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A TECHNIQUE FOR APPROXIMATING A TUBULAR HELICAL SURFACE WITH STRIPS OF TORUSES

Andrii Nesvidomin
PhD, Associate Professor*
Serhii Pylypaka
Doctor of Technical Sciences, Professor, Head of Department*
Victor Nesvidomin
Doctor of Technical Sciences, Professor*
Vitaliy Babka
Corresponding author
PhD, Associate Professor*
E-mail: babkavitaliy@ukr.net
Olga Shoman
Doctor of Technical Sciences, Professor
Department of Geometric Modeling and Computer Graphics
National Technical University "Kharkiv Polytechnic Institute"
Kyrpychova str., 2, Kharkiv, Ukraine, 61002
Oleksandr Savoiskyi
PhD, Head of Department
Department of Transport Technologies**
Taras Pylypaka
PhD, Associate Professor
Department of Agricultural Engineering
National University of Water and Environmental Engineering
Soborna str., 11, Rivne, Ukraine, 33028
Mykola Lokhonia
PhD
Department of Educational Management and Pedagogy of Higher School
Sumy State Pedagogical University named after A. S. Makarenko
Romenska str., 87, Sumy, Ukraine, 40002
Svetlana Semirnenko
PhD, Associate Professor
Department of Technical Systems Design**
Yana Borodai
Senior Lecturer
Department of Architecture and Engineering Surveying**
*Department of Descriptive Geometry, Computer Graphics and Design
National University of Life and Environmental Sciences of Ukraine
Heroiv Oborony str., 15, Kyiv, Ukraine, 03041
**Sumy National Agrarian University
Herasyma Kondratieva str., 160, Sumy, Ukraine, 40021

This study's object is the approximation of a non-swept helical tubular surface by strips of sweeping surfaces (toruses) and the construction of sweeps of these strips.

Approximating non-swept tubular surfaces by sections of sweep ones is a common practice in the design of various types of pipelines. A clear example of such an approximation is a sports ball whose outer shell consists of a certain number of separate elements. These elements must fit most tightly to the non-swept surface along its certain lines. Such lines are the lines of curvature. The task is to find these lines on the surface in order to subsequently analytically describe the torus strip, which is tangent to the non-swept surface along this line.

As is known, there are two families of mutually perpendicular lines of curvature on surfaces. This paper considers a family of curvature lines that has advantages over another one in terms of approximation. This explains the results reported here. Their special feature is that in order to find the desired family of curvature lines, it is necessary to solve a differential equation. The solution to this equation was borrowed from a scientific article and used for further calculations. The results were visualized in the form of an approximated tubular surface with four and six strips.

The sweeps of these strips were constructed for a tubular surface, in which the center line is a helical line $r = 1$. All dimensions are given in linear units. Instead of a circle generatrix, it is given by the radius of the cylinder $a = 2$, which hosts it, and the helical parameter $b = 1.5$ (step $H = 9.4$). The radius of the circle generatrix of the tubular surface of the original tubular surface in the approximated surface in the given examples is a polygon (square or equilateral hexagon)

Keywords: line of curvature, tangent strip, geodesic curvature, sweeping surface, numerical integration

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1. Introduction

Existing approximation methods (for example, based on B-splines [1]) do not provide surface continuity, which complicates the manufacture of pipelines because of the need to connect individual sections. This leads to an increase in the number of seams, a decrease in the strength and tightness of structures.

With the advent of 3D scanning, surface approximation based on point clouds [2] is becoming more widespread.

Researchers focus on constructing accurate and smooth surfaces from discrete data. Another area is surface reconstruction based on a set of images.

A cylinder with a straight center line can be considered a tubular surface. If the center line is curved, then the tubular surface is non-swept, that is, it cannot be manufactured by simply bending sheet material like a cylinder. A relevant task is to simplify the technology for manufacturing a tubular helical surface. Such a simplification could be achieved by approximating it with pieces of unfolded surfaces in cases when this does not compromise operational properties. The method is especially relevant for:

– pipelines (oil, gas, chemical industry), where surface continuity is critical for tightness;

– sports equipment (for example, balls), where reducing the number of seams improves aerodynamics and strength;

– aircraft structures, where lightness and strength are key requirements.

