This study's object is the process of cargo movement along the helical surface of an oblique open helicoid under the action of its natural weight. Such movement takes place in gravity chutes where the cargo descends under the action of its natural weight. Gravity (screw) chutes are used for transportation, separation, and enrichment of material.

For a given surface, the problem is solved by composing differential equations of motion of a mathematical point, which is conditionally replaced by cargo, in projections onto the axis of the spatial coordinate system. If the surface is helical, then after stabilization of the motion, it is possible to find the parameters of the helical line – the trajectory of cargo movement. The task implies solving the inverse problem – constructing a helical surface along a given trajectory of cargo descent, which is a helical line.

The results are attributed to the use of two accompanying trihedra of the trajectory with a common vertex and tangent orts to the trajectory, which coincide. One of them is a Frenet trihedron whose position is determined by the differential characteristics of the curve, and the second is a Darboux trihedron, the position of which depends on the point of the trajectory on the surface. In addition to the two coincident orts, the remaining four orts are located in the plane normal to the trajectory. The use of these two orts makes it possible to compose differential equations of motion of the load in projections onto a moving Darboux trihedron, one of the planes of which is tangent to the surface.

A feature of the solution to the problem is that the trajectory of the load, i.e., the helix, is given by radius r of the cylinder on which it is located and velocity V of the load. Using these data, angle β of its ascent is determined. For example, at $r = 0.5$ m, $V = 2.5$ m/s, the angle of elevation is $\beta = 20.7°$. Then, a helical linear surface is constructed that passes through the given trajectory

Keywords: Frenet and Darboux trihedra, arc length, applied forces, differential equations, helix

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DESIGNING A GRAVITY CHUTE BASED ON THE GIVEN TRAJECTORY OF CARGO MOVEMENT

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1. Introduction

Screw chutes are used for transporting bulk materials [1, 2] and various cargoes [3, 4], for separation [5] and enrichment of material in the mining industry [6], for preparing feed mixtures [7, 8], etc. Particles of technological material move down the screw surface in the form of a flow. In addition to gravity, centrifugal forces act on them. These forces force particles of different nature to move along different screw trajectories, which makes it possible to separate the material in the lower part of the surface. When analytically describing such a movement, the screw surface is first specified, and then the movement parameters are found. It is logical when designing a screw surface to be guided by these parameters as initial ones, in particular the speed of movement of particles or cargo. It is obvious that the speed of movement is affected by the pitch of the surface. After stabilization of the movement, the particle or cargo moves at a certain distance from the axis of the surface. This speed and distance can be specified and then a screw surface that would provide these parameters can be found.

Therefore, scientific research into this area is relevant because it makes it possible to purposefully find a helical surface based on the given parameters for the motion of a particle or load.

2. Literature review and problem statement

Numerous scientific papers consider the design of screw conveyors. Thus, in work [9], conveyors used in the transportation of bulk materials were analyzed. The main attention is paid to solutions based on a vertical or angular screw shaft. At the same time, issues related to the modeling of particle motion remain unresolved.

In [10], the results of studies on the feasibility of using a vertical static screw conveyor for transporting granular fertilizers are reported and the indicated drawback is eliminated. The authors use the discrete element method to model particle motion, which makes it possible to evaluate various structures of screw conveyors. The work investigated the granulometric and mechanical properties of fertilizers, carried out measurements to calculate bulk density and tests to assess the behavior of particles under the action of compression and shear forces. From a scientific point of view, the dependence between the design features of the working elements and the behavior of particles when moving along them is important.

The authors of [11] emphasize that the process of transporting material by screw feeders and conveyors appears mechanically simple but the physics of such a process in the system is quite complex. In this regard, the authors' attention was focused on studying the behavior of the flow of granular materials during their transportation and mixing.

The authors of [12] built on this scientific area by considering the design of screw conveyors from the point of view of determining the operational parameters of such devices using the discrete element method. They investigated the influence of some input parameters of the model on the simulation results. The key parameters that determine the operational characteristics of the screw conveyor were determined: particle size, internal and external friction coefficients. However, issues related to controlling the behavior of particles by changing the design features of the working body remained unresolved. The reason for this may be objective difficulties associated with the complexity of such a process using the traditional approach.

