number of information flows entering their input;

– an improved genetic algorithm – for simultaneous accelerated search for solutions in several directions;

– neuro-fuzzy expert systems – for collective resolution of disputes among models that differ in origin and units of measurement.

Table 1 presents the composition of the heterogeneous model field and the methods of model representation.

Modeling of the operation of the polymodel complex was carried out based on an active operational group of troops (forces) according to the wartime organization, based on the peacetime operational command structure. During the modeling process, typical computational tasks that are solved when processing heterogeneous information for decision-making by the commander and staff officers were used.

Modeling of the operation of the proposed polymodel complex was performed in the Microsoft Visual Studio 2022 (USA) software environment. The hardware used for the research process was an AMD Ryzen 5 processor.

Table 1

Models included in the heterogeneous model field

Model	Class of model and its characteristics	Implementation
Artificial neural networks	Artificial neural network (ANN) (search for hidden dependencies in statistical data and prediction of plan execution) – functional element. ANN with an evolving structure. Neuron transfer function: sigmoid. The number of inputs during experiments varied from 3 to 30, and the number of outputs from 1 to 10. The number of hidden neurons ranged from 1 to 8. Neuron transfer function: sigmoid. ANN training method – as in [2]. Average training error – 9%. Training sequence – 60 test tasks	Author's algorithm written in Microsoft Visual Studio 2022. Total code volume: 250 lines
Improved genetic algorithm (GA)	Improved GA [19] for solving an optimization problem – functional element. Population of 100 chromosomes. Evolution: crossover and mutation. Selection: combination of panmixia and ranking. Fitness (in %) – when fitness is below 50%, half of the population is eliminated and regenerated. If for ten generations fitness does not change but exceeds 92%, the best individual is considered the solution	Author's algorithm written in Microsoft Visual Studio 2022. Visualization algorithm implemented. Total code volume: 300 lines
Neuro-fuzzy expert systems	Production knowledge model for finding the decisive subgraph on an AND/OR graph. Forward reasoning. Knowledge base size of functional elements – 6–48 productions, and for the IDSS element – 15 productions. Fact base – up to 15 facts. Knowledge of experts and decision-makers was extracted by protocol analysis	Author's algorithm written in Microsoft Visual Studio 2022. Forward chaining used

5. Development of the polymodel complex for resource management of intelligent decision support systems

5. 1. Conceptual model of intelligent decision support systems

A conceptual model is a model of the subject domain that defines a set of concepts, properties, and characteristics for describing this domain, as well as the laws of the processes occurring within it. The conceptual model, on the one hand, delimits the subject domain as a set of objects, connections, and relationships among them, as well as the procedures for transforming these objects during problem-solving. On the other hand, it introduces the developer's subjective views in the form of their knowledge and experience – concepts – into the modeling process.

Conceptual models of entities, for example, of tasks and intelligent decision support systems (IDSS), are constructed based on a conceptual model scheme containing 11 categories of concepts C, of which the following five are used:

Definition 1. A resource is a concept denoting an object that is at the disposal of the control subject for accomplishing tasks. The set of resources is denoted as $RES = C^{res} \subseteq C$.

Definition 2. A property is everything that is not within the boundaries of a given object. It is that which, while characterizing objects, does not form new objects. The set of properties is denoted as $PR = C^{pr} \subseteq C$.

Definition 3. An action is a concept denoting relation among resources as a result of activity, actions, and behavior. The set of actions is denoted as $ACT = C^{act} \subseteq C$.

Collective effects in intelligent decision support systems (IDSS) are presented in Table 2.

Table 2

Collective effects in IDSS

Effect	Brief description	Positive impact	Negative impact
Adaptation	Adjustment to the external environment or its modification for effective operation of the IDSS	Expands the range of tasks solved by the IDSS	Complicates analysis of IDSS performance
Boomerang	When information is distrusted, an opposite opinion to that contained in it arises	Unreliable information is not perceived or is considered deliberately false	Reliable information from an unreliable source may be regarded as false
Wave	Dissemination of ideas within the IDSS that correspond to the interests of its members	Collective refinement of ideas	Prolonged work of experts on unpromising ideas
Homeostasis	Maintenance of system parameters within limits away from critical values	Ensures long-term viability of the IDSS	Sometimes the IDSS in borderline states generates higher-quality decisions than under normal conditions
Group Egoism	The goals of the collective are more important than those of its members or society	None	The efficiency of the collective's activity may harm society
Conformism	The common opinion is truth; the opinion of an individual is nothing	None	Hinders the emergence of new approaches to problem-solving
Fashion (Imitation)	Voluntary adoption of the viewpoints on problems established within the collective	Basis for self-learning among collective members; facilitates mutual adaptation	Reduces the likelihood of original viewpoints and approaches to problem-solving
Ringelmann Effect	As the group size increases, the individual contribution to joint work decreases	Reduces the workload on individual IDSS participants	Decreases expert motivation for effective teamwork
Self-learning	Work of IDSS participants to improve their knowledge based on experience	Maintains the knowledge of IDSS participants in an up-to-date state	The acquired knowledge may be unsystematized, nonverbalized, or erroneous
Self-organization	Relationships among experts are dynamic and change during the work process	Adaptation to the external environment; each time a new relevant method is developed. Emergence of original approaches and synergy	Complicates analysis and external management of the collective
Synergy	Attainment of a collective result that individual experts cannot achieve independently	Emergence of a qualitatively superior collective result	Possible occurrence of negative synergy (dissynergy)
Social Facilitation	Enhancement of dominant reactions in the presence of others	Accelerates solutions to simple tasks for which the individual knows the answer	In complex tasks, increases the probability of erroneous responses

Definition 4. Value is a concept or number that indicates the quantity of measurement units. The set of values is denoted by $VAL = C^{val} \subseteq C$.

Definition 5. State is a concept that denotes the manifestation of processes occurring in a resource at a certain time. The set of states is denoted by $ST = C^{st} \subseteq C$.

A set of relations R is established between the concepts of these categories.

Definition 6. A relation is that which forms a thing from given elements (properties or other things). A relation is that which, being established between things, forms new things.

The fact of a relation being established between concepts is denoted by $r^{\alpha\beta}\left(c_1^{\alpha}, c^{\beta}\right)$. It is possible to distinguish relations between different categories of concepts: $R^{\alpha\ \beta} \subseteq R$ – the set of relations between concepts from the set C^{α} and the set C^{β}, where $\alpha, \beta \in \left\{"res", "pr", "act", "val"\right\}$.

Thus, a fragment of the conceptual model schema sch_1 for structuring knowledge about the subject domain of the modeled task can be represented as follows

$$\begin{aligned} sch_1 = {} & R^{res\ res}\left(RES, RES\right) \circ R^{pr\ pr}\left(PR, PR\right) \circ \\ & \circ R^{act\ act}\left(ACT, ACT\right) \circ R^{val\ val}\left(VAL, VAL\right) \circ \\ & \circ R^{st\ st}\left(ST, ST\right) \circ R^{res\ pr}\left(RES, PR\right) \circ \\ & \circ R^{pr\ res}\left(PR, RES\right) \circ R^{res\ act}\left(RES, ACT\right) \circ \\ & \circ R^{act\ res}\left(ACT, RES\right) \circ R^{res\ st}\left(RES, ST\right) \circ \\ & \circ R^{st\ res}\left(ST, RES\right) \circ R^{pr\ act}\left(PR, ACT\right) \circ \\ & \circ R^{act\ pr}\left(ACT, PR\right) \circ R^{pr\ val}\left(PR, VAL\right) \circ \\ & \circ R^{val\ pr}\left(VAL, PR\right) \circ, \end{aligned} \qquad (1)$$

where the symbol $\circ$ – denotes concatenation.

The micro-level conceptual model of the intelligent decision support system (IDSS) can be expressed as follows

$$\widetilde{dss} = R^{res\ res}\left(prt^{dm}, env\right) \circ R^{res\ res}\left(PRT, PRT\right), \qquad (2)$$

where prt^{dm} – the knowledge model of the decision-maker (DM); $env \in RES$ – external environment; $PRT = \left\{prt_1, \ldots, prt_n, prt^{dm}\right\}$, $PRT \subseteq RES$ – the set of participants of the IDSS, including the decision-maker (DM) prt^{dm}; $R^{res\ res}$ – the set of "resource-resource" relations among the participants of the IDSS, as well as between the decision-maker (DM) and the external environment.

In work [13], it is noted that each participant $prt \in PRT$ of the IDSS has its own objective pr^{gsu}, which may coincide with or contradict the objectives of other participants. During the discussion, the experts exchange data pr^{dat}, knowledge pr^{knw}, explanations pr^{exp} and partial solutions pr^{dec} of the joint task. Thus, they perform a set of actions related to the transmission ACT^{itr} and reception ACT^{iac} of information, a set of professional functions ACT^{prt}, and exert influence on other participants of the IDSS and members of the surrounding environment by performing actions ACT^{conf}. Each expert has their own model res^{mod} of the external environment, including the control object, as well as their own set of methods RES^{met} for problem solving. Considering the heterogeneous nature of complex tasks, for their successful solution the IDSS must include experts of various specializations, with different sets of problem-solving methods, that is $RES_i^{met} \neq RES_j^{met}$, where $i, j = 1, \ldots, n$, $i \neq j$ – the index of a participant in the set PRT.

The conceptual model of an IDSS expert is expressed as follows

$$\begin{aligned} prt_i = {} & r_1^{res\ pr}\left(prt, pr^{gsu}\right) \circ r_1^{res\ pr}\left(prt, pr^{dat}\right) \circ \\ & \circ r_1^{res\ pr}\left(prt, pr^{knw}\right) \circ r_1^{res\ pr}\left(prt, pr^{\exp}\right) \circ r_1^{res\ pr}\left(prt, pr^{dec}\right) \circ \\ & \circ r_2^{res\ act}\left(prt, ACT^{prt}\right) \circ r_2^{res\ act}\left(prt, ACT^{itr}\right) \circ \\ & \circ r_2^{act\ pr}\left(ACT^{itr}, pr^{dat}\right) \circ r_2^{act\ pr}\left(ACT^{itr}, pr^{knw}\right) \circ \\ & \circ r_2^{act\ pr}\left(ACT^{itr}, pr^{\exp}\right) \circ r_2^{act\ pr}\left(ACT^{itr}, pr^{dec}\right) \circ \\ & \circ r_2^{res\ act}\left(prt, ACT^{iac}\right) \circ r_2^{act\ pr}\left(ACT^{iac}, pr^{dat}\right) \circ \\ & \circ r_2^{act\ pr}\left(ACT^{iac}, pr^{knw}\right) \circ r_2^{act\ pr}\left(ACT^{iac}, pr^{\exp}\right) \circ \\ & \circ r_2^{act\ pr}\left(ACT^{iac}, pr^{dec}\right) \circ r_2^{res\ act}\left(prt, ACT^{conf}\right) \circ \\ & \circ r_3^{res\ res}\left(prt, res^{\mathrm{mod}}\right) \circ r_3^{res\ res}\left(prt, RES^{\mathrm{m}et}\right), \end{aligned} \qquad (3)$$

where $r_1^{res\ pr}$ – the "have property" relation, which establishes the correspondence between an IDSS participant and their properties; $r_2^{res\ act}$ – the "perform" relation, which links a subject and the action they perform; $r_2^{act\ pr}$ – the "have property" relation, which links an action with its property; $r_3^{res\ res}$ – the "include" relation, which links a whole and its parts.

Many relations $R^{res\ res}$ in (2) consist of subsets of relations of various classes: cooperation $R_{coop}^{res\ res}$, competition $R_{comp}^{res\ res}$, neutrality $R_{neut}^{res\ res}$, trust $R_{trus}^{res\ res}$, pressure and conformism $R_{conf}^{res\ res}$, coordination $R_{coor}^{res\ res}$, dispute $R_{disp}^{res\ res}$, and others $R_{oth}^{res\ res}$. The subset of relations $R_{oth}^{res\ res}$ is introduced into the model to make it complete and extensible. Thus, the set $R^{res\ res}$ can be represented by the expression

$$\begin{aligned} R^{res\ res} = {} & R_{coop}^{res\ res} \cup R_{comp}^{res\ res} \cup R_{neut}^{res\ res} \cup R_{trus}^{res\ res} \cup R_{conf}^{res\ res} \cup \\ & \cup R_{coor}^{res\ res} \cup R_{disp}^{res\ res} \cup R_{oth}^{res\ res}. \end{aligned} \qquad (4)$$

The composition of relations from the set $R^{res\ res}$ and its subsets is not known in advance and is determined during the operation of the IDSS in accordance with the interaction rules $INT \subseteq RES$ because of its self-organization. Owing to the dynamism of the links among experts and self-organization, the IDSS is capable of generating a new solution method relevant to the prevailing conditions, and the conceptual model of the IDSS as a self-organized entity, a method for solving a complex task, can be represented by the expression

$$\begin{aligned} & RES_{dss}^{met} = \\ & = R^{res\ res}\left(RES_1^{met}, RES_2^{met}\right) \circ \ldots \circ R^{res\ res}\left(RES_1^{met}, RES_n^{met}\right) \circ \\ & \circ R^{res\ res}\left(RES_2^{met}, RES_1^{met}\right) \circ \ldots \circ R^{res\ res}\left(RES_2^{met}, RES_n^{met}\right) \circ \\ & R^{res\ res}\left(RES_n^{met}, RES_1^{met}\right) \circ \ldots \circ R^{res\ res}\left(RES_n^{met}, RES_{n-1}^{met}\right), \end{aligned} \qquad (5)$$

where the method RES_{dss}^{met}, generated by the IDSS in the process of solving a current task, represents an interconnected set of method sets RES_i^{met}, $i = 1, \ldots, n$, used by the experts in solving their partial tasks. In solving the current task, the intensity and orientation of the relations $R^{res\ res}$ among the IDSS experts, and consequently among the methods they employ, change, leading to the development, in accordance with (5), of a new method relevant to the complex task, that is, a synergistic effect arises. The external manifestation of this effect is that the IDSS produces solutions of higher quality compared to the opinions of individual experts.

Taking the above into account, the macro-level model of the IDSS can be represented as follows

$$\widehat{dss} = \left(PRT, env, INT, \widetilde{DSS}, EFF\right), \qquad (6)$$

where PRT – the set of IDSS participants described by the conceptual model (3); env – the environment in which the IDSS operates; INT – the set of elements structuring the interactions among experts; $\widetilde{DSS}$ – the set of IDSS micro-level models (5) corresponding to the specific functions of the experts within the IDSS and to the relations established among them; EFF is the set of conceptual models of macro-level (collective) effects in the IDSS (Table 1): adaptation ad, boomerang bo, wave wa, homeostasis ho, group egoism ge, groupthink gt, fashion fa, Ringelmann effect re, self-learning sl, self-organization so, synergy se, and social facilitation sf. Let's consider in more detail the models of these macro-level effects.

Two types of adaptation are distinguished: passive and active. In the first case, the adapting system changes so as to perform its functions in the given environment in the best possible way. The conceptual model of such adaptation is expressed as follows

$$ad_p = r_3^{res\ res}\left(dss, PRT\right) \circ r_2^{res\ act}\left(PRT, ACT^{iac}\right) \circ$$
$$\circ r_1^{act\ res}\left(ACT^{iac}, env\right) \circ R_1^{res\ res}\left(\widetilde{DSS}, \widetilde{DSS}\right) \circ$$
$$\circ r_1^{res\ pr}\left(\widetilde{DSS}, PR^{cr}\right) \circ r_1^{pr\ val}\left(PR^{cr}, VAL^{cr}\right) \circ$$
$$\circ r_1^{val\ val}\left(VAL^{cr}, VAL^{cr\ go}\right), \tag{7}$$

where PR^{cr} – the set of criteria for evaluating the effectiveness of the IDSS; VAL^{cr} – the set of values of critical parameters of the IDSS for micro-level models; $VAL^{cr\,go}$ – the set of target values of critical parameters of the IDSS; $r_1^{pr\ val}$ – the "have value" relation; $r_1^{val\ val}$ – the relation of proximity between two values.

Active adaptation implies a change of the environment in order to maximize the efficiency criterion or an active search for such an environment. The conceptual model of active adaptation for the IDSS is as follows

$$ad_a = r_3^{res\ res}\left(dss, PRT\right) \circ r_2^{res\ act}\left(PRT, ACT^{iac}\right) \circ$$
$$\circ r_1^{act\ res}\left(ACT^{iac}, env\right) \circ R_1^{res\ res}\left(ENV, ENV\right) \circ$$
$$\circ r_1^{res\ pr}\left(\widetilde{DSS}, PR^{cr}\right) \circ r_1^{pr\ val}\left(PR^{cr}, VAL^{cr}\right) \circ$$
$$\circ r_1^{val\ val}\left(VAL^{cr}, VAL^{cr\ go}\right) \circ r_2^{res\ act}\left(dss, ACT^{inf}\right) \circ$$
$$\circ r_1^{act\ res}\left(ACT^{inf}, env\right), \tag{8}$$

where $ENV \subseteq RES$ – the set of external environments suitable for the operation of the IDSS; ACT^{inf} – the set of IDSS influences on the application environment.