The task of approximating non-swept surfaces by sweeping ones may arise while using an article and when the working surface wears our. The results of appropriate studies are needed by practitioners because they could simplify the manufacture or repair of non-swept surfaces [3, 4]. This predetermined the relevance of our study.

2. Literature review and problem statement

In [5], the results of studies on the reconstruction of sweeping surfaces using neural implicit models are reported. It is shown that the use of a regularization term based on second-order derivatives makes it possible to achieve zero Gaussian curvature. This provides smooth deformations of surfaces with infinite resolution, overcoming the limitations of discrete representations. However, issues related to the approximation of complex 3D shapes, in particular tubular helical surfaces, where it is necessary to take into account the specificity of two families of curvature lines, remain unresolved. A likely reason is difficulties associated with the need for an analytical description of the strips of sweeping surfaces, which makes neural models insufficiently effective for practical application in engineering.

In [6], a method for calculating a piecewise linear surface that approximates an arbitrary free-form mesh surface is proposed. It is shown that optimizing a given mesh shape to an approximately piecewise linear shape allows for improved alignment with the target shape. However, the method requires a preliminary representation of the surface in the form of parametric models, which complicates its application for tubular helical surfaces. The reason is the computational cost, as well as the inability to ensure the continuity of the strip approximation, which makes the study impractical for industrial tasks.

In [7], a method for discretizing linear surfaces for their manufacture is described. It is shown that discretization makes it possible to create complex shapes from linear surfaces while preserving their sweep. However, issues related to the approximation of tubular helical surfaces remain unresolved. The reason is the limitation of the method in ensuring the accuracy of approximation for surfaces with complex geometry.

In [8], an approximation method is reported, which uses a classical definition from differential geometry. It is shown that the iterative process of thinning the Gaussian image of the surface and its deformation make it possible to achieve high accuracy. However, the issue of approximating tubular helical surfaces by strips of sweeping surfaces, rather than by separate sections, remains unresolved. This limits the application of the method to surfaces that require a continuous description, for example, in the production of pipelines.

In [9], an approach to segmentation and approximation of triangular meshes is proposed. It is shown that the detection of exact regions and their approximation makes it possible to calculate parametric equations for milling on machine tools. However, the method does not take into account the specificity of tubular helical surfaces, where it is necessary to ensure the continuity of the strips of sweeping surfaces along the lines of curvature. The reason is the limitation of the method with respect to the topology of the surface, which makes it unsuitable for approximating complex helical shapes.

A separate area is the approximation of non-swept surfaces by pieces of sweeping ones. The main idea is to automatically divide a complex 3D model into a set of small sections, each of which can be replaced by the corresponding section of the sweeping surface. This process helps in production when complex shapes are created from flat materials, such as sheet metal. An example of such an approach is work [10], in which the division of a non-swept surface into separate sections is proposed based on the search for geodesic lines on it.

In [11], the approximation of a sphere is considered not by pieces, as is done on sports balls, but by a continuous strip of a sweeping surface. In this case, the line of contact of the strip with the surface is a line of curvature. Since for a sphere every line on it is a line of curvature, the search was reduced to finding a line of rational form. However, for a tubular helical surface, where the lines of curvature have a more complex geometry, this approach requires adaptation. The reason is difficulties associated with the need to take into account two families of curvature lines, which makes the corresponding studies impractical without the development of special algorithms.

One option to overcome the difficulties is to apply classical methods of differential geometry for finding the lines of curvature and constructing tangent strips of sweeping surfaces. This is the approach used in [12], but it does not take into account the possibility of approximating a tubular helical surface by continuous strips.

All this allows us to state that it is advisable to conduct a study aimed at devising a method for approximating a tubular helical surface not by pieces but by strips of sweeping surfaces.

3. The aim and objectives of the study

The aim of our research is to devise a method for approximating a helical tubular surface by strips of sweeping surfaces. This will simplify the technology of its manufacture and repair.

