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DETERMINING THE RATIONAL ENERGY LEVEL FOR PROCESSING ENVIRONMENTS OF DIFFERENT STRUCTURES

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Vibration processing underlies many technological processes in various sectors of the national economy. The object of the study is a single common wave process of motion of the system, and the subject is the parameters and modes that provide a rational level of energy. At the same time, attention is focused on the use of energy-saving technologies. Most processes use energy-consuming technologies and equipment, the calculations of which are based on the application of formulas for determining energy on discrete and empirical dependencies. Such approaches do not reveal the essence of the material processing processes and cannot accurately estimate energy costs. This problem is solved in the work by determining the parameters and modes based on the conditions for implementing the synergy of the "vibration exciter – processing medium" system. This is the peculiarity and difference of the obtained research results. Discrete-continuous models have been developed taking into account the rheological properties of the media, and analytical solutions and experimental studies have allowed to determine the parameters of low-frequency and high-frequency actions on the processing medium. The work uses a method by which the combination of frequency and amplitude of vibrations determines the intensity of the vibration effect on the processing environment. The resistance of the technological environment to the movement of the working body of the vibration exciter, which consisted of inertial, elastic and dissipative parts, was determined. The study determined the qualitative and quantitative picture of the change in energy dissipation in specific materials and environments under different laws of their loading and processing. The energy level for processing technological environments in low-frequency and high-frequency modes was determined. The research methodology and analytical dependence of energy determination can be used for various environments under load

Keywords: vibration exciter, environment, discrete-continuous model, energy dissipation, amplitude of vibrations, frequency

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1. Introduction

Modern trends in the development of technologies for processing technological media (grinding, mixing, compaction, emulsification, etc.) are aimed at finding new effective solutions. This is due to the fact that in modern conditions there are requirements for reducing energy consumption, improving the quality of the new materials obtained, ensuring the environmental friendliness of processes. These requirements are not fully met by the low-frequency and high-frequency technologies for processing media that are widespread in various industries. Thus, in vibration systems operating at low frequencies, the potential for increasing efficiency has been exhausted. They operate in a non-resonant mode, which is characterized by high energy consumption. Existing methods for determining energies during vibration effects on material processing processes are based on the use of discrete models, which are reliable only within the framework of the research conducted. Such approaches do not reveal the essence of material processing processes and the search for solutions to accurately estimate energy consumption. The absence of refined models does not solve the problems of reliable assessment of the impact of vibration actions on the processes of processing technological environments.

High-frequency actions are widespread in cavitation technologies for the implementation of dispersion, emulsification, homogenization processes. Energy is determined using a separate approach to the energy converter and energy for the flow of the technological process. This approach excludes the search for solutions to determine the rational energy level. The work considers a joint study of the above technologies in terms of the general description of the motion of the system "vibration exciter – processing environment" based on the use of generally accepted provisions of the classical theory of mechanical vibrations and the theory of continuous media. An identical method of transferring energy from the exciter to the environment with the subsequent assessment and analysis of specific physical and rheological properties of the media is the key to solving the problem of obtaining and implementing an energy-saving operating mode. Therefore, studies devoted to determining energy consumption for vibration processes when processing various technological environments are relevant.

2. Literature review and problem statement

In [1], the method and status of statistical energy analysis of the theory of sound and vibration are described. The results of research into the concept of entropy as a measure of information lost in the transition from the classical theory of sound and vibration to statistical energy analysis are presented. The authors propose an approach for describing the internal energy of nonlinear systems. However, the lack of prerequisites and assumptions does not allow assessing the effectiveness of this method in terms of the argumentation of the adopted laws of change in energy indicators. In [2], the oscillations of a solid body on kinematic bases are considered, the main elements of which are rolling bearings limited by high-order surfaces of rotation during horizontal movement of the base. The explanations of the laws of dissipative forces and the analytical dependencies for determining energy remain unresolved. In [3], studies of the motion of a pendulum are presented, which is taken as an analogue of real movements. The authors propose a method for finding the dynamic equilibrium of oscillatory systems through the synchronization of integrators. Such a solution allows to obtain a positive result with minimal energy consumption for damping. However, in the given second term of the equation, which characterizes energy losses during oscillations, there is no argumentation for its writing. That is, such an approach cannot serve as a generally accepted one, since there are other types of friction. In work [4], damping is taken as viscous friction, which facilitates the solution, but there is no evidence of its application for other processes. And in work [5], a study is presented in which damping is taken as frequency independent, that is, a function of stress, which is valid in certain processes. This is especially important for predicting and applying resonances as energy-saving modes. The study given in work [6] was performed on the basis of a mathematical model represented by discrete parameters. The study presents a study of the thermomechanical behavior of high-energy vibrating mills, which are widely used in modern technological processes for processing media. The authors consider a model of a multicomponent medium, and the analysis of changes in dynamic and thermodynamic parameters based on the laws of mechanics and thermodynamics. In this case, the internal energy of the medium is characterized by heat release and an increase in internal energy. The efficiency of the medium processing process is determined by the temperature of the grinding bodies. The lack of justification for such a model is not given. The reason for this decision is to simplify the studied process of material grinding. And the lack of a dependence for determining energy complicates the analysis of the obtained results. In work [7], a concrete damage model was developed, which can reflect the complex phenomena of concrete hysteresis under cyclic loading. The issues of the physical aspects of possible crack formation with the determination of the law of change in energy consumption remained unresolved. In work [8], the vibration behavior of sleepers during the passage of a train was investigated and the physical causes of sleeper vibration were determined. But the specific frequencies at which large vibrations are observed are not given, but are explained mainly by the function of the train geometry. Work [9] is devoted to the use of a nonlinear energy absorber, but there is no assessment of the degree of influence of the components of the nonlinear energy absorber.

In work [10], a formula for an integral economic indicator is proposed, which represents the synergy of the indicators of the transporting mechanism. But a full justification of all components of the criterion is not given. In work [11], two possible ways of increasing the intensity of ultrasonic impact on biological objects are presented. But the issues related to the concentration of ultrasonic energy during cavitation treatment in a thin layer on the radiation surface remain unresolved. The use of vibration action in mixing processes is given in work [12], which, according to the authors, allowed to reduce the duration of mixing by 2.5–3 times. But there are no comparisons regarding the energy level for the process and for the movement of the mixer, the reliability of the operation of the units, due to the vibration of the machine body and energy consumption are not indicated. In [13], a qualitative and quantitative transformation of the force action of the ultrasonic device on the processing environment is given. But the issue of energy distribution and possible solutions for controlling the cavitation process are not fully resolved. The influence of ultrasonic extraction in pulsed and continuous mode on energy consumption is given in [14]. At the same time, there is no data on the consideration of the joint energy process of the device and the environment.