The boomerang effect (bo) is the ignoring of, or identification as false, information originating from unreliable sources

$$bo = r_3^{res\ res}\left(dss, PRT\right) \circ r_2^{res\ act}\left(PRT, ACT_{trus}^{iac}\right) \circ$$
$$\circ R_{trus}^{res\ res}\left(PRT, env\right) \circ R_{trus}^{res\ res}\left(PRT, PRT\right), \tag{9}$$

where ACT_{trus}^{iac} – a set of actions for obtaining information that considers the relations of trust among the IDSS participants, as well as between the participants and information sources from the external environment.

According to Table 1, the wave effect (wa) is a mechanism for the dissemination of ideas and objectives within the IDSS that correspond to the interests of its members, transmitted to IDSS participants primarily from the "inner circle" of the source expert. Subsequently, these participants may modify the idea and transmit it to the IDSS participants within their own "inner circle". The wave effect is formally expressed as follows

$$wa = r_3^{res\ res}\left(dss, PRT\right) \circ r_2^{res\ act}\left(PRT, ACT_{trus}^{itr}\right) \circ$$
$$\circ R_{trus}^{res\ res}\left(PRT, PRT\right), \tag{10}$$

where ACT_{trus}^{itr} – the set of actions for transmitting information that considers the relations of trust among the IDSS participants.

The conceptual model of homeostasis (ho) in the IDSS is expressed as follows

$$ho = r_3^{res\ res}\left(dss, PRT\right) \circ r_2^{res\ act}\left(PRT, ACT^{iac}\right) \circ$$
$$\circ r_1^{act\ res}\left(ACT^{iac}, env\right) \circ R_1^{res\ res}\left(\widetilde{DSS}, \widetilde{DSS}\right) \circ$$
$$\circ r_1^{res\ pr}\left(\widetilde{DSS}, PR^{cr}\right) \circ r_1^{pr\ val}\left(PR^{cr}, VAL^{cr}\right) \circ$$
$$\circ r_1^{val\ val}\left(VAL^{cr}, VAL^{cr\ all}\right), \tag{11}$$

where $VAL^{cr\ all}$ – the set of permissible values of the critical parameters of the IDSS.

The group egoism effect (ge) consists in the IDSS disregarding the objectives of society and of individual members of the IDSS

$$ge = r_3^{res\ res}\left(dss, PRT\right) \circ r_2^{res\ act}\left(PRT, ACT^{conf}\right) \circ$$
$$\circ r_1^{act\ res}\left(ACT^{conf}, PRT\right) \circ$$
$$\circ r_1^{act\ res}\left(ACT^{conf}, env\right) \circ R_{conf}^{res\ res}\left(PRT, PRT\right), \tag{12}$$

where $r_1^{act\ res}$ – the "have as object" relation, which links an action with the object toward which it is directed.

The groupthink effect (gt) is the suppression of opinions of IDSS participants that differ from the opinions of the majority of the IDSS members

$$gt = r_3^{res\ res}\left(dss, PRT\right) \circ r_2^{res\ act}\left(PRT, ACT^{conf}\right) \circ$$
$$\circ r_1^{act\ res}\left(ACT^{conf}, PRT\right) \circ R_{conf}^{res\ res}\left(PRT, PRT\right). \tag{13}$$

The fashion effect (fa) consists in the voluntary adoption of the viewpoint on a problem that has become established within the collective

$$fa = r_3^{res\ res}\left(dss, PRT\right) \circ r_2^{res\ act}\left(PRT, ACT^{iac}\right) \circ$$
$$\circ r_2^{act\ pr}\left(ACT^{iac}, PR^{dec}\right). \tag{14}$$

According to Table 1, the Ringelmann effect (re) is the decrease in the intensity of individual work as the group size increases

$$re = r_3^{res\ res}\left(dss, PRT\right) \circ r_2^{res\ act}\left(PRT, ACT^{prt}\right) \circ$$
$$\circ r_2^{act\ pr}\left(ACT^{prt}, PR^{efi}\right) \circ$$
$$\circ r_2^{act\ pr}\left(ACT^{prt}, PR^{efc}\right) \circ r_1^{pr\ pr}\left(PR^{efi}, PR^{efc}\right), \tag{15}$$

where PR^{efi} – the efficiency of performing an action in individual work, determined individually for each IDSS and each task. In general, efficiency is understood as an indicator that considers the assessment of the speed of decision-making and the quality of proposed solutions; PR^{efc} – the efficiency of performing an action during collective work; $r_1^{pr\ pr}$ – the "be greater than" relation.

The conceptual model of decision-maker (*DM*) self-learning sl_{dm} is expressed as follows

$$\begin{aligned} sl_{dm} = {} & r_3^{res\ res}\left(dss, PRT\right) \circ r_2^{res\ act}\left(PRT, ACT^{iac}\right) \circ \\ & \circ r_1^{act\ res}\left(ACT^{iac}, env\right) \circ R_1^{res\ res}\left(\widetilde{DSS}, \widetilde{DSS}\right) \circ \\ & \circ r_1^{res\ pr}\left(\widetilde{dss}, PR^{cr}\right) \circ r_1^{pr\ val}\left(PR^{cr}, VAL^{cr\ pl}\right) \circ \\ & \circ r_1^{pr\ val}\left(PR^{cr}, VAL^{cr\ fct}\right) \circ r_3^{res\ res}\left(prt^{dm}, res^{fdb}\right) \circ \\ & \circ r_1^{val\ val}\left(VAL^{cr\ pl}, VAL^{cr\ fct}\right) \circ r_2^{pr\ act}\left(res^{fdb}, ACT^{lrn}\right) \circ \\ & \circ r_1^{act\ res}\left(ACT^{lrn}, res^{rul}\right) \circ r_3^{res\ res}\left(res^{rul}, res^{ienv}\right) \circ \\ & \circ r_3^{res\ res}\left(res^{rul}, res^{idss}\right) \circ r_3^{res\ res}\left(res^{rul}, res^{ifct}\right), \end{aligned} \tag{16}$$

where $VAL^{cr\ pl}$ – the set of planned values of the IDSS efficiency criteria for the selected micro-level model $\widetilde{dss}$; $VAL^{cr\ fct}$ – the set of actual values of the IDSS efficiency criteria for the selected micro-level model $\widetilde{dss}$; res^{fdb} – the fuzzy knowledge base of the decision-maker (DM) for selecting micro-level IDSS models from the set $\widetilde{DSS}$; ACT^{lrn} – learning and adjustment of the rules of the decision-maker's (DM's) fuzzy knowledge base res^{fdb}; res^{rul} – a rule of the decision-maker's (DM's) fuzzy knowledge base for selecting micro-level IDSS models from the set $\widetilde{DSS}$; res^{ienv} – information about the external environment; res^{idss} – information about the micro-level model $\widetilde{dss}$; res^{ifct} – information about the actual values of the IDSS efficiency criteria corresponding to the selected model $\widetilde{dss}$.

Self-organization of the IDSS (*so*) is a specific effect in which the IDSS collective, without apparent external causes, creates or modifies the interrelations among participants and the organizational structures

$$\begin{aligned} so = {} & r_3^{res\ res}\left(dss, PRT\right) \circ r_2^{res\ act}\left(PRT, ACT^{iac}\right) \circ \\ & \circ r_1^{act\ res}\left(ACT^{iac}, env\right) \circ R_1^{res\ res}\left(\widetilde{DSS}, \widetilde{DSS}\right), \end{aligned} \tag{17}$$

where $r_1^{act\ res}$ – the "have as object" relation between an action and its resources; $R_1^{res\ res}$ – the set of relations between the preceding micro-level model and the subsequent one in the course of their transformation.

The synergy effect (*se*) is the result of the interrelations among the IDSS participants during their collaborative work on a task, that is, the generation of an organizational structure relevant to the problem being solved. This effect in the IDSS is manifested in obtaining a collective solution of higher quality than any of the individual ones

$$\begin{aligned} se = {} & r_3^{res\ res}\left(dss, PRT\right) \circ r_2^{res\ act}\left(PRT, ACT^{iac}\right) \circ \\ & \circ r_1^{act\ res}\left(ACT^{iac}, env\right) \circ R_1^{res\ res}\left(\widetilde{DSS}, \widetilde{DSS}\right) \circ \\ & \circ r_4^{res\ res}\left(PRT, RES^{dec}\right) \circ r_1^{act\ pr}\left(RES^{dec}, PR^{qua}\right) \circ \\ & \circ r_4^{res\ res}\left(dss, res_{dss}^{dec}\right) \circ r_1^{res\ pr}\left(dss, pr_{dss}^{qua}\right) \circ \\ & \circ r_1^{pr\ pr}\left(pr_{dss}^{qua}, PR^{qua}\right), \end{aligned} \tag{18}$$

where RES^{dec} – the set of solutions to the task assigned to the IDSS, proposed by the experts as a result of individual work; PR^{qua} – the set of quality indicators of the experts' individual solutions; res_{dss}^{dec} – the solution produced by the IDSS as a result of the experts' collaborative work; pr_{dss}^{qua} – the quality of the solution produced by the IDSS; $r_4^{res\ res}$ – the relation that links an expert or the IDSS with the solution produced.

As shown in Table 1, social facilitation (SF) involves the enhancement of dominant responses in the presence of other experts; that is, it contributes to the acceleration of decision-making

$$\begin{aligned} sf = {} & r_3^{res\ res}\left(dss, PRT\right) \circ r_2^{res\ act}\left(PRT, ACT^{prt}\right) \circ \\ & \circ r_1^{act\ pr}\left(ACT^{prt}, PR^{spi}\right) \circ \\ & \circ r_2^{act\ pr}\left(ACT^{prt}, PR^{spc}\right) \circ r_1^{pr\ pr}\left(PR^{spc}, PR^{spi}\right), \end{aligned} \tag{19}$$

where PR^{spi} – the speed of performing an action during individual work, PR^{spc} – the speed of performing an action during collective work.

Analysis of expressions (7)–(19) has shown that certain macro-level effects are interrelated. For example, expressions (7), (11), (16), and (18) can be transformed using expression (17) as follows:

$$\begin{aligned} ad_p = {} & so \circ r_1^{res\ pr}\left(\widetilde{DSS}, PR^{cr}\right) \circ r_1^{pr\ val}\left(PR^{cr}, VAL^{cr}\right) \circ \\ & \circ r_1^{val\ val}\left(VAL^{cr}, VAL^{cr\ go}\right), \end{aligned} \tag{20}$$

$$\begin{aligned} ho = {} & so \circ r_1^{res\ pr}\left(\widetilde{DSS}, PR^{cr}\right) \circ r_1^{pr\ val}\left(PR^{cr}, VAL^{cr}\right) \circ \\ & \circ r_1^{val\ val}\left(VAL^{cr}, VAL^{cr\ all}\right), \end{aligned} \tag{21}$$

$$\begin{aligned} sl_{dm} = {} & so \circ r_1^{res\ pr}\left(\widetilde{dss}, PR^{cr}\right) \circ r_1^{pr\ val}\left(PR^{cr}, VAL^{cr\ pl}\right) \circ \\ & \circ r_1^{pr\ val}\left(PR^{cr}, VAL^{cr\ fct}\right) \circ r_3^{res\ res}\left(prt^{dm}, res^{fdb}\right) \circ \\ & \circ r_1^{val\ val}\left(VAL^{cr\ pl}, VAL^{cr\ fct}\right) \circ r_2^{pr\ act}\left(res^{fdb}, ACT^{lrn}\right) \circ \\ & \circ r_1^{act\ res}\left(ACT^{lrn}, res^{rul}\right) \circ r_3^{res\ res}\left(res^{rul}, res^{ienv}\right) \circ \\ & \circ r_3^{res\ res}\left(res^{rul}, res^{idss}\right) \circ r_3^{res\ res}\left(res^{rul}, res^{ifct}\right), \end{aligned} \tag{22}$$

$$\begin{aligned} se = {} & so \circ r_4^{res\ res}\left(PRT, RES^{dec}\right) \circ \\ & \circ r_1^{act\ pr}\left(RES^{dec}, PR^{qua}\right) \circ r_4^{res\ res}\left(dss, res_{dss}^{dec}\right) \circ \\ & \circ r_1^{res\ pr}\left(dss, pr_{dss}^{qua}\right) \circ r_1^{pr\ pr}\left(pr_{dss}^{qua}, PR^{qua}\right). \end{aligned} \tag{23}$$

Expressions (20)–(23) show that self-organization plays a special and fundamental role among the collective effects in the IDSS – it is the prerequisite for the emergence of other effects that positively influence the performance of the IDSS, such as adaptation, homeostasis, self-learning, and synergy [18].

5. 2. Models of coordination in hybrid intelligent decision support systems

5. 2. 1. Mathematical model for solving a complex computational task

Within the systems approach, tasks are traditionally considered as systems [12, 14] composed of individual interrelated subtasks that are connected and interact with one another. The order of interconnection and interaction among elements in an HIDSS (Hybrid Intelligent Decision Support System) is determined by its structure.

Let's denote the task-system as prb^u, an individual task – prb^h. Then $PRB^h = \{prb_1^h, \ldots, prb_{N_h}^h\}$ – the set of individual tasks included in prb^u; $\widehat{PRB}^u = \{\widehat{prb}_1^u, \ldots, \widehat{prb}_{N_u}^u\}$ – the set of decompositions of tasks prb^u [5]; $R^h = \{r_{wq} \mid w, q = 1, \ldots, N_h; q \neq w\}$ – the set of relations among individual tasks; N_h – s the cardinality of the set PRB^h.

The model of a computational task of the IDSS can be represented as

$$prb^u = \langle PRB^h, \widehat{PRB}^u, R^h \rangle, \tag{24}$$

and the model of each partial computational task as [5]

$$prb^h = \langle GL^h, DAT^h, MET^h \rangle, \tag{25}$$

where GL^h – the final goal; DAT^h – input data; MET^h – conditions that specify how DAT^h are transformed into GL^h.

Model (24) satisfies all the properties of an IDSS:

– it consists of a set of elementary tasks PRB^h, among which relations R^h; are established; the connections are organized, which is reflected in the set of decompositions $\widehat{PRB}^u$;

– when solving the overall system task, the individual elementary tasks are predominantly isolated from the environment or its state is fixed, that is, the requirement is met that the internal connections within the system are much stronger than those with the external environment;

– a simple summation of the solutions of individual tasks does not yield a solution to the overall task as a whole [9, 10].

Model (25) has certain shortcomings. The main one is the inadequate representation of relations among the elements of the IDSS. Considering only the set of relations Rh among the partial tasks is insufficient. Studies of IDSSs have shown that, in most cases, experts are unable to provide professional solutions to partial tasks while taking into account the data on the complex task specified by the decision-maker (DM). Typically, there is a shortage of resources, particularly time, and errors occur in the formulation of the goal. Modification of the initial conditions of model (24) is impossible due to the absence of a crucial element – the image of the DM, which performs the function of a coordinator and reformulates the experts' goals depending on the situation.

The problem-solving process is thus considered as a system with a coordinator prb^k, which function is to monitor and manage the process of solving individual partial tasks $prb_1^h,\ldots,prb_{N_h}^h$ by the experts during collective discussion. The coordinator is linked by relations $R^{hk} = \{r^{kw} \mid w = 1,\ldots,N_h\}$ with each task prb^h in the IDSS prb^u, through which information is collected about the state of the process of solving an individual task by an expert. At certain moments in time, it also issues coordinating influences to modify the input data set-resources and goals. In this case, the model of a complex task with coordination is expressed as follows

$$prb^{uk} = \langle PRB^h, \widehat{PRB}^u, prb^k, R^h, R^{hk} \rangle, \tag{26}$$

where prb^k – the coordinator; $R^{hk} = \{r^{kw} \mid w = 1,\ldots,N_h\}$ – the sets of relations between the coordinator and the individual tasks.

A comparison of (24) and (26) shows that (26) is of a more general nature and reduces to (24) when the coordinator task is omitted from model (26), that is, in the case when the decision-maker (DM) in the IDSS does not perform coordination during the process of solving a complex task. The coordinator element may be represented as a "coordinating task" (k-task), which should be "added" to the decomposition $\widehat{prb}^u \in \widehat{PRB}^u$ of the complex task prb^u, to adequately represent the specific features of planning tasks in the model.

