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DEVELOPMENT OF A MODEL OF THE USEFUL EFFECT OF THE SYSTEM AT DIFFERENT LEVELS OF SUBSYSTEM INTERCHANGEABILITY IN THE PROBLEM OF OPTIMIZING THE ALLOCATION OF A LIMITED RESOURCE

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The object of this study is the process of optimizing the allocation of limited resources among the subsystems of a large complex system. The problem addressed lies in the insufficient adequacy of existing utility models, which fail to account for the degree of subsystem interchangeability and therefore cannot accurately predict optimal resource distribution. The core result is the development of a universal utility model based on logistic dependencies and a flexible scalar convolution constructed using a modified marginal Kolmogorov-Gabor polynomial with the coefficient k_{Add}, which governs interchangeability. Complete trajectories of optimal solutions were obtained for various values of total resources and interchangeability levels. It was found that variations in these parameters alter the optimal effect by 25–35% under conditions of significant resource scarcity. The problem was resolved by combining the universality of logistic models with the flexibility of the new convolution, which generalizes additive, multiplicative, and minimizing forms and enables continuous adjustment of the model to reflect subsystem interaction characteristics. The observed patterns indicate that at low interchangeability, resources are distributed more evenly, while at high interchangeability, they are concentrated in the most efficient subsystem, significantly enhancing the overall effect under resource deficit conditions. The results are applicable to the design and management of large complex systems – such as IT security, crisis management, and organizational design – where informed decisions are required regarding the allocation of limited resources while considering structural flexibility. This approach is particularly suitable when statistical data are available for calibrating the logistic parameters of subsystems and assessing their interchangeability.

Keywords: scalar convolution, resource optimization, utility effect, optimal solution trajectories, cybersecurity, information systems

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1. Introduction

The challenges of the modern world largely stem from the problems of large complex systems. Small, simple systems typically do not cause significant issues – and when they do, closer examination often reveals that they exhibit certain characteristics of complex systems. One of the defining properties of large complex systems (hereinafter referred to as systems) is their ability to produce a useful effect. If the effect is not beneficial, it assumes a negative value. The useful effect generated by a system depends in specific ways on the resources supplied at its input. Large systems produce immense effects and consume equally vast resources. Global problems mainly arise within the context of large complex systems (LCS), as their structural and functional complexity gives rise to nonlinearity, multistability, and extreme sensitivity to initial conditions. A key characteristic of LCS is their capacity to generate a useful effect that depends on both the quantity and quality of input resources. At the same time, even minor errors in design, management, or resource allocation can lead to catastrophic outcomes due to the system's inherent scalability. Because field experiments are costly, mathematical and simulation modeling serves as an effective tool for exploring the relationships between resources and the resulting beneficial effects. This highlights the importance of developing accurate models for optimizing the resource allocation of large complex systems. Errors in creating, managing, or planning resource provision for such systems can be extremely costly. Therefore, mathematical and simulation modeling represents a more practical and cost-effective alternative to field experimentation when determining optimal resource allocation. The synergy within complex systems lies in their ability to replicate the qualitative behavior of their own components (subsystems). However, the total effect of a system is not merely the sum of the effects produced by its subsystems. The way in which the beneficial effects of subsystems combine to form the overall system effect depends on the system's properties, the characteristics of its subsystems, and must therefore be reflected in the model. One of the most critical characteristics of any system is the degree of interchangeability among its subsystems. Variations in this parameter alter the relationship by which subsystem effects are aggregated into the overall system effect. This, in turn, determines how resources should be distributed among subsystems to achieve the maximum overall benefit. Consequently, developing models of system utility under varying levels of subsystem interchangeability represents a relevant and significant task – one that forms part of the broader problem of optimizing the distribution of a system's limited resources among its subsystems.

2. Literature review and problem statement

In [1], a model was proposed for assessing the level of software information security (the effect) as a function of available resources. However, the specific dependence of the effect on resources was not established, as the study primarily focused on conceptual aspects of evaluating software information security under limited computational resources. To identify more concrete patterns relevant to a specific domain, a more narrowly defined problem formulation was needed. This was accomplished in [2], where dependencies describing the efficiency of regional production process development were identified. Nevertheless, these dependencies were not linked to the inherent characteristics of the system itself, which limited the generalizability of the findings to other industries. The reason was that the researchers did not intend to extend their study beyond their particular field. For an effective analysis of how the useful effect depends on input resources – and to enable the transfer of results across different domains – it is advisable to begin by determining the fundamental dependencies that govern system development. This approach was adopted in [3], where an analysis of computer system information security in the context of malicious software was conducted. In that work, the S-shaped form of the typical dependence between the coefficient of information utility and its volume was identified. However, no precise formalization was provided that could link this relationship with standard development modeling dependencies, such as exponential or logistic ones. The dynamic nature of computer epidemic development, expressed through logistic dependencies in differential form as a system of ordinary differential equations, was explored in [4, 5]. This made it possible to forecast the progression of epidemics under various countermeasure strategies. The main relationships demonstrated a clear logistic character. However, the differential form of these logistic dependencies was not connected to their integral form, which is more straightforward to apply and extend to other fields. This disconnect stemmed from the heterogeneity of the model, which combined both logistic and non-logistic components. Such a connection was later established in [6], where the historical development of epidemic models was examined systematically and in detail. A logical framework for the family of such models was developed, and a link between the differential and integral forms of logistic equations was established. However, the application of logistic equations in that study was limited exclusively to computer epidemics. The generalization of the logistic modeling approach to other fields was accomplished in [7], which introduced a model describing organizational development during the transformation stage aimed at adapting to new projects. In this study, logistic models were applied to represent both organizations and projects. Moreover, unlike previous research, it provided a rigorous mathematical derivation of the integral logistic form by solving the corresponding ordinary differential logistic equation. The limitation, however, was that the organization was treated as a monolithic entity rather than as a complex system composed of interacting subsystems.

