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IMPROVING A METHOD FOR DETERMINING THE SPATIAL PARAMETERS IN THE MATHEMATICAL MODEL OF A DISTRIBUTED AUTOMATED INFORMATION-MEASURING SYSTEM FOR REAL-TIME CONTROL OVER THE QUALITY OF IRON ORE RAW MATERIALS

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This study investigates processes of irradiating iron ore raw materials by a source of low-energy gamma quanta and registering radiation scattered as a result of the Compton effect, as well as parameters of the irradiation geometry. This work addressed the task of improving the accuracy of existing nuclear-physical methods of control over the quality of iron ore raw materials, which make it possible to promptly determine the material composition of the crushed rock mass.

The results essentially indicate that when using a centrally shifted irradiation geometry, the sensitivity of registration of the integral backscatter flux increases compared to the side and central irradiation geometries. This is attributed to improved visibility conditions of the detector, and the formation of a scattering angle close to the optimal.

The constructed mathematical model has made it possible to identify the main geometric parameters for the system of operational control over the quality of iron ore raw materials. A formula has been derived that connects the basic parameters in the system of operational control over iron content in ore with the use of centrally shifted irradiation geometry.

The studies demonstrated changes in the sensitivity of the registration of the integral backscatter flux when changing the vertical location of the gamma-ray source. The results revealed a maximum sensitivity with a value of 6.08·10-7 at a minimum distance of the radiation source from the single crystal and a distance of 110 mm from the irradiated surface. The value of the correlation coefficient between the model and experimental data is 0.981.

The findings could be practically applied to improve the accuracy of methods for operational control over the content of a usable component in iron-containing ores under industrial conditions at ferrous metallurgy enterprises

Keywords: Compton effect, albedo, gamma quantum, irradiation, rock, operational control

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1. Introduction

Ukraine's transition to market relations, especially taking into account the European integration processes, has led to the need to improve the quality of industrial products. This, in particular, has affected the formation of stricter requirements for the quality of iron ore raw materials. At the same time, the quality of iron ore products directly affects its cost – for example, according to [1], in 2021, the cost of a ton of concentrate with an iron content of 62% was USD 160 on world markets.

The issue of control over the quality of iron ore raw materials implies the need to determine its exact parameters, and in real time. One of the most important parameters in the enrichment of iron ore raw materials, as is known, is the content of a usable component – iron. At the same time, determining this parameter is one of the most labor-intensive processes, which is associated with the peculiarities of its measurement. The methods of control over the quality of iron ore raw materials that were previously used were based on the preparation of special samples with their subsequent processing under laboratory conditions. This approach to measuring the iron content in ore, although it met the required measurement accuracy, had a significant drawback – long time spent on analysis. It is not satisfactory under modern conditions of raw material processing at ferrous metallurgy enterprises because of the impossibility of timely use of the information received to manage technological processes of processing iron ore raw materials. This causes fluctuations in the content of the usable component in the end product – concentrate.

This makes it relevant to use operational control methods that allow an-order-of-magnitude decrease in the time between the start of analysis and its result. The use of these methods contributes to improving the information support for the grinding, classification, and magnetic separation processes. This, in turn, opens up wider possibilities for implementing control effects that could ensure stabilization of the mill loading and stabilization of the mass fraction of iron entering the magnetic separation stage [2]. In addition, operational control methods do not require sampling operations, which allows for direct analysis of the quality of crushed raw materials on the conveyor belt. This makes it possible to automate the process, thereby reducing the impact of the human factor on the analysis result, and decreasing the workload on the employees at the technical control department.

Thus, research and improvement of express methods for controlling the material composition of crushed ore mass is an urgent task; solving it would make it possible to improve the quality of the concentrate and increase its competitiveness in the world market of iron ore raw materials.

2. Literature review and problem statement

In addition to the conventional methods for chemical analysis of iron content in ore, which are widely used and the duration of which can reach two hours [3], several methods of operational control can be distinguished. They include magnetometric, ultrasonic, nuclear-physical, and other methods. Methods of operational control, the principle of which is based on the assessment of change in relative magnetic permeability when interacting with iron ores, are based on the magnetic properties of iron minerals, primarily magnetite (Fe_3O_4). For example, in [4], the use of the Satmagan-135 magnetic materials analyzer [5] is considered, in which the sample is weighed alternately in gravitational and magnetic fields. The authors determined the magnetite content both in comparison with standard samples and according to calibration graphs. The analysis time is about one minute. At the same time, it should be noted that the use of this device involves sampling. In addition, magnetometric methods, despite their ease of use and efficiency, do not make it possible to obtain information about the content of iron that does not exhibit ferromagnetic properties, for example, in the composition of hematite (Fe_2O_3), carbonates, silicates, and other minerals. This does not make it possible to obtain a holistic picture of the content of total iron, which includes both magnetic and non-magnetic components.