Let $MET^* = \{met_1,\ldots,met_{N_{MET^*}}\}$ – be the set of conditions. Then, a correspondence ψ_1 can be defined

$$\psi_1 : SOL_1^h \otimes SOL_2^h \otimes MET^* \to SOL^u. \tag{27}$$

The elements of the correspondence ψ_1 – are tuples $((sol_\alpha^{h\,1} sol_\beta^{h\,2} met_\gamma), sol_\eta^u)$, where $\alpha = 1,\ldots,N_{sh1}$; $\beta = 1,\ldots,N_{sh2}$; $\gamma = 1,\ldots,N_{\mathrm{MET}}$; $\eta = 1,\ldots,N_{su}$, with the first component being a three-component vector consisting of the solution $sol_\alpha^{h\,1} \in SOL_1^h$ of task prb_1^h, the solution $sol_\beta^{h\,2} \in SOL_2^h$ of task prb_2^h and the coordinating condition $met_\gamma \in MET^*$, and the second component being the solution sol_η^u of the task prb^u.

The correspondence ψ_1 is not a function; it cannot be written analytically or computed, since the coordination conditions and the results of solving individual partial tasks are most often represented in natural language.

Let, as a result of solving the partial tasks prb_1^h and prb_2^h the solutions $sol_1^{h\,1} \in SOL_1^h$, and $sol_1^{h\,2} \in SOL_2^h$, and $\{(sol_1^{h\,1}, sol_1^{h\,2})\} \otimes \otimes MET^* \nrightarrow SOL^u$, and let, that is, the obtained solutions $sol_1^{h\,1}$, and $sol_1^{h\,2}$ for all $met_\gamma \in MET^*$ do not lead to the solution of task prb^u. The symbol "$\nrightarrow$" denotes the absence of a mapping from the set on the left-hand side of the symbol to the set on its right-hand side. In this case, it is necessary to re-solve tasks prb_1^h and prb_2^h. However, in the IDSS, there is often insufficient time to re-solve the tasks, so reasoning about the prb^u complex task is divided into separate, logically complete intermediate stages [13, 14], and at the end of these stages, the integrated result of solving the complex task is systematically verified that is, an iterative process is organized. Consequently, the solutions of the partial task prb^h (the experts' lines of reasoning) are also divided into parts.

In this example, during the process of solving tasks prb_1^h and prb_2^h the following intermediate results will be obtained

$$\begin{aligned} &sol_{1\,1}^{h\,1} \Rightarrow sol_{1\,2}^{h\,1} \Rightarrow \ldots \Rightarrow sol_{1\,s-1}^{h\,1} \Rightarrow sol_{1\,N_{sol}}^{h\,1} = sol_1^{h\,1}, \\ &sol_{1\,1}^{h\,2} \Rightarrow sol_{1\,2}^{h\,2} \Rightarrow \ldots \Rightarrow sol_{1\,s-1}^{h\,2} \Rightarrow sol_{1\,N_{sol}}^{h\,2} = sol_1^{h\,2}, \end{aligned} \tag{28}$$

where N_{sol} – the number of iteration steps into which the partial tasks are divided; and $sol_1^{h\,1}$ and $sol_1^{h\,2}$ – are the results of solving the partial tasks prb_1^h and prb_2^h, respectively, obtained through the sequence of steps 1,..., N_{sol}.

Based on the coordinator's verification of the results obtained at a particular step, the relevance of influencing the course of solving the individual partial tasks prb_1^h prb_2^h is determined, so that the process of solving the complex task leads to the desired result the goal. This influence is referred to as coordinating, and for simplicity, let's further denote the result of an intermediate stage without the first lower index, that is, $sol_l^{h\,1}$ and $sol_l^{h\,2}$, where $l = 1, \ldots, N_{sol}$.

Following [17], it is possible to introduce the set of coordinating influences

$$E = \left\{ e_1^\alpha, \ldots, e_{N_{prt}}^\alpha \right\}, \tag{29}$$

where α – the type of coordinating influence, $\alpha = 1,\ldots,6$. Let's consider each of the six types.

Integral coordination ($\alpha = 1$) – the decision-maker (DM) establishes various constraints (standards) on the input parameters $in_i^{h\,1} \in IN^{h\,1} \subseteq DAT^h$ of the partial task prb_1^h for a certain period of time

$$\int_0^T \left(in_i^{h\,1}(t) \right) \mathrm{d}t = in_i^{h\,1\,H}, \tag{30}$$

where $in_i^{h\,1\,H}$ – the standard for the input parameter $in_i^{h\,1} \in IN^{h\,1}$, $i = 1,\ldots,N_{inh1}$; IN^{h1} – the set of input parameters of the partial task prb_1^h; $[0,T]$ – the time interval.

Precise coordination ($\alpha = 2$) imposes constraints on the input parameters of the partial task so that at each moment of time t they are equal to the specified value

$$in_i^{h\,1}(t) = in_i^{h\,1\,Set}, \tag{31}$$

where $in_i^{h\,1}(t)$ – the input parameter; $in_i^{h\,1\,Set}$ – the specified value of the parameter; t – an arbitrary moment in time when the fulfillment of the condition is verified $t \in [0, +\infty]$.

Interval coordination ($\alpha = 3$) requires that the input parameter $in_i^{h\,1}$ of the partial task (input data) belong to a specified interval

$$in_i^{h\,1} \in \left[val_{\min}^{h\,1\,i}, val_{\max}^{h\,1\,i}\right], \tag{32}$$

where $val_{\min}^{h\,1\,i}$, $val_{\max}^{h\,1\,i}$ – the interval boundaries.

Linguistic coordination ($\alpha = 4$) is a condition specified in natural language. Temporal coordination, or synchronization of the solution of partial tasks ($\alpha = 5$), to determine after what period an intermediate result must be provided. $sol_l^{h\,1}$, where $l = 1, ..., N_{sol}$ the results of solving the partial tasks are issued at certain time intervals

$$sol_l^{h\,1} \overset{\tau}{\Rightarrow} sol_{l+1}^{h\,1}, \tag{33}$$

where τ – the time interval after which the solution is issued; $sol_l^{h\,1}$ and $sol_{l+1}^{h\,1}$ – the results of solving the task prb_1^h after the i-th and $i+1$-th stages of solving the complex task, respectively.

Let's denote the situation in which the expert's line of reasoning does not change as a "null action," $\alpha = 6$. For example, the decision-maker (DM) considers that it is unnecessary to influence the course of solving the partial tasks by the expert.

Since the results of solving the partial tasks are most often issued in natural language, the coordinating influences $e_1^\alpha, \ldots, e_{N_{prt}}^\alpha$ are also most often presented in the same way.

Then, taking the above into account, let's establish the correspondence

$$\psi_2 : \left\{\left(sol_l^{h\,1}, sol_l^{h\,2}\right)\right\} \otimes MET^* \to E, \tag{34}$$

where $l = 1, ..., N_{sol} - 1$. The maximum value of the index l is taken as $N_{sol} - 1$, since after stage N_{sol} it is no longer possible to coordinate the solution of the partial tasks the final result has been obtained.

The elements of the correspondence ψ_2 – are pairs $((sol_l^{h\,1}, sol_l^{h\,2}, met_\gamma), e_q^\alpha)$, for $l = 1, ..., N_{sol} - 1$; $\gamma = 1, ..., N_{MET}$; $q = 1, ..., N_{prt}$, where the first component is a three-component vector consisting of the solution $sol_l^{h\,1} \in SOL_1^h$ of the task prb_1^h, the solution $sol_l^{h\,2} \in SOL_2^h$ the task prb_2^h, and the coordinating condition $met_\gamma \in MET^*$, and the second component is the coordinating influence $e_q^\alpha \in E$. Analogous to (26), the correspondence (34) is not a function. It is multivalued, since it is possible to apply to the same partial task prb^h to apply several coordinating actions $e_q^\alpha \in E$.

Since there is a limit on the number of steps, when $l = N_{sol}$, there must be a correspondence

$$\psi_3 : \left\{\left(sol_l^{h\,1}, sol_l^{h\,2}\right)\right\} \otimes MET^* \to SOL^u. \tag{35}$$

The elements of the correspondence ψ_3 – pairs of the form $((sol_l^{h\,1}, sol_l^{h\,2}, met_\gamma), sol_\eta^u)$, where $l = 1, ..., N_{sol}$ -1, $\gamma = 1, ..., N_{MET}$; $\eta = 1, ..., N_{su}$ with the first component being a three-component vector consisting of the solution of task, the solution $sol_l^{h\,1} \in SOL_1^h$ of task prb_1^h, the solution $sol_l^{h\,2} \in SOL_2^h$ the task prb_2^h and the coordinating condition $met_\gamma \in MET^*$, and the second component being the solution $sol_\eta^u \in SOL^u$ task met^u. If ψ_3 is absent, that is, if, as a result of the search for the elements of ψ_3, It is found that $\psi_3 = \varnothing$, then the decision-maker (DM) must modify the set of coordination conditions MET*: introduce new conditions and remove some of the old ones.

The correspondence ψ_3 – a subset of the set $\psi_1, \psi_3 \subseteq \psi_1$, since the only difference is that ψ_3 specifies the concrete results of solving tasks prb_1^h and prb_2^h.

Since not all elements of the correspondence ψ_3 have as their second component $sol_\eta^u \in SOL^u$, hat, that satisfy the objectives of solving prb^u, let's denote by DAT_{ψ_3} the set of elements of the correspondence ψ_3, which second component satisfies the objectives of solving prb^u $DAT_{\psi_3} \in \psi_3$.

Taking the above into account, and considering model (25), the model of the k-task can be written as follows

$$prb^k = < SOL_1^h, SOL_2^h, \psi_2, DAT_{\psi_3} >, \tag{36}$$

where SOL_1^h, SOL_2^h – the input data for the coordinator task prb^k, expressed as a combination of numbers, words, and expressions; $DAT_{\psi_3} \in \psi_3$ – the final goal of solving the coordinator task prb^k; ψ_2 – the set of conditions that specify how the coordinating influences (34) are formed after each step, as a result of the application of which, after the final step, DAT_{ψ_3} can be obtained.

On the basis of the above, let's give the following definition of the coordination process: an iterative (multistage) process during which, after each iteration, the decision-maker (DM) analyzes the integrated result of solving the set of partial tasks. A coordinating influence is selected for the line of reasoning of each expert so that, upon completion of the process of solving the complex task, a maximally comprehensive overall result of its solution is obtained.

It may also be noted that as the number of partial tasks increases, the relevance of coordinating their solutions grows, since the number of relations (such as information exchange, use of common variables, or common constraints) among the task elements increases combinatorially.

5. 2. 2. Conceptual model of coordination in intelligent decision support systems

In the previous section, the coordinator model (36) was obtained. In an IDSS, the decision-maker (DM) functions as this element: it decomposes a complex task into a series of partial tasks, provides input data to the experts, and collects the solution results.

The drawback of existing IDSSs lies in the fact that coordination is performed only once – at the end of the problem-solving process – when the DM, after aggregating the results of solving the partial tasks into a single solution, draws a conclusion about its adequacy. If the integrated result is assessed as unsatisfactory, the possibility of solving the task anew may be lost. Therefore, it is relevant to develop IDSSs in which coordination occurs continuously throughout the process of solving a complex task.

Based on the IDSS model [6] and the model of a complex task with coordination, let's construct the model of an IDSS with coordination

$$DSS = < PRT, prt^{dm}, R^{dm} >, \tag{37}$$

where $PRT = \{prt_q | q = 1, ..., N_{prt}\}$ – a set of expert models; prt^{dm} – the decision-maker model; $R^{dm} = \{r^{dm\,q} | ..., N_{prt}\}$ – the relations between the decision-maker and the experts, for example, relations of information exchange. Each expert works strictly within his or her own domain of knowledge $S_{prtq} \in S$, where S – the set of all domains of knowledge necessary for solving a complex task, and does not engage in any partial tasks outside his or her own domain $S_q \cap S_w = \varnothing$, for $q,w = 1,...,N_{prt}$; $q \neq w$. Based on the considerations in [5] and taking into account that in real tasks the partial tasks are solved by experts step by step, the expert model can be expressed as

$$prt_q = \left\langle \begin{matrix} B_{prof}, B_{theor}, B_{prec}, B_{facts}, \\ MET_{prtq}, S_{prtq}, In_{prtq}, \Delta t \end{matrix} \right\rangle, \quad (38)$$

where B_{prof} – production base of professional knowledge; B_{theor} – production base of theoretical knowledge; B_{prec} – case base (experience); B_{facts} – fact base; MET_{prtq} – set of reasoning methods; S_{prtq} – description of the expert's domain of knowledge, for example, in mathematics this includes the description of the mathematical language, basic concepts, and operations; In_{prtq} – interpreter that ensures the execution of a sequence of rules for solving a problem based on facts and rules stored in the databases and knowledge bases; Δt – the period during which experts provide intermediate solutions.

The decision-maker model can be constructed by analogy with (38)

$$prt^{dm} = \left\langle \begin{matrix} B_{prof}, B_{theor}, B_{prec}, B_{facts}, B_{ext}, \\ MET_{prtdm}, S_{prtdm}, In_{prtdm}, E, T \end{matrix} \right\rangle, \quad (39)$$

where B_{ext} – production knowledge base concerning how to perform reduction, aggregation, comparison, and coordination; E – the set of coordinating processes; T – the time required to solve the complex task.

Expression (39), in comparison with (38), has significant differences. The production knowledge base B_{ext} concerns how the decision-maker manages the process of solving a complex task. This knowledge comes from other experts. The set E describes how the decision-maker can coordinate the work of the experts. In the present work, the decision-maker models do not consider: B_{prof} – the base of professional knowledge; B_{theor} – the base of theoretical knowledge; B_{prec} – the case base (experience).

Let's consider how the IDSS functions according to (37). Let the decision-maker be given a task prb^u, which he or she reduces to partial tasks $prb_1^h,\ldots,prb_{N_h}^h$. By analyzing (23) and (35), the following conclusions can be made: GL^h is contained in B_{prec} and B_{facts} – experience combined with facts allows the expert to determine what result should be obtained; MET^h is contained in B_{prof}, B_{theor}, B_{prec}, MET_{prti}, S_{prti} and In_{prti}; DAT^h is contained in B_{facts}.

In traditional IDSSs, described, for example, in [2], each expert, prt_q, $q = 1,...,N_{prt}$ receiving his or her partial task $prt_j^h, j = 1,\ldots,N_h$, finds its solution using his or her professional knowledge B_{prof} and theoretical knowledge B_{theor}. After completing the solution process, the expert provides the result $sol^{h\,j} \in SOL_j^h$, where SOL_j^h – the set of results obtained from solving the task prt_j^h, which can be represented as the correspondence ψ_4

$$\psi_4 : DAT^h \otimes B^U \to SOL^h,\ B^U = B_{prof} \cup B_{theor}. \quad (40)$$

The elements of the correspondence ψ_4 – are tuples $(\{dat_\sigma^h\},\{b_\beta^u\},sol_\gamma^h)$, where $\sigma = 1,\ldots,N_{dath}$; $\beta = 1,\ldots,N_b$; $\gamma = 1,\ldots,N_{sh}$, in which the first component is a two-component vector consisting of the list of input data $\{dat_\sigma^h\}, dat_\sigma^h \in DAT^h$ and the list of knowledge used by the expert $\{b_\beta^u\}, b_\beta^u \in B^U$ (professional knowledge – production rules; theoretical knowledge – analytical dependencies), and the second component is the result $sol_\gamma^u \in SOL^h$ of solving the task prb^h.

The correspondence ψ_4 is not a function (it cannot be represented analytically or computed by numerical methods), since the expert's knowledge and the results of solving the task element can be expressed in natural language. It is ambiguous, because with an incomplete set of input data, the expert may propose several alternative results; it is subjective, since each solution of task prb^h corresponds to at least one element from and it is not injective, as not every element of $DAT^h \otimes B^U$ corresponds to a solution of task prb^h.

Let denote the number of stages into which the experts divide the process of solving partial tasks, and let ΔN_{sol}, and sol_l^h – be the result of solving the partial task at the l-th stage, $l = 1,..., N_{sol}$. A time interval Δt, is allocated to each stage, since in practical tasks the total time T for solving the complex task prb^u, is strictly limited, and with Δt being constant, the number of stages is determined by the formula

$$N_{sol} = T / \Delta t. \quad (41)$$

It should be noted that in the process of solving a partial task prb^h, due to the coordinating influences of the decision maker, the input data DAT^h in (40) may be modified – additional information may be introduced, or outdated information may be replaced with new information. Let DAT_l^h the input data for the l-th stage, $l = 1,...,N_{sol}$. So DAT_1^h – the output data obtained from the decision maker, where $DAT_1^h = DAT^h$, and DAT_l^h, $l = 2,...,N_{sol}$ – the output data of the subsequent stages. The index l denotes the stage number at which the output data are used. Let's define DAT_{l+1}^h – the output data of the $(l+1)$-th stage obtained after the coordinating influences of the decision maker concerning the modification of the data of the l-th stage.