To achieve this goal, the following tasks were set:

– to find parametric equations for strips of sweeping surfaces that uniformly wrap around a tubular helical surface;

– to construct strip sweeps that approximate the surface.

4. The study materials and methods

The object of our study is the process of approximating a helical tubular surface by strips of sweeping surfaces and constructing their sweeps. One family of curvature lines of a tubular surface is a set of circles – curves of the normal cross-section of the surface. It was hypothesized that the tubular surface should be approximated by strips tangent to a second family of curvature lines, which are perpendicular to the first. It was assumed that when approximating the surface along its curvature lines, the strip width would be constant. A simplification is to ignore the thickness of the sheet material when bending the sweep into the desired shape.

The axis of the tubular surface is the curve of the centers of the set of circles perpendicular to this curve. For a tubular helical surface, this is a helix, which is given by radius a of the cylinder on which it is located, and the helix parameter b, through which its pitch $H = 2\pi b$ is determined. If a circle of radius ρ is placed at each point of the helix, then the set of circles will form the frame of a tubular surface. Its parametric equations take the following form [12]:

$$X = a\cos\alpha - \rho\left(\cos\alpha\cos v - \frac{b\sin v}{\sqrt{a^2+b^2}}\sin\alpha\right);$$

$$Y = a\sin\alpha - \rho\left(\sin\alpha\cos v + \frac{b\sin v}{\sqrt{a^2+b^2}}\cos\alpha\right);$$

$$Z = b\alpha + \frac{a\rho\sin v}{\sqrt{a^2+b^2}}, \quad (1)$$

where α and v are independent surface variables that have a physical meaning: α is the angle of rotation of a point around the axis of the helix when it moves along it; v is the angle of rotation of a point around the center of the circle when it moves along it.

Parametric equations (1) describe a helical tubular surface, in which only one family is a family of curvature lines, namely a frame of circles of radius ρ. The second family of curvature lines, perpendicular to the first, is of interest. As a result of solving the differential equation in [12], a transition to new surface variables is given, which provide the assignment of the surface to two families of coordinate lines, which are curvature lines

$$\alpha = \frac{s}{\sqrt{a^2+b^2}}; \; v = w - \frac{bs}{a^2+b^2}, \quad (2)$$

where s is a new independent surface variable – the length of the arc of the center line; u is the second independent variable – also an angle that varies within $w = 0...2\pi$, which provides closed circle generatrixes of the surface.

In Fig. 1, a, one revolution of the surface is constructed according to equations (1), and in Fig. 1, b – when switching to new variables according to formulas (2).

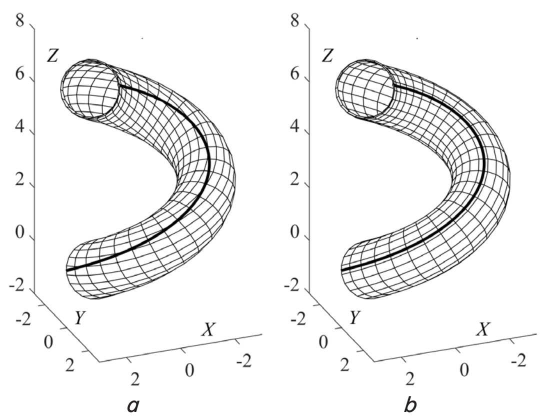

Fig. 1. Tubular helical surface at $a = 2$; $b = 1.1$; $\rho = 1$ with a selected line of curvature on it: a — a family of lines of curvature is only the frame of the circle generatrixes; b — both families of coordinate lines are lines of curvature

In the first case, the grid of coordinate lines is not orthogonal, and in the second – orthogonal, because both families of coordinate lines are lines of curvature. If we take $w = \text{const}$ in the second dependence (2), then a line of curvature of the second family will stand out on the surface. From Fig. 1, a, it is clear that it is not one of the lines of curvature, but in Fig. 1, b – it is.

The next stage is the construction of a strip of a sweeping surface tangent to the helical tubular surface along the line of curvature, highlighted in Fig. 1 by a thick line. For this purpose, the apparatus of differential geometry was used.