One way to overcome such difficulties may be to take into account a wide range of input parameters of the transportation process. This is the approach used in [13], which investigates the mixing process of a mixture of coal and cylindrical biomass particles in a screw feeder. In contrast to [12], the authors took into account the influence of some input parameters of the process and established the relationship between the biomass feed rate, feed rate, screw rotation speed, and feed efficiency.

In [14], the process of distributing fertilizers by a screw distributor was investigated from the point of view of the influence of the geometry of its outlet (its length and angle, as well as the interaction of these two factors) on the accuracy of application. Thus, the authors raise the issue of solving the inverse problem: changing the design features of the working body to achieve the required behavior of particles when moving along it. Similar in logic, although with a different purpose, is work [3] in which the authors devised a methodology for calculating the structures of screw transport and technological mechanisms in order to improve their technical and economic characteristics. It should be noted that in [3, 14], an analytical description of the constructed surfaces is not provided, which could be used for further development of this area of research.

Therefore, we systematized locally unresolved issues and problems that require further research as follows:

– lack of effective methods for modeling particle motion in screw conveyors, especially when transporting loose and granular materials [9, 10, 12];

– lack of a comprehensive approach to determining and optimizing the operating parameters of screw conveyors [3, 12, 13];

– lack of methodologies for solving the inverse problem – changing the design features of the working body to achieve the required particle behavior [3, 14].

Thus, it is advisable to devise a methodology for designing screw surfaces of screw conveyors along a predetermined trajectory of material lowering, taking into account the analytical description of the surfaces and optimization of structural parameters to achieve the required particle behavior. This will allow us to optimize the structure of working bodies for specific operating conditions and change it to achieve the required particle behavior.

3. The aim and objectives of the study

The purpose of our study is to devise an analytical description of the helical surface. This will make it possible to provide a given speed of lowering the load at a given distance from its axis.

To achieve this goal, the following tasks were set:

– to derive an equation of the gravity chute surface as a result of solving the inverse problem for the given parameters of the trajectory and speed;

– to verify the result by solving the direct problem of finding the trajectory and speed of movement of the load along the given surface.

4. The study materials and methods

The object of our study is the process of cargo movement along the helical surface of an oblique open helicoid under the action of its natural weight. The hypothesis of the study is the assumption that using the accompanying Frenet and Darboux trihedra of the trajectory, it is possible to solve the inverse problem of constructing a surface according to the given parameters of cargo movement. In this case, a simplification is introduced, in which the cargo is replaced by a mathematical point, which is practiced in similar problems.

When solving the problems set, the methods of mechanics and differential geometry were used. When a mathematical point moves along a curvilinear trajectory, a centrifugal force arises. Its magnitude, according to the provisions of mechanics, depends on the mass, speed of movement, and curvature of the trajectory. This force is directed along the main normal of the accompanying Frenet trihedra of the trajectory, which has no relation to the surface. To establish the connection between the action of centrifugal force and the surface, the Darboux trihedron is used, which has a line tangent to the trajectory, common with the Frenet trihedron, and the other two lines form a plane tangent to the surface. Based on the established relationship between these trihedrons when they move along the trajectory of the point's movement, the problem of finding the construction of the desired surface is solved. In this case, the formulas for the connection between the moving trihedrons and the fixed coordinate system are used. For analytical transformations, simplifications, differentiation, and to solve the system of equations, the capabilities of symbolic mathematics in the "Mathematica" software package were used.