The ultrasonic control method, which involves measuring and evaluating the parameters of the propagation processes of ultrasonic bulk and surface waves, as well as high-energy ultrasound in liquid and solid media, is widely used in mining. Thus, in work [6] a method for identifying mineralogical varieties of iron ore raw materials using ultrasonic measurements is considered. The authors propose a combination of an acoustic indicator of sample quality and a characteristic coefficient based on the dispersion and average amplitude of the ultrasonic signal for unambiguous identification of ores. It should be noted that this method makes it possible to attribute the studied sample to a certain type of ore (for example, magnetite quartzite) with a certain probability. However, it is not intended for direct quantitative determination of the percentage of iron in the studied samples. At the same time, it should be noted that ultrasonic methods are quite critical to the granulometric composition of the studied medium and may require additional measures to ensure the quality of crushing of iron ore raw materials [7]. Therefore, their use for quality control over crushed ore is difficult.

A method for estimating the propagation of elastic waves is considered in [8]. This method for determining the content of a usable component in iron ore pulp is based on the use of Love surface waves under the simultaneous influence of acoustic and magnetic fields. The work notes that the method was used for operational control of iron in heterophase media. Therefore, its use for analyzing the quality of dry crushed ore is difficult due to changes in the propagation of elastic waves outside the liquid medium of the carrier.

There is a gravitational method [9] for estimating the content of a usable component in iron ore raw materials, which is based on the conversion of the bulk density of the sample into specific gravity, but this method requires the selection and processing of samples.

An alternative technique for measuring the content of iron is the nuclear physical method, in particular the gamma-gamma method. This method is based on the assessment of the result of the interaction of γ-radiation with iron ore. An important point is that the implementation of this method is feasible, which allows for prompt control over the quality of iron ore raw materials on the conveyor in ore processing plants. Thus, in [10], the effects of backscattering of gamma rays (γ) from single-element and multi-element materials are studied. The work is aimed at establishing relationships between the albedo coefficients of γ-photons and their average reflection energy depending on the type and thickness of the material, the energy of the incident γ-rays, and the angle of incidence by using the Monte Carlo method. The study models the behavior of γ-rays when reflected from predetermined materials with a known composition, while iron ore is a heterogeneous mixture.

In [11], the reflection coefficients of γ-photons from various materials, including iron, are studied. This allows for an assessment of change in albedo depending on the thickness of the material, the energy of the photons, and the atomic number of the element. Study [12] also demonstrated that the albedo factors of the samples decrease with increasing average atomic number. It should be noted that the determination of albedo is an important step in the assessment of the material composition of the studied material, but the reviewed works do not describe methods for using its values to determine the iron content in the studied samples. Work [13] describes a spectral analysis method that uses Gaussian convolution to effectively eliminate background counts caused by Compton scattering and improve the accuracy of calculating the peak position and area. However, the study was focused on the analysis of noble gases and did not include the determination of the composition of solids.

One of the popular nuclear physics methods is PGNAA. This method is based on the registration of instantaneous gamma radiation emitted by the nuclei of matter at the moment of neutron capture. The elemental composition of the sample is determined by the characteristic energy lines of gamma quanta, which makes it possible to quantitatively assess the concentration of individual elements. In the mining industry, PGNAA is considered an effective method of elemental analysis due to the provision of information on both the composition of the ore and the concentration of elements in it [14]. Thus, in work [15], PGNAA is used to identify lead-zinc ores in combination with machine learning. However, the described set of methods was used to determine the content of the usable component in drill holes, while monolithic rocks are studied, which causes a potential discrepancy between the results and the analysis of crushed ore. In [16], PGNAA is applied to determine the content of copper in powder samples but, in this case, the granulometric composition of the material under study also does not meet the specified requirements. The main drawback in the analysis of the material composition of ore using PGNAA is the problem of spectral interference when energy peaks overlap. The iron spectrum graph is superimposed with lines of other elements, in particular silicon (Si), which is contained in ferruginous quartzites in the form of quartz (SiO_2). This can distort the results of iron determination, especially when using detectors with low energy resolution (Na(Tl)), which are often used in industrial PGNAA complexes.