5. Mathematical description of strips of sweeping surfaces that approximate a tubular helical surface, their sweeps

5. 1. Parametric equations of strips tangent to the tubular surface along lines of curvature

A tangent sweeping surface is located as the boundary surface of a set of tangent planes along a given line of curvature. A separate tangent plane is perpendicular to the normal of the tubular surface, constructed at a separate point of the line of curvature. Therefore, it is necessary to determine the coordinates of the normal to the tubular surface (1) along the line of curvature. The normal vector $\bar{N}$ is the vector product of vectors tangent to the coordinate lines. These vectors are partial derivatives from equations (1) taking into account (2) with respect to variables s and w. Thus, we can write:

$$\bar{N} = \begin{vmatrix} X & Y & Z \\ X'_s & Y'_s & Z'_s \\ X'_w & Y'_w & Z'_w \end{vmatrix}. \quad (3)$$

The subscripts in (3) indicate the variable in which the partial derivatives of the equations of the tubular surface are found.

Differentiation of equations (1) in variables s and w, expansion of the determinant (3) was carried out using the symbolic mathematics product "Mathematica". The vector obtained in projections onto the coordinate axis was reduced to the unit vector. Its components take the following form:

$$N_x = \cos\alpha\cos v - \frac{b}{\sqrt{a^2+b^2}}\sin\alpha\sin v;$$

$$N_y = \sin\alpha\cos v + \frac{b}{\sqrt{a^2+b^2}}\cos\alpha\sin v;$$

$$N_z = -\frac{a}{\sqrt{a^2+b^2}}\sin v, \quad (4)$$

where the expressions of angles α and v in terms of independent variables s and w are given in (2).

The direction vector $\bar{I}$ of a rectilinear generating of sweeping surface, which passes through the point of its contact with the tubular surface, is determined by the vector product of vector (4) by its derivative vector [11]:

$$\bar{I} = \begin{vmatrix} X & Y & Z \\ N_x & N_y & N_z \\ N'_{xs} & N'_{ys} & N'_{ys} \end{vmatrix}. \quad (5)$$

The projections of the normal vector (4) must be differentiated with respect to variable s because for the chosen line the curvature $w = const$. After differentiation, expansion of the determinant (5) and reduction of vector $\bar{L}$ to the unit, one can find its projections:

$$I_x = \cos\alpha \sin v + \frac{b}{\sqrt{a^2+b^2}} \sin\alpha \cos v;$$

$$I_y = \sin\alpha \sin v - \frac{b}{\sqrt{a^2+b^2}} \cos\alpha \cos v;$$

$$I_z = \frac{a}{\sqrt{a^2+b^2}} \cos v, \qquad (6)$$

where the expressions of angles α and v in terms of independent variables s and w are given in (2).

The straight-line generatrixes of the tangential sweeping surface intersect the line of curvature parallel to the direction vector (6). In this regard, we can write the parametric equations of the sweeping surface tangent to the tubular surface:

$$X_s = x(s) + uI_x;$$

$$Y_s = y(s) + uI_y;$$

$$Z_s = z(s) + uI_z, \qquad (7)$$

where $x(\gamma)$, $y(\gamma)$, $z(\gamma)$ are parametric equations (1) taking into account (2) the line of curvature. They are marked with lowercase letters because at w = const these equations are no longer equations of the surface but equations of a line on it, which is the line of curvature. u is the second independent variable of the surface – the length of the straight-line generatrix of torus; I_x, I_y, I_z are the coordinates of the direction vector (6) of the straight-line generatrix of sweeping surface.

Based on equations (7), Fig. 2, *a* shows a tangent strip to the turn of the tubular surface along its line of curvature. The width of the strip depends on the limits of change in parameter u.