5. Surface construction according to given kinematic parameters for the motion of a mathematical point

5. 1. Results of solving the inverse problem of surface construction using Frenet and Darboux trihedra

Let a mathematical point move along a spatial curve on the surface. In its current position, two accompanying trihedra can be constructed: the Frenet trihedra and the Darboux trihedra. The direction of orts of the first trihedra $\tau n b$ is determined by the differential characteristics of the curve itself, and the direction of the orts of the second trihedra TPN is determined by the position on the surface (Fig. 1, *a*). The orts tangent to the trajectory τ and T of trihedra coincide. The remaining orts are located in the normal plane θ of the trajectory. Orts T and P form a plane tangent to the surface. Between the orts b, N and n, P in plane θ there is an angle γ, which can change when the trihedra move along the curve (trajectory).

At each time point, when the trihedra move, the forces acting on the point must be balanced. The balance of these forces must be projected onto the orts of the Darboux trihedron since its plane, formed by orts T and P, is tangent to the surface. This means that the sliding of a point on the surface in its vicinity can be considered as sliding along the tangent plane. The sliding of a point occurs under the action of the force of gravity mg (m is the mass of the point, $g = 9.81$ m/s^2), namely due to component $mg\sin\beta$, which is the projection of the force of gravity onto the ort tangent T (Fig. 1, *b*). Angle β is the angle between the direction of tangent T and the horizontal plane. In the general case, the point can move with acceleration $\frac{dV}{dt}=\frac{dV}{ds}\frac{ds}{dt}=V\frac{dV}{ds}$, where t is time, V is speed, s is the length of the arc of the trajectory). The balance of forces acting on ort T is written as follows

$$mV\frac{dV}{ds}=mg\sin\beta - fR, \tag{1}$$

where f is the friction coefficient, R is the surface reaction force. The product fR is the friction force, which is directed in the opposite direction of the point's motion (Fig. 1, *b*).

The balance of forces must be ensured in normal plane θ. The weight component $mg\cos\beta$ is projected onto it, which in turn can be decomposed into component 1 in the projection onto ort P and component 2 in the projection onto ort N, which is normal to the surface (Fig. 1, *b*). In addition to the weight component, the centrifugal force mV^2k acts in normal plane θ, where k is the curvature of the trajectory. The direction of view on normal plane θ was chosen so that ort T of the tangent was projected onto the point (Fig. 2).

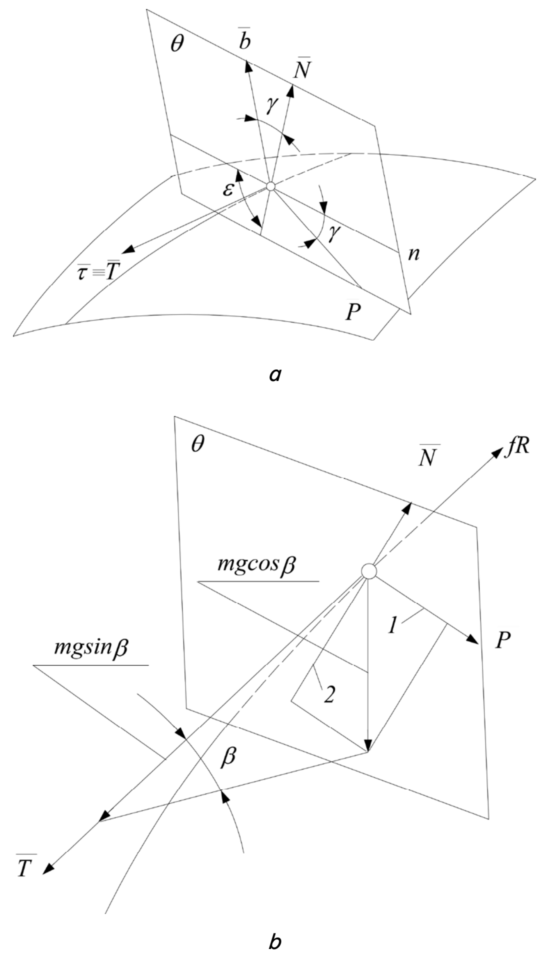

Fig. 1. The accompanying Frenet and Darboux trihedra of the trajectory of a point's motion on the surface: *a* – the mutual position of the trihedra is determined by angle γ in the normal plane between their corresponding orts; *b* – projection of the acting forces onto the ort of the tangent and the normal plane