Thus, our review of the literature revealed that existing methods for controlling the material composition do not meet the requirements set forth in the context of quality control of crushed iron ore. They either do not provide the necessary speed of analysis or are not designed to control the content of components in crushed solid materials, or do not provide the necessary accuracy when controlling the composition of quasi-binary media. At the same time, nuclear-physical methods are the most suitable for solving the task of operational control over iron in the rock mass under the conditions of processing enterprises. However, for their effective application, it is necessary to construct a mathematical model of the process of irradiation of iron ore raw materials with a source of $\boldsymbol{\gamma}$-quanta and subsequent registration of the integral backscatter flux. This will make it possible to isolate and assess the influence of the main factors on the process of measuring the iron content, excluding secondary ones.

3. The aim and objectives of the study

The aim of our study is to improve the method of controlling the iron content, which is based on the assessment of the value of the integral flux of $\boldsymbol{\gamma}$-radiation arising as a result of the Compton effect. This will make it possible to improve the accuracy of operational control over the quality of iron ore raw materials under industrial conditions when using the gamma-gamma method by using the optimal parameters for arranging a control system's elements.

To achieve the goal, the following tasks were set:

– to substantiate the geometry of $\boldsymbol{\gamma}$-radiation in systems of operational control over the material composition of iron ore raw materials;

– to construct a mathematical model of the system of operational control over the iron content in ore taking into account the geometric parameters of the location of the $\boldsymbol{\gamma}$-quantum source, the recording single crystal, and the irradiated surface;

– to carry out a computational experiment in order to obtain a dependence of the sensitivity of the method of $\boldsymbol{\gamma}$-irradiation of the rock on changes in the geometric parameters of the system.

4. The study materials and methods

The object of our study is the processes of irradiation of iron ore raw materials by a source of low-energy γ-quanta, registration of radiation scattered as a result of the Compton effect, as well as the parameters of the irradiation geometry.

The principal hypothesis assumes that the accuracy of operational quality control over mineral raw materials on the conveyor belt depends on both the physicochemical and physicomechanical parameters of the investigated rock mass, as well as on the geometric parameters of the control system. In addition, the measurement results are directly affected by the parameters of the γ-radiation source, such as activity (number of decays per unit time) and γ-radiation energy. When the conveyor belt moves, the ore mass is irradiated by a γ-radiation source, and depending on the iron content in it, the integral flux of backscattered γ-quanta changes. The γ-radiation source (isotope Am-241) is placed in a protective lead container in the collimation channel. In this case, the choice of the optimal distance between the NaJ single crystal and the protective container ensures the maximum visibility and registration of the γ-quanta reflected by the single crystal, which creates the prerequisites for improving the accuracy of control.

The geometry of the installation enables the irradiation of the controlled rock mass with γ-radiation at angles of up to 120° relative to the normal to its surface. The maximum informativeness is given by quanta falling at an angle of 90°, which is associated with the optimal ratio of the penetration depth and the level of the backscattered signal. Therefore, it is important that the γ-quanta, after being reflected from the rock mass, fall on the single crystal. Thus, the implementation of the model of the operational ore quality control system makes it possible to proceed to the optimization of its geometric parameters to maximize the registration of the integral flux of scattered γ-radiation. The magnitude of the integral flux of backscattered γ-quanta forms a dependence on the average iron content in the rock mass.

In order to assess the accuracy of operational control, it is necessary to investigate the sensitivity of the operational control system to the iron content. The sensitivity is characterized by dependence $E = dN / dFe$, i.e., the change in the number of pulses registered by the detector relative to one percent change in iron concentration. It is obvious that the value of E depends both on the parameters of the radiation source and on the average iron content in the controlled rock mass.

In the process of research, a rock mass with an iron content in the range of 40–70% was used. To illustrate the advantage of the proposed centrally-displaced geometry of measuring the iron content on the conveyor over the conventionally used central and lateral geometries, the family of dependences $N = f(q)$ was considered.

The Mathcad software package [17] and Microsoft Excel spreadsheets [18] were used in the calculations.