In [15], the results on the optimization of the efficiency of hydrogen production by ultrasonic action are presented. These studies are presented exclusively within the framework of the influence of ultrasound in the auxiliary process of electrolysis. The study of the dynamics and characteristics of the destruction of cavitation bubbles near the surfaces of rocks under the influence of ultrasound with the determination of energy is presented in [16]. However, the work does not indicate whether the assessment was carried out under the conditions of interaction of the apparatus and the processes of crushing the rock.

In [17], a method of computer determination for automatic processing of images of cavitation bubbles for the classification of petroleum products with different octane numbers using an artificial neural network is presented. At the same time, the energy level for these processes is not indicated. In [18], the prediction of the response variance is presented using statistical analysis of the energy of transient processes, which reflect only a specific study. In [19], the addition of a high-frequency transducer (1174 kHz) showed that bubble fragments and satellite bubbles induced by a low-frequency transducer (24 kHz) were able to extend their life cycle and increase their spatial distribution, thus expanding the boundaries of the cavitation zone. Regarding the determination of energy, the energy component in the range of these frequencies is not indicated, since it is the frequency that is a very significant parameter of analytical dependencies. In [20], information is provided on the use of the potential of acoustic cavitation for large-scale application in the food industry. In [21], an assessment and justification of methods for studying the parameters of acoustic processing of technological environments were carried out. Functional relationships between the acoustic parameters of the cavitation apparatus and the rheological properties of technological processing environments were revealed. This is the approach used in the study as the basis for further research. However, they lack studies of the synergy of the system.

Therefore, taking into account the current state of the issue of dynamic processes that occur during the processing of media of different structures, the impact on energy consumption, specific energy consumption and the quality of technological processes gives grounds to argue that it is advisable to conduct a study devoted to the use of low-frequency and high-frequency technologies for processing different media based on the synergy of similar processes. The solution to the problem is to use discrete-continuous models taking into account the rheological properties of the media and determine the parameters and modes based on the identified conditions for the implementation of synergy.

3. The aim and objectives of the study

The aim of the study is to determine the rational energy level in the processes of vibration processing of media of different composition and practical use. This will allow to realize the synergy of the vibration system with ensuring a rational mode of processing technological media.

To achieve the stated aim, the following objectives were solved:

– to substantiate the effectiveness of the use of vibration processing of media of different structures;

– to develop a method for studying energy dissipation in processed media;

– to study the modes and parameters of media and materials that ensure a rational level of energy distribution.

4. Materials and methods

The object of the study is a single common wave process of motion of the system, and the subject is the parameters and modes that provide a rational level of energy. A rational level of energy is an action on the processed medium at which the desired result is achieved with minimal energy expenditure. The work considers a system of a complex structure under the action of dynamic influences as a single system with its corresponding dynamic characteristics. Ensuring the given law of motion of the studied systems is due to the correct consideration of the arising resistance forces based on the use of discrete-continuous models.

The following assumptions are made in the work:

– the energy emitter is represented by a discrete model, and the working technological medium is represented by a continuous model;

– the calculation models reflect the elastic-inertial and dissipative physical-mechanical properties and parameters;

– in the contact zone of energy transfer, there is a continuous mode of motion of the vibration exciter and the medium.

Theoretical studies are performed on the basis of a discrete-continuous model, which reflects the real processes of motion of dynamic systems.

The results of the study were obtained using MATLAB software.

Experimental studies were performed on developed units for studying the processing of media by ultrasonic cavitation technology and the vibration effect on the concrete mixture.

Methods of processing experimental data: spectral analysis, natural frequencies and forms of oscillations.

The paper considers a joint study of the above technologies in terms of the general description of the motion of the system "vibration exciter – processing medium" based on the use of generally accepted provisions of the classical theory of mechanical vibrations and the theory of continuous media. The justification and choice of the model are based on classical approaches to the use of a continuous system of media motion taking into account their rheological properties.

5. Results of the study of the energy level for the treatment of technological environments

5. 1. Justification of the effectiveness of the use of vibration treatment of environments of different structures.

Vibration treatment, used in the work, is due to a significant impact on the compaction process. This process is used to improve the composition of existing liquids and emulsions or create new materials, such as concrete mix, solutions, suspensions. Typically, the vibration effect is implemented at low and high frequencies. Low-frequency and medium-frequency oscillations have a frequency in the range of 5…100 Hz, which is used to create materials, such as concrete mix, solutions [10]. High-frequency vibrations have a frequency in the range of 20,000…45,000 Hz. Such frequencies are used in ultrasonic cavitation technologies for the processes of dispersion, emulsification, homogenization, degassing, extraction, crystallization, sterilization and others in the chemical industry and processing industries [11]. The vibration effect is characterized by pressure (Pa), intensity (W/m^2), frequency (Hz) and amplitude (m) of vibrations. The physics of the cavitation technology process for processing media for the above methods of use is as follows. Under the action of external forces, compression and stretching waves arise in the processing medium, which implement a particular technological process for processing the technological medium. In the half-period of stretching, vapor-gas bubbles are formed in the deformation wave, which collapse in the half-period of compression in the deformation wave [11]. At the stage of bubble bursting, a high-pressure pulse is released in the form of a shock wave or, if a hard or elastic surface is nearby, a powerful cumulative microjet is formed in the direction of this surface.

The vibration effect on the concrete mixture is of great practical importance and is the basis of all modern technology for compacting mixtures. The essence of the vibration effect is that during vibrations, the concrete mixture acquires fluidity properties due to the disruption of the connection between the particles. The particles, which receive increased mobility, are mixed and, under the action of gravity, tend to take a more stable position. In this case, the air between the particles is squeezed out upwards and the mixture, in the end, is significantly compacted. The process of vibration compaction of the concrete mixture is complex and takes place in several stages: rearrangement of components with intensive air displacement, particle convergence and final air displacement, as well as possible additional compaction due to some additional, for example, static, pressure. The specified stage is called compression and is carried out both during vibration of the mixture and after the completion of the vibration process. In the first case, the positive effect of increasing the density and strength of concrete is achieved by a small static pressure for several minutes. In the second case, the same effect can be obtained only due to a significant specific pressure of several megapascals. In both cases, the effect is achieved due to partial compression and a more uniform distribution of pore water, as well as the sealing of contacts between the grains of aggregates. Studies of the processes of processing technological environments (Table 1) by high-frequency ultrasonic technology were carried out on the unit (Fig. 1).