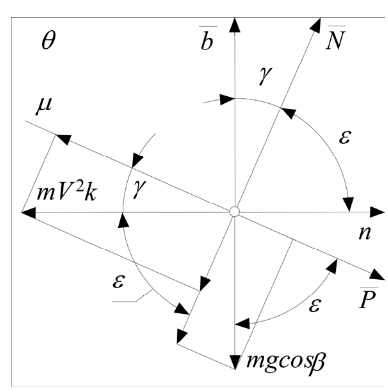

Fig. 2. Decomposition of the acting forces in normal plane θ of the trajectory

The plane of the Darboux trihedron formed by orts $\overline{T}$ and $\overline{P}$, which is tangent to the surface, is projected onto straight line μ. The centrifugal force mV^2k acts along ort n of the main normal of the Frenet trihedron in the opposite direction to its direction. The component of gravity force $mg\cos\beta$ and the centrifugal force mV^2k must be projected onto the lines of the Darboux trihedron orts. For this purpose, angle ε between orts $\overline{N}$ and n of the trihedrons is used. After projection onto ort $\overline{P}$, i.e., onto the tangent plane, we obtain

$$mV^2k\sin\varepsilon = mg\cos\beta\cos\varepsilon. \quad (2)$$

By projecting these forces onto ort $\overline{N}$, that is, normal to the surface, we can obtain the pressure force, which is balanced by reaction R of the surface. Hence,

$$R = mg\cos\beta\sin\varepsilon + mV^2k\cos\varepsilon. \quad (3)$$

After substituting (3) into (1) and reducing the resulting equation, as well as equation (2) by mass m, we can derive a system of equations that describes the motion of a mathematical point on a surface under the action of its natural weight

$$\begin{cases} V\dfrac{dV}{ds} = g\sin\beta - f\left(g\cos\beta\sin\varepsilon + V^2k\cos\varepsilon\right); \\ V^2k\sin\varepsilon = g\cos\beta\cos\varepsilon. \end{cases} \quad (4)$$

System (4) includes four unknown dependences: $V = V(s)$, $\varepsilon = \varepsilon(s)$, $k = k(s)$ and $\beta = \beta(s)$. This means that two dependences need to be specified, and the rest can be found from system (4). Let the surface be linear. This means that in the normal plane of the accompanying Frenet trihedron of the trajectory, it is necessary to draw a straight-line surface generatrix at angle γ to main normal $\overline{n}$ (Fig. 2). This generatrix together with the tangent's ort will form plane μ, tangent to the surface.

Let the surface be helical, i.e., the curvature of trajectory k and its ascent angle β are constant quantities. In this case, after stabilization of the motion, velocity V of the point's descent will also be constant [15]. In this case, the left-hand side of the first equation in system (4) will be equal to zero. As a result, solving it gives the following result:

$$\sin\varepsilon = f\mathrm{ctg}\beta;$$

$$V^2 = \frac{g\sin\beta}{fk}\sqrt{1 - f^2\mathrm{ctg}^2\beta}. \quad (5)$$

According to the first equation in (5), the value of angle ε depends on friction coefficient f and angle β of the trajectory ascent. The second equation in (5) includes three quantities that affect the motion of the point: β, k, and V. Two of them must be specified, the third is determined. When a load moves along a surface, there is a limit on the sliding speed to 2.5 m/s to prevent its damage [16]. It is obvious that this parameter must be specified. Curvature k of the trajectory was also specified since it determines distance r from the surface axis at which the load will move. There is a relationship between curvature k of the helix and distance r: $k=\cos^2\beta/r$. After substituting this relationship into the second equation in (5) and after simplifications, it takes the following form

$$V^2 = \frac{gr}{f\cos^2\beta}\sqrt{1 - \left(1 + f^2\right)\cos^2\beta}. \quad (6)$$

Solving (6) with respect to $\cos\beta$ yields the following result

$$\cos\beta = \sqrt{\frac{gr\sqrt{\left(1+f^2\right)^2 g^2r^2 + 4f^2V^4} - \left(1+f^2\right)g^2r^2}{2f^2V^4}}. \quad (7)$$

Given the desired speed V and distance r with known friction coefficient f, we can find angle of elevation β of the helix. Distance r and angle β completely determine it. The next step is to draw a set of straight-line generatrices through this helix, i.e., the trajectory of the cargo, the location of which is determined by angle ε.