5. Results of investigating parameters for arranging structural elements in a distributed automated information and measurement system

5. 1. Substantiating the geometry of irradiation in the systems of operational control over the quality of rock

One of the most important issues for improving the accuracy of analysis is the selection of optimal geometric parameters for the system of operational control over the quality of iron ore raw materials. When considering nuclear physics methods, one of the promising techniques is the use of a centrally-shifted geometry of γ-irradiation of iron-containing ore.

To measure the total iron content in ore, the gamma-gamma method is used, which is based on the effect of the interaction of low-energy gamma quanta with matter. The controlled mass of iron ore is irradiated, and then the intensity of the integral flux of scattered γ-radiation is recorded by the detector. The peculiarity of modeling the measurement process is to build a model that, with a centrally shifted geometry, would take into account the dependence of value of the reflected γ-radiation on a number of geometric parameters. It includes the distance between the γ-radiation source and the ore breakdown, as well as the distance between the detector and the γ-radiation source.

The synthesis of a mathematical model as a measure of rock quality naturally begins with the well-known and tested formulas describing propagation of the γ-radiation flux in the environment [19, 20].

The lateral and central geometries of interaction between γ-radiation and rocks are shown in Fig. 1.

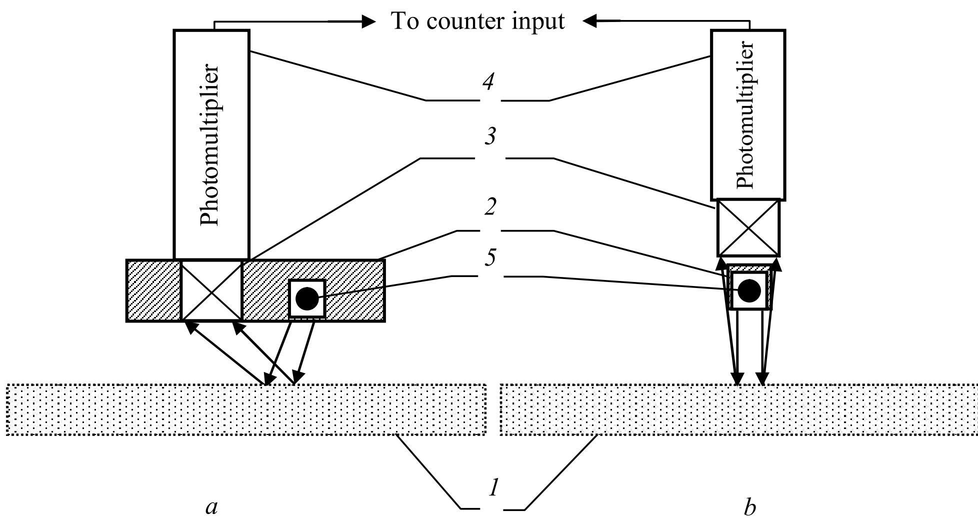

Fig. 1. Geometries of interaction between γ-radiation and rocks: *a* — lateral, *b* — central. Structural elements: 1 — irradiated rock; 2 — lead protection from the radiation source; 3 — NaJ single crystal; 4 — detector (photomultiplier); 5 — radiation source (Am — 241)

Conventional methods of operational control over the quality of rock use lateral geometry (*a*), where source 5 of γ-radiation with protective container 2 is located on the same horizontal line with single crystal 3 of the γ-radiation receiver. In this case, all scattered γ-quanta fall on receiver 3 repeatedly. It is known that the most informative are γ-quanta falling on the controlled rock at an angle of 90 degrees, so the lateral geometry does not fully ensure the accuracy of operational control over the quality of rock on the conveyor.

To improve the accuracy of operational control over the quality of rock, the central geometry (*b*) is also used, where receiver 3 of reflected γ-radiation with a single crystal, protective container 2 with the radiation source and controlled rock 1 are located vertically on the same axis. In this case, part of sensor 3 of single crystal is shielded by protective lead container 2, which also reduces the accuracy of operational control over the quality of rock on the conveyor [21].

In order to improve the accuracy of operational control over the quality of rock, another geometry is proposed – centrally shifted. In this case, to enable full visibility of the reflected γ-quanta by the single crystal sensor, the protective container with the radiation source is shifted horizontally by half the diameter of the single crystal.