The sequence scheme of the stages of the expert's work on finding a solution to the partial task πh can be described as follows

$$DAT_l^h \otimes B^U \otimes \left\{sol_1^h\right\} \otimes,\ldots,\otimes \left\{sol_{l-1}^h\right\} \Rightarrow \left\{sol_l^h\right\},$$
$$l = 1,...,N_{sol}. \quad (42)$$

Output data DAT_l^h, $l = 1,...,N_{sol}$ at each stage are supplemented by coordinating influences $e^\alpha \in E$, issued by the decision maker to the expert, which are determined based on the integrated result of the task solution prb^u at the $(l-1)$-th stage. In some cases, the decision maker may issue several coordinating influences to each expert. Let's assume that each expert receives one coordinating influence of a single type. Let's define the correspondence ψ_5

$$\psi_5 \left\{sol_l^u\right\} \otimes B_{ext} \to E,\ l = 1,...,N_{sol} - 1. \quad (43)$$

The maximum value of l equals $N_{sol} - 1$, because after N_{sol} the stage, it is no longer possible to apply coordination, since the final result has been obtained; sol_l^u – the integrated result of the task solution prb^u at the l-th stage; $E = \{\varepsilon_1,\ldots,\varepsilon_{N_\varepsilon}\}$ – a set of vectors of the form $(e_1^1,\ldots,e_{N_{prt}}^6)$, each component of which is a coordinating action for the expert, $e_q^\alpha \in E$, $q = 1,..., N_{prt}$.

Since the knowledge about integration is included in B_{ext} the decision maker (40), the integrated result sol_l^u of solving the complex task prb^u can be expressed as follows

$$\left\{sol_l^{h\,1}\right\}\otimes\ldots\otimes\left\{sol_l^{h\,N_h}\right\}\otimes B_{ext}\to\left\{sol_l^u\right\},\tag{44}$$

where $sol_l^{h\,1},\ldots,sol_l^{h\,N_h}$ – of solving the partial tasks $prt_1^h,\ldots,prt_{N_h}^h$ accordingly.

The elements of the correspondence ψ_5 – tuples $((sol_l^u,\{b_{ext}^\mu\}),\varepsilon_p)$, so $l=1,\ldots,\ N_{sol},\ \mu=1,\ldots,\mu_{N_\mu},\ p=1,\ldots,\ N_\varepsilon$ where the first component is a two-component vector consisting of the integrated result sol_l^u solution of the task prb^u at the l-th stage and the list of the DM's knowledge used concerning how to perform comparisons, and $e\in E$ for the expert.

At the N_{sol}-th stage ($l=N_{sol}$) the vector of coordinating influences $(e_1^1,\ldots,e_{N_{prt}}^6)$, $\alpha=6$, that is, the DM does not issue coordinating influences to the experts but only aggregates (performs the integration of the solutions to the prt^h tasks into a single, integrated solution sol_l^u of the complex task prt^u) the results of their work. If the obtained integrated result sol_l^u does not satisfy the DM, it must revise the initial data of the task prb^u. It is necessary to change DAT_l^h for all prt^h or change the list of its knowledge B_{ext} and the experts' knowledge B_{prof} (models (40) and (39)), and after that, initiate the repeated operation of the DSS. The correspondence ψ_5 is not a function (cannot be expressed analytically or computed), since the DM's knowledge and the integrated result of the task solution prb^u can be represented in natural language. It is unambiguous since each expert is assigned a specific coordinating influence e_q^α, and therefore, the correspondence ψ_5 uniquely determines only one vector $e\in E$. It is subjective, because to each vector $e\in E$ there corresponds at least one element $\{sol_l^u\}\otimes B_{ext}$, and not injective, because not every element $\{sol_l^u\}\otimes B_{ext}$ corresponds to a vector $e\in E$.

The analysis of the above-described model of the IDSS with coordination allows the following conclusion to be drawn. In this case, the errors in solving the complex task will be detected and corrected before obtaining the result of solving the complex task prb^u. Previously, this required repeated solutions.

5. 2. 3. Mathematical model of the functional hybrid system with coordination

In [5], the following conceptual model of the IDSS, based on the automaton approach [7–9], is presented, designed for solving a complex task prb^u

$$\begin{aligned}
res_A^u &= R_1^{res\ met}\left(res_A^u,met^u\right)\circ R_1^{res\ pr}\left(res_A^u,pr^{ui}\right)\circ\\
&\circ R_1^{res\ pr}\left(res_A^u,pr^{uo}\right)\circ R_1^{res\ st}\left(res_A^u,st^u\right)\circ\\
&\circ R_1^{st\ st}\left(st^u(t),st^u(t+1)\right)\circ R_1^{pr\ st}\left(pr^{ui}(t),st^u(t+1)\right)\circ\\
&\circ R_1^{st\ pr}\left(st^{up}(t),pr^{uo}(t)\right)\circ R_1^{res\ res}\left(RES^e,RES^e\right)\circ\\
&\circ R_1^{pr\ pr}\left(pr^{ui},PR^{ei}\right)\circ R_2^{pr\ st}\left(PR^{eo},pr^{uo}\right),
\end{aligned}\tag{45}$$

where t – model time, $t\in\mathbb{N}$; $\circ$ – concatenation symbol; res_A^u – the IDSS-aggregate as a resource for solving a heterogeneous task; met^u – the integrated method for solving a heterogeneous task; pr^{ui} – output data DAT^u [5] solution of a complex task prb^u, that are transmitted to the input of one or several elements res^e, constructed according to scheme (45) in accordance with the decomposition $\widehat{prb}^u$ task prb^u; pr^{uo} – the output of one or several elements res^e, constructed according to scheme (45) in accordance with $\widehat{prb}^u$, which is the goal GL^u of solving the task prb^u; $st^u(t)$ – the state of the IDSS at time t; RES^e – a nonempty set composed of elements res^e, constructed in accordance with scheme (45); PR^{ei}, PR^{eo} – the set of properties "input" and "output" of the elements from RES^e accordingly; $R_1^{st\ st}$, $R_1^{pr\ st}$, $R_1^{st\ pr}$ – relations of the functioning of the IDSS; $R_1^{res\ res}$ – relations of integration [5] of the elements; $R_1^{pr\ pr}$ – relations between the inputs of the IDSS and the inputs of the elements; $R_2^{pr\ pr}$ – relations between the outputs of the elements and the outputs of the IDSS.

The element res^e models the solution of a homogeneous partial task or performs auxiliary operations, constructed according to an autonomous method met^e and possesses the properties $PR^e\subseteq PR$, the most important of which are "input" pr^{ei}, "output" pr^{eo} i "state" st^i.

Conceptual model of an IDSS element

$$\begin{aligned}
res^e &= R_1^{res\ met}\left(res^e,met^e\right)\circ R_1^{res\ pr}\left(res^e,pr^{ei}\right)\circ\\
&\circ R_1^{res\ pr}\left(res^e,pr^{eo}\right)\circ R_1^{res\ st}\left(res^e,st^e\right)\circ\\
&\circ R_1^{st\ st}\left(st^e(t),st^e(t+1)\right)\circ R_1^{pr\ st}\left(pr^{ei}(t),st^e(t+1)\right)\circ\\
&\circ R_1^{st\ pr}\left(st^e(t),pr^{eo}(t)\right),
\end{aligned}\tag{46}$$

where $R_1^{st\ st}$, $R_1^{pr\ st}$, $R_1^{st\ pr}$ – the "state-state", "input-state", and "state-output" relations, respectively. Among the set of $MET^e=\{met_y^e\mid y=1,\ldots,N_{met}\}$ autonomous methods, it is possible to distinguish met_1^e – analytical computations, met_2^e – neurocomputations, met_3^e – fuzzy computations, met_4^e – reasoning based on experience, evolutionary computations, met_6^e – statistical computations, met_7^e – and logical reasoning. If between an element res^e and an autonomous method, met_y^e a relation is established $R_1^{res\ met}(res^e,met_y^e)$, it is possible to denote the element $res^{e\ y}$.

Relations $R_1^{pr\ pr}$, $R_2^{pr\ pr}$ (45) are defined on sets of variables DAT^u, GL^u, and on sets of variables DAT^h, GL^h of the partial tasks included in the complex task.

In [5], three possible cases are given:

1) a set of variables for prb^u coincides with the set of variables for prb^h, so $DAT^u=DAT^h$, $GL^u=GL^h$;

2) the set of variables for prb^h – a subset of the corresponding set prb^u, so $DAT^h\subset DAT^u$, $GL^h\subset GL^u$;

3) the set of variables of a subset of the corresponding set prb^h, so $DAT^h\subset DAT^u$, $GL^h\subset L^u$.

Since the automaton approach is used for modeling, the state of the automaton is influenced only by the input signal. The output signal depends on the state of the automaton at the previous moment of automaton time and on the input signal.

The extension of models (43) and (45) is carried out based on the following considerations. In the process of coordination, the intermediate states of the solutions to partial tasks are monitored [11]. In the adopted notations (43), (45), these states are understood as the states (solution results) of the functional elements res^e, that simulate the solutions of partial tasks prb^h. From the analysis of these states, the properties of the "input" change during coordination pr^{ei} of one or several elements res^e.

To take this fact into account, let's introduce into the conceptual model of the IDSS (43), (45) the triple $R_1^{st\ pr}\left(st^u(t),pr^{ui}(t+1)\right)$. In other words, based on the state of the IDSS $st^u(t)$ at time t, the output data change $pr^{ui}(t+1)$ for the IDSS, but already now in time $t+1$, that is, for the next iteration. Many $R_1^{st\ pr}$ establish relationships between the state $st^u(t)$ hybrid res_A^u (43) at the current model time t and the state of the inputs of one or several elements res^e at the next step. To make the necessary change to the inputs pr^{ei} of one or several functional elements res^e for (45) let's introduce the triple $R_1^{st\ act}(st^u, act^{ek})$, where $ACT^{ek}=\{act_1^{ek\ \alpha},\ldots,act_{N_{prt}}^{ek\ \alpha}\}$ – a set of concepts denoting coordinating actions, which is identical to the set of coordinating actions E (30), where α – the type of coordinating influence, $\alpha=1,\ldots,6$.

The modified conceptual model for the IDSS with coordination

$$res_A^u = res_A^u \circ R_1^{st\ pr}\left(st^u\left(t\right), pr^{ui}\left(t+1\right)\right), \tag{47}$$

and the modified model of the IDSS element

$$res^e = res^e \circ R_1^{st\ act}\left(st^u, act^{ek}\right). \tag{48}$$

Relationships $R_1^{st\ pr}$ and $R_1^{st\ act}$ are not predetermined, just as $R_1^{st\ st}$, $R_1^{pr\ st}$, $R_1^{st\ pr}$ are recorded in the course of the IDSS operation and are the result of solving the k-task prb^k (33).

Let's consider an example of an IDSS consisting of three elements $res_1^{e\ 1}$, $res_2^{e\ 6}$, $res_3^{e\ 7}$ for solving partial tasks, which it is possible to call functional [5], and one coordinating (technological) element $res_k^{e\ 7}$ for solving the k-task, which determines the order of interaction of the functional elements. The input of the IDSS receives the initial data DAT^u, divided among the functional elements according to the decomposition $\widetilde{prb_u} \in \widetilde{PRB^u}$ of solving a complex task prb^u. At the output, let's obtain the results of the operation of the functional elements $res_1^{e\ 1}$, $res_2^{e\ 6}$, $res_3^{e\ 7}$, integrated into the overall solution SOL^u of the complex task prb^u.

At each moment in time, t_l the state of all elements is recorded (polled) $res_q^{e\ y}$. After that, $res_k^{e\ 7}$ based on the state $st^u(t_l)$, the IDSS issues a coordinating action $act_q^{ek\ \alpha} \in ACT^{ek}$ for each element $res_q^{e\ y}$. In the process of processing by the technological element $res_k^{e\ 7}$ state $st^u\left(t_l\right)$ of the IDSS, that is, the solution of the k-task prb^k the state changes of the technological element $res_k^{e\ 7}$. Moreover, the time τ', allocated for such processing, must not exceed the period after which the state of the IDSS is recorded

$$\tau' \le T / N_{sol}, \tag{49}$$

where T – the time allocated for solving the complex task prb^u; N_{sol} – the total number of stages. The transitions between the states of the functional elements of the IDSS occur abruptly, since between the moments in time t_l this state $res_q^{e\ y}$ does not change.

Below is the conceptual model of the operation of the IDSS constructed according to (46) and (48)

$$\begin{aligned}
&st^{e\ k}\left(t_0'\right) \Rightarrow \begin{Bmatrix} st^{e\ 1}\left(t_0\right) \\ st^{e\ 2}\left(t_0\right) \end{Bmatrix} \to \begin{Bmatrix} st^{e\ 1}\left(t_1\right) \\ st^{e\ 2}\left(t_1\right) \end{Bmatrix} \Rightarrow \\
&\Rightarrow st^{e\ k}\left(t_0'\right) \to st^{e\ k}\left(t_1'\right) \Rightarrow \\
&\Rightarrow \begin{Bmatrix} st^{e\ 1}\left(t_1\right) \\ st^{e\ 2}\left(t_1\right) \end{Bmatrix} \to \begin{Bmatrix} st^{e\ 1}\left(t_2\right) \\ st^{e\ 2}\left(t_2\right) \end{Bmatrix} \Rightarrow \\
&\Rightarrow st^{e\ k}\left(t_1'\right) \to st^{e\ k}\left(t_2'\right) \Rightarrow \ldots \Rightarrow \\
&\Rightarrow \begin{Bmatrix} st^{e\ 1}\left(t_{p-1}\right) \\ st^{e\ 2}\left(t_{p-1}\right) \end{Bmatrix} \to \begin{Bmatrix} st^{e\ 1}\left(t_p\right) \\ st^{e\ 2}\left(t_p\right) \end{Bmatrix} \Rightarrow \\
&\Rightarrow st^{e\ k}\left(t_{p-1}'\right) \to st^{e\ k}\left(t_p'\right),
\end{aligned} \tag{50}$$

where "$\Rightarrow$" denote the relations $R^{st\ st}$, that link states from different subspaces and define the transition from one homogeneous space to others during the functioning of the IDSS; "$\to$" – the transition between states within the corresponding subspace. The transitions "$\Rightarrow$" from the subspace of the technological element $res_q^{e\ 7}$ model the issuance of coordinating actions from the decision maker to the experts. And the set of transitions "$\Rightarrow$" and "$\to$" allows modeling and tracing the process of self-organization during the operation of the IDSS.

In (50), curly brackets denote the beginning and completion of the parallel operation of the functional elements. From the model, it is evident that after each fixation of "$\}\Rightarrow$" of states, functional element $res_q^{e\ y}$ control is transferred to the technological element $res_q^{e\ 7}$, and after it changes its state, control is transferred to a group of functional elements of the IDSS.

This model is related to the conceptual model presented in [5]

$$\begin{Bmatrix} st^{e\ 1}\left(t\right) \to st^{e\ 1}\left(t+1\right) \to \ldots \to st^{e\ 1}\left(t+n\right) \\ st^{e\ 2}\left(t\right) \to st^{e\ 2}\left(t+1\right) \to \ldots \to st^{e\ 2}\left(t+n\right) \end{Bmatrix}. \tag{51}$$

The model (50) is based on the idea that the same homogeneous task can be solved in parallel by different functional elements of the IDSS. The relations of integration among the elements arise as internal nonverbal images in the user's memory, allowing them to compare the dynamics of modeling a complex task from different viewpoints, which makes it possible to perceive aspects that cannot be revealed through modeling with a single model. In model (49), another assumption is developed: the inclusion of the DM model within the mathematical model of the IDSS leads to the emergence of a self-organization effect.

5. 3. Mathematical models of consistency in intelligent decision support systems

5. 3. 1. Conceptual model of consistency in intelligent decision support systems

Consistency is understood as the degree of similarity among the goals of the IDSS participants. According to [13], a goal is a state of affairs that the decision-maker (DM) seeks to achieve, and which has a certain subjective value for them. In [3], a goal is defined as an ideal anticipation of the result of activity that acts as its regulator, while in [7] it is described as a situation or set of situations that must be achieved during the functioning of the system within a specified time frame. Generalizing these definitions, it is possible to identify the main characteristics of a goal: it represents the state of the control object, acts as a regulator of activity, has a temporal nature (a function of time), and is subjectively valuable to the DM.

Definition 7. Goal $pr^{\ gsu}$ of the expert as a control subject res^{su} – state st^{pou} of the control object res^{su}, which has value (utility) for the expert pr^{csu}, that determines its activity (sequence of actions) act^{dsu}, which must be achieved within a period of time pr^t.