The analysis of complex systems is almost always multifactorial, while their optimization tends to be multicriteria. To simplify the analysis of such complex processes or systems, it is often useful to decompose them into components. For instance, this approach is applied to radio signals using the Fourier transform [8]. However, that method has a rather narrow and specialized range of applicability. When decomposing a complex system – such as an organization – into components, it is necessary to thoroughly examine its structure, as well as its internal and external connections, within the context of a specific domain. This approach was demonstrated in [9, 10]. In [9], multicriteria optimization was applied to ensure fair service delivery in outpatient care. In [10], a multicriteria problem was solved for managing digital social channels, where the task was to improve the criteria used in decision-support procedures for relevant management tasks. However, neither of these works [9, 10] provided generalizations applicable to other domains. Moreover, the issue of scalarizing multivector criteria and multivector assessments was not addressed. This omission stems from the fact that, without a detailed analysis of the object's specific characteristics, it is difficult to justify the preference for one type of scalar convolution over another. Therefore, it is necessary to evaluate the adequacy of the model based on a particular form of scalar convolution. Such an evaluation was carried out in [11, 12]. In [11], additive convolutions with weighting coefficients were used to assess the likelihood of multiple attack implementations targeting a selected security service. In [12], an additive convolution with time-varying weighting coefficients was employed to scalarize the vector criterion in an information system survivability model. Both models presented in [11, 12] demonstrated high adequacy in representing the processes being modeled. However, other forms of scalar convolution – such as those capable of accounting for potential changes in subsystem or subcriterion interchangeability – were not examined. To address this limitation, it would have been necessary to consider, at a minimum, multiplicative and minimizing convolutions. A comprehensive analysis of scalar convolution types was conducted in [13], which examined additive, multiplicative, minimizing, and various hybrid hierarchical convolutions. Nonetheless, the properties of these combined convolutions were not explored in detail, as such an analysis extended beyond the intended scope of that publication.

A logical next step in the development of combined convolutions is the application of the Kolmogorov-Gabor polynomial, which is employed in the group argument accounting method [14]. However, in that work, the Kolmogorov-Gabor polynomial was used exclusively for solving group regression problems within the framework of the group argument accounting method. While the study established the main theoretical foundations, it paid insufficient attention to solving specific applied problems. Applied aspects of using the Kolmogorov-Gabor polynomial for water resource forecasting were examined in [15, 16]. Mathematically, the models in those works can be regarded as analogous to combined convolutions. However, the approach of using a scalar convolution based on the Kolmogorov-Gabor polynomial was not formulated, making it impossible to modify the method to create a more flexible tool for building multifactor models. It is therefore necessary to formalize such an approach, incorporating the ability to smoothly adjust the subsystem interchangeability indicator within the convolution – one of the objectives of this study.

Modeling is essential for predicting the consequences of management decisions, particularly those related to resource management. In practice, project and organizational resource management is typically performed by managers who rely on their own experience or on established best practices summarized in guidelines and manuals such as [17, 18]. However, these manuals are largely general and do not account for the specifics of particular industries. They must be supplemented with standards and recommendations specific to individual domains, such as information security and cybersecurity [19, 20]. Even then, the absence or oversimplification of models that could help predict outcomes remains a significant limitation. These documents are designed primarily for managers rather than applied mathematicians. Yet, to make truly optimal decisions, one needs adequate models describing the development of the systems being managed, along with methods for optimizing relevant processes based on those models.

An example of resource allocation optimization – specifically for human resources (programmers) with different qualification levels – was presented in [21]. The optimization criteria were minimal project duration and minimal project cost. However, this approach was not extended to other domains due to the high specificity of the problem and the corresponding mathematical model. Nonetheless, the idea proposed in that study regarding the interaction of processes with opposing trends can be applied more broadly.

This concept was implemented in [22], which focused on assessing information security effectiveness in relation to economic indicators. It was shown that the total economic losses of an information system consist of two components: losses from attacks, which decrease hyperbolically with increasing investment in information security, and security costs, which increase linearly. The resulting dependency has an optimal point (a minimum) that must be determined through optimization procedures. However, the model's main limitation lies in its simplicity and its failure to account for various other types of losses and expenses.

This limitation was addressed in [23], which focused entirely on constructing the relationship between information system losses and the level of information security expenditures. In addition to the previously considered components, this study also accounted for secondary attack effects, costs of consequence mitigation, system recovery, and resource diversion from primary functions. As a result, the model provided a more accurate and comprehensive solution. However, the issue of resource allocation among subsystems remained unexamined.

An analysis of the literature revealed that existing studies do not offer a universal approach to modeling the effective performance of large, complex systems while considering subsystem interchangeability. Most works are confined to specific types of scalar convolutions or fail to relate the system's structural properties to the form of effect aggregation. Flexible mechanisms enabling continuous model adjustment according to the degree of subsystem interchangeability have not been explored, nor have complete trajectories of optimal solutions for variations in key parameters been developed. These shortcomings underscore the need for a new model that integrates empirically derived logistic relationships with a flexible scalar convolution based on a modified Kolmogorov-Gabor polynomial.

3. The aim and objectives of the study

The aim of the study is to develop a general model of the useful effect of the system at different levels of subsystem interchangeability in the task of optimizing the distribution of a limited total resource between subsystems. This will allow to improve the quality of applied systems for supporting management decisions regarding the resource provision of large complex systems.

To achieve this aim, the following objectives were accomplished:

– to determine the basic models of the useful effects of systems and subsystems;

– to determine the nature of the dependence of the useful effect on resources for convolution based on the modified Kolmogorov-Gabor polynomial;

– to search for optimal useful effects of the system depending on the total resource provision of the system;

– to search for the optimal distribution of the limited total resource of the system between its subsystems taking into account the level of subsystem interchangeability.

4. Materials and methods

The object of the study is the process of optimizing the distribution of a limited resource between subsystems of a large complex system.

The subject of the study is the model of the useful effects of the system, which will be used in the procedure for optimizing the resource provision of a large complex system.

The main hypothesis of the study is the possibility of taking into account the level of interchangeability of subsystems in the model of the useful effect of the system by using a hybrid convolution based on the Kolmogorov-Gabor polynomial with the appropriate additional weight coefficient.

The following assumptions were made.

It was believed that to form the useful effects of the system and subsystems, all available opportunities for obtaining the best solution are used. In this case, the corresponding dependencies will approach the classical ones, that is, statistically tested on a large number of similar problems, namely, linear, exponential and logistic dependencies.

The useful effect of the system can have a negative and positive sign depending on the success of the system or subsystem.

The effect of the system is not a simple sum of the effects of subsystems and depends on the level of interchangeability of subsystems.

The study made certain simplifications, which are given below.

The dependences of useful effects on the resources spent are an approximation of real dependencies. Approximation allows to get rid of random changes in indicators and highlight the main patterns of processes.

The study analyzed statistical data on the dependence of the useful effects of systems and subsystems on resource provision. The analysis was performed using regression analysis methods, on the basis of which the parameters of the regression logistic dependence were determined for the subsystems of the objects under study.

The scale of measurement of the values of resources and useful effects was reduced to a relative one. A resource that corresponds to the normatively complete provision of the subsystem was taken as a unit of resource. Similarly, the value of the effect that corresponds to the normatively complete useful effect of the subsystem was taken as a unit of useful effect.