5. 2. Construction of a mathematical model for the system of operational control over the quality of rock under conditions of centrally displaced geometry

The functional scheme of the centrally displaced geometry of operational control over the quality of rock on the conveyor is shown in Fig. 2.

According to the scheme demonstrated in Fig. 2, the γ-radiation source, shown as a black rectangle, emits a flux of γ-radiation that falls on an ore sample located at distance H. This flux of γ-radiation, which has the shape of a cone with the apex inside the lead container, forms a circle of radius R on the ore sample, which is illustrated in Fig. 2. When the conveyor belt moves, the rock on the conveyor is irradiated by the γ-radiation source.

The integral flux of γ-radiation reflected from the rock carries information about the iron content in the rock. When reflecting the γ-radiation flux, it is necessary to take into account the attenuation of the flux, which is carried out by introducing a albedo coefficient [20]. The reflected γ-radiation flux falls on the detector, which records its value, which is related to the iron content in the ore sample. A characteristic feature used in the centrally shifted geometry of rock irradiation is that the detector is shifted relative to the center of the γ-radiation source to the left along the axis by the width of the lead container containing the γ-radiation source. This displacement of the detector determines the nature of the centrally shifted geometry of rock irradiation. An important point when calculating the scattered γ-radiation absorbed by the detector is that it is necessary to take into account the absorption of this radiation by the lead container containing the γ-radiation source.

Fig. 2 shows the type of the scheme under consideration, with centrally shifted geometry. The dashed lines depict a lead container containing a γ-radiation source that absorbs the reflected γ-radiation. The first step in the calculations is to determine the radius of the irradiation circle of the rock sample. To do this, it is necessary to derive an equation for the rays passing through the edges of the cell containing the γ-radiation source.

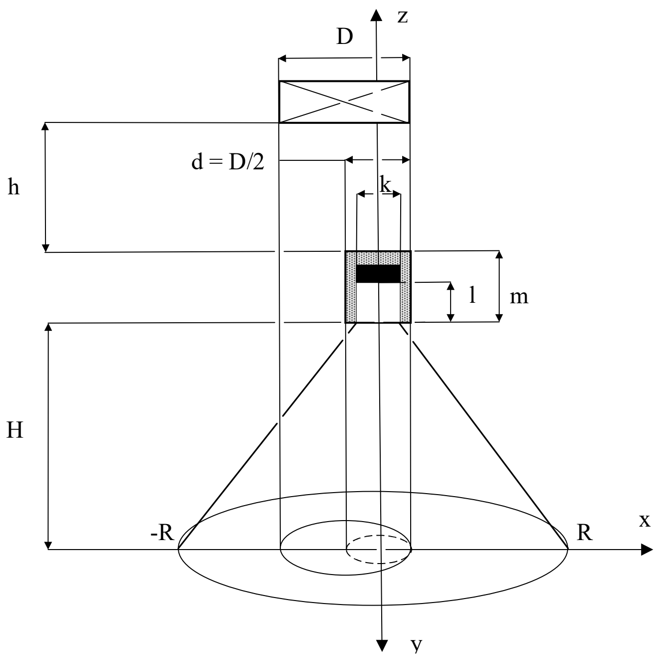

Fig. 2. Functional diagram of the centrally shifted geometry of operational control over the quality of rock on the conveyor

Fig. 3 shows a diagram of the geometry of the rock irradiation element, according to Fig. 2, and the beams passing through the corresponding points.

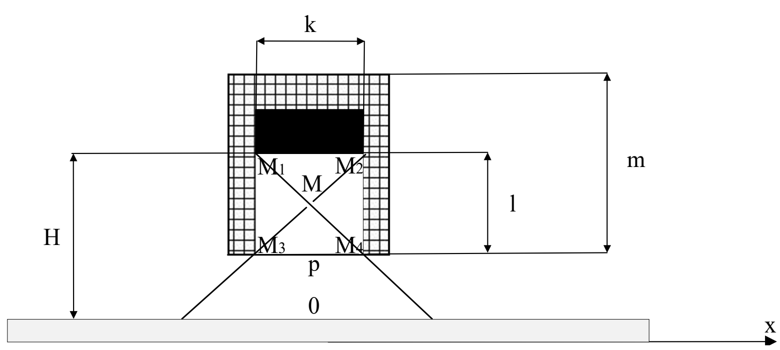

Fig. 3. Geometry diagram of a rock irradiation element

Equation of a beam passing through points $M_1\left(-\frac{k}{2};0;H\right)$ and $M_4\left(\frac{k}{2};0;H-l\right)$

$$\frac{x+\frac{k}{2}}{k}=\frac{y-0}{0}=\frac{z-H}{-l}, \quad (1)$$

where k is the diameter of the collimation window;

l is the depth of immersion of the single crystal of the γ-radiation source into the container.