With known distance r and angle of elevation β, the parametric equations of the helix as a function of the length of its own arc s are written:

$$x = r\sin\left(\frac{\cos\beta}{r}s\right);$$

$$y = -r\cos\left(\frac{\cos\beta}{r}s\right);$$

$$z = -s\sin\beta. \quad (8)$$

Each point of the helical line (trajectory) (8) is passed by a straight-line generatrix, which in the Frenet trihedron has the following coordinates (Fig. 2):

$$\tau = 0;$$

$$n = -\sin\varepsilon;$$

$$b = \cos\varepsilon. \quad (9)$$

Using formulae for a transition from the Frenet trihedron system to the fixed coordinate system, the coordinates of the directional unit vector W of the rectilinear generatrix in the fixed coordinate system were determined. The end result takes the following form:

$$W_x = \sin\beta\cos\varepsilon\cos\left(\frac{\cos\beta}{r}s\right) + \sin\varepsilon\sin\left(\frac{\cos\beta}{r}s\right);$$

$$W_y = \sin\beta\cos\varepsilon\sin\left(\frac{\cos\beta}{r}s\right) - \sin\varepsilon\cos\left(\frac{\cos\beta}{r}s\right);$$

$$W_z = \cos\beta\cos\varepsilon. \quad (10)$$

Parallel to vector (10), through each point of curve (9), a straight-line generatrix of the surface was drawn. After that, the parametric equations of the helical linear surface take the following form:

$$X = r\sin\left(\frac{\cos\beta}{r}s\right) + u\left[\begin{matrix} \sin\beta\cos\varepsilon\cos\left(\frac{\cos\beta}{r}s\right) + \\ +\sin\varepsilon\sin\left(\frac{\cos\beta}{r}s\right) \end{matrix}\right];$$

$$Y = -r\cos\left(\frac{\cos\beta}{r}s\right) + u\left[\begin{matrix} \sin\beta\cos\varepsilon\sin\left(\frac{\cos\beta}{r}s\right) - \\ -\sin\varepsilon\cos\left(\frac{\cos\beta}{r}s\right) \end{matrix}\right];$$

$$Z=-s\sin\beta+u\cos\beta\cos\varepsilon, \tag{11}$$

where u, the second independent variable of the surface, is the length of the straight-line generatrix, the count of which starts from the points of curve (8).

When $u=0$, the equations of surface (11) are transformed into the equations of trajectory (8). The surface (11) must ensure the movement of the load along it at given speed V, and the load must slide along trajectory (8).

5. 2. Verification of the result by solving a direct problem of the motion of a point on the surface

The differential equations of the motion of a point on the surface under the action of the force of its natural weight in projections onto a fixed coordinate system take the following form:

$$mx''=-fRT_x+RN_x;$$

$$my''=-fRT_y+RN_y;$$

$$mz''=-fRT_z+RN_z-mg, \tag{12}$$

where R is the surface reaction force; T_x, T_y, T_z are projections of the unit vector tangent to the trajectory; N_x, N_y, N_z are projections of the unit vector normal to the surface along the trajectory, xx'', y'', z'' are projections of the acceleration of the point.

It was assumed that the point moves along trajectory (8). Its velocity and acceleration are functions of time t, i.e., $s=s(t)$. The first derivatives, projections of the velocity of the point taking into account dependence $s=s(t)$, take the following form:

$$\frac{dx}{dt}=\frac{ds}{dt}\cos\beta\cos\left(\frac{\cos\beta}{r}s\right);$$

$$\frac{dy}{dt}=\frac{ds}{dt}\cos\beta\sin\left(\frac{\cos\beta}{r}s\right);$$

$$\frac{dz}{dt}=-\frac{ds}{dt}\sin\beta. \tag{13}$$

In velocity projections (13) the velocity itself is present because $ds/dt=V$. If expressions (13) are divided by velocity V, then projections of the unit tangent vector T_x, T_y, T_z will be obtained.