The analysis of the properties of convolutions of various types was performed: additive, multiplicative, minimizing, and convolutions based on the Kolmogorov-Gabor polynomial were proposed. In the scalar convolution based on the Kolmogorov-Gabor polynomial, an additional weighting factor was introduced for flexible control of the level of interchangeability of subsystems in the model of the useful effect of the system.

The search for optimal solutions for the distribution of resources between subsystems was performed using the method of complete direct search with a decrease in the search step as the optimal solution is approached. The step reduction was performed to ensure the given accuracy of the solution.

The simulation models were created using the algorithmic language Matlab.

Analysis of the results of existing studies [1–7] showed that the most adequate and at the same time universal dependence of the useful effect Ef_1 on the resources R_1 for each of the subsystems is the logistic dependence, which can be represented in the differential

$$\frac{dEf_i}{dt} = m_i\left(a_i - R_i\right)\left(R_i - d_i\right), \quad (1)$$

or integral form

$$Ef_i = SL_i\left(R_i\right) = d_i + \frac{a_i - d_i}{1 + exp\left(-\frac{2}{T_i}\left(R_i - s_i\right)\right)}, \; i = \overline{1,3}, \quad (2)$$

where i – the number of the subsystem to which a certain value belongs; d_i, a_i – the ordinates of the lower and upper asymptotes of the dependence; T_i – the constant value of the logistic curve (asymptotes segments that are parallel to the abscissa); s_i – the abscissa of the point of symmetry of the logistic curve; m_i – the coefficient of the growth rate of the useful effect

$$m_i = \frac{2}{T_i\left(a_i - d_i\right)}. \quad (3)$$

The value $m_i(a_i - d_i)s_i$ – essentially a constant of integration, which depends on the initial conditions for solving the differential equation, which, in turn, directly affect the value s_i. The logistic dependences of the useful effects of subsystems are uniquely determined by the set of parameters d_i, a_i, T_i, s_i, which consisted of three subsystems each. The parameters d_i, a_i, T_i, s_i for these subsystems are given in MATLAB notation.

Organizational structures and IT projects:

```
K = { % d a T s
[ 0 1.3 0.1 0.25 ] 'HR'
[ 0 1.4 0.15 0.3 ] 'Tools'
[ 0 1.5 0.15 0.15 ] 'Management' }.
```

Group of unmanned aerial vehicles for emergency response:

```
K = { % d a T s
[ 0 1.5 0.3 0.5 ] 'Wing1'
[ 0 1.5 0.2 0.6 ] 'Wing2'
[ 0 1.5 0.4 0.4 ] 'Wing3' }.
```

Information system survivability system:

```
K = { % d a T s
[ 0 1 0.3 0.5 ] 'Integrity'
[ 0 1 0.2 0.6 ] 'Availability'
[ 0 1 0.3 0.4 ] 'Confidentiality' }.
```

To choose a method of adequate scalar convolution (aggregation) of useful effects of subsystems into the effect of the system, let's evaluate the behavior of the system.

The behavior of the system significantly depends on the nature of the interaction of subsystems, in particular on the indicators of their interchangeability. The level and features of the implementation of subsystem interchangeability were taken into account in previous studies by the authors of the article [11–13] by means of the appropriate choice of one or another type of scalar convolution. The following notation was introduced: i – subsystem number (used as a subscript of other parameters related to a certain subsystem), R_i – input resource, $SL_i(R_i)$ – logistic dependence for finding the useful effect, β_i – weight coefficient.

If the subsystems are completely interchangeable, then the model of the useful effect of the system based on additive convolution will be adequate

$$Ef_{sys} = \sum_{i=1}^{3} \beta_i SL_i\left(R_i\right). \quad (4)$$

If the subsystems are completely non-interchangeable, then the most adequate model of the useful effect of the system based on minimizing convolution will be

$$Ef_{sys} = \min_{i=1}^{3} \frac{R_i}{\beta_i}. \quad (5)$$

In this case, the weight coefficient is usually determined by the number of resource elements required to form one complete set. For example, it can be the number of charges in the ammunition set, the number of crew members or the amount of fuel sufficient to fully refuel the vehicle. Also, such sets include the number of training hours required to complete the training cycle, and other similar units. $\beta_i = R_{norm}$. In this case, resources are measured in sets. And the useful effect is not linearly dependent, as in the previous case, but by logistic

$$Ef_{sys} = \min_{i=1}^{3} SL_i\left(\frac{R_i}{\beta_i}\right). \quad (6)$$

In some cases, it is not resources that are rationed, but the beneficial effects of subsystems

$$Ef_{sys} = \min_{i=1}^{3} \beta_i SL_i\left(R_i\right). \quad (7)$$

Additive and minimizing convolutions correspond to limiting cases regarding the level of interchangeability of subsystems. To take into account intermediate options of interchangeability, multiplicative convolution can be used

$$Ef_{sys} = \prod_{i=1}^{3}\left(SL_i\left(R_i\right)\right)^{\beta_i}. \quad (8)$$

To increase the adequacy of models based on additive and multiplicative convolutions, it is also necessary to satisfy the condition of normalization of weight coefficients

$$\sum_{i=1}^{3}\beta_i = 1. \quad (9)$$

Multiplicative convolutions occupy a fixed intermediate position between additive and minimizing convolutions. To increase the flexibility of creating models with different levels of subsystem interchangeability, the Kolmogorov-Gabor marginal polynomial can be used [14–16]

$$p\left(x_1, x_2, \ldots x_n\right) = \sum_{i=1}^{n}\beta_i\, x_i + \beta_{1,2,\ldots n}\prod_{l=1}^{n} x_l. \quad (10)$$

Logistic dependencies and weight coefficients were introduced into this polynomial not only for additive but also for multiplicative convolution, and an approximation coefficient was introduced to the properties of the additive convolution k_{Add}, i.e. to the properties of complete interchangeability of subsystems in the system being modeled

$$Ef_{sys} = k_{Add}\sum_{i=1}^{3}\beta_i\, SL_i\left(R_i\right) + \left(1 - k_{Add}\right)\prod_{i=1}^{3}\left(SL_i\left(R_i\right)\right)^{\beta_i}. \quad (11)$$

In fact, using the coefficient k_{Add}, additional weighting coefficients for additive and multiplicative convolution were introduced, which in sum should also be equal to unity. It is this coefficient that determines the level of interchangeability of subsystems ($k_{Add} = 1$ – full interchangeability, $k_{Add} = 0$ – non-interchangeability at the level of multiplicative convolution).

To find optimal useful effects, the criteria of optimal resource were used.