The equation of the beam passing through points $M_2\left(\frac{k}{2};0;H\right)$ and $M_3\left(-\frac{k}{2};0;H-l\right)$

$$\frac{x-\frac{k}{2}}{-k}=\frac{y-0}{0}=\frac{z-H}{-l}. \quad (2)$$

The point of intersection of these beams is found as a solution to the system of these two equations. When finding the coordinates of this point, it was taken into account that at the intersection of the same coordinates the coordinates would be equal.

$$\frac{x+\frac{k}{2}}{k}=\frac{x-\frac{k}{2}}{-k}. \quad (3)$$

With a sequential solution of this equation, the following is obtained:

$$x+\frac{k}{2}=-x+\frac{k}{2},\ \ 2x=0,\ \ x=0. \quad (4)$$

For the second coordinate, respectively, the value $y = 0$ was obtained.

To find the third coordinate, it was taken into account that $x = 0$. Then, according to (1)

$$\frac{0+\frac{k}{2}}{k}=\frac{z-H}{-l},\ \ \frac{1}{2}=\frac{z-H}{-l},\ \ z=H-\frac{l}{2}. \quad (5)$$

Given the obtained coordinates, the point of intersection of beams (1) and (2) will be written in the form

$$M\left(0;0;H-\frac{1}{2}\cdot l\right). \quad (6)$$

This point of intersection of beams (1) and (2) is shown in Fig. 3.

To find the radius of the circle of rock irradiation, we shall use the beam equations. To this end, it will suffice to set coordinate z to zero in equation (1). As a result of such a substitution, we derive the equation

$$\frac{R+\frac{k}{2}}{k}=\frac{0-H}{-l}. \quad (7)$$

When solving this equation with respect to radius R, the radius of the irradiance circle of the rock was found

$$R+\frac{k}{2}=\frac{k}{l}H,\ \ R=-\frac{k}{2}+\frac{k}{l}H,\ \ R=\frac{k}{2}\left|1-2\cdot\frac{H}{l}\right|. \quad (8)$$

In formula (8) the modulus sign is used since the radius of the circle is a non-negative quantity.

In the next step, it is necessary to calculate the intensity of γ-radiation falling on the circle of rock irradiation.

The radiation intensity of the elementary region of the circle of rock irradiation is determined from the formula

$$dN(x,y,H)=\frac{q}{\left(x^2+y^2+H^2\right)^{\frac{3}{2}}}dxdy, \quad (9)$$

where q is the intensity of the γ-radiation source, 1/sec, *dxdy* is the area element in the Cartesian coordinate system, m^2.

Then the γ-radiation flux is reflected from the irradiation circle of the ore sample. Moreover, part of the γ-radiation flux falls on the detector. It should be noted that only a smaller part of the γ-radiation flux is reflected, which is taken into account using the albedo coefficient. The reflected part of the γ-radiation flux that fell into the detector can be written using the formula

$$dN_1\left(P_1\left(x_1,y_1,H+h+m-l\right);P\left(x,y,H\right)\right)= =\frac{A\cdot dN(x,y,H)}{\left[\left(x_1-x\right)^2+\left(y_1-y\right)^2+\left(H+h+m-l\right)^2\right]^{\frac{3}{2}}}dx_1dy_1, \quad (10)$$

where A is the albedo.

Formula (10) determines the intensity of γ-radiation at point $P_1(x_1, y_1, H+h+m-l)$, located on the detector. According to formula (10), the source of γ-radiation is point $P(x, y, H)$, located in the circle of rock irradiation, in which the radiation intensity determined from formula (9) is generated.