The scheme of conceptual goal models can be represented in the form of

$$\begin{aligned}
&pr^{gsu} = R^{res\ st}\left(res^{ou}, st^{pou}\right) \circ \\
&\circ R^{res\ pr}\left(res^{su}, pr^{csu}\right) \circ R^{res\ act}\left(res^{ou}, act^{dsu}\right), \\
&act^{dsu} = R^{act\ act}\left(ACT^{su}, ACT^{su}\right) \circ R^{act\ pr}\left(ACT^{su}, PR^t\right),
\end{aligned} \tag{52}$$

where $R^{res\ st}$ – the "resource-state" relations, which assign to the control object its state; $R^{res\ pr}$ – the "resource-property" relations, which determine the subjective usefulness of the state of the control object for the expert (the control subject);

$R^{pr\ act}$ – the "property-action" relations, which assign to the target state a sequence of actions act^{dsu}; ACT^{su} – the set of possible actions of the expert; $R^{act\ act}$ – the "action-action" relations that determine the order of actions $act^{dsu} \in ACT^{su}$ in the sequence act^{dsu}; $R^{act\ pr}$ – the "action-property" relations between actions with CT^{su} and the time of their execution PR^{t}.

The state st^{pou} of the control object res^{ou} is determined by the values of its properties

$$st^{pou} = R^{res\ pr}\left(res^{ou}, PR^{ou}\right) \circ R^{pr\ val}\left(PR^{ou}, VAL^{ou}\right),$$

where $R^{res\ pr}$ – the "resource-property" relations, which define the set of properties of the control object, and $R^{pr\ val}$ – the "property-value" relations, where each property of the control object is associated with a set of values. One of the properties in the set Pr^{ou} may represent the time associated with the functioning of the control object. In this case, the expert's goal also becomes dynamic and changes over time.

Since, as noted above, the properties of the control object are considered variable when recording cause-and-effect relationships in one or another modeling method, several tools may be used in goal setting. This leads to the complexity of modeling decision-making when it is necessary to compare partial goals described by different methods. Such a situation arises, for example, when Pareto-optimal solutions exist, and it is necessary to select only one of them. Let's assume that there is a control object with two properties pr_1^{ou} and pr_2^{ou}, as well as two states of the control object st_1^{pou}, and st_2^{pou}, so st_1^{pou} closer to the target state, st^{pou}, than st_2^{pou}, according to the first criterion pr_1^{ou} and st_2^{pou}, according to the second one pr_2^{ou}.

If the properties are represented by different variables (for example, stochastic and fuzzy linguistic ones) processed by different methods, it will be difficult to select one of the solutions. However, if the properties are represented by variables of the same type, it is possible to define a metric in a two-dimensional space of vectors representing the admissible states of the control object and determine the distance between st_1^{pou} and st^{pou}, and also st_2^{pou} and st^{pou}, after which they can be compared with one another. To avoid such situations, it is possible to choose a single method for representing all properties that define the state of the control object, and consequently, those used in describing the goals of the decision-maker (DM) and the experts. Analysis has shown that the apparatus of fuzzy set theory [10] is relevant for this purpose.

Definition 8. A fuzzy goal of an expert pr^{gsu} – a fuzzy set defined on the set of states of the control object $ST^{pou} \subseteq ST$, with a membership function $\mu^{pr^{gsu}}(st^{pou})$, or, for brevity $\mu^{gsu}(st^{pou})$.

The membership function $\mu^{gsu}(st^{pou})$ takes values on the set of real numbers within the interval [0; 1]. The greater its value, the closer the state of the control object st^{pou} is to the expert's goal st^{gsu}. The state st^{pou} of the control object is described by a set of its properties $PR^{ou} = \{pr_1^{ou}, \ldots, pr_{N_{pr}^{ou}}^{ou}\}$, represented by variables belonging to one of the classes listed in [5], that is,

$$\mu^{gsu}\left(st^{pou}\right) = \mu^{gsu}\left(pr_1^{ou}, \ldots, pr_{N_{pr}^{ou}}^{ou}\right). \tag{53}$$

The value of the membership function is determined by substituting into (53) the values from the set VAL^{ou} of the control object's properties corresponding to this state, that is, it is described by the expression $\mu^{gsu}(val_1^{ou}, \ldots, val_{N_{val}^{ou}}^{ou})$.

A fuzzy goal of an expert can be represented using one of the methods for constructing membership functions of fuzzy sets considered in [7]. The choice of method is determined by the IDSS developer. Below, to describe the causal relationships between goals and the interaction relations of experts, direct methods [7] for constructing fuzzy goals are used.

When the experts' goals are formalized, pairwise comparison can be performed, and the degree of closeness can be determined. One of the options for determining the degree of closeness between experts' goals is the calculation of the Euclidean or Hamming distance between fuzzy sets [4, 5].

However, their application to determining the degree of similarity of experts' goals is problematic: they are computed only under the condition of convergence of the series or integrals used in them. Otherwise, when $val_{\min}^{ou} = -\infty$ or $val_{\max}^{ou} = \infty$ where $val_{\min}^{ou}$ and $val_{\max}^{ou}$ the minimum and maximum values of the property pr^{ou}, that describes the state st^{pou}, the distance will be equal to infinity, even if one set includes another. In this case, a measure of similarity of fuzzy goals is proposed [6, 10, 12]

$$s\left(A,B\right) = 0.5 \cdot \left(\frac{\int_{val_{\min}^{ou}}^{val_{\max}^{ou}} \mu_{A \cap B}^{gsu}\left(pr^{ou}\right) \mathrm{d}\left(pr^{ou}\right)}{\int_{val_{\min}^{ou}}^{val_{\max}^{ou}} \mu_{A}^{gsu}\left(pr^{ou}\right) \mathrm{d}\left(pr^{ou}\right)} + \frac{\int_{val_{\min}^{ou}}^{val_{\max}^{ou}} \mu_{A \cap B}^{gsu}\left(pr^{ou}\right) \mathrm{d}\left(pr^{ou}\right)}{\int_{val_{\min}^{ou}}^{val_{\max}^{ou}} \mu_{B}^{gsu}\left(pr^{ou}\right) \mathrm{d}\left(pr^{ou}\right)} \right). \tag{54}$$

Analysis shows that, unlike the Euclidean or Hamming distance, relation (53) should be considered a measure of similarity between fuzzy sets rather than a distance between them, since it does not satisfy some of the conditions (specifically, (55) and (56)) required of a distance function in mathematics:

$$d\left(X,Y\right) \geq 0,$$

$$d\left(X,Y\right) = d\left(Y,X\right),$$

$$d\left(X,Z\right) \leq d\left(X,Y\right) + d\left(Y,Z\right), \tag{55}$$

$$d\left(X,X\right) = 0. \tag{56}$$

After determining the measure of similarity between the experts' goals, it becomes possible to define the type of relations among them based on the level of consistency. Let's represent this as fuzzy sets on the universe of values of the similarity measure of goals s (on the set of real numbers within the interval [0;1]). The study identifies three types of relations according to the degree of consistency: competition, neutrality, and cooperation. The greater the value of the measure of similarity of the experts' goals (54), the closer their interaction.

Thus, the membership function of the fuzzy set "cooperation" should attain its maximum value at $s = 1$, while the membership function of the fuzzy set "competition" should attain its maximum at $s = 0$. The maximum of the membership function of the fuzzy set "neutrality" should be equidistant from these maxima, that is, located at the point $s = 0.5$. The membership functions of the fuzzy sets representing the relations of competition $\mu_{\text{"competition"}}(s)$, of neutrality $\mu_{\text{"neutrality"}}\left(s\right) = \left(1 + \left(6 \cdot \left(s - 0.5\right)\right)^{8}\right)^{-1}$, and cooperation $\mu_{\text{"cooperation"}}\left(s\right) = \left(1 + \left(6 \cdot \left(s - 0.5\right)\right)^{8}\right)^{-1}$.

Let's represent the relations between the participants of the IDSS according to the degree of consistency of the linguistic variable cl – "type of relation"

$$cl = \left\langle \beta^{cl}, T^{cl}, U^{cl}, G^{cl}, M^{cl} \right\rangle, \tag{57}$$

where β^{cl} = "type of relation" – the designation of the linguistic variable; T^{cl} = {"competition"; "neutrality"; "cooperation"} – the set of names of the linguistic values of the variable (term set), which constitute the designations of the fuzzy variable; $U^{cl}=[0;1]$ – the domain of definition (universe) of fuzzy variables included in the definition of the linguistic variable; $G^{cl}=\varnothing$ – a syntactic procedure that describes the process of formation of new terms from the elements of the set T; $M^{cl}=\{\mu_{\text{"competition"}}(s), \mu_{\text{"neutrality"}}(s), \mu_{\text{"cooperation"}}(s)\}$ – a semantic procedure that assigns to each term of the set T and to the terms formed by the procedure G a fuzzy set [6, 10, 12].

The value of the linguistic variable *cl* (type of relations) is the term with the maximum value of the membership function. To calculate it, it is necessary to determine the value of the membership function for each fuzzy set representing the relations and compare them with one another.

The fuzzy set with the maximum value of the membership function corresponds to the type of relations established between the pair of experts. It is possible to define the mapping "relation classifier" $rcl:\left(prt_i, prt_j\right) \to T^{cl}$, ag_i, $ag_j \in AG^*$, $i \neq j$, which assigns to each pair of participants of the IDSS (prt_i prt_j), one of the terms t_k^{cl} of the linguistic variable *cl*, that is, the type of relation. The mapping is defined as follows

$$rcl:\left(prt_i, prt_j\right) = \underset{t_k^{cl} \in T^{cl}}{\arg\max}\left(\mu_{t_k^{cl}}\left(s\left(pr_p^{gsu}, pr_q^{gsu}\right)\right)\right), \tag{58}$$

so $r_1^{res\ pr}(prt_i, pr_p^{gsu}), r_1^{res\ pr}(prt_j, pr_q^{gsu}), i \neq j$.

Many values of this mapping form the matrix RCL, which classifies the relations among the participants of the IDSS. The rows and columns of the matrix represent the participants, and the elements $rcl_{i\,j} = rcl(prt_i, prt_j)$ – the class of relations among them. This matrix is used to identify the collective decision-making situation for the complex task.

Depending on the classes of relations present in the IDSS, three collective decision-making situations (micro-level IDSS models) can be distinguished for the task:

1. The cooperation situation $\widetilde{dss}_{coop}$, when the IDSS consists of cooperative and neutral participants and there are no competitive relations.

2. The neutrality situation $\widetilde{dss}_{neut}$ occurs when all relations in the IDSS are neutral.

3. The competition situation $\widetilde{dss}_{comp}$ occurs when the IDSS contains at least one pair of experts with a competitive relationship.

In such IDSSs, neutral and cooperative participants may also be present. In the presence of cooperative participants, they are regarded as a single notional participant; in this case, all remaining participants are either competitive or neutral.

Thus, the process of self-organization based on goal analysis can be divided into two parts: identification of the current collective decision-making situation (the micro-level model of the IDSS) and selection, from the set of possible situations, of the desired collective decision-making situation that is relevant to the conditions of the given task. Taking this into account, the self-organization model (27) can be rewritten as follows

$$\begin{aligned} so^{goa} &= r_2^{res\ act}\left(dss, ACT^{sen}\right) \circ r_1^{act\ res}\left(ACT^{sen}, env\right) \circ \\ &\circ R_1^{res\ res}\left(\widetilde{DSS}, \widetilde{DSS}\right) \circ r_3^{res\ res}\left(dss, prt^{dm}\right) \circ \\ &\circ r_2^{res\ act}\left(prt^{dm}, act_{ia}\right) \circ r_1^{act\ res}\left(act_{ia}, \widetilde{dss}_{cur}\right) \circ \\ &\circ r_2^{res\ act}\left(prt^{dm}, act_{ac}\right) \circ r_1^{act\ res}\left(act_{ac}, \widetilde{DSS}\right) \circ \\ &\circ r_2^{act\ res}\left(act_{ac}, \widetilde{dss}_{des}\right), \end{aligned} \tag{59}$$

where act_{ia} – the DM's action "identification of the current collective decision situation"; act_{ac} – the DM's action "selection of the desired collective decision situation from the set of possible ones"; $\widetilde{dss}_{cur}$ – the current collective decision situation (micro-level model of the IDSS); $\widetilde{dss}_{des}$ – the collective decision situation desired by the DM in terms of the task parameters and its knowledge about the effectiveness of a particular situation from the set $\widetilde{DSS}$ of possible in the IDSS; $r_2^{res\ act}$ – the "performs" relation, which links a subject and the action it performs; $r_1^{act\ res}$ – the "has as an object" relation, which links an action and its resource; $r_2^{act\ res}$ – the "has as a result" relation, which links an action and the result of its execution.

The first stage of identification act_{ia} (59) collective decision situations – formalization of the experts' goals considering the definition of the fuzzy goal (53). After the fuzzy goals of all experts have been determined, the next stage of identification is performed, act_{ia} (59), collective decision situations – pairwise comparison of goals and determination of their degree of consistency using measure (54). Next, the type of relations between the experts is determined according to the degree of consistency using the linguistic variable cl "type of relation" (57).

The final stage of identification act_{ia} (59) collective decision situations – recognition of the collective decision situation using the matrix CL′. Depending on the classes of relations present in the matrix CL′, three collective decision-making situations are distinguished: cooperation $\widetilde{dss}_{coop}$, neutrality $\widetilde{dss}_{neut}$, and competition $\widetilde{dss}_{comp}$.

After identifying the current collective decision-making situation, the decision-maker (DM) selects act_{ac} (59) from the set of possible collective decision-making situations that correspond to the conditions of the given task. Depending on the task parameters and their knowledge of the effectiveness of a particular collective decision situation, the decision-maker (DM) may seek to establish one of them. This is necessary to increase the efficiency of the IDSS operation or to attempt to change it if the discussion reaches an impasse.

5. 3. 3. Model of a hybrid intelligent multi-agent IDSS with self-organization based on the analysis of goal consistency of agents

Self-organization of the IDSS is understood as the process of changing the architecture of the IDSS by an agent that simulates the DM and is part of the system, based on the analysis of interactions among other agents, in order to improve the quality of decisions. The model of a hybrid intelligent multi-agent IDSS with self-organization based on the analysis of goal consistency of agents is described as follows:

$$himas = \left(AG^*, env, INT^*, ORG, \{so, sl\}\right), \tag{60}$$

$$AG^* = \left\{ag_1, \ldots, ag_n, ag^{dm}\right\}, \tag{61}$$

$$INT^* = \left\{prot, lang, ont, rcl\right\}, \tag{62}$$

$$ORG = ORG_{coop} \cup ORG_{neut} \cup ORG_{comp},$$

$$ORG_{coop} \cap ORG_{neut} = \varnothing,$$

$$ORG_{coop} \cap ORG_{comp} = \varnothing, ORG_{comp} \cap ORG_{neut} = \varnothing, \quad (63)$$

$$act_{himas} = \left(\bigcup_{ag \in AG^*} act_{ag} \right) \cup act_{ia} \cup act_{ac} \cup act_{col}, \quad (64)$$

$$act_{ag} = \left(MET_{ag}, IT_{ag} \right), ag \in AG^*, \left| \bigcup_{ag \in AG^*} IT_{ag} \right| \geq 2, \quad (65)$$

$$ag = ag \vee himas, \quad (66)$$

where AG^* – the set of IDSS agents, including the decision-making agent ag_{dm}; n – the number of expert agents in the IDSS; env – the environment in which the IDSS operates; INT^* – the set of elements for structuring interactions among agents; ORG – the set of basic organizational structures of the IDSS corresponding to specific functions (roles) of the agents and the established relations among them, which includes the following subsets; ORG_{coop}, ORG_{neut}, ORG_{comp} – the set of architectures with cooperating, neutral, and competing agents, respectively; $prot$ – the protocol of agent interaction, which defines the format of the messages exchanged by the agents; $lang$ – the message exchange language, which defines the lexicon and syntax of the transmitted messages; ont – the domain model, on the basis of which the semantics of the messages transmitted by the agents are defined; rcl – the classifier of relations between the IDSS agents; act_{himas} – the overall function of the IDSS; act_{ag} – the function of the agent $ag \in AG^*$; act_{ia} – the "interaction analysis" function; act_{ac} – the "architecture selection" function; MET_{ag} – the set of task-solving methods implemented by the agent $ag \in AG^*$; IT_{ag} – the set of intelligent technologies of the IDSS [6], within which the set of methods is implemented MET_{ag}.

As noted, from the set of IDSS agents (61), one agent is distinguished as the decision-making agent. This agent, among other functions, organizes the interactions of agents in the hybrid intelligent multi-agent system (HIMAS), using two functions (64):

1) act_{ia} – the "interaction analysis" function;

2) act_{ac} – the "architecture selection" function.

The function "interaction analysis" act_{ia} is applied to monitor the interaction of agents and to identify the IDSS architecture based on the analysis of the consistency of the agents' goals. Based on its results, the DM, by means of the "architecture selection" function act_{ac} may determine the necessity of changing the type of agent relations and adjusting their goals. For this purpose, the DM must possess a knowledge base used by the "architecture selection" function act_{ac} and reflects the dependence of the efficiency of the IDSS operation on the consistency of the agents' goals.