Table 1

Main technological environments

No.	Environment	Density, kg/m^3	Viscosity, 10^{-3}*Pa/s
1	Water	1000	8272
2	Ethyl alcohol	798	222
3	Transformer oil	900	530
4	Olive oil	950	853
5	Motor oil	980	400

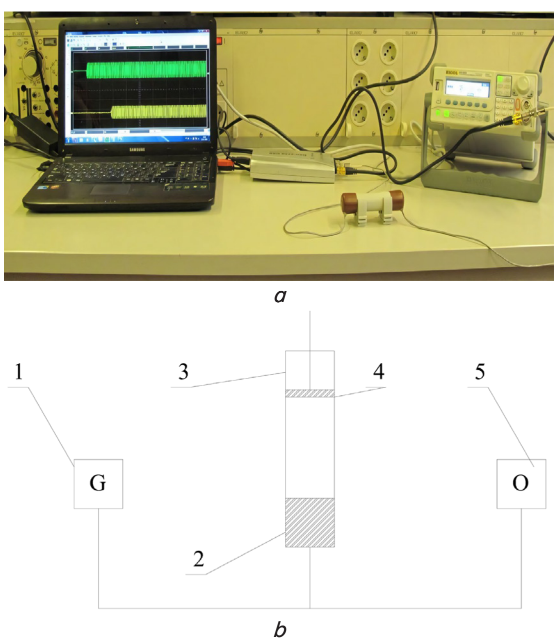


Fig. 1. Unit for studying the treatment of media by ultrasonic cavitation technology: *a* – general view; *b* – diagram; 1 – pulse generator, 2 – emitter; 3 – tube with technological medium; 4 – reflector; 5 – computer

During the research, the process parameters were determined and recorded on the computer display.

Research on low-frequency treatment was carried out by vibration compaction of concrete mixtures (Table 2) on the unit (Fig. 2).

Table 2

Composition of concrete mixtures

No.	Composition in %: C:S:CS	W/C	Hardness, s
1	1:3:0	0.33	80…100
2	1:1.4:2.6	0.35	100…120
3	1:1.82:3.38	0.41	30…60

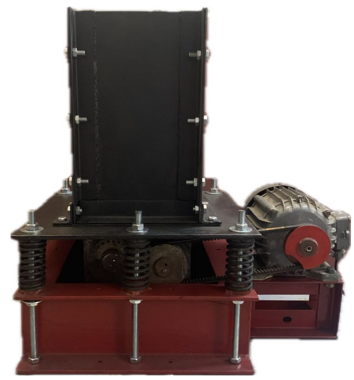


Fig. 2. General view of the vibration unit

The unit structure consists of a welded frame made of channels, which rests on vibration-isolating supports. A vibration exciter is attached to the working frame from below, and a mold is attached to the upper part.

The mold for the concrete mixture consists of a bottom part and detachable sides, which allowed changing the geometric dimensions of the mold without changing its weight. During the compaction process, the amplitude of oscillations, pressure and compaction time were measured. And the values of the parameters were recorded on the display.

5. 2. Development of a method for studying energy dissipation in materials and environments.

The procedure for developing methods for studying energy dissipation in materials and environments consists in choosing a physical model with a subsequent mathematical description and assessing the influence of parameters on determining the required energy level. From a physical point of view, the choice of the model is determined by the main characteristics: deformation and stress, which arise during any technological operation and form a stress-strain state. The phase shift between stress and strain in a dissipative environment is due to energy absorption and can be determined by various physical phenomena. The main ones are: viscous friction, mechanical hysteresis, plastic fluidity of the material, relaxation of the material. Viscous or fluid friction caused by friction of particles of a substance between themselves. The greater the relative velocity of the particles, the more energy is required for its movement. Therefore, the loss coefficient in viscous friction is proportional to the frequency, while dependent or independent of other parameters, including frequency. When a force acts on a stressed medium, irreversible micro-structure changes occur in solids (rotation and destruction of crystals in metals, destruction of molecular chains in plastics, etc.). Known calculation relations for determining energy dissipation (the method of damped oscillations, the phase angle method, energy, and others) are based on the assumption of regularities in the change of these resistance forces, proportional to the first degree of the oscillation velocity. A different approach is used in the work. The combination of frequency and amplitude X of oscillations determines the intensity of the vibration effect, which can be represented in the forms: intensity relative to acceleration I_a, intensity relative to energy I_e and intensity relative to power I_p

$$I_e = X\omega^2, I_e = X^2\omega^2, I_p = X^2\omega^3, \quad (1)$$

where X – the amplitude of oscillations (m), ω – the frequency of oscillations (rad/s).

The choice of the type of intensity is determined by the formulation of the research tasks and the technological features of the mode: so, criterion I_a has the dimension m/s^2 and characterizes the dynamics of the process. Criterion I_e has the dimension m^2/s^2 and characterizes the specific energy per unit area of the medium. Criterion I_p has the dimension m^2/s^3 and characterizes the specific energy per unit volume of the medium. From the point of view of this problem, the importance of all criteria can be noted, since in any case it is necessary to determine both the amplitude and the frequency of oscillations. And based on the adopted main hypothesis of the work, the adopted research method must take into account the effective coordination of forces in the contact zone of the "vibration exciter-environment" system. That is, regardless of the form of the load, amplitude and frequency of oscillations, it is necessary to determine the resistance of the technological medium to the movement of the working body of the vibration exciter. This resistance consists of inertial, elastic and dissipative parts. The inertial-elastic components determine the reactive resistance, and the dissipative component determines the active resistance. In analytical form, the resistance is represented as impedance

$$Z = Z_p + iZ_a, \quad (2)$$

where Z_p – the reactive part of the resistance, which is associated with the processes of periodic exchange of kinetic energy of motion with the potential energy of the medium; Z_a – the active part of the resistance, which is associated with dissipative losses in the medium of the oscillating system, i is an imaginary unit indicating the rotation of Z_p relative to the angle $\pi/2$.