Considering (9), formula (10) takes the form

$$dN_1\left(P_1\left(x_1,y_1,H+h+m-l\right);P\left(x,y,H\right)\right)= =\frac{q\cdot A\cdot dxdy\cdot dx_1dy_1}{\left(x^2+y^2+H^2\right)^{\frac{3}{2}}\left[\left(x_1-\frac{d}{2}-x\right)^2+\left(y_1-y\right)^2+\left(H+h+m-l\right)^2\right]^{\frac{3}{2}}}. \quad (11)$$

The intensity of γ-radiation received by detector point $P_1(x_1, y_1, H+h+m-l)$ of radiation from the rock irradiation circle is found by integrating over the plane of the rock irradiation circle

$$dN_1\left(P_1\left(x_1,y_1,H+h+m-l\right)\right)= =q\cdot A\iint_S\left(\frac{dx_1dy_1}{\left(x^2+y^2+H^2\right)^{\frac{3}{2}}\left[\left(x_1-\frac{d}{2}-x\right)^2+\left(y_1-y\right)^2+\left(H+h+m-l\right)^2\right]^{\frac{3}{2}}}\right)\mathrm{d}x\mathrm{d}y, \quad (12)$$

where S is the plane of the rock irradiation circle.

In turn, the intensity of the detector receiving γ-radiation from the rock irradiation circle is found by integrating over the detector plane

$$N_1=q\cdot A\iint_S\left[\iint_{S_1}\frac{\mathrm{d}x_1\mathrm{d}y_1}{\left(x^2+y^2+H^2\right)^{\frac{3}{2}}\left[\left(x_1-\frac{d}{2}-x\right)^2+\left(y_1-y\right)^2+\left(H+h+m-l\right)^2\right]^{\frac{3}{2}}}\right]\mathrm{d}x\mathrm{d}y, \quad (13)$$

where S_1 is the detector plane.

It should be emphasized that quantity (13) will not change from the rearrangement of the order of integration. This feature was used in further calculations.

The feature of further calculations is that during calculations it is necessary to pass from double integrals to repeated integrals, placing the corresponding integration limits. Taking into account the complexity of the integration limit of the internal integral (13), associated with the presence of a lead container as an absorber of scattered γ-radiation, it was divided into the sum of two integrals

$$\iint_S\frac{\mathrm{d}x\mathrm{d}y}{\left(x^2+y^2+H^2\right)^{\frac{3}{2}}\left[\left(x_1-\frac{d}{2}-x\right)^2+\left(y_1-y\right)^2+\left(H+h+m-l\right)^2\right]^{\frac{3}{2}}}= =B_1\left(x_1,y_1\right)+B_2\left(x_1,y_1\right), \quad (14)$$

where

$$B_1\left(x_1,y_1\right)= =\int_{-R}^{-d}\mathrm{d}x\int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}}\frac{\mathrm{d}y}{\left(x^2+y^2+H^2\right)^{\frac{3}{2}}\left[\left(x_1-\frac{d}{2}-x\right)^2+\left(y_1-y\right)^2+\left(H+h+m-l\right)^2\right]^{\frac{3}{2}}}, \quad (15)$$

$$B_2\left(x_1,y_1\right)= =\int_{d}^{R}\mathrm{d}x\int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}}\frac{\mathrm{d}y}{\left(x^2+y^2+H^2\right)^{\frac{3}{2}}\left[\left(x_1-\frac{d}{2}-x\right)^2+\left(y_1-y\right)^2+\left(H+h+m-l\right)^2\right]^{\frac{3}{2}}}. \quad (16)$$

Then formula (13), when moving to the repeated integral, will take the form

$$N_1=q\cdot A\times \times\int_{-D+d}^{d}\mathrm{d}x_1\int_{-\sqrt{D^2-(x_1-d)^2}}^{\sqrt{D^2-(x_1-d)^2}}\left(\begin{array}{l}B_1\left(x_1,y_1\right)+\\+B_2\left(x_1,y_1\right)\end{array}\right)\mathrm{d}y_1. \quad (17)$$

Thus, formulas (15) to (17) determine the dependence of the integral flux of backscattered γ-radiation on parameters, i.e.

$$N_1=f(q, A, h, H, D, d, m, l). \quad (18)$$

A feature of the constructed mathematical model of the centrally-displaced geometry of γ-irradiation of rock (15) to (17) is the presence of integrals, the calculation of which requires the use of numerical methods for calculating integrals and appropriate software [17, 18].

Equations (15) to (17) were solved numerically using the Mathcad software package.