A change in architecture means that, from a certain moment, the IDSS modifies its functioning algorithm, exhibiting the property of self-organization and transitioning to a basic architecture of a different type. The algorithms of these functions will be discussed in detail below.

The set INT* is supplemented with another element for structuring interactions among agents – the relation classifier rcl. It analyzes the fuzzy goals pr^{gsu} of the agents and evaluates the degree of their *consistency* using the similarity measure $s(A, B)$ (54). Then, using the mapping (58), it constructs the matrix RCL, classifying the relationships among the agents into one of the following types: competition, neutrality, or cooperation. Based on this matrix, the consistency of the IDSS agents and the class of its basic architecture are determined.

The set of basic IDSS architectures ORG is divided by the degree of consistency into three pairwise disjoint subsets (63):

1) the IDSS architecture with cooperating agents, which consists only of cooperative and neutral agents, with competitive relationships completely absent ($ORG_{coop} \subseteq ORG$);

2) the IDSS architecture with neutral agents, in which only neutral relationships are present ($ORG_{neut} \subseteq ORG$);

3) the IDSS architecture with competing agents, in which there is at least one pair of agents with a competitive relationship.

In such IDSS architectures, neutral and cooperative agents may also be present; when cooperative agents exist, they are treated as a single "super-agent," and in this case, all agents become either competing or neutral ($ORG_{comp} \subseteq ORG$).

During operation, the IDSS may transition from one basic architecture to another to improve the quality of decisions by establishing relationships among agents that are relevant to the conditions of the task being solved. The decision quality criteria are defined either by the IDSS developer at the design stage or by the user during operation. This, however, implies the presence of an external control source. The architecture of a self-organizing IDSS can and should change even if the quality criterion remains unchanged.

According to (65), each agent function is represented by a two-component tuple (MET_{ag}, IT_{ag}). Because of (65), an IDSS must employ at least two intelligent technologies in order to be considered an IDSS and to remain relevant for modeling complex tasks. In addition, according to (64), two functions of the decision-maker (DM) are distinguished for managing the interaction of the IDSS agents: act_{ia} – the function "interaction analysis" and act_{ac} – the function "architecture selection." Let's consider the algorithms of these functions.

As a result of the execution of the function "interaction analysis," the basic architecture of the IDSS is identified, which enables the DM, through the "architecture selection" function f_{ac} to determine the necessity (or absence thereof) of its modification to improve the quality of decisions. Thus, the architecture of the IDSS changes to one from the set ORG (60), and the system self-organizes.

After determining the type of IDSS architecture, the decision-making agent, by means of the "architecture selection" function act_{ac} may determine the necessity of replacing one type of agent relations with another and adjusting their goals. However, it is evident that for deciding regarding the replacement of one IDSS architecture with another, information about only the current type of IDSS architecture is insufficient. For such a decision, the DM requires a knowledge base concerning which of the IDSS architecture types is more effective under particular conditions or for a specific class of tasks.

As a result of the analysis of methods for constructing knowledge bases for the implementation of the "architecture selection" function act_{ac} fuzzy inference systems were selected.

The output data of the "architecture selection" function act_{ac} – the conditions of the task being solved. The result is the value of the degree of confidence [17] in the selection of each IDSS architecture, as well as the adjustment of the DM's knowledge base. The fuzzy inference model of the DM mod_{ac} can be formulated as follows

$$mod_{ac} = < RUL, X, Y, F^{\mu}, F^{y}, F^{sl}, I^{fsl} >, \quad (68)$$

where RUL – the knowledge base as a set of symbolic rules rul_k, $k = 1,...N_{RUL}$, which use fuzzy sets as premises and conclusions A_i^k, B_j^k, respectively, such that $A_i^k \subseteq X_i$, $B_j^k \subseteq Y_j$,

$i=1,\ldots,n$, $j=1,\ldots,m$; X, Y – a set of input and output linguistic variables, respectively; $F^{\mu}=F_X^{\mu}\cup F_Y^{\mu}$ – a set of membership functions of fuzzy variables of the input and output linguistic variables, respectively; F^{y} – a set of defuzzification functions; F^{sl} – a set of functions for adjusting the parameters of the membership functions of fuzzy variables from the set F^{μ}.

The interpreter I^{fsl} is defined as follows

$$I^{fsl}=\left\{I^{fsl1},I^{fsl2},I^{fsl3},I^{fsl4},I^{fsl5},I^{fsl6},I^{fsl7},I^{fsl8},I^{fsl9}\right\},$$

where $I^{fsl1},\ldots,I^{fsl9}$– processes of the standard Mamdani fuzzy inference system [5]: I^{fsl1} – fuzzification, I^{fs2} – aggregation, I^{fsl3} – activation, I^{fsl4} – accumulation, I^{fsl5} – defuzzification; I^{fsl6} – architecture selection; I^{fsl7} – decision; I^{fsl8} – error determination; I^{fsl9} – learning. The processes $I^{fsl1},\ldots,I^{fsl9}$ are performed iteratively

$$I^{fsl1}\to I^{fsl2}\to I^{fsl3}\to I^{fsl4}\to$$
$$\to I^{fsl5}\to I^{fsl6}\to I^{fsl7}\to I^{fsl8}\to I^{fsl9}. \tag{69}$$

Model (68) is a Mamdani fuzzy inference system with self-learning. The self-learning processes $I^{fsl6},\ldots,I^{fsl9}$ enable the system to modify the domain knowledge in order, first, to correct possible developer errors in its definition and, second, to automatically maintain the knowledge base in an up-to-date state over time and with changing task conditions.

Moreover, due to the use of Mamdani fuzzy inference, the knowledge base remains transparent and interpretable by humans, unlike in Takagi-Sugeno fuzzy inference systems or neuro-fuzzy hybrids such as ANFIS or GARIC, where variables are often hidden. This approach is consistent with [1, 2]; however, in the proposed system, learning is performed according to the backpropagation error algorithm. Let's consider the essence of these processes $I^{fsl1},\ldots,I^{fsl9}$ (69).

Fuzzy inference $I^{fsl1},\ldots,I^{fsl5}$ is implemented using the Mamdani system [1]. Its input receives information (dimensionality) about the task being solved in the form of a deterministic variable. The output of the system is the values of the confidence degrees in selecting the IDSS architecture, represented as deterministic variables. In the rules of the fuzzy knowledge base, the input linguistic variable is "IDSS dimensionality" sz, and the output variables are "confidence degree in selecting an IDSS with cooperating agents" pcp, "confidence degree in selecting an IDSS with neutral agents" pnt, and "confidence degree in selecting an IDSS with competing agents" pcn.

Let's define the linguistic variable sz "IDSS dimensionality"

$$sz=\left\langle \beta^{sz},T^{sz},U^{sz},G^{sz},M^{sz}\right\rangle,$$

where β^{sz} = "dimension of the IDSS" – the designation of the linguistic variable; T^{sz} = {"small"; "medium"; "large"} – the set of its values (term set), the designations of the fuzzy variable; $U^{sz}=[3;100]$ – the domain of definition (universe) of fuzzy variables included in the definition of the linguistic variable; $G^{sz}=\varnothing$ – a syntactic procedure that describes the process of formation from the elements of the set T^{sz} new terms; $M^{sz}=\{\mu_{low}^{sz}(u^{sz}),\mu_{medium}^{sz}(u^{sz}),\mu_{high}^{sz}(u^{sz})\}$ – a semantic procedure that assigns to each term of the set T^{sz}, as well as to each new term formed by the procedure G^{sz}, a meaningful content through the formation of the corresponding fuzzy set.

The membership functions of fuzzy sets with M^{sz} are described by the expressions

$$\mu_{low}^{sz}\left(u^{sz}\right)=\left(1+e^{-a_{sz1}\left(u_{sz}-c_{sz1}\right)}\right)^{-1},$$

where $a_{sz1}=-0.375$, $c_{sz1}=15$,

$$\mu_{medium}^{sz}\left(u^{sz}\right)=e^{\frac{\left(u_{sz}-c_{sz2}\right)^2}{2\sigma_{sz2}^2}}$$

where $\sigma_{sz2}=10$, $c_{sz2}=30$,

$$\mu_{high}^{sz}\left(u^{sz}\right)=\left(1+e^{-a_{sz3}\left(u_{sz}-c_{sz3}\right)}\right)^{-1},$$

where $a_{sz3}=0.375$, $c_{sz3}=45$, the parameter values a_{sz1}, c_{sz1}, σ_{sz1}, σ_{sz2}, c_{sz2}, a_{sz2}, c_{sz3} are specified approximately and must be adjusted during self-learning of the fuzzy inference system. Let's define the linguistic variables pcp, pnt, pcn by the expressions:

$$pcp=\left\langle \beta^{pcp},T^{pcp},U^{pcp},G^{pcp},M^{pcp}\right\rangle,$$

$$pnt=\left\langle \beta^{pnt},T^{pnt},U^{pnt},G^{pnt},M^{pnt}\right\rangle,$$

$$pcn=\left\langle \beta^{pcn},T^{pcn},U^{pcn},G^{pcn},M^{pcn}\right\rangle,$$

where the designations of the linguistic variables β^{pcp} = "the degree of confidence in the selection of the IDSS with cooperating agents org_{coop}"; β^{pnt} = "the degree of confidence in the selection of the IDSS with neutral agents org_{neut}"; β^{pcn} = "the degree of confidence in the selection of the IDSS with competing agents org_{conc}"; $T^{pcp}=T^{pnt}=T^{pcn}$ = {"low"; "medium"; "high"} – sets of their values (term set), each of which is the designation of a fuzzy variable; $U^{pcp}=U^{pnt}=U^{pcn}=\left[0;1\right]$ – domains of definition (universes) of fuzzy variables included in the definition of linguistic variables; $G^{pcp}=G^{pnt}=G^{pcn}=\varnothing$ – syntactic procedures that describe the processes of formation from the elements of the sets T^{pcp},T^{pnt},T^{pcn} new terms;

$$M^{pcp}=\{\mu_{low}^{pcp}(u^{pcp}),\mu_{medium}^{pcp}(u^{pcp}),\mu_{high}^{pcp}(u^{pcp})\},$$

$$M^{pnt}=\{\mu_{low}^{pnt}(u^{pnt}),\mu_{medium}^{pnt}(u^{pnt}),\mu_{high}^{pnt}(u^{pnt})\},$$

$$M^{pcn}=\{\mu_{low}^{pcn}(u^{pcn}),\mu_{medium}^{pcn}(u^{pcn}),\mu_{high}^{pcn}(u^{pcn})\}$$

– semantic procedures that assign to each term of the set T^{pcp}, T^{pnt}, T^{pcn}, as well as to each new term formed by the procedure G^{pcp}, G^{pnt}, G^{pcn}, a meaningful content through the formation of the corresponding fuzzy set.

Membership functions of fuzzy sets M^{pcp}, M^{pnt}, M^{pcn} are described by the expressions

$$\mu_{low}^{pcp}\left(u^{pcp}\right)=\left(1+e^{-a_{pcp1}\left(u_{pcp}-c_{pcp1}\right)}\right)^{-1},$$

where $a_{pcp1}=-25$, $c_{pcp1}=0.34$,

$$\mu_{medium}^{pcp}\left(u^{pcp}\right)=e^{\frac{\left(u_{pcp}-c_{pcp2}\right)^2}{2\sigma_{pcp2}^2}},$$

where $\sigma_{pcp2}=0.13$, $c_{pcp2}=0.5$,

$$\mu_{high}^{pcp}\left(u^{pcp}\right)=\left(1+e^{-a_{pcp3}\left(u_{pcp}-c_{pcp3}\right)}\right)^{-1},$$

where $a_{pcp3}=25$, $c_{pcp3}=0.66$,

$$\mu_{low}^{pnt}\left(u^{pnt}\right)=\left(1+e^{-a_{pnt1}\left(u_{pnt}-c_{pnt1}\right)}\right)^{-1},$$

where $a_{pnt1}=-25$, $c_{pnt1}=0.34$,

$$\mu_{medium}^{pnt}\left(u^{pnt}\right)=e^{\frac{\left(u_{pnt}-c_{pnt2}\right)^2}{2\sigma_{pnt2}^2}},$$

where $\sigma_{pnt2}=0.13$, $c_{pnt2}=0.5$,

$$\mu_{high}^{pnt}\left(u^{pnt}\right)=\left(1+e^{-a_{pnt3}\left(u_{pnt}-c_{pnt3}\right)}\right)^{-1},$$

where $a_{pnt3}=25$, $c_{pnt3}=0.66$,

$$\mu_{low}^{pcn}\left(u^{pcn}\right)=\left(1+e^{-a_{pcn1}\left(u_{pcn}-c_{pcn1}\right)}\right)^{-1},$$

where $a_{pcn1}=-25$, $c_{pcn1}=0.34$,

$$\mu_{medium}^{pcn}\left(u^{pcn}\right)=e^{\frac{\left(u_{pcn}-c_{pcn2}\right)^2}{2\sigma_{pcn2}^2}},$$

where $\sigma_{pcn2}=0.13$, $c_{pcn2}=0.5$,

$$\mu_{high}^{pcn}\left(u^{pcn}\right)=\left(1+e^{-a_{pcn3}\left(u_{pcn}-c_{pcn3}\right)}\right)^{-1},$$

where $a_{pcn3}=25$, $c_{pcn3}=0.66$, in which the values of the parameters a_{pcp1}, c_{pcp1}, σ_{pcp2}, c_{pcp2}, a_{pcp3}, c_{pcp3}, a_{pnt1}, c_{pnt1}, σ_{pnt2}, c_{pnt2}, a_{pnt3}, c_{pnt3}, a_{pcn1}, c_{pnt1}, σ_{pcn2}, c_{pcn2}, a_{pcn3}, c_{pcn3} are specified approximately and are adjusted during the self-learning process of the fuzzy inference system. The considered linguistic variables are connected by a set RUL of rules of the Mamdani fuzzy inference system [1]. The rules have the form: $ant_i \xrightarrow{rul_i} cns_i$, where ant_i – denotes the antecedent of the i-th rul_i, cns_i – the consequent of the rul_i, $i = 1,..., N_{rul}$, where N_{rul} – the number of rules in the fuzzy knowledge base of the decision-making agent.

The Mamdani fuzzy inference with self-learning consists of the following stages:

1) Fuzzification I^{fsl1} for establishing the correspondence between the values of $u^{sz}\in U^{sz}$ and the value of the membership function of the corresponding term of the linguistic variable sz: for each term, $t^{sz}\in T^{sz}$ the value $y_{tsz}^{fuz}=\mu_{tsz}^{sz}\left(u^{sz}\right)$ is calculated;

2) Aggregation I^{fsl2} degenerates, since only one input fuzzy variable is used. The truth degree y_i^{agr} of the premise ant_i of the rule rul_i corresponds to the value of the truth degree y_{tsz}^{fuz} of the term t_{sz} of the input variable from ant_i;

3) Activation I^{fsl3} – for each term of $t_{pcp}\in T_{pcp}$, $t_{pnt}\in T_{pnt}$, $t_{pcn}\in T_{pcn}$ the output linguistic variables pcp, pnp, pcn, that is included in the conclusion at the degree determined during the computation cns_i of rule rul_i, the function of the min-activation method is defined as follows:

$$\mu_{tpcp\ i}^{pcp}\left(u^{pcp}\right)=\min\left(y_i^{agr},\mu_{tpcp}^{pcp}\left(u^{pcp}\right)\right),$$

$$\mu_{tpnt\ i}^{pnt}\left(u^{pnt}\right)=\min\left(y_i^{agr},\mu_{tpnt}^{pnt}\left(u^{pnt}\right)\right),$$

$$\mu_{tpcn\ i}^{pcn}\left(u^{pcn}\right)=\min\left(y_i^{agr},\mu_{tpcn}^{pcn}\left(u^{pcn}\right)\right);$$

4) Accumulation I^{fsl4} – for each output linguistic variable, the final membership functions μ^{pcp}, μ^{pnt}, μ^{pcn} are determined by combining the membership functions of the terms $\mu_{tpcp\ i}^{pcp}$, $\mu_{tpnt\ i}^{pnt}$, $\mu_{tpcn\ i}^{pcn}$, activated at the previous step. The type of the membership functions is determined according to the rules: $\mu^{pcp}(u^{pcp})=\max_i(\mu_{tpcp\ i}^{pcp}(u^{pcp}))$, $\mu^{pnt}(u^{pnt})=\max_i(\mu_{tpnt\ i}^{pnt}(u^{pnt}))$, $\mu^{pcn}(u^{pcn})=\max_i(\mu_{tpcn\ i}^{pcn}(u^{pcn}))$;

5) Defuzzification I^{fsl5}, the results of defuzzification of the variables pcp, pnt, pcn – real numbers u^{pcp}, u^{pnt}, u^{pcn} respectively, which are obtained using the centroid defuzzification method:

$$u^{pcp}=\frac{\int_{U^{pcp}} u^{pcp}\mu^{pcp}\left(u^{pcp}\right)\mathrm{d}u^{pcp}}{\int_{U^{pcp}} \mu^{pcp}\left(u^{pcp}\right)\mathrm{d}u^{pcp}},$$

$$u^{pnt}=\frac{\int_{U^{pnt}} u^{pnt}\mu^{pnt}\left(u^{pnt}\right)\mathrm{d}u^{pnt}}{\int_{U^{pnt}} \mu^{pnt}\left(u^{pnt}\right)\mathrm{d}u^{pnt}},$$

$$u^{pcn}=\frac{\int_{U^{pcn}} u^{pcn}\mu^{pcn}\left(u^{pcn}\right)\mathrm{d}u^{pcn}}{\int_{U^{pcn}} \mu^{pcn}\left(u^{pcn}\right)\mathrm{d}u^{pcn}};$$

or

$$u_{pcp}=\frac{\int_{U_{pcp}} u_{pcp}\mu_{pcp}\left(u_{pcp}\right)\mathrm{d}u_{pcp}}{\int_{U_{pcp}} \mu_{pcp}\left(u_{pcp}\right)\mathrm{d}u_{pcp}};$$

as a result, crisp values of the output linguistic variables are computed:

– "Confidence degree in selecting an IDSS with cooperating agents" pcp;

– "Confidence degree in selecting an IDSS with neutral agents" pnt;

– "Confidence degree in selecting an IDSS with competing agents" pcn.