The criterion of optimality of resource distribution between subsystems is the maximum useful effect of the system, which has the form

$$Ef_{opt} = \max_{R_1, R_2, R_3}\left(Ef_{sys}\right), \quad (12)$$

for a given total number of subsystem resources

$$R_{sum} = R_1 + R_2 + R_3. \quad (13)$$

The total resource of the system was taken as a constant for each case under consideration. Then the resources of one of the subsystems can be found from the expression

$$R_3 = R_{sum} - R_1 - R_2. \quad (14)$$

This allows the number of unknowns in the optimization problem to be reduced from three to two

$$\left(R_{1opt}, R_{2opt}\right) = \arg\max_{R_1, R_2}\left(Ef_{sys}\right). \quad (15)$$

5. Results of research on the beneficial effect model of the system

5. 1. Definition of basic models of beneficial effects of systems and subsystems

The system models were created based on the logistic basic subsystem models (Fig. 1) with parameters that exhibit the most characteristic features of systems of this type:

K = { % d a T s
[0 1.0 0.2 0.15] 'Subsystem 1'
[0 1.3 0.25 0.35] 'Subsystem 2'
[0 1.8 0.35 0.45] 'Subsystem 3'}.

In Fig. 1 it is seen that the dependences of the useful effects of all subsystems have an s-shaped logistic character. But in a given range of resource provision, not all characteristics of the logistic dependencies are clearly visible. In all subsystems, the zone of slow initial growth of the effect is very reduced, which has the character of an exponent in the growth zone. This is due to the specifics of the objects being modeled. Namely, the subsystems are not created from scratch and, in principle, have passed the stage of preliminary preparation for use. At the same time, it is clearly visible that all subsystems have limitations in the capabilities of the technologies they use. This is manifested in the presence of upper asymptotes to which the useful effect asymptotically approaches.

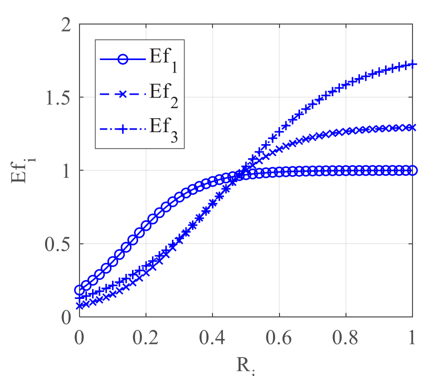

Fig. 1. Dependences of useful effects of subsystems Ef_i on resources R_i, $i = \overline{1,3}$

5. 2. The nature of the dependence of the useful effect on resources for convolution based on the modified Kolmogorov-Gabor polynomial

For clarity and understanding of the qualitative picture regarding the possible location of the optimal solution, it is advisable to construct a three-dimensional graph for the dependence that implements the scalar convolution of the useful effects of the subsystems $Ef_{sys}(R_1, R_2)$. Examples of such graphs (surfaces for searching for optimal solutions) for different values of R_{sum} and k_{Add} are shown in Fig. 2, 3. It can be seen that the surfaces significantly change their appearance when k_{Add} is varied.

Fig. 2 shows that at a small level of subsystem interchangeability $k_{Add} = 0.4$ the surface of search for optimal solutions has one global optimum (maximum). Since subsystems cannot fully replace each other, each system has to work for the overall effect. For this, resources between subsystems are distributed in a balanced manner according to the capabilities of subsystems to create a useful effect Fig. 3.

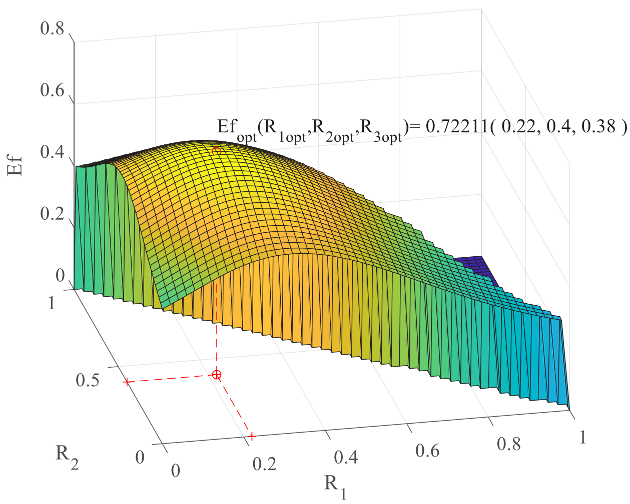

Fig. 2. Dependence of the useful effect on resources $Ef_{sys}(R_1, R_2)$ for convolution based on the marginal Kolmogorov-Gabor polynomial with $R_{sum} = 1$ and $k_{Add} = 0.4$

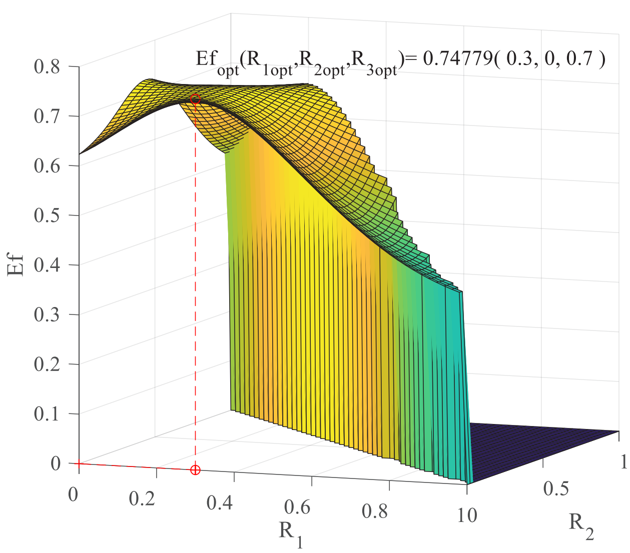

Fig. 3. Dependence of the useful effect on resources $Ef_{sys}(R_1, R_2)$ for convolution based on the marginal Kolmogorov-Gabor polynomial with $R_{sum} = 1$ and $k_{Add} = 0.9$

Increasing the subsystems' interchangeability $k_{Add} = 0.9$ (Fig. 3) leads to the fact that subsystems can relatively easily replace each other. In this case, it is advisable to give all resources to the most efficient subsystem, and exclude other subsystems from resource provision and, accordingly, from work in the system. The danger of such an approach is that over time some of the subsystems will completely lose their ability to create a useful effect. The optimal solution depends not only on the level of subsystem interchangeability, but also on the value of the total resource provision of the system R_{sum} as a whole.