Another differentiation of expressions (13) gives projections of acceleration:

$$\frac{d^2x}{dt^2}=\frac{d^2s}{dt^2}\cos\beta\cos\left(\frac{\cos\beta}{r}s\right)-\left(\frac{ds}{dt}\right)^2\frac{\cos^2\beta}{r}\sin\left(\frac{\cos\beta}{r}s\right);$$

$$\frac{d^2y}{dt^2}=\frac{d^2s}{dt^2}\cos\beta\sin\left(\frac{\cos\beta}{r}s\right)+\left(\frac{ds}{dt}\right)^2\frac{\cos^2\beta}{r}\cos\left(\frac{\cos\beta}{r}s\right);$$

$$\frac{d^2z}{dt^2}=-\frac{d^2s}{dt^2}\sin\beta. \tag{14}$$

It remains to find projections N_x, N_y, N_z of the unit normal vector. The normal vector is determined from the vector product of two vectors: the unit tangent vector (13) and the unit rectilinear generatrix vector (10). It should be borne in mind that it can be directed in one or the opposite direction relative to the surface. It is necessary to ensure that it coincides with the direction of the surface reaction, i.e., in the considered case, the vertical component must be positive. The result is given below:

$$N_x=\sin\beta\sin\varepsilon\cos\left(\frac{\cos\beta}{r}s\right)-\cos\varepsilon\sin\left(\frac{\cos\beta}{r}s\right);$$

$$N_y=\sin\beta\sin\varepsilon\sin\left(\frac{\cos\beta}{r}s\right)+\cos\varepsilon\cos\left(\frac{\cos\beta}{r}s\right);$$

$$N_z=\cos\beta\sin\varepsilon. \tag{15}$$

It was assumed that the velocity of point $V=ds/dt$ is constant. Then in acceleration expressions (14), $x''=y''=z''=0$, due to which these expressions are significantly simplified. Taking this into account, the substitution into equation (12) of the corresponding expressions from (14), (13), and (15) makes it possible to derive a system of differential equations of the point motion on the surface (11):

$$-\frac{mV^2}{r}\cos^2\beta\sin\left(\frac{\cos\beta}{r}s\right)=-fR\cos\beta\cos\left(\frac{\cos\beta}{r}s\right)+R\left[\sin\beta\sin\varepsilon\cos\left(\frac{\cos\beta}{r}s\right)-\cos\varepsilon\sin\left(\frac{\cos\beta}{r}s\right)\right];$$

$$\frac{mV^2}{r}\cos^2\beta\cos\left(\frac{\cos\beta}{r}s\right)=-fR\cos\beta\sin\left(\frac{\cos\beta}{r}s\right)+R\left[\sin\beta\sin\varepsilon\sin\left(\frac{\cos\beta}{r}s\right)+\cos\varepsilon\cos\left(\frac{\cos\beta}{r}s\right)\right];$$

$$0=fR\sin\beta+R\cos\beta\sin\varepsilon-mg. \tag{16}$$

“Mathematica” software gives the following solutions to system (16):

$$V^2=\frac{gr}{f\cos\beta}\sqrt{\mathrm{tg}^2\beta-f^2};$$

$$\sin\varepsilon=f\mathrm{ctg}\beta;$$

$$R=\frac{mg}{f}\sin\beta. \tag{17}$$

If we substitute $k=\cos^2\beta/r$ into the second equation in (5), then, after the transformations, we shall obtain the first expression in (17). The second expression in (17) coincides with the first expression in (5). It is convenient to show that the third expression in (17) is identical to expression (3). It consists of two terms. The first term of expression (3) (it was marked with the letter "A") after substituting $\sin\varepsilon = f\mathrm{ctg}\beta$ into it, takes the form