A change in the reactive component of the medium load Z_p leads to a change in the resonant frequency of the ultrasonic oscillating system. The active component Z_a determines the active losses in the processing medium (including useful work) and is associated with a decrease in the amplitude of the oscillation velocity. At the resonance of the oscillating system, mutual compensation of inertial and elastic forces occurs. The reactive resistance of the system becomes equal to zero, and the impedance takes on a minimum value. Further, a model of the studied system is determined and its analytical description is carried out to determine the amplitude of oscillations and energy dissipation.

The description of the motion of the vibration exciter, taking into account the motion of the medium, is carried out on the basis of a discrete-continuous model, in which the ratio of stresses σ and deformation ε has the form

$$\sigma = E\varepsilon\left(1+i\gamma\right), \quad (3)$$

where i – an imaginary unit; E – the modulus of elasticity of the medium; γ – the energy loss coefficient.

The wave equation of the continuous system in accordance with equation (3) is taken as

$$\frac{\partial^2 u}{\partial z^2} = \frac{1}{c^2\left(1+i\gamma\right)} \cdot \frac{\partial^2 u}{\partial t^2}, \quad (4)$$

where u – the longitudinal displacement, which depends on the coordinate z and on the time t; c – the speed of wave propagation in the medium. The calculation scheme of the "vibration exciter-medium" system is shown in Fig. 3.

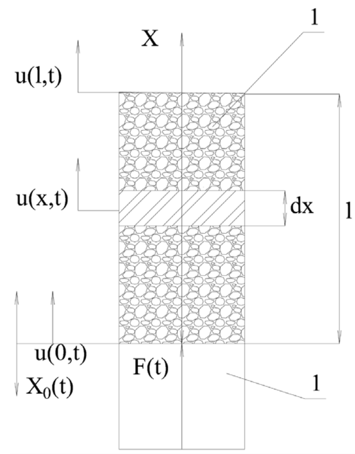


Fig. 3. Calculation scheme of the "vibration exciter-environment" system: 1 — vibration exciter; 2 — processing environment

The solution to equation (4) is defined as

$$u\left(x,t\right) = \left[C_1 e^{x(\alpha+i\beta)} + C_2 e^{-x(\alpha+i\beta)}\right] e^{i\omega t}, \quad (5)$$

where C_1 and C_2 – constants of integration determined from the boundary conditions; α and β are parameters determined by substituting (5) into (6).

The corresponding partial derivatives of the solutions (5) after simple transformations have the form

$$\left(\alpha+i\beta\right)^2 \left[C_1 e^{x(\alpha+i\beta)} + C_2 e^{-x(\alpha+i\beta)}\right] e^{i\omega t} = \\ = \frac{\omega^2}{c^2\left(1+i\gamma\right)} \left[C_1 e^{x(\alpha+i\beta)} + C_2 e^{-x(\alpha+i\beta)}\right] e^{i\omega t}, \quad (6)$$

from where $\left(\alpha+i,\beta\right)^2 = -\frac{\omega^2}{c^2} \cdot \frac{1}{\left(1+i\gamma\right)}$.

Having divided (6) into real and imaginary parts, it is possible to obtain

$$\alpha^2 - \beta^2 + 2\alpha\beta i = -\frac{\omega^2}{c^2} \cdot \frac{\left(1-i\gamma\right)}{\left(1+i\gamma\right)},$$

or

$$\alpha^2 - \beta^2 + 2\alpha\beta i = -\frac{\omega^2}{c^2} \cdot \frac{1}{\left(1+\gamma^2\right)} + i\frac{\omega^2}{c^2} \cdot \frac{\gamma}{\left(1+\gamma^2\right)},$$

from where:

$$\alpha^2 - \beta^2 = -\frac{\omega^2}{c^2} \frac{1}{\left(1+\gamma^2\right)},$$

$$\alpha^2 - \beta^2 = -\frac{\omega^2}{c^2} \frac{1}{\left(1+\gamma^2\right)}. \quad (7)$$

The solution (7) is the expression for determining the coefficients

$$\alpha=\mu\frac{\omega}{A};\ \beta=\gamma\frac{\omega}{A}, \tag{8}$$

where

$$\mu=\sqrt{\frac{\sqrt{1+\gamma^2}-1}{2\left(1+\gamma^2\right)}};\ \nu=\sqrt{\frac{\sqrt{1+\gamma^2}+1}{2\left(1+\gamma^2\right)}}.$$

To determine the amplitude of the oscillations of the exciter, the following boundary conditions are adopted:

– at $z=0$

$$F_0e^{i\omega t}-m_2\ddot{u}(t,0)=F(t,0); \tag{9}$$

– at $x=h$

$$F(t,h)=0, \tag{10}$$

where

$$F(t,0)=p_0S=E\varepsilon_0\left(1+i\gamma\right)S, \tag{11}$$

p_0 – the pressure of the medium on the contact surface of the oscillation exciter at $x=0$; $\varepsilon_0(\partial u/\partial z)^u_{z=0}$ – the relative deformation of the medium in the contact zone.

The analytical dependence for the forces is determined

$$F(t,0)=ES(1+i\gamma)\left(\frac{\partial u}{\partial z}\right)_{x=0}=$$
$$=ES(1+i\gamma)\left[C_1e^{x(\alpha+i\beta)}-C_2e^{-x(\alpha+i\beta)]}\right]_{x=0}^{(\alpha+i\beta)}e^{i\omega t}=$$
$$=ES(1+i\gamma)e^{i\omega t}\left(C_1-C_2\right)\left(\alpha+i\beta\right)=ES\frac{C_2-C_1}{\alpha+i\beta}k^2e^{i\omega t}.$$

Considering that:

$$E=c^2\rho;$$

$$k^2=\frac{\omega^2}{c^2};\ \rho Sh=m_c, \tag{12}$$

obtained

$$F(t,0)=\rho c^2S\frac{\left(C_1-C_2\right)}{(\alpha+i\beta)}\frac{\omega^2}{c^2}e^{i\omega t},$$

or in the final form

$$F(t,0)=\frac{m_c\omega^2}{h}\left(C_2-C_1\right)\frac{e^{i\omega t}}{\left(\alpha+i\beta\right)}=$$
$$=-\frac{m_c\omega^2}{h}\cdot\frac{e^{i\omega t}}{\left(\alpha+i\beta\right)}\left(C_1-C_2\right). \tag{13}$$