5. 3. Finding optimal beneficial effects of the system

The optimal values of the useful effect were found by the method of complete forward search [24] for all possible values of the total resource of the system R_{sum} and the coefficients of the level of interchangeability of the subsystems k_{Add} (Fig. 4). Fig. 4 shows graphs of the dependence of the effect on the spent resources for individual subsystems Ef_1, Ef_2, Ef_3, as well as for the system as a whole for different values of k_{Add}.

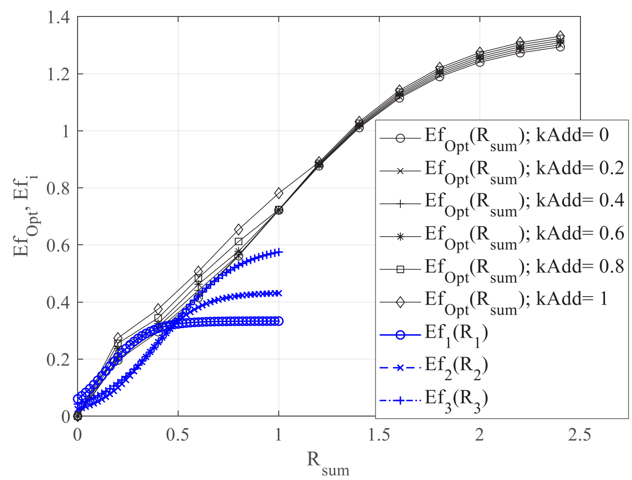

Fig. 4. Dependence of the optimal useful effect on the total system resource $Ef_{opt}(R_{sum})$ for different values of k_{Add} ($R_{sum} > 2.5$)

The dependence in Fig. 4 allows to conclude that the resulting dependence of the useful effect of the system on the total resource of the system $Ef_{opt}(R_{sum})$ has a form close to the logistic one. The variation of k_{Add} from 0 to 1 leads to a noticeable variation of the dependence $Ef_{opt}(R_{sum})$. With a significant resource deficit (R_{sum} from 0.3 to 0.5), the variation of $Ef_{opt}(R_{sum})$ is especially noticeable (25–35%). With a slight resource deficit (R_0 from 0.5 to 0.9), the variation of $Ef_{opt}(R_{sum})$ is less significant (10–15%). In the absence of resource deficit ($R_{sum} > 1$), the variation of $Ef_{opt}(R_{sum})$ practically disappears (1–3%).

5. 4. Finding the optimal allocation of resources between subsystems

When solving the optimization problem by the method of complete forward search, optimal control influences were found, which in this problem are the values of resources assigned to subsystems. To choose the correct system control strategy, it is useful to understand the general patterns of changes in optimal solutions when changing the control parameters of the system. For this, based on the models considered above, the trajectories of the optimal points (Ef_{opt}, $R1_{opt}$, $R2_{opt}$) were constructed when changing the parameters R_{sum} and k_{Add} (Fig. 5).

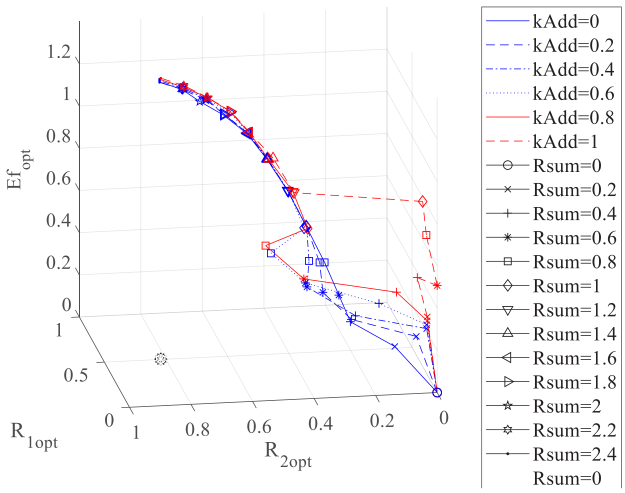

Fig. 5. Trajectories of optimal points (Ef_{opt}, R_{1opt}, R_{2opt}) for k_{Add} fixed in the range 0 to 1 when R_{sum} changes from 0 to 2.4

Fig. 5 shows that when the resource provision is more than a certain value $R_{sum}>1$, all the trajectories of the optimal solutions for different values of the parameter k_{Add} are attracted to a certain highway, which in qu alitative nature resembles a logistic dependence. When $R_{sum}>1$, the trajectories $Ef_{opt}(R_{1opt}, R_{2opt})$ demonstrate sharp switching between the resource provision of different subsystems. There is a complete disconnection of individual subsystems from resources in favor of others. The logic is that if there are not enough resources for all subsystems, then the dispersion of resources will lead to inefficient operation of the system. It is necessary to concentrate resources on those subsystems that provide the maximum effect. This requires not only disconnecting certain subsystems from resource provision, but also a sudden change in the subsystems to which resources are directed when R_{sum} and k_{Add} change. In general, the specified switching of provision has a non-trivial behavior that is difficult to predict based on experience. It is proposed to predict possible resource switching using models based on logistic dependencies and convolutions, which are selected according to the properties of the systems being modeled.

The useful effect of the system can be controlled not only through the resource provision R_{sum}, R_{1opt}, R_{2opt}, R_{3opt}, but also through changing the properties of the system regarding the interchangeability of subsystems using the parameter k_{Add}. The trajectories of the optimal points $Ef_{opt}(R_{1opt}, R_{2opt})$ at constant values of R_{sum} and variations of k_{Add} are shown in Fig. 6.

It can be seen that the trajectories have a rather simple appearance, but at the same time they reflect a significant change in R_{1opt}, R_{2opt} when varying k_{Add} in conditions of insufficient resource provision. Controlling the effect of the system using k_{Add} is possible and expedient. However, with satisfactory resource provision $R_{sum}>1$, the possibility of controlling using k_{Add} is reduced to almost zero.

Three-dimensional pictures of the trajectories of optimal solutions are clear and convenient for studying the patterns of optimal solutions. In this case, one coordinate belongs to the optimality criterion Ef_{opt}, and the other two to the resource provision values of the two subsystems R_{1opt}, R_{2opt}. The optimal resource provision of the third subsystem is found from the expression

$$R_{3\,opt}=R_{sum}-\left(R_{1\,opt}+R_{2\,opt}\right). \tag{16}$$

Increasing the number of subsystems in the analysis is limited by the capabilities of three-dimensional representation. However, if the optimal effect of the Ef_{opt} system is represented, for example, by the size of a node at a point in space (Fig. 10), then this allows to visually analyze systems consisting of four subsystems. At the same time

$$R_{4\,opt}=R_{sum}-\left(R_{1\,opt}+R_{2\,opt}+R_{3\,opt}\right). \tag{17}$$

Fig. 7 demonstrates the possibilities of the approach to increasing the number of subsystems in the system without losing the clarity of the graphical representation of optimal trajectories.