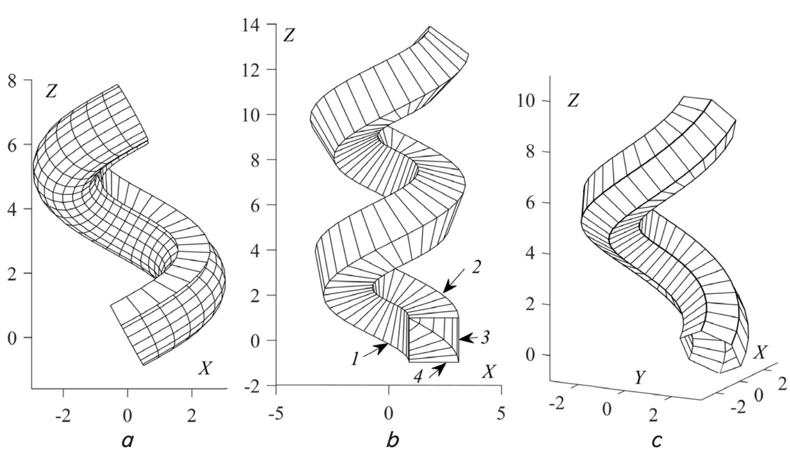

Fig. 2. Surfaces constructed at $a = 2$; $b = 1$; $\rho = 1$: *a* — a turn of a tubular surface with a strip of the sweeping surface tangent to the line of curvature; *b* — two turns of a tubular surface approximated by four strips; *c* — a turn of a tubular surface approximated by six strips

One can choose the desired number of strips. By placing them alternately at equal intervals of parameter w, one can wrap a tubular surface. For example, with four strips, the interval of the parameter is $w = 2\pi/4 = \pi/2$. In Fig. 2, *b*, the numbers indicate the strip numbers: 1 – at $w = 0$; 2 – at $w = \pi/2$; 3 – at $w = \pi$; 4 – at $w = 3\pi/2$. The length of the straight-line generatrix of the strip varied within $u = -1...1$, which corresponds to the side of a square circumscribed around a circle of radius $\rho = 1$. In Fig. 2, *c*, the tubular surface is approximated by six strips, which corresponds to the interval of the parameter: $w = \pi/3$.

5. 2. Mathematical description of the sweeps of strips tangent to the tubular surface, as well as their construction

To construct the sweeps, the well-known provision from differential geometry was taken as the basis, according to which the geodesic curvature k_g of the curve on the sweeping surface and on its sweep remain unchanged. Since the line of curvature on the tubular surface is the common line of tangency of the strip of the swept surface, they will have a common geodesic curvature. It is determined from the following determinant:

$$k_g = \left(\frac{ds}{dl}\right)^3 \begin{vmatrix} N_x & N_y & N_z \\ x' & y' & z' \\ x'' & y'' & z'' \end{vmatrix}, \qquad (8)$$

where N_x, N_y, N_z are projections of the unit normal vector (4); x', y', z', x'', y'', z'' – the first and second derivatives are the derivatives from the equations of the curvature line (1), (2) with respect to variable s at w = const; $ds/dl = 1:(dl/ds)$. The expression dl/ds is the derivative of the arc length l of the tangent line, i.e. the curvature line. It is determined through the first derivatives of the curvature line using the following formula

$$dl/ds = \sqrt{x'^2 + y'^2 + z'^2} = $$
$$= 1 - \frac{a\rho}{a^2+b^2} \cos\left(w - \frac{bs}{a^2+b^2}\right). \qquad (9)$$

After calculations, the expression for geodesic curvature k_g takes the form

$$k_g = -\frac{a \sin\left(w - \frac{bs}{a^2+b^2}\right)}{a^2+b^2 - a\rho\cos\left(w - \frac{bs}{a^2+b^2}\right)}. \qquad (10)$$

According to the known dependence of geodesic curvature (10) of the curve on the surface, its equation on the sweep can be derived using the following expressions:

$$x_0 = \int \cos\left(\int k_g dl\right) dl;$$

$$y_0 = \int \sin\left(\int k_g dl\right) dl. \qquad (11)$$

The expression in parentheses (11) is the angle γ of rotation of the tangent to the curve on the sweep. It can be integrated taking into account the dl expression from (9)

$$\gamma = \int k_g dl = \int \frac{a}{a^2+b^2} \sin\left(w - \frac{bs}{a^2+b^2}\right) ds = $$
$$= -\frac{a}{b} \cos\left(w - \frac{bs}{a^2+b^2}\right). \qquad (12)$$