Introduced notations

$$h\left(\alpha+i\beta\right)=\delta. \tag{14}$$

After transformations (6), (7), (12), (13) and application of boundary conditions (9) and (11) and taking into account (14), a system of algebraic equations is obtained for finding the integration constants C_1, C_2

$$F_0e^{i\omega t}+m_v\left(C_1+C_2\right)\omega^2e^{i\omega t}=\frac{m_c\omega^2}{h}\frac{e^{i\omega t}}{\left(d+i\beta\right)}\left(C_1-C_2\right),$$

which after simplification takes the following form

$$\left(\frac{m_c\omega^2}{\delta}-m_v\omega^2\right)C_1+\left(\frac{m_c\omega^2}{\delta}-m_v\omega^2\right)C_2=F_0. \tag{15}$$

To find the constants C_1 and C_2, the dependence (15) was used, for which the determinant has the form

$$\Delta=\left(m_v\omega^2+\frac{m_c\omega^2}{\delta}\right)e^{-\delta}-\left(m_v\omega^2-\frac{m_c\omega^2}{\delta}\right)e^{-\delta}=$$
$$=e^{-\delta}\left[\left(m_v\omega^2\right)+\left(m_v\omega^2\right)\frac{m_c}{\delta}-\frac{m_c^2\omega^2}{\delta^2}\right]-$$
$$-e^{\delta}\left[\left(m_v\omega^2\right)+\left(m_v\omega^2\right)\frac{m_c}{\delta}-\frac{m_c^2\omega^2}{\delta^2}\right]=$$
$$=-\left(m_v\omega^2\right)\left(e^{\delta}-e^{-\delta}\right)+\left(e^{\delta}+e^{-\delta}\right)-$$
$$-\left(m_v\omega^2\right)\frac{m_c}{\delta}\left(e^{\delta}+e^{-\delta}\right)+\frac{m_c\omega^2}{\delta^2}\left(e^{\delta}-e^{-\delta}\right)=$$
$$=2\text{ch}\delta\left\{-\left(m_v\omega^2\right)+\frac{m_c^2\omega^2}{\delta^2}\text{ch}\delta-\frac{m_c}{\delta}\left(m_v\omega^2\right)\right\}. \tag{16}$$

Then C_1 and C_2 are defined as:

$$C_1=\frac{F_0}{\Delta}\left(\frac{m_c}{\delta}\right)e^{-\delta};\ C_2=-\frac{F_0}{\Delta}\left(\frac{m_c}{\delta}\right)e^{\delta}. \tag{17}$$

Based on the value of the integration constants (17), a general solution of the differential equation is obtained, which represents the displacement of any element of the system "vibration exciter – processing medium"

$$u(x,t)=-\left[\left(\frac{m_c}{\delta}\right)e^{(h-x)(\alpha+i\beta)}-\left(\frac{m_c}{\delta}\right)e^{-(h-x)(\alpha+i\beta)}\right]\frac{e^{i\omega l}}{\Delta}F_0. \tag{18}$$

To determine law of motion of the exciter, taking into account the physical and mechanical properties of the medium, an analytical expression of its oscillation amplitude was found. For this purpose, in the general solution (18), it is assumed that $x=0$, i.e., the condition for considering the joint motion of the contact zone of the exciter and the medium.

Under this condition, expression (18) has the form

$$u(0,t)=-\frac{2\text{ch}\delta}{\Delta\delta}F_0m_ce^{i\omega t}, \tag{19}$$

where

$$\Delta\delta=2\text{ch}\delta\left\{\left[\left[\left(m_v\omega^2\right)-\frac{m_c^2\omega^2}{\delta^2}\right]\delta\text{ch}\delta+m_c\left(m_v\omega^2\right)\right]\right\}, \tag{20}$$

or taking into account (20)

$$U(0,t)=\frac{F_0e^{i\omega t}}{\left[\left(m_v\omega^2\right)+\frac{m_c^2\omega^2}{\delta^2}\right]\delta\text{sh}\delta}. \tag{21}$$

To determine the amplitude of the oscillations of the exciter, it is necessary to expand the real and imaginary parts in (21)

$$u(0,t)=A_1+iA_2. \tag{22}$$

It can be shown that

$$\frac{\mathrm{ch}\delta}{\delta}=b+ia;\ \delta\mathrm{ch}\delta=a+ib, \tag{23}$$

where

$$a=\frac{\alpha\mathrm{sh}2\alpha h+\beta\sin 2\beta h}{h\left(\alpha^2+\beta^2\right)\left(\mathrm{ch}2\alpha h+\cos 2\beta h\right)};$$

$$d=\frac{\alpha\sin 2\beta h-\beta\mathrm{sh}2\alpha h}{h\left(\alpha^2+\beta^2\right)\left(\mathrm{ch}2\alpha h+\cos 2\beta h\right)}. \tag{24}$$

Then expression (21) taking into account the coefficients (24) will be written as follows

$$X_0=\frac{F_0}{\sqrt{\left(m_v\omega^2+m_c\omega^2 a\right)^2+\left(m_c\omega^2 d\right)^2}}. \tag{25}$$

To determine the analytical dependence of energy, the pressure of the medium on the exciter of oscillations is determined

$$F_c\left(0,t\right)=ES\left(1+i\gamma\right)\frac{\partial u}{\partial z}\bigg|_{Z=0}=ES\left(1+i\gamma\right)\left[\frac{\left(C_2-C_1\right)}{\left(\alpha+i\beta\right)e^{i\omega t}}\right]=$$
$$=ES\left[\frac{\left(C_2-C_1\right)}{\left(\alpha+i\beta\right)e^{i\omega t}}\right]=-\frac{m_c\omega^2}{h}\left[\left(C_1-C_2\right)\frac{e^{in\omega t}}{\left(\alpha+i\beta\right)}\right]. \tag{26}$$

Then the expression for determining the contact force has the form

$$F_c\left(0,t\right)=m_c X_o\omega^2\sqrt{a^2+d^2}. \tag{27}$$

The pressure of the medium on the exciter of oscillations is obtained by relating (27) to the cross-sectional area of the medium S

$$p_c=\frac{m_c\omega^2 X_o}{S}\sqrt{a^2+d^2}. \tag{28}$$

The obtained expression for the energy accumulated in the contact zone, which is transformed into the processing medium

$$E_{k.z.}=\int_0^T F_c\dot{x}\sin\alpha\cos\omega t dt, \tag{29}$$

where F_c – a component of the total force that overcomes the energy component; α – the angle of displacement of this force relative to the displacement x; $\dot{x}$ – the speed of oscillations.