$$A = mgf\cos\beta\mathrm{ctg}\beta. \quad (18)$$

In the second term of expression (3) (it was marked with the letter "B"), instead of V^2, it is necessary to substitute the corresponding expression from (17), as well as expressions $k = \cos^2\beta/r$ and $\cos\varepsilon = \sqrt{1 - f^2\mathrm{ctg}^2\beta}$. After transformations and simplifications, we obtain

$$B = \frac{mg}{f}\sin\beta - mgf\cos\beta\mathrm{ctg}\beta. \quad (19)$$

The sum of terms A (18) and B (19) exactly coincides with the expression of reaction force R in (17), which indicates the reliability of our solution to the direct and inverse problems.

Within the framework of this study, options for constructing a helical linear surface with given speed $V = 2$ m/s of sliding of the load along it and with different distances r of the trajectory from the surface axis were considered.

Let $r = 0.5$ m. The friction coefficient was considered known: $f = 0.3$. From expression (7), angle β of the slide trajectory ascent was found: $\beta = 20.67°$. To construct the surface according to equations (11), it is necessary to know the value of angle ε. From the middle expression in (17), it was determined: $\varepsilon = 52.67°$. The surface according to these data is constructed in Fig. 3, *a*.

A surface of normals is constructed along the trajectory of the sliding load, from which it can be seen that all normals are directed towards the reaction force of the surface along which the load slides. Surfaces for $r = 0.25$ m (Fig. 3, *b*) and $r = 0.75$ m (Fig. 3, *c*) are constructed using the same algorithm. For all three cases, the length of the trajectory is the same and is equal to 6 m.

6. Discussion of results based on the inverse and direct problem of constructing a helical surface of the gravity chute

In [3, 11, 12, 14] on the design of gravity chutes, their calculation is carried out in a conventional way. It implies that a point or a material particle falls onto a given helical surface, after which the kinematic parameters of its sliding are calculated. Unlike [3, 11, 12, 14], our results are attributed to the fact that an inverse calculation is proposed: to set the sliding parameters and, based on them, build a surface that would provide these parameters. This becomes possible due to the use of the so-called internal geometry of curved lines and surfaces, i.e., the moving Frenet trihedron of the spatial curve, which is the trajectory of the particle sliding, and the moving Darboux trihedron are combined (Fig. 1, *a*). Both trihedra have a common tangent ort and move along the trajectory. They also have a common normal plane perpendicular to the trajectory. In the Darboux trihedron, a straight-line generatrix is given in its normal plane, the position of which can change during the movement of the trihedra. The set of these positions forms a linear surface. The position of the straight-line generatrix in the normal plane of the Darboux trihedron is determined by the equilibrium of the forces acting on the particle (Fig. 2). It is this approach that resolves the task and makes it possible to construct a surface according to the given parameters of load movement.

In the proposed approach to solving the problem of calculating a helical gravity chute, an important parameter is the speed of lowering a particle (load). In Fig. 3, helical surfaces are constructed that ensure the sliding of the load at the same speed. Given that the length of the sliding trajectory at a given speed is the same in all cases and is equal to 6 m, we can conclude that the load has a different speed of lowering. In the same time, the load travels the same sliding distance but at a different height of descent. From Fig. 3 it is seen that with a decrease in distance r, the height of descent in the same time increases.