Further substitution (9) and (12) in (11) yields the following expressions:

$$x_0 = \int \cos\gamma \left[1 - \frac{a\rho}{a^2+b^2} \cos\left(w - \frac{bs}{a^2+b^2} \right) \right] \mathrm{d}s;$$

$$y_0 = \int \sin\gamma \left[1 - \frac{a\rho}{a^2+b^2} \cos\left(w - \frac{bs}{a^2+b^2} \right) \right] \mathrm{d}s. \qquad (13)$$

To construct curve (13), which is the line of contact of the strip of the swept surface on its sweep, it is necessary to apply numerical integration. Through this curve on the sweep, a straight-line generatrix of the strip passes perpendicular to it. Based on this, we can write the parametric equations of the sweep:

$$X_0 = \int \cos\gamma \left[1 - \frac{a\rho}{a^2+b^2} \cos\left(w - \frac{bs}{a^2+b^2} \right) \right] \mathrm{d}s - u\sin\gamma;$$

$$Y_0 = \int \sin\gamma \left[1 - \frac{a\rho}{a^2+b^2} \cos\left(w - \frac{bs}{a^2+b^2} \right) \right] \mathrm{d}s + u\cos\gamma. \qquad (14)$$

There is a technique to check the correspondence of the equations of a swept surface in space and its sweep in a plane. A sufficient condition for such correspondence is their common first quadratic form. It was found for surface (7) and its sweep (14) but is not given because of its cumbersome form.

According to equations (14), strip sweeps were constructed that approximate a tubular surface. For two turns of the folded surface (Fig. 2, *b*), the four-strip sweeps take the following form (Fig. 3).

It should be noted that the shape of the strip sweeps depends on the ratio of the structural parameters of the surface a, b, and ρ. When the pitch of the axial line of the tubular surface decreases, that is, when the helical parameter decreases from $b = 1$ to $b = 0.9$, the strips will begin to overlap themselves, which will lead to the need to break them into parts. This also applies to increasing radius ρ of the circle generatrix, as well as changing parameter a.

To check the reliability of our results, a full-scale model was made from paper by connecting the sweeps of three strips. The sweep under number 4 was not included in the model (Fig. 4) since the straight-line generating strips along which the bending is carried out and which are located inside the approximated surface would not be visible.

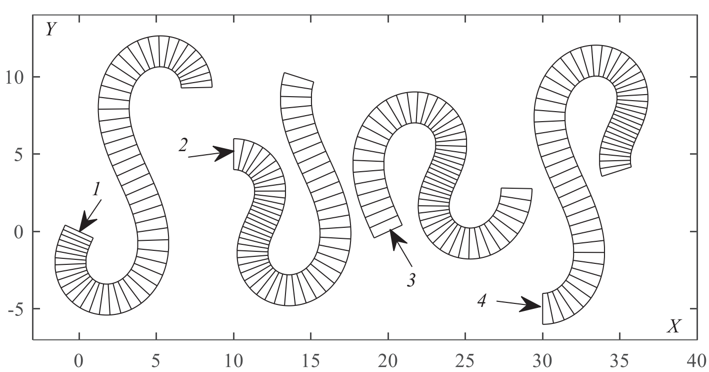

Fig. 3. The sweeps of strips corresponding to the folded surface in Fig. 2, *b* (the numbers indicate corresponding strips on the surface and on the sweep)

If we find the sweep of individual turns of the approximated surface as their number increases, we can see that individual turns of a smaller size (Fig. 3) can be cut out of the general strip (Fig. 5).