That is, for each IDSS architecture, the confidence value in its selection is determined;

6) selection of the architecture I^{fsl6}. The confidence degrees for selecting the architectures from the previous step are normalized so that their sum equals one: $u^{pcp}+u^{pnt}+u^{pcn}=1$ according to the rules: $u^{pcp\prime}=u^{pcp}/(u^{pcp}+u^{pnt}+u^{pcn})$, $u^{pnt\prime}=$ $=u^{pnt}/(u^{pcp}+u^{pnt}+u^{pcn})$, $u^{pcn\prime}=u^{pcn}/(u^{pcp}+u^{pnt}+u^{pcn})$, and the IDSS architecture org is selected randomly from among org_{coop}, org_{neut}, org_{conc} according to the normalized values of the confidence degrees $u^{pcp'}$, $u^{pn't}$, $u^{pcn'}$, accordingly;

7) the solution of a complex task I^{fsl7}. The operation of the IDSS is simulated using the brainwriting method [13] with the selected IDSS architecture; in this process, the decision-maker (DM) "observes" the course of solving the complex task and records the presence (syn = 1) or absence (syn = 0) of the manifestation of the synergistic effect, that is, whether the IDSS has obtained a higher-quality solution compared to the solutions of individual experts.

8) extermination of the absolute error of the fuzzy inference I^{fsl8}. It is performed according to the expression

$$er=0.5\cdot\left(\begin{array}{l}\left(d^{pcp}-u^{pcp}\right)^2+\\+\left(d^{pnt}-u^{pnt}\right)^2+\left(d^{pcn}-u^{pcn}\right)^2\end{array}\right), \tag{70}$$

where er – the absolute error of the fuzzy inference; d^{pcp}, d^{dnt}, d^{pcp} – the desired values of the confidence degrees in the selection of architectures as defined by the decision-maker (DM); u^{pcp}, u^{dnt}, u^{pcp}, that is, the values at which the probability of selecting an architecture that produces a synergistic effect under the given task conditions would be higher;

9) study I^{fsl9} of the fuzzy inference system. Here, the parameters of the membership functions of the input linguistic variable sz and the output linguistic variables pcp, pnt, and pcn are adjusted; during training, the gradient descent method with a variable learning rate coefficient [14] is used

$$par(t+1)=par(t)-\eta\frac{\partial\, er}{\partial\, par}, \tag{71}$$

where par – any of the parameters of the membership functions (a_{sz1}, c_{sz1}, σ_{sz1}, σ_{sz2}, c_{sz2}, a_{sz3}, c_{sz3}, a_{pcp1}, c_{pcp1}, σ_{pcp2}, c_{pcp2}, a_{pcp3}, c_{pcp3}, a_{pnt1}, c_{pntp1}, σ_{pnt2}, c_{pnt2}, a_{pnt3}, c_{pnt3}, a_{pcn1}, c_{pcn1}, σ_{pcn2}, c_{pcn2}, σ_{pcn3}, c_{pcn3}), that are included in the definition of the input or output linguistic variables; t is the ordinal number of the task being solved (iteration of the fuzzy inference system training); η – the correction step, $0<\eta<1$, $\eta(t)=t^{-1}$; for the parameters par_{pcp}, par_{pnt}, par_{pcn} for the membership functions of the output variables pcp, pnt, and pcn, expression (71) is written as:

$$par_{pcp}(t+1)=par_{pcp}(t)-\eta\frac{\partial\, er}{\partial\, u^{pcp}}\frac{\partial\, u^{pcp}}{\partial\, par_{pcp}}=$$
$$=par_{pcp}(t)+\eta\left(d^{pcp}-u^{pcp}\right)\frac{\partial\, u^{pcp}}{\partial\, par_{pcp}},$$

$$par_{pnt}(t+1)=par_{pnt}(t)-\eta\frac{\partial\, er}{\partial\, u^{pnt}}\frac{\partial\, u^{pnt}}{\partial\, par_{pnt}}=$$
$$=par_{pnt}(t)+\eta\left(d^{pnt}-u^{pnt}\right)\frac{\partial\, u^{pnt}}{\partial\, par_{pnt}},$$

$$par_{pcn}(t+1)=par_{pcn}(t)-\eta\frac{\partial\, er}{\partial\, u^{pcn}}\frac{\partial\, u^{pcn}}{\partial\, par_{pcn}}=$$
$$=par_{pcn}(t)+\eta\left(d^{pcn}-u^{pcn}\right)\frac{\partial\, u^{pcn}}{\partial\, par_{pcn}},$$

for the parameters par_{sz} of the membership functions of the input variable sz, expression (71) is written as

$$par_{sz}(t+1)=par_{sz}(t)+ +\eta\left(\begin{array}{l}\left(d^{pcp}-u^{pcp}\right)\sum\limits_{i=1}^{N_{rul}}\frac{\partial\, u^{pcp}}{\partial\, y_i^{agr}}\frac{\partial\, y_i^{agr}}{\partial\, par_{sz}}+\\ +\left(d^{pnt}-u^{pnt}\right)\sum\limits_{i=1}^{N_{rul}}\frac{\partial\, u^{pnt}}{\partial\, y_i^{agr}}\frac{\partial\, y_i^{agr}}{\partial\, par_{sz}}+\\ +\left(d^{pcn}-u^{pcn}\right)\sum\limits_{i=1}^{N_{rul}}\frac{\partial\, u^{pcn}}{\partial\, y_i^{agr}}\frac{\partial\, y_i^{agr}}{\partial\, par_{sz}}\end{array}\right). \tag{72}$$

After the adjustment of the parameters of the membership functions of the input linguistic variable, the training stage and the operation of the fuzzy inference system are completed.

When solving a new computational task, the fuzzy inference system begins operation from the first stage, using new values for all the parameters of the membership functions.

5. 4. Fuzzy system of model disputation in an intelligent decision support system

Autonomous models are models that simulate the reasoning lines of experts when solving partial tasks obtained because of stratification. While solving partial tasks and obtaining partial (intermediate) solutions, an autonomous model interacts during disputation with other models, during which integration of the partial solutions occurs and the final solution is obtained. Thus, the dispute serves as a mechanism for integrating the partial solutions of tasks obtained by autonomous models.

Therefore, disputation in an IDSS represents a hybrid model of knowledge integration among experts in situations where their opinions diverge. A dispute is a mechanism of communication and interaction among autonomous models in the IDSS. The strategy of their interaction in a dispute is expressed through "Cooperation-Compromise-Consensus," and the method of forming the final solution from the partial ones is integration as an aggregating and combining element.

When solving a task in an IDSS that includes model disputation, two levels of knowledge integration are distinguished:

1. Integration at the point of divergence when a model solves a subtask with a certain objective function "collides" in a dispute with another model, each having its own "viewpoint," objective function, and set of parameters. At the point of divergence, knowledge comparison takes place (identification of the "stronger" knowledge), their combination according to the main objective function, and the development of the subsequent path of problem solving. The knowledge integration formula in this case is represented by production rules of the form "if … then …".

2. Integration during the formation of the resulting solution of the task from the results of solving partial tasks at the points of divergence, where the overall solution is represented as a sequence of solutions from different models, leading to the desired result. The knowledge integration formula in this case takes the form of a set-theoretic union of the experts' solutions obtained at different stages of problem solving $dec_1^u \cup dec_2^u \cup dec^u$, where dec_1^u, dec_2^u, dec^u – solutions obtained at different stages by different models.

Horizontal integration is ensured by a sequence of disputes during the process of solving a complex task.

Integration at the point of divergence (in depth) is achieved through the willingness of experts (models) to make compromises and engage in discussions to ultimately reach consensus. Each model specifies a region of compromise solutions, a certain function f^t of trust, or a subjective assessment of the quality of the solution, which indicates the extent to which the model trusts a particular solution from the set of possible ones.

The trust function f^t resembles the membership function used in fuzzy systems [2]. With the help of this function, each model defines, for every parameter, an admissibility interval to the left and right of the solution it has obtained.

The trust function is specified by each model for every parameter of the task being solved. The trust function of a model for each parameter is determined by the ranges of its trust in that parameter and by the parameter value corresponding to the solution adopted by the model at a specific stage of autonomous operation. That is, the trust function of model j for parameter i is defined as $f_j^{t\,i}(p_i, a_{j\,i}, b_{j\,i}, c_{j\,i})$, where:

– a_{ji} – the trust range of model j for parameter i to the left of the value b_{ji};

– b_{ji} – the value of parameter i for the solution proposed by model j at the point of divergence;

– c_{ji} – the trust range of model j for parameter i to the right of the value b_{ji}. To establish all trust functions before the beginning of the task execution, each model must determine the trust intervals for every parameter.

As an example, let's consider a dispute between models at the point of divergence. Let two models (M_1, M_2) solve a task $Z=\{p_1\}$, which solution is the value of a single parameter p_1; b_{11} and b_{21} – the values of parameter p_1 for the first and second models, respectively, with which they have reached the point of divergence; a_{11} and c_{21} – are the trust ranges for the first model; a_{21} and c_{21} – the trust ranges for the second model; $f_1^{t\,1}(p_1, a_{1\,1}, b_{1\,1}, c_{1\,1})$ – the trust function for the first model

$$f_1^{t\,1}\left(p_1,a_{1\,1},b_{1\,1},c_{1\,1}\right)=$$
$$=\begin{cases}0,\ b_{2\,1}+c_{2\,1}<p_1\le b_{1\,1}-a_{1p},\\ \left(p_1-b_{1\,1}+a_{1\,1}\right)\left(a_{1\,1}\right)^{-1},\ b_{1\,1}-a_{1\,1}<p_1\le b_{1\,1},\\ \left(b_{1\,1}+c_{1\,1}-p_1\right)\left(c_{1\,1}\right)^{-1},\ b_{1\,1}<p_1\le b_{1\,1}+c_{1\,1},\end{cases}\qquad(73)$$

and $f_2^{t\,1}(p_1,a_{2\,1},b_{2\,1},c_{2\,1})$ – for the second model

$$f_2^{t\,1}\left(p_1,a_{2\,1},b_{2\,1},c_{2\,1}\right)=$$
$$=\begin{cases}0,\ b_{2\,1}+c_{2\,1}<p_1\le b_{2\,1}-a_{2p},\\ \left(p_1-b_{2\,1}+a_{2\,1}\right)\left(a_{2\,1}\right)^{-1},\ b_{2\,1}-a_{2\,1}<p_1\le b_{2\,1},\\ \left(b_{2\,1}+c_{2\,1}-p_1\right)\left(c_{2\,1}\right)^{-1},\ b_{2\,1}<p_1\le b_{2\,1}+c_{2\,1},\end{cases}\qquad(74)$$

where p^* – the parameter value adopted because of the dispute. Each model, at the point of divergence, provides its own solution and its own trust function. In this case, the following variants are possible:

1) The value b_{11} coincides with the value b_{21}; that is, the dispute is redundant. This parameter value (identical for both models) becomes the starting point for the further solution of the partial tasks by each model

$$p^*=b_{1\,1}=b_{2\,1};$$

2) The parameter values of the experts' solutions do not coincide

$$b_{1\,1}\ne b_{2\,1}.$$

Several variants are possible here:

a) the parameter value of the solution of one model falls within the trust region of the other, but not vice versa;

b) the parameter value of the solution of one model falls within the trust function region of the other and vice versa;

c) the trust function regions of the models intersect, but the parameter values of the models' solutions lie outside the intersection zone;

d) the trust functions of the models do not intersect.

Let's consider in more detail the situations of disputation listed in item 2.

The situation in which the parameter value of one model falls within the trust region of the other, but not vice versa, is expressed as

$$\begin{cases}b_{l\,1}\notin\left(b_{j\,1}-a_{j\,1},b_{j\,1}+c_{j\,1}\right),\\ b_{j\,1}\in\left(b_{l\,1}-a_{l\,1},b_{l\,1}+c_{l\,1}\right),\end{cases}$$

where $j,l\in\mathbb{N},\ j,l\in\left[1,2\right],\ l\ne j$.

By analogy with a dispute between experts in an IDSS, this situation corresponds to a case where the arguments of the first expert persuade the second to reach a compromise. If one of the experts has defined their trust region and the result of the other falls within this region, it is reasonable to state that the argumentation of the latter expert proved to be stronger.

The degree of agreement of the second model with the first k_2^t – the value of the trust function of the second model for the given parameter value p_1, which corresponds to the solution of the first model. In the case of the degree of agreement of the second model with the first, it is necessary to find the value of the trust function for the second model (74) for $p_1 = b_{11}$

$$k_2^t=f_2^{t\,1}\left(b_{1\,1},a_{2\,1},b_{2\,1},c_{2\,1}\right)=\left(b_{1\,1}-b_{2\,1}+a_{2\,1}\right)\left(a_{2\,1}\right)^{-1}.$$

As a result of the dispute, integration of the models' knowledge took place, and the second model agreed with the first. The outcome of the dispute is a collective solution corresponding to the solution of the first model ($p_1 = b_{11}$), and the coefficient of agreement of the second model with this solution is k_2^t.

The situation in which the parameter values of the models' solutions, along with their respective arguments, fall within each other's trust function regions is expressed as

$$\begin{cases}b_{l\,1}\in\left(b_{j\,1}-a_{j\,1},b_{j\,1}+c_{j\,1}\right),\\ b_{j\,1}\in\left(b_{l\,1}-a_{l\,1},b_{l\,1}+c_{l\,1}\right),\end{cases}$$

where $j,l\in\mathbb{N},\ j,l\in\left[1,2\right],\ l\ne j$.

The trust interval of the second model includes the solution b_{11} of the first model, and the trust interval of the first model includes the solution b_{21} of the second; that is, the second model trusts the solution of the first and vice versa. It remains to determine the degree of trust each model has in the opponent's solution.

The degree of agreement of the first model with the second k_1^t is the value of the trust function of the first model for the parameter value p_1 corresponding to the solution of the second model.

The degree of agreement of the second model with the first k_2^t is the value of the trust function of the second model for the parameter value p_1 corresponding to the solution of the first model. To find the degree of agreement of the second model with the first, it is necessary to determine the value of the trust function for the second model (74) for $p_1 = b_{11}$

$$k_2^t=f_2^{t\,1}\left(b_{1\,1},a_{2\,1},b_{2\,1},c_{2\,1}\right)=\left(b_{1\,1}-b_{2\,1}+a_{2\,1}\right)\left(a_{2\,1}\right)^{-1}.$$

It is then necessary to compare the coefficients k_1^t and k_2^t. As a result, the solution with the higher trust coefficient will be selected

$$p^*=\begin{cases}b_{1\,1},\ k_1^t<k_2^t,\\ b_{2\,1},\ k_1^t>k_2^t.\end{cases}$$

The situation like $k_1^t=k_2^t$ should be resolved using one of the conflict resolution strategies in the dispute. At the current stage of work, the chosen strategy for resolving such contradictions is the priority strategy. To implement it, it is necessary that, before the beginning of the problem-solving process, one of the models possesses a higher priority. Then, when a conflict arises in the dispute, the advantage in selecting the final solution p^* will be given to the model, which priority is higher.