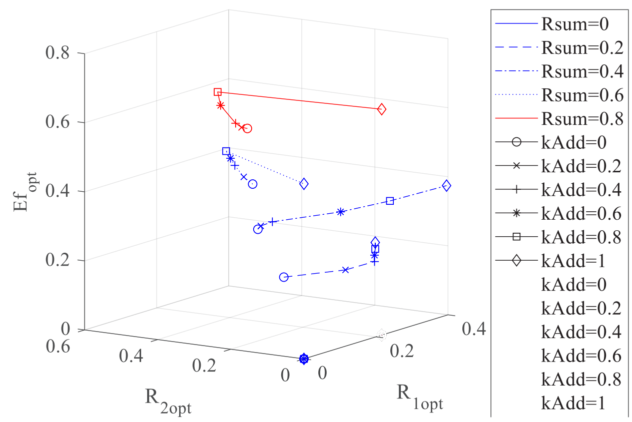

Fig. 6. Trajectories of optimal points (Ef_{opt}, R_{1opt}, R_{2opt}) for R_{sum} fixed in the range 0 to 0.8 when k_{Add} changes from 0 to 1

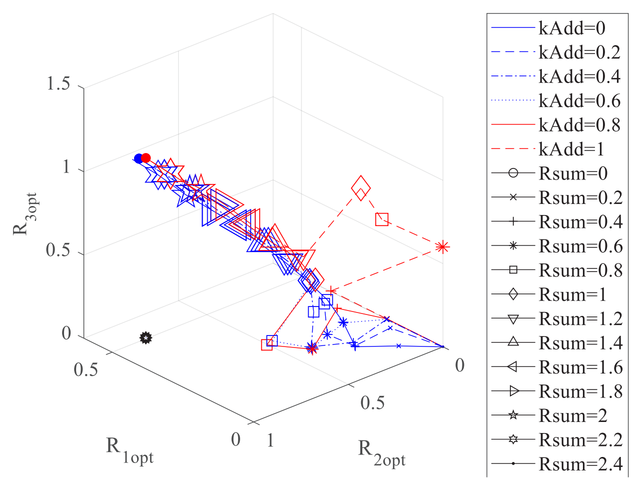

Fig. 7. Trajectories of optimal points (Ef_{opt}, R_{1opt}, R_{2opt}, R_{3opt}) for a system including 4 subsystems

Each trajectory corresponds to a fixed value of k_{Add} in the range from 0 to 1 with a step of 0.2, which covers the entire range of possible levels of subsystem interchangeability. Each point on the trajectory corresponds to its value of the total resource provision of the system R_{sum} in the range from 0 to 2.4. In this case, the three coordinates on the graph represent the values of resource provision that should be assigned to the corresponding subsystems to achieve the optimal effect of the system. The value of the corresponding useful effect is indicated by the size of the marker at the corresponding point of the optimal solution (Ef_{opt}, R_{1opt}, R_{2opt}, R_{3opt}). Despite the fact that the graph is supposedly illustrative in nature, it allows to draw conclusions that the best values of useful effects are on the highway, which corresponds to the values of $R_{sum} > 1$ for arbitrary values of the parameter k_{Add}. That is, the optimal solutions that fall on the highway practically do not depend on the level of interchangeability of subsystems in the system k_{Add}. If there is a need for a more precise determination of the optimal solution in a given area $R_{sum} > 1$, the highway can be used as a reference solution. Such a reference solution allows, with a high probability of convergence, to refine the result within the framework of iterative procedures. For example, gradient optimization methods can be used for this.

The goal of optimization is to find such an allocation of resources that provides the maximum useful effect of the system. But is the maximum effect necessary at any cost? An additional criterion for making a correct decision on resource allocation is efficiency, which is equal to the effect divided by the resources that were spent on obtaining it (Fig. 8)

$$Efc = \frac{Ef_{opt}}{R_{sum}}. \qquad (18)$$

The dependence $Ef_C(R_{1opt}, R_{2opt})$ shows that at $R_{sum} > 0.8$ the efficiency of the system begins to decrease first slowly and then intensively. Although the effect is close to the best. The effect Ef_{opt} and the efficiency Ef_C become contradictory indicators. To make the right decision regarding the optimal parameters R_{sum}, R_{1opt}, R_{2opt}, R_{3opt}, k_{Add}, it is necessary to add a compatible analysis of the criteria Ef_{opt} and Ef_C. If there is a minimum permissible useful effect of the system $Ef_{\min}$, then the corresponding criterion can be transformed into the constraint $Ef_{opt} > Ef_{\min}$ and then maximize the efficiency criterion $Ef_C \to \max$. Or, conversely, introduce a similar constraint $Ef_{Copt} > Ef_{C\min}$ and then maximize the efficiency criterion $Ef \to \max$. The dependence shows that the best values of Ef_C are achieved approximately at $R_{sum} < 0.2$. But at the same time, the magnitude of the useful effect has too small a value. The best combinations of effect and efficiency occur in the zone of R_{sum} values from 1.4 to 1.8. A more complete picture of the patterns of optimal solutions is provided by a three-dimensional graph of efficiency trajectories (Fig. 9), which allows to analyze the general picture of the change in R_{1opt}, R_{2opt} for the trajectories $Ef_C(R_{1opt}, R_{2opt})$.

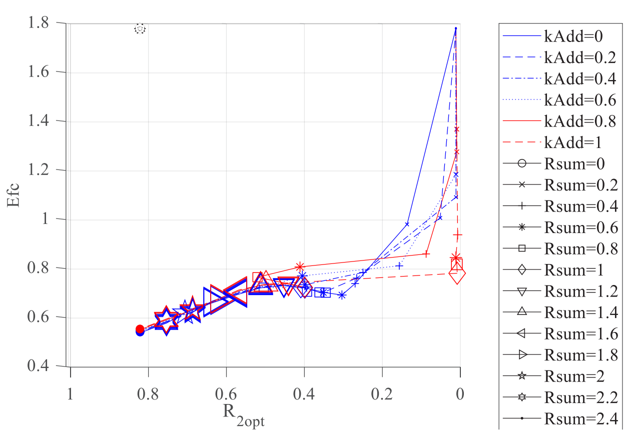

Fig. 8. Trajectories of movement of optimal points (Ef_C, R_{1opt}, R_{2opt}) for k_{Add} fixed in the range 0 to 1 when changing R_{sum} from 0 to 2.4 (view in projection from the side of the R_{1opt} axis)