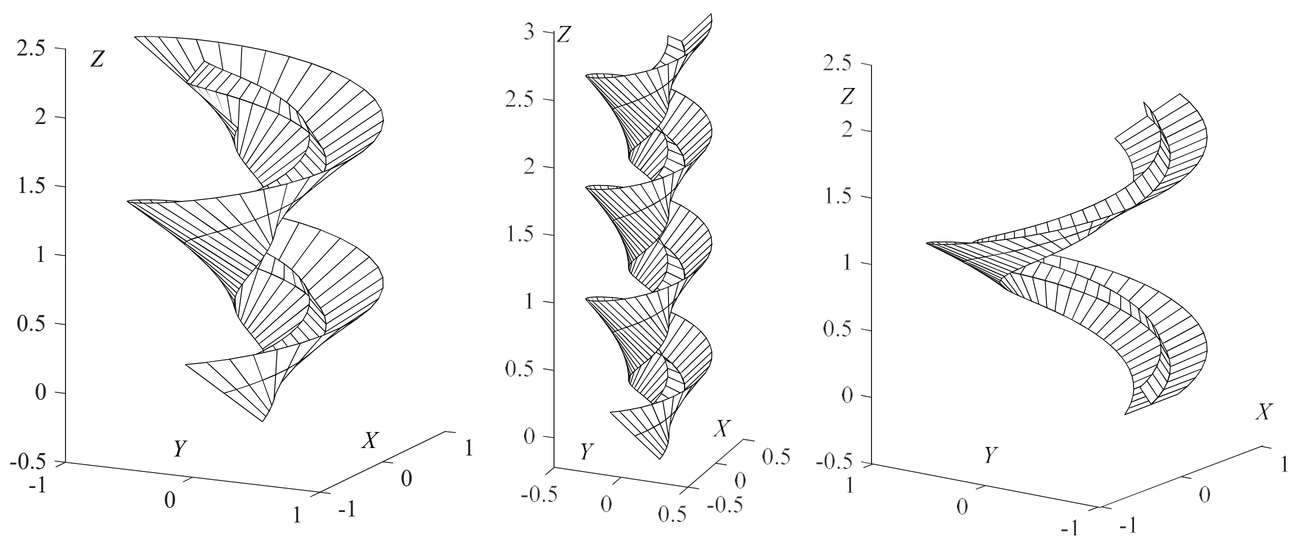

Fig. 3. Helical linear surfaces that provide a load sliding speed along them at speed $V = 2$ m/s: *a* – $r = 0.5$ m; *b* – $r = 0.25$ m; *c* – $r = 0.75$ m

According to the proposed algorithm, the formed surface is linear. In principle, it may not be linear if the generatrix of the surface is not a straight generatrix in the normal plane of the trihedra but a curved one. However, in both the first and second cases, the generatrix will not intersect the axis of the helical surface. This is a limitation of the proposed approach since in the direct problem of particle sliding, the surface is given by the curve of its axial section, i.e., a straight line or curve that intersects the axis of the surface.

The disadvantage of our research on the movement of a load on a helical linear surface is that the load is taken as a mathematical point or a material particle. Such a model only approximately reflects the real process, but it makes it possible, with a certain approximation, to identify the regularities of the movement of the load depending on the design parameters of the surface. The findings can be verified and refined as a result of the experiment.

Future research involves the use of Frenet and Darboux trihedra to determine the kinematic parameters of a particle on a moving surface, i.e., in complex particle motion.

7. Conclusions

1. The use of the Frenet and Darboux trihedra allowed us to solve the inverse problem of constructing the surface of a helical descent according to the given kinematic parameters of the sliding of the load. The trihedra have a common tangent's ort to the given trajectory and a common normal plane to the trajectory. Based on the relationship between the trihedra during their movement, the equilibrium of the forces acting on the load was compiled in the projections onto the orts of the Darboux trihedra. That has made it possible to find a set of rectilinear generatrices of the surface in the Frenet trihedra system. Through the direction cosines of the Frenet trihedra, the position of this set of straight lines in a fixed coordinate system was found and the parametric equations of the surface were compiled.

2. Taking the resulting surface as given, the direct problem of finding the kinematic parameters of the movement of the load along it was solved. The differential equations of motion were compiled in projections onto the axes of a fixed coordinate system, provided that the motion was stabilized and the sliding speed of the load was constant. As a result of solving the equations, the kinematic parameters of the load motion were derived, which completely coincided with the similar parameters obtained when using the moving Frenet and Darboux trihedra.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study, as well as the results reported in this paper.

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Data availability

All data are available, either in numerical or graphical form, in the main text of the manuscript.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

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