The force component balances the reaction of the medium, which is the key dependence, which, unlike the discrete model, is a mixed discrete-continuous

$$F_0\sin\alpha=m_c X_o\omega^2 d, \tag{30}$$

where d – the wave coefficient, which takes into account the energy component of the resistance of the technological medium to the oscillations of the vibration exciter and is dependent on the acoustic and geometric parameters and rheological properties of the medium

$$d=\frac{\alpha\sin 2\beta h-\beta\mathrm{sh}2\alpha h}{h\left(\alpha^2+\beta^2\right)\left[\mathrm{ch}2\alpha h+\cos 2\beta h\right]}. \tag{31}$$

Then the expression for the energy accumulated in the contact zone

$$E_{k.c.}=\pi m_c X_0^2\omega^3 d. \tag{32}$$

Reduced to the mass of the medium. m_c the energy expression will take the form

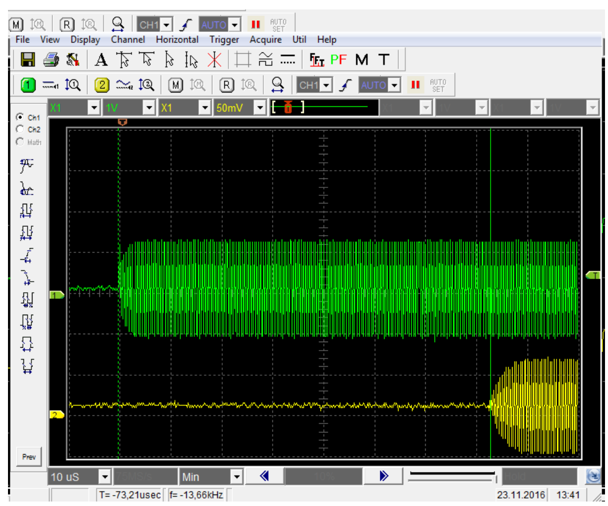
$$\bar{E}_k=\pi X_0^2\omega^3 d. \tag{33}$$

The proposed analytical dependence for determining energy (33) can be used for various media or materials that have elastic-viscous properties and are under load.

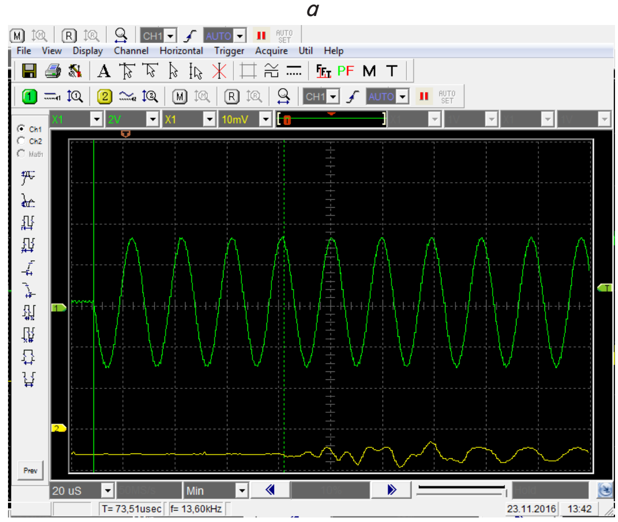
5. 3. Research of modes and parameters of media and materials that ensure a rational level of energy distribution

5. 3. 1. Vibroacoustic processing of media

As a result of the experimental studies performed, vibrograms were obtained (Fig. 4), the processing of which allowed the construction of graphs of changes in wave coefficients (Fig. 5) and the establishment of parameters (Fig. 6) and determination of intensities (Table 3) of a rational level of energy.



a



b

Fig. 4. Characteristic vibrograms of the amplitude of milk vibrations at different frequencies: *a* — frequency of vibrations f = 2.58 MHz; *b* — frequency of vibrations f = 52 kHz

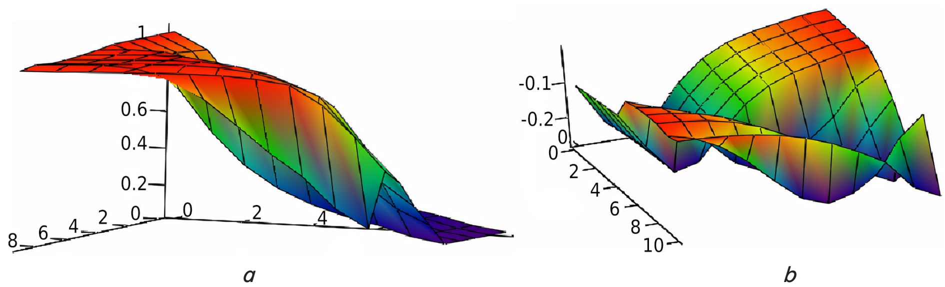


Fig. 5. Graphs of changes in wave coefficients: *a* – reactive resistance of the medium; *b* – active resistance of the medium

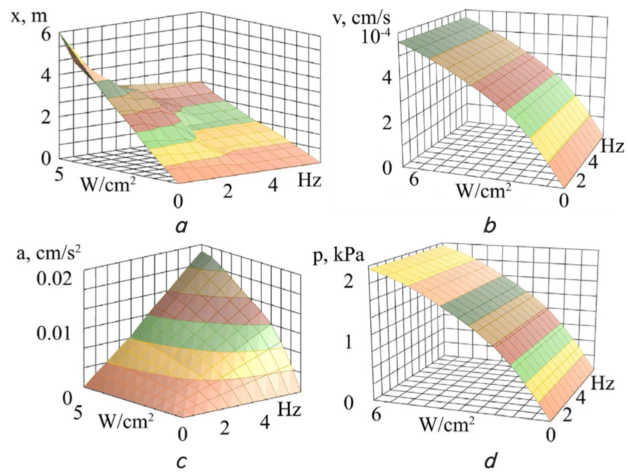


Fig. 6. Change in parameters of the cavitation process depending on frequency and intensity: *a* – amplitude of oscillations; *b* – velocity of oscillations; *c* – acceleration; *d* – acoustic pressure

Table 3

Intensity thresholds for media of different viscosities

Viscosity, η, 10^{-3} Pa	20	40	80	100	150	250	300	400
Intensity, I_{min}, W/cm^2	1.75	2.00	2.45	2.55	3.75	7.50	12.50	16.00
Intensity, I_{max}, W/cm^2	4.35	6.00	7.55	7.75	10.35	17.00	21.55	35.00

The constructed graphs demonstrate a complex form of change in wave coefficients.