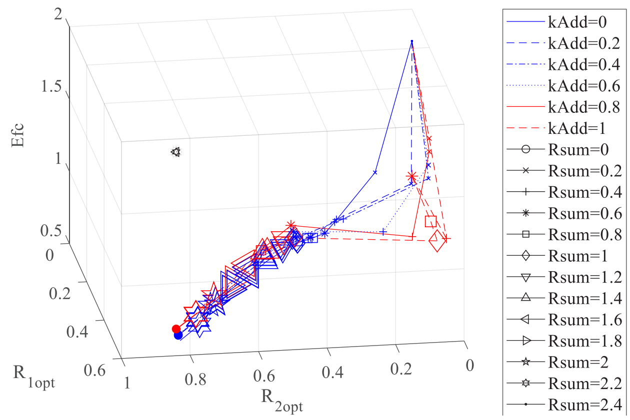

Fig. 9. Trajectories of movement of optimal points (Ef_C, R_{1opt}, R_{2opt}) for k_{Add} fixed in the range 0 to 1 when changing R_{sum} from 0 to 2.4. (view in three-dimensional version)

The three-dimensional plot of the dependence (Ef_C, R_{1opt}, R_{2opt}) also confirms the presence of a highway to which all trajectories in the parameter space (Ef_C, R_{1opt}, R_{2opt}) are attracted in the zone $R_{sum} > 1$ for all fixed values of k_{Add} in the range 0 to 1. Detection of the highway allows to significantly reduce the consumption of computational resources when solving the optimization problem.

6. Discussion of the results of the useful effect of the system at different levels of subsystem interchangeability

The optimal dependence of the useful effect of the system on the total resource of the system $Ef_{opt}(R_{sum})$ is similar to the logistic one (Fig. 4). This is explained by the fact that the limitations of the levels of development of subsystems, which are determined by the upper and lower asymptotes of their dependencies of useful effects (Fig. 1), are projected onto the system. The system cannot but have limitations of development if its subsystems have such limitations.

The dependence of the useful effect (Fig. 4) significantly changes its appearance when the coefficient of the level of interchangeability of subsystems varies. At different levels of resource provision, the variation of the coefficient of the level of interchangeability of subsystems leads to a variation of the values of the useful effect of the system $Ef_{opt}(R_{sum})$ from 1–3% to 25–35% of the current values. At the same time, an increase in the coefficient of the level of interchangeability of subsystems leads to a significant increase in the useful effect (25–35%) in the zone of incomplete resource provision. This is explained by the fact that with a high level of subsystem interchangeability, it is quite easy to switch work between subsystems in favor of the most effective subsystem. Since all subsystems are in a state of insufficient resource provision, any subsystem is ready to accept resource provision and convert it into the appropriate level of useful effect.

In the absence of a resource deficit, the variation of the useful effect of the system when the coefficient of the level of subsystem interchangeability varies practically disappears (1–3%). This is explained by the fact that all subsystems are already provided with resources and operate at higher levels of their own capabilities. The dependencies of the useful effects of all subsystems are in the saturation zone. Additional resource provision no longer leads to a noticeable increase in the useful effect. The capabilities of the technology are almost exhausted. If further increase in the useful effect is required, then it is necessary not to add resources, but to change the technology to a more progressive one.

The results of finding optimal solutions for the distribution of the total system resource between subsystems were presented in the form of trajectories of optimal solution points. The control parameters were the total resource provision of the system R_{sum} and the coefficient of the level of interchangeability of subsystems k_{Add}. Each trajectory corresponded to the fixed value of one of the control parameters and the change of the other control parameter in the entire range of permissible values. The trajectories for the fixed k_{Add} are presented in Fig. 5. The trajectories for the fixed R_{sum} are presented in Fig. 6. Knowledge of the trajectories of the optimal points (Ef_C, R_{1opt}, R_{2opt}) when changing the parameters R_{sum} and k_{Add} allows to choose the correct system control strategy. When the system resource provision $R_{sum} > 1$, all the trajectories of the optimal solutions for different values of the parameter k_{Add} are attracted to the main line, which in its qualitative nature resembles a logistic dependence. Along the highway are the best values of useful effects, which almost do not change when changing the parameter k_{Add}. For a more accurate finding of the optimal solution in the highway area, the latter can be taken as a reference solution, which can then be refined with good convergence by iterative computational methods.

When $R_{sum} < 1$, the trajectories $Ef_{opt}(R_{1opt}, R_{2opt})$ abruptly switch between the resource provision of different subsystems. Individual subsystems are disconnected from resources in favor of others. Resources are supplied to subsystems that provide the maximum effect.

An additional tool for controlling the useful effect of the system is changing the level of interchangeability of subsystems (parameter k_{Add}). The greatest efficiency of such control is achieved in conditions of insufficient resource provision. In other conditions, the possibility of control using k_{Add} is reduced to almost zero.

When $R_{sum} > 0.8$, the efficiency of the system decreases noticeably. The effect is close to the best. To make a correct decision regarding the optimal parameters R_{sum}, R_{1opt}, R_{2opt}, R_{3opt}, k_{Add}, it is necessary to perform a combined analysis of the Ef_{opt} and Ef_C criteria. The best Ef_C values are achieved approximately at $R_{sum} < 0.2$. But the useful effect is too small. The best combination of effect and efficiency is achieved in the R_{sum} value range from 1.4 to 1.8.

The use of logistic dependencies as basic models of subsystem development is not something particularly new, since logistic dependencies have been known for more than one century. But the peculiarity of this study, unlike, for example, [22], is the deliberate use of these dependencies as regression dependencies for a large number of systems development processes. At the same time, 4 parameters are clearly distinguished that uniquely determine a specific logistic dependence: the ordinates of the lower and upper asymptotes, the constant of the logistic dependence and the abscissa of the symmetry point.

A feature of the use of scalar convolutions, in contrast to [9, 10], is a clear mathematical formalization and the choice of the type of convolution based on the indicators of the level of interchangeability of subsystems in the system being modeled. It is well known that the most popular is the additive convolution. But this does not mean that it is in all cases sufficiently adequate to the multifactor process being modeled. In some cases, in contrast to work [11], other types of convolutions could be more adequate, for example, multiplicative or minimizing, which were analyzed in work [13]. But in work [13], in contrast to this current study, combined convolutions were not sufficiently analyzed. Unlike works [14–16], in this study the marginal Kolmogorov-Gabor polynomial is used to form a new type of convolution, which, using an additional weighting factor, allows taking into account the level of interchangeability of subsystems in the system.