Fig. 4. A full-scale model made by connecting the sweeps of individual strips

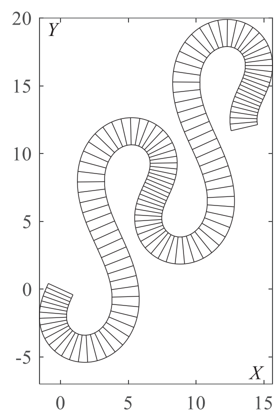

Fig. 5. Periodicity of the contours of the strip sweep of the approximated surface as the number of its turns increases

Individual elements of the strip (Fig. 5) are periodically repeated.

6. Results of approximating a helical tubular surface by strips of sweeping surfaces: discussion

Each non-swept surface can be approximated by pieces of sweeping surfaces. In this case, the question arises about choosing such pieces of sweeping surfaces and their size so that the approximation of a non-swept surface by pieces of sweeping surfaces most accurately reproduces it. In work [13], the construction of a helical surface from sections of a sweeping helicoid was proposed, in [14] – the construction of a sweeping surface passing through a helical line of variable pitch. Unlike [13, 14], in this study, we have confirmed the hypothesis that the surface should be approximated along the lines of curvature perpendicular to the family of circle generatrixes, which are also lines of curvature.

For this purpose, a transition was made from equations (1), in which the lines of curvature were only one family of coordinate lines – the frame of circle generatrixes – to equations with two families of curvature lines. To this end, a transition was made from independent variables α and v to the new independent variables s and w according to expressions (2). For the value $w = \text{const}$ on the surface, a line of curvature corresponds (Fig. 1). Along it, a tangent strip of the sweeping surface is constructed (Fig. 2, *a*), the equation for which is given in (7). By dividing the value $w = 2\pi$ into an equal number of parts, we obtain a division of the circle generatrix of the tubular surface into equal parts. Each part corresponds to its own line of curvature, along which strips of tangent sweeping surfaces are constructed. In Fig. 2, *b*, there are four such strips, and in Fig. 2, *c*, there are six.

To find the parametric equations (7) of the tangent strip from the sweeping surface, the means of differential geometry and vector algebra were used. They allowed us to find the projections of the unit vector of the rectilinear generating sweeping surface by expanding the determinant (5). Owing to this approach, strips were constructed that approximate the helical tubular surface on any number of its turns. This gives an advantage over other approximation techniques in which the section of the sweeping surface is not a strip but a small piece [10]. In addition, the research does not end there but continues with the construction of the sweeping tangent strips. For this purpose, the differential geometry proposition that the geodesic curvature of the strip's tangent line is the same in space and on its sweeping surface was again used. It was found by expanding the determinant (8), which made it possible to derive the parametric equations (14) of the strip sweep.

The limitations of our study relate to the fact that the strip cannot be made continuous at a certain ratio of design parameters a, b, and ρ. For example, in Fig. 3, the sweeps are constructed for the extreme position, when their contours do not overlap each other. When the surface pitch is reduced or the circle generatrix is increased, overlapping occurs and the strip cannot be continuous. The disadvantage is that numerical integration methods must be used to construct the sweeps according to equations (14), which are depicted in Fig. 3.

Future studies should involve approximating surfaces with a variable value of the circle generatrix, which are termed channel surfaces.

7. Conclusions

1. A tubular helical surface is formed by a frame of circles of the same radius with centers on the helical line. In this case, the set of circles is located in planes perpendicular to this helical line. If a second family of lines perpendicular to the family of circles is found, then along them it is possible to approximate the surface by continuous strips. The strips have a constant width, which depends on their number. After such an approximation, the cross-section of the surface by a plane perpendicular to the axis is a regular polygon, the number of sides of which is equal to the number of strips. In particular, with four strips, the cross-section is a square.

2. Parametric equations of the sweeps of strips have been derived; their contours were constructed when the number of strips is four. The basis for finding the sweep is the provision from differential geometry implying that the geodesic curvature and the length of the arc of a line on the surface and on its sweep are the same. To construct the sweeps based on our equations, numerical integration was performed. To verify the reliability of the results, the individual obtained sweeps of strips were connected into an approximated helical tubular surface.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study, as well as the results reported in this paper.

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Data availability

All data are available, either in numerical or graphical form, in the main text of the manuscript.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

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