In this case, the coefficient (Fig. 5, *a*) determines the influence of reactive resistance (elastic-inertial properties) in a certain region of the accepted numerical values approaches zero, i.e. under such conditions it is near the resonant mode. The coefficient (Fig. 5, *b*) determines the influence of active resistance (dissipative properties) has a value in one region, i.e. affects the reduction of both pressure and the intensity of oscillations.

The influence of the parameters of the working process on the energy of processing technological media is determined (Fig. 3).

As follows from the above graphs, the amplitude of oscillations, as well as acceleration, have a complex nature in antiphase with each other. Velocity and pressure coincide in phase of oscillations. The obtained limit values of intensity for media of different viscosity are given in Table 3.

The analysis of the numerical values given in Table 3 shows a significant influence of viscosity on the intensity as a key energy parameter. The obtained result served as the basis for the development of classification features of media when determining the energy level for the stages of ultrasonic cavitation treatment [21].

5. 3. 2. Low-frequency treatment of media

To calculate the amplitude of vibrations of the vibration platform, the above formula (25) was refined due to the presence of supports that are vibration isolators (Fig. 2)

$$X_0 = \frac{F_0}{\sqrt{\left(m_v\omega^2 + m_c\omega^2 a\right)^2 + \left(m_c\omega^2 d\right)^2}}. \quad (34)$$

where C_v – the stiffness coefficient of the vibration platform supports.

Its influence on the dynamics of motion is felt only in the start-up and resonance modes, since it is calculated from the condition of vibration isolation from the foundation. As a result of the research, vibrograms were obtained (Fig. 7), the processing of which allowed the calculation of parameters (Table 4) for energy calculation (Table 5).

The given vibrograms testify to the complex form of vibrations, both at the frequency of vibrations 157 s^{-1} and at the frequency of vibrations 314 s^{-1}, but the vibrations are characterized by a stable mode of their change.

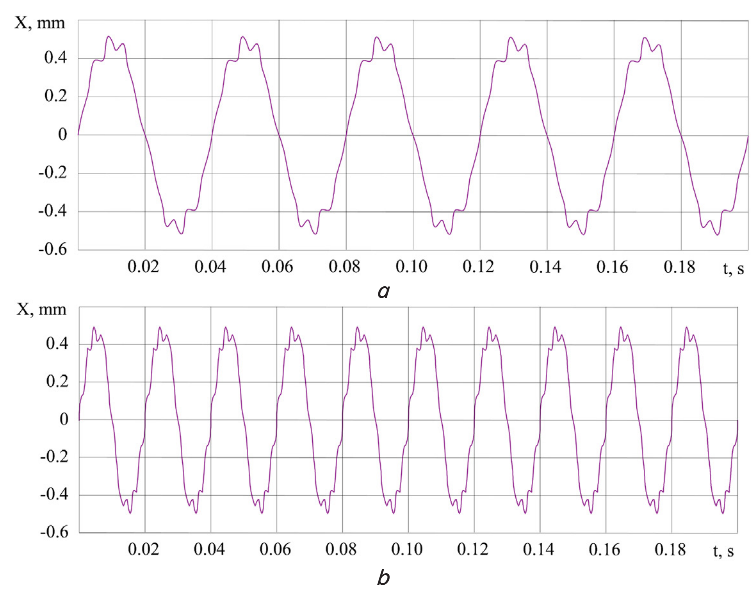


Fig. 7. Vibrograms of steady vibrations of the vibrating platform: *a* – mode 157 s^{-1}; *b* – mode 314 s^{-1}

Table 4

Numerical values of the parameters of the working process of compaction of the concrete mixture

Experiment series No.	Concrete mix stiffness, s	Amplitude of relative deformation, $\varepsilon_0 \cdot 10^{-3}$	Frequency, ω, s^{-1}	Energy dissipation coefficient, ψ	Resistance coefficient, γ	Dynamic modulus of elasticity, $E \cdot 10^4$, N/m	Velocity of vibrations in the concrete mix, C, m/s
1	90	1.332	314	0.0200	0.0040	482	46
2		1.670	314	0.0434	0.0069	482	46
3		2.582	314	0.0942	0.0150	490.5	46.4
4		2.582	219.8	0.0942	0.0150	234	32.2
1	60	2.062	314	0.3460	0.0550	446	44
2		2.062	157	0.3460	0.0550	105	21.6
3		1.423	314	0.3150	0.0502	450	44.2

The important results, which are given in the table, are the numerical values of the energy dissipation coefficients, which are functions of the amplitude of oscillations. The obtained values of the amplitude of oscillations were compared theoretically and experimentally by the vibrating platform depending on the height (Fig. 8). The discrepancy was 14%.

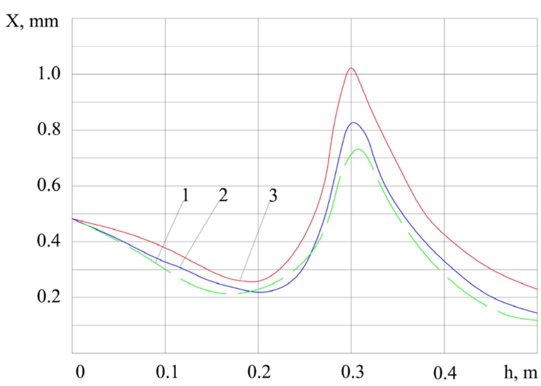


Fig. 8. Curves of changes in the amplitude of vibrations of the vibrating platform depending on the height: 1 — experimental; 2 — calculated by formula (25); 3 — by formula (34)

The constructed graphs (Fig. 9) of the change in the energy spent on compaction of the concrete mixture depending on the height for the vibration frequencies $\omega = 200\ s^{-1}$ and $\omega = 314\ s^{-1}$ made it possible to propose numerical values of the specific energy depending on the consistency of the concrete mixture (Table 5).