Unlike a number of studies on resource optimization [21–23], in this study not only resource optimization of the system is performed, but also the determination of optimal values of resource provision of subsystems is performed. Unlike the solution of the problem of optimal allocation of resources between subsystems in works [13, 25], in this current study a complete set of solutions for all permissible values of control parameters is found, which allows predicting and correcting the optimal allocation of resources depending on the current and future scenarios of the system development.

The results obtained can be explained by the fundamental dependence of the structure of aggregation of effects on the level of interchangeability of subsystems. At a low value of the coefficient k_{Add} (low interchangeability), the system behaves as a set of weakly connected components, where the total effect is limited to the least efficient subsystem – this is reflected in the convolution approaching the minimizing one. In this mode, the optimal strategy involves a balanced allocation of resources to avoid "bottlenecks". On the contrary, at a high k_{Add} (high interchangeability), the system acquires the properties of an additive model, where the effects of subsystems are summed up, and the optimal solution is to concentrate resources in the most productive subsystem. This behavior is especially pronounced in conditions of resource scarcity, when the return on an additional resource is maximum precisely in the most efficient subsystem. In the saturation zone (excess of resources), all subsystems reach the upper asymptotes of their logistic characteristics, so further redistribution of resources has practically no effect on the total effect – this is what explains the slight variation (1–3%) of the optimal useful effect when changing k_{Add} in this region. Thus, the observed patterns are a consequence of the interaction of two factors: the nonlinear (logistic) nature of the efficiency of individual subsystems and the structurally determined way of aggregation of their effects, which is governed by the level of interchangeability. But the numerical indicators can differ significantly. Therefore, before applying the methods of this work, it is necessary to carefully study the statistical data on the objects that need to be modeled and select the correct regression dependence and the correct type of scalar convolution for the useful effects of the subsystems.

Statistical data for this were obtained by the method of complete forward search and the gradient search method. It was found that in the area of satisfactory resource provision, the gradient method has good convergence at the level of 15 iteration steps for an accuracy of 3 significant digits after the decimal point. But in the area of insufficient resource provision and with high levels of subsystem interchangeability, the gradient method can lead to local maxima and not see global maxima. The way out of this situation is to run the gradient search method multiple times from pseudo-random starting points.

However, there is a limitation on the number of subsystems in the system of three with the possibility of expansion only to 4. Increasing the number of subsystems will destroy the clarity of solutions, but will not prevent finding optimal solutions by other methods, for example, gradient ones. The above limitations have shown that ordinary office computing equipment allows finding optimal solutions with the number of subsystems from 30 to 150 (depending on the complexity of the development scenario).

Further research directions may include the development of methods that allow finding optimal solutions for systems consisting of a larger number of subsystems and that are less demanding on computational resources. Difficulties in this regard may be associated with gaining access to relevant and representative statistical data. The data require careful preprocessing: cleaning, filtering, filling gaps, extracting representative indicators through indirect calculations, etc.

7. Conclusions

1. Logistic dependencies were defined as the basic dependencies of the useful effect on resources. The peculiarities of the obtained logistic dependencies are that they allow modeling all stages of the dependence of the useful effect on the resource (slow start, intensive growth, saturation zone). This is explained by the fact that abstract logistic dependencies in integral form correspond to logistic dependencies in differential form with specific physical content. Compared to linear models, logistic dependencies more adequately model the processes of creating a useful effect from resources. And compared to exponential ones, they link all stages of the growth of the useful effect in a single dependency.

2. To flexibly take into account an arbitrary level of subsystem interchangeability, it was proposed to use convolutions based on the marginal Kolmogorov-Gabor polynomial. A feature of this polynomial was the introduction of an additional weighting factor, which determines the level of subsystem interchangeability and allows to increase the adequacy of the model. This is explained by the fact that real objects in most cases do not correspond to the model in the pure form of an additive or multiplicative convolution. Models of real objects combine the properties of the specified convolutions. At the same time, the level of additivity or multiplicativeness for different objects may differ with a very small step. And for one object, this indicator may change over time. The interchangeability factor allows this to be taken into account. The variation of the coefficient of the level of interchangeability of subsystems leads to a variation in the dependence of the optimal beneficial effects of the system on resources at the level of 25–35% in the area of significant resource shortage, 10–15% in the area of minor resource shortage, and 1–3% in the area of no resource shortage.

3. The optimal dependences of the useful effect of the system on the total resource provision of the system were found. The search for optimal useful effects was performed by the method of complete forward search with a decrease in the step as the optimum point is approached. This approach simplifies the selection of the global maximum among local extrema and provides the required accuracy. A feature of the resulting dependence is that it turned out to be similar in appearance to the logistic dependence. This is explained by the design of asymptotic constraints of the subsystems on the formation of the upper and lower constraints of the dependence of the useful effect of the system. The greatest similarity to the logistic dependence is observed in the zone of satisfactory resource provision. With unsatisfactory resource provision, the dependence differs from the logistic one the more the greater the value of the level of interchangeability of the subsystems. But even these differences do not overshadow the similarity with the logistic dependence.

4. The optimal distribution of the total system resource between subsystems was found. The optimization was performed by the method of complete forward search with a decrease in the search step to achieve the specified search accuracy. The optimization results were presented in the form of a three-dimensional graph of the dependence of the optimal useful effect on the optimal resource provision of the subsystems. Each point on the graph corresponded to specific values of the total resource provision of the system and the coefficient of interchangeability of the subsystems, which were considered control parameters. Also, trajectories of system efficiencies were obtained, which corresponded to the optimal values of the subsystem resources and the system effect. When finally choosing the optimal solution, it is necessary to simultaneously analyze the criteria of effect and efficiency, which are contradictory. The graphs were presented in the form of a set of trajectories of optimal values. Each trajectory was formed for one fixed control parameter and changing the other in the full range of permissible values. A feature of the results obtained is that the presence of such visual graphs led to the emergence of new opportunities for controlling the optimal effect of the system. Namely, to expand the possibility of controlling the effect of the system, it was proposed to include the total resource of the system and the level of interchangeability of subsystems in the control parameters of the system. The emergence of new opportunities is explained by the possibility of analysis using a single three-dimensional graph of optimal solutions for all possible sets of control parameters. Compared with the analysis of optimal solutions using more traditional two-dimensional representations, the three-dimensional representation allows, along with the optimal value of the useful effect, to track the optimal distribution of resources between subsystems in a single qualitative picture, which is convenient for analyzing and predicting optimal solutions.

Conflict of interest

The authors declare that they have no conflict of interest regarding this study, including financial, personal, authorship, or other, that could influence the study and its results presented in this article.

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The study was conducted without financial support.

Data availability

The manuscript has no associated data.

Using artificial intelligence tools

The authors confirm that they did not use artificial intelligence technologies when creating the presented work.

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