The situation in which the trust function regions of the models intersect, but the parameter values of the solutions of both models lie outside the intersection zone, can be expressed as

$$\begin{cases}b_{l\,1}\notin\left(b_{j\,1}-a_{j\,1},b_{j\,1}+c_{j\,1}\right),\\ b_{j\,1}\notin\left(b_{l\,1}-a_{l\,1},b_{l\,1}+c_{l\,1}\right),\\ b_{j\,1}-a_{j\,1}<b_{l\,1}+c_{l\,1},\end{cases}\qquad(75)$$

where $j,l\in\mathbb{N},\ j,l\in\left[1,2\right],\ l\ne j$.

In this case, the models can produce a new solution that satisfies both and lies within the intersection area of their trust functions

$$p^* \in \left(b_{21} - a_{21}, b_{11} + c_{11}\right).$$

Thus, the intersection of the models' trust functions defines the discussion interval – the region within which the models can produce a solution that satisfies them.

In practice, the IDSS experts argue, presenting and justifying their viewpoints, and eventually reach a common decision, even if it differs from the positions they initially held. This decision will be a compromise that satisfies both experts, and in modeling the dispute, it will lie within the discussion interval.

Let's define the discussion function as the operation of the intersection of the models' trust functions

$$f_{12}^{d\,1} = f_1^{c\,1} \cap f_2^{c\,1}.$$

In the graphical interpretation, the discussion function is a triangle formed by the intersection of the trust functions, the base of which is the interval $(b_{21} - a_{21}, b_{11} + c_{11})$. To find the vertex of the triangle defined by the discussion function, it is necessary to determine the point of intersection of the trust functions. Taking into account the condition $b_{21} - a_{21} < b_{11} + c_{11}$(75) and knowing the trust functions of the models (73) and (74), for the value p_1^u parameter p_1 at the point of intersection of the trust functions, the following equality can be written

$$\left(p_1^u - b_{21} + a_{21}\right)\left(a_{21}\right)^{-1} = \left(b_{11} + c_{11} - p_1^u\right)\left(c_{11}\right)^{-1},$$

from which the value follows

$$p_1^u = \left(b_{21}c_{11} + b_{11}a_{21}\right)\left(a_{21} + c_{11}\right)^{-1}. \tag{76}$$

Thus, the discussion function for the interaction of two models can be defined as

$$f_{12}^{d\,1}\left(p_1, a_{21}, b_{21}, c_{21}\right) = \\ = \begin{cases} 0,\ b_{11} + c_{11} < p_1 \le b_{21} - a_{2p}, \\ \dfrac{p_1 - b_{21} + a_{21}}{a_{21}},\ b_{21} - a_{21} < p_1 \le \dfrac{b_{21}c_{11} + b_{11}a_{21}}{a_{21} + c_{11}}, \\ \dfrac{b_{11} + c_{11} - p_1}{c_{11}},\ \dfrac{b_{21}c_{11} + b_{11}a_{21}}{a_{21} + c_{11}} < p_1 \le b_{11} + c_{11}. \end{cases} \tag{77}$$

The discussion function, which defines the discussion interval within which the task for a single parameter at a given stage will be solved, has been determined. However, this does not yet constitute the solution of the task. Let's consider how, as a result of the discussion, a solution is produced that to some extent satisfies both models. For this purpose, let's refer to the theory of fuzzy systems, in which the concept of defuzzification [1] is introduced – the transformation of a fuzzy set into a single (crisp) value that can be transmitted to the control object either directly to the executive mechanisms or through the decision-maker (DM).

Let's consider two methods of defuzzifying the discussion function:

1) the center of gravity of the discussion function;
2) the maximum of the discussion function.

The "center of gravity of the discussion function" method is based on the principles of mechanics and proceeds from the assumption that the crisp value p_1^d parameter p_1 and, accordingly, the solution p^*, produced as a result of the discussion of the models is located at the center of gravity of the discussion function (77). The center of gravity of the discussion function is defined as follows

$$p_1^d = \left(\int p_1 \cdot f_{12}^{d\,1}\left(p_1, a_{21}, b_{21}, c_{21}\right) \mathrm{d}p_1\right) \times \\ \times \left(\int f_{12}^{d\,1}\left(p_1, a_{21}, b_{21}, c_{21}\right) \mathrm{d}p_1\right)^{-1}. \tag{78}$$

Substituting (77) into (78), let's obtain

$$p_1^d = \frac{\displaystyle\int_{b_{21}-a_{21}}^{\frac{b_{21}c_{11}+b_{11}a_{21}}{a_{21}+c_{11}}} p_1 \frac{p_1 - b_{21} + a_{21}}{a_{21}} \mathrm{d}p_1 + \int_{\frac{b_{21}c_{11}+b_{11}a_{21}}{a_{21}+c_{11}}}^{b_{11}+c_{11}} p_1 \frac{b_{11} + c_{11} - p_1}{c_{11}} \mathrm{d}p_1}{\displaystyle\int_{b_{21}-a_{21}}^{\frac{b_{21}c_{11}+b_{11}a_{21}}{a_{21}+c_{11}}} \frac{p_1 - b_{21} + a_{21}}{a_{21}} \mathrm{d}p_1 + \int_{\frac{b_{21}c_{11}+b_{11}a_{21}}{a_{21}+c_{11}}}^{b_{11}+c_{11}} \frac{b_{11} + c_{11} - p_1}{c_{11}} \mathrm{d}p_1}.$$

After calculating the integrals and simplifying the expression, let's obtain

$$p_1^d = \frac{\left(\begin{array}{l} \left(a_{21} + c_{11}\right)^2 \left(\begin{array}{l} c_{11}\left(b_{21} - a_{21}\right)^3 + \\ + a_{21}\left(b_{11} + c_{11}\right)^3 \end{array}\right) - \\ - \left(b_{21}c_{11} + b_{11}a_{21}\right)^3 \end{array}\right)}{3\left(a_{21} + c_{11}\right)\left(\begin{array}{l} \left(a_{21} + c_{11}\right)\left(\begin{array}{l} c_{11}\left(a_{21} - b_{21}\right)^2 + \\ + a_{21}\left(b_{11} + c_{11}\right)^2 \end{array}\right) - \\ - \left(b_{21}c_{11} + b_{11}a_{21}\right)^2 \end{array}\right)}.$$

In this case, the center of gravity of the figure will represent a certain compromise solution adopted by both models in the course of the dispute $p^* = p_1^d$, which was produced as a result of the discussion and differs from the variants proposed by the models. In this situation, the agreement coefficients of the first and second models are calculated according to the formulas:

$$k_1^t = f_1^{t\,1}\left(p_1^d, a_{11}, b_{11}, c_{11}\right) = \left(b_{11} + c_{11} - p_1^d\right)\left(c_{11}\right)^{-1},$$

$$k_2^t = f_2^{t\,1}\left(p_1^d, a_{21}, b_{21}, c_{21}\right) = \left(p_1^d - b_{21} + a_{21}\right)\left(a_{21}\right)^{-1}.$$

Since, in the case of two experts, the discussion function (77) has the form of a triangle, the maximum of this function will be located at its vertex formed by the intersection of the experts' trust functions (76). This value will be the value of the parameter adopted at the given stage of problem-solving during the discussion

$$p^* = p_1^u = \left(b_{21}c_{11} + b_{11}a_{21}\right)\left(a_{21} + c_{11}\right)^{-1}.$$

The agreement coefficients of the first and second models are equal and are computed according to the expression

$$k_1^t = k_2^t = p^* = p_1^u = \left(b_{1\,1} - b_{2\,1}\right)\left(a_{2\,1} + c_{1\,1}\right)^{-1} + 1.$$

The situation in which the trust functions of the models do not intersect can be expressed as follows

$$b_{j\,1} - a_{j\,1} > b_{l\,1} + c_{l\,1},$$

where $j,l \in \mathbb{N},\ j,l \in [1,2],\ l \neq j.$

In such a situation, there is no dispute, since the models have no interaction points; that is, the discussion function is not defined. In this case, the following resolution options are possible:

1) the decision-maker model independently selects the solution p^*, since the models at this point of divergence, being unable to produce a solution, do not engage in discussion;

2) the decision-maker model proposes that one or both models expand their confidence intervals to resolve the issue at the current point of divergence, so that the conflict situation can be resolved;

3) the decision-maker model halts the solution process and selects other models with confidence functions that allow for effective task resolution.

In any case, all conflict situations and methods for their resolution are recorded so that, during repeated solving of this or a similar task, the decision-maker model can select models in such a way that no conflicts arise between them in the course of problem-solving or that their number is minimized.

Thus, when modeling the dispute process at the point of divergence, three dispute resolution strategies are possible:

1. Acceptance strategy. One of the models accepts the decision proposed by the other, where the degree of agreement with this decision p^* is expressed by the agreement coefficient kt. For this, it is necessary that the confidence functions of the models intersect and that the decision adopted by one of the models lies within the confidence function of the other, but not vice versa. In this case, the final value of the parameter adopted at this stage, p^*, is the value proposed by one of the models, and kt is the agreement coefficient of the other model. Thus, when the acceptance strategy is applied, a single alternative is chosen for the continuation of the discussion – the decision proposed by one of the models.

2. Mutual acceptance strategy. Both models accept the decision proposed by the other model, with the degree of agreement with this decision expressed by the agreement coefficients of the first and the second models – k_1^t, k_2^t. Therefore, it is necessary that the confidence functions of the models intersect and that the decision of each model is accepted by the other; that is, the decisions of the models must mutually fall within each other's confidence functions. In this case, it can be stated that when using the mutual acceptance strategy, there are two alternatives for continuing the dispute: the decision proposed by the first model b_{11} with the confidence coefficient of the second model for this decision k_2^t, so there are alternatives $(b_{1\,1}, k_2^t)$ and $(b_{2\,1}, k_1^t)$. In this case, the final value of the parameter adopted at this stage is – p^* – the value of the parameter proposed by one of the models, which confidence coefficient from the other model is higher.

3. Discussion strategy. Neither models accept the decision proposed by the third model. In this case, the discussion function is considered, and a new decision is developed that satisfies the requirements of both models. For the continuation of the dispute, there is one alternative – a jointly accepted decision – p^*, where each model will have its own confidence coefficient for this decision k_1^t and k_2^t. It is important that this decision satisfies both models.

Let's consider how the task is solved in this case. Let, at a certain point of divergence, the decision-maker model define a set of alternatives $A = \{A_1, A_2, \dots, A_S\}$, each of which contains the values of the task parameters corresponding to that alternative

$$A_S = \left\{p_{1\,A_s}, \dots, p_{n\,A_s}\right\},$$

where s – the alternative number, $s = \{1, 2, \dots, S\}$, and i – the number of the task parameter, $i = \{1, 2, \dots, n\}$. The number of experts solving the task is two, and the number of parameters is n.

For each parameter, a discussion function of the experts must be established. Then one can speak of a set of discussion functions for all parameters at the point of divergence

$$F^d = \left\{f^{d\,1}, f^{d\,1}, \dots, f^{d\,n}\right\}.$$

Naturally, during the task-solving process, only those alternatives will be considered which parameter values are included in the discussion functions corresponding to those parameters. Let's denote the set of analyzed alternatives as a subset of the main set A, that is, $A' \subset A$, under the condition that for all elements of A' the following condition holds: $\forall p_{i\,A_s} \in f^{d\,i}$. Thus, the set A' contains only those alternatives that can be selected during the dispute between the two experts A', followed by the condition

$$A' = \left\{A'_1, A'_2, \dots, A'_M\right\},$$

where m – amount of alternatives, $m = \{1,2,\dots,M\}$.

Since each parameter value proposed by an alternative lies within the scope of the discussion function, it has its own evaluation by each model, expressed through a confidence coefficient. This allows each alternative to be represented as a set of pairs – the experts' confidence coefficients for each alternative

$$\overline{A'_m} = \left\{\left(k_{1\,A_m}^{t\,p_1}, k_{2\,A_m}^{t\,p_1}\right), \dots, \left(k_{1\,A_m}^{t\,p_n}, k_{2\,A_m}^{t\,p_n}\right)\right\}.$$

For each parameter of every alternative, there are two evaluations – the confidence coefficients of each expert. When selecting alternatives, the method of the sum of the smallest coefficients for each parameter will be used. In this case, the comparison of alternatives reduces to comparing these sums. The evaluation of an alternative is calculated according to the following formula

$$\overline{OA'_m} = \sum_{i=1}^{n} \min\left(k_{1\,A_m}^{t\,p_i}, k_{2\,A_m}^{t\,p_i}\right),$$

and the best alternative has the highest evaluation

$$A^* = \max_m\left(\overline{OA'_m}\right).$$

Let's proceed to the discussion of the research results on the development of the polymodel complex.

6. Discussion of results on the development of the polymodel complex for resource management

The advantages of the proposed method of the polymodel complex are as follows:

– it provides a comprehensive description of the functioning process of intelligent decision support systems (1)–(78), compared to [6]. This is achieved through the use of an integrated mathematical description of the processes occurring within IDSS. This enables higher modeling accuracy of intelligent decision support systems for subsequent managerial decision-making;

– it allows the description of both static and dynamic processes occurring in intelligent decision support systems (1)–(78), compared to [7]. This is achieved through the development of corresponding mathematical expressions presented in the study;

– it enables modeling of both individual processes occurring in intelligent decision support systems and comprehensive modeling of the processes taking place within them (1)–(78), compared to [16]. That is, for solving computational tasks, both individual models and the polymodel complex as a whole can be applied;

– it establishes conceptual dependencies of the functioning process of intelligent decision support systems (Table 1, (1)–(23)). This makes it possible to describe the interaction among individual models at all stages of solving computational tasks, compared to [6, 10]. This is achieved through the development of the conceptual model;

– it describes coordination processes in hybrid intelligent decision support systems (expressions (24)–(44)), thereby increasing the reliability of managerial decision-making compared to [9]. This is achieved through the development of a corresponding mathematical coordination model;

– it models the processes of solving complex computational tasks in intelligent decision support systems (expressions (45)–(51)) through the conceptual description of the specified process, compared to [11];

– it coordinates computational processes in intelligent decision support systems (expressions (52)–(59)), thereby reducing the number of computational resources of the systems compared to [4, 10]. This is achieved through the development of a mathematical consistency model within the proposed polymodel complex;

– it provides a comprehensive resolution of disputes through a set of corresponding mathematical models (expressions (72)–(78)), compared to [9, 14]. This is achieved through the development of the corresponding mathematical model.

The drawbacks of the proposed polymodel complex include:

– the lack of the ability to account for the degree of uncertainty in the data circulating within intelligent decision support systems;

– higher computational complexity of operations in intelligent decision support systems compared to known studies.

The proposed polymodel complex will make it possible to:

– conduct comprehensive modeling of the functioning process of intelligent decision support systems;

– determine effective measures to improve the operational efficiency of intelligent decision support systems;

– increase the speed of solving computational tasks in intelligent decision support systems while maintaining the specified reliability of decision-making during data processing;

– reduce the use of computational resources of intelligent decision support systems.

The proposed scientific results are advisable for use in information and automated troop control systems such as "Delta", "Dzvin-AS", "Oreanda-PS", or their analogues. They are particularly useful under conditions of rapid and dynamic changes in the number of computational tasks circulating within different computational contours of such automated and information systems.

The effect of applying the proposed complex is an increase in the efficiency (criterion: number of operations per unit time/type of computational task) of using computational resources in IDSS.

The limitations of the study include the necessity of considering the delay time for collecting and transmitting information from the components of intelligent decision support systems.

Future research directions should focus on integrating the developed models into existing automated and information systems of general and special purpose.

7. Conclusions

1. The study proposes a conceptual model of intelligent decision support systems. The distinction of the proposed model lies in the comprehensive description of the functioning process of intelligent decision support systems and the establishment of conceptual dependencies of their functioning process. This enables improved modeling accuracy of intelligent decision support systems for subsequent managerial decision-making.

2. A set of coordination models in hybrid intelligent decision support systems has been developed. Their distinguishing feature is the presence of interconnections among them, which allows comprehensive modeling of the coordination process in IDSS.

3. A mathematical model of consistency in intelligent decision support systems has been proposed. The proposed mathematical model allows the harmonization of the application of individual mathematical models for solving complex computational tasks. This is achieved through mathematical expressions that enable the coordination of solutions of individual models when forming a generalized solution. It allows new mathematical models to be integrated into existing ones for their further improvement.

4. A fuzzy system for resolving model disputes in intelligent decision support systems has been developed. This mathematical model enables the resolution of conflict situations in IDSS when solving complex computational problems. This improves both the timeliness and efficiency of solving computational tasks in IDSS.

Conflict of interest

The authors declare that they have no conflict of interest regarding this study, including financial, personal, authorship-related, or any other conflicts that could have influenced the study and its results presented in this work.

Financing

The study was conducted without financial support.

Data availability

The manuscript has associated data in a data repository.

Use of artificial intelligence

The authors confirm that no artificial intelligence technologies were used in the creation of the presented work.

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