Table 5

Value of specific energy depending on the consistency of concrete mixture

Concrete mix stiffness, (cm), s	5...7 s	30 s	60 s	90 s	120 s
Specific energy value, w_k, kPa · m	56	112	180	254	320

These specific energy values practically cover the most commonly used concrete mix stiffnesses and are those that determine the required level of energy for compaction.

6. Discussion of the results of determining the rational energy level

The obtained results are reliable, which is confirmed by the justification and choice of the equation of motion of the medium (4), its solution by the Fourier method (5) and wave coefficients (24). The complex motion of the system (Fig. 5) and the adequacy of theoretical and experimental solutions (Fig. 8) confirm the stated hypothesis and research results. The research method and results used in the work are different in comparison with the cited works. The peculiarity of the proposed method is the reduction of complex discrete-continuous systems to a discrete form while maintaining the consideration of wave phenomena in the processing media. Unlike [6], where the result is given for a discrete model that does not take into account wave phenomena. And the proposed approach allows to take into account wave phenomena, due to the presence of the coefficient d in formula (30), which reveals the features of motion and energy (Fig. 6, 8).

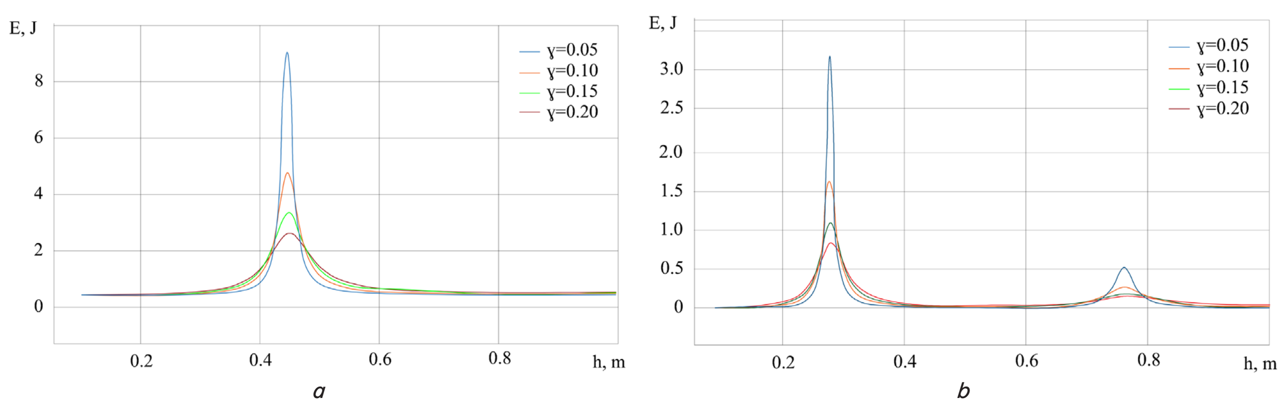


Fig. 9. Graphs of changes in energy spent on compaction of concrete mixture depending on the height and frequency of oscillations: a — $\omega = 200\ s^{-1}$; b — $\omega = 314\ s^{-1}$

The second feature is to consider the joint energy process of the apparatus, since there is a mutual influence of subsystems during their interaction. In work [14], such an approach is absent, since energy consumption is considered during ultrasonic extraction of the medium without taking into account the mutual influence. The developed method for studying energy dissipation in the processed media allowed to obtain numerical data based on the dependencies (25), (27), (29), (32) and to analyze them. Thus, the problem of determining rational parameters and energy levels in practical calculations of high-frequency and low-frequency processes of processing media of different structures was solved. Thus, for high-frequency vibration actions (Table 3), the limiting values of intensity for media of different viscosities were obtained. And for low-frequency vibration actions (Tables 4, 5) the specific energy values were obtained depending on the consistency of the concrete mixture. The results of experimental studies confirm the data obtained theoretically (Fig. 4, 7, 8). The proposed research method and the obtained dependencies can be applied to implement practical recommendations for processing media of different structures, used in the calculation and design of other similar vibration systems. These are the processes of grinding materials, creating new solutions, vegetable milk, which are increasingly used.

However, there are certain limitations. The results obtained are valid in the range of force action on the processing media within the limits of the performed studies. Thus, experimental vibroacoustic processing was carried out for milk, and low-frequency processing for concrete mixture with stiffness of 60 and 90 s. Under other conditions, the results require verification, especially this applies to data obtained experimentally. The disadvantages include the lack of research on the variable mode of vibration action at the stages of material processing and the analysis of transient processes, which can be implemented in further research.

7. Conclusions

1. The areas of application of vibration processing are determined and the effectiveness of vibration processing of media of various structures is substantiated. With high-frequency action on the medium, a high-pressure pulse is released in the form of a shock wave. Low-frequency action occurs during compaction, as a result of partial compression and more uniform distribution of water, as well as compaction of contacts between aggregate grains.

2. Substantiation and development of methods for studying energy dissipation in materials and media is based on the adopted hybrid model of the "vibration exciter – processing medium" system. In the performed studies, the model is considered as a common computational system, which implements the whole directional movement with a rational level of energy distribution. An analytical solution to the problem was performed, which allowed obtaining analytical dependencies for determining the energy for the flow of the technological process of processing media.

3. The modes and parameters of media and materials that ensure a rational level of energy distribution are investigated and determined. Thus, the range of changes in the intensity of high-frequency action within $I_{\min} = 1.65...16.0$ W/cm^2 is established as the main energy criterion for liquid media with a viscosity of $20 \cdot 10^{-3}$ Pa…$400 \cdot 10^{-3}$ Pa. The value of the specific energy for low-frequency action is in the range of 112…320 kPa · m for concrete mixtures with a stiffness of 30…120 s.

Conflict of interest

The authors declare that they have no conflict of interest regarding this study, including financial, personal, authorship or other nature, which could affect the study and its results presented in this article.

Financing

The study was conducted without financial support.

Data availability

The manuscript has no related data.

Use of artificial intelligence

The authors used artificial intelligence technologies within the permissible framework to provide their own verified data, which is described in the research methodology section.

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