The radiation sensitivity, according to our mathematical model (15) to (17), is calculated from the following formula

$$\frac{dN_1}{dq} = A \cdot \int_{-D+d}^{d} dx_1 \int_{-\sqrt{D^2-(x_1-d)^2}}^{\sqrt{D^2-(x_1-d)^2}} \left(B_1(x_1,y_1)+B_2(x_1,y_1)\right)dy_1. \quad (19)$$

or, taking into account the linear dependence of the integral flux of backscattered γ-radiation on parameter q, we can write

$$\frac{N_1}{q} = A \cdot \int_{-D+d}^{d} dx_1 \int_{-\sqrt{D^2-(x_1-d)^2}}^{\sqrt{D^2-(x_1-d)^2}} \left(B_1(x_1,y_1)+B_2(x_1,y_1)\right)dy_1. \quad (20)$$

One can see from formula (20) that the considered sensitivity of irradiation characterizes the probability of registration by the detector of a γ-quantum emitted by the source, which was reflected from the irradiated surface.

5. 3. Computational experiment on the application of mathematical modeling of the centrally-displaced geometry of γ-irradiation of rock

According to the functional scheme for the centrally-displaced geometry of control over the quality of rock, shown in Fig. 2, the following parameters were set to conduct a computational experiment to evaluate the results of mathematical modeling using the centrally-displaced geometry of γ-irradiation of rock:

$$d = 30 \text{ mm}, D = 60 \text{ mm}, m = 50 \text{ mm},$$

$$l = 25 \text{ mm}, k = 6 \text{ mm}, \quad (21)$$

which were constant values. The distance from the γ-radiation source to the rock H was chosen as the variable parameters at fixed values of the specified distances from the detector to the upper edge of the protective container of the γ-radiation source h.

When mathematically modeling the centrally-displaced geometry of the γ-radiation of the rock, the relative sensitivity was calculated, which was defined as the ratio of the sensitivity to the albedo value, i.e., the sensitivity value per unit of albedo. This approach can be explained by the fact that under the modeling conditions the albedo value is unknown, but this value, being constant, does not affect the nature of the studied dependence. Therefore, taking into account (21), the value defined as the relative sensitivity was applied

$$\hat{N}_1 = \frac{N_1}{q \cdot A}, \quad (22)$$

or, according to (20), we can write

$$\hat{N}_1 = \int_{-D+d}^{d} dx_1 \int_{-\sqrt{D^2-(x_1-d)^2}}^{\sqrt{D^2-(x_1-d)^2}} \left(B_1(x_1,y_1)+B_2(x_1,y_1)\right)dy_1. \quad (23)$$

Calculations were performed for different values of distances from the detector to the upper edge of the protective container of the γ-radiation source h = 0.5, 10, 15, 20 mm. The results of our calculations are given in Table 1.

Fig. 4 shows a graphical representation of the results given in Table 1.

Table 1

Relative radiation sensitivity depending on the distance of the γ-radiation source to the rock at different distances of the detector to the upper edge of the protective container of the γ-radiation source

H \ h	0	5	10	15	20
75	0	0	0	0	0
80	1.49E-07	1.31E-07	1.16E-07	1.03E-07	9.24E-08
85	3.17E-07	2.81E-07	2.50E-07	2.24E-07	2.01E-07
90	4.45E-07	3.97E-07	3.55E-07	3.18E-07	2.86E-07
95	5.31E-07	4.75E-07	4.26E-07	3.84E-07	3.47E-07
100	5.81E-07	5.21E-07	4.70E-07	4.24E-07	3.85E-07
105	6.04E-07	5.44E-07	4.92E-07	4.46E-07	4.05E-07
110	6.08E-07	5.50E-07	4.98E-07	4.53E-07	4.13E-07
115	5.98E-07	5.43E-07	4.93E-07	4.50E-07	4.11E-07
120	5.80E-07	5.27E-07	4.81E-07	4.40E-07	4.03E-07
125	5.56E-07	5.07E-07	4.64E-07	4.25E-07	3.91E-07
130	5.28E-07	4.83E-07	4.43E-07	4.07E-07	3.75E-07
135	4.99E-07	4.58E-07	4.21E-07	3.88E-07	3.58E-07
140	4.70E-07	4.32E-07	3.98E-07	3.67E-07	3.40E-07
145	4.41E-07	4.06E-07	3.75E-07	3.47E-07	3.21E-07
150	4.12E-07	3.81E-07	3.52E-07	3.26E-07	3.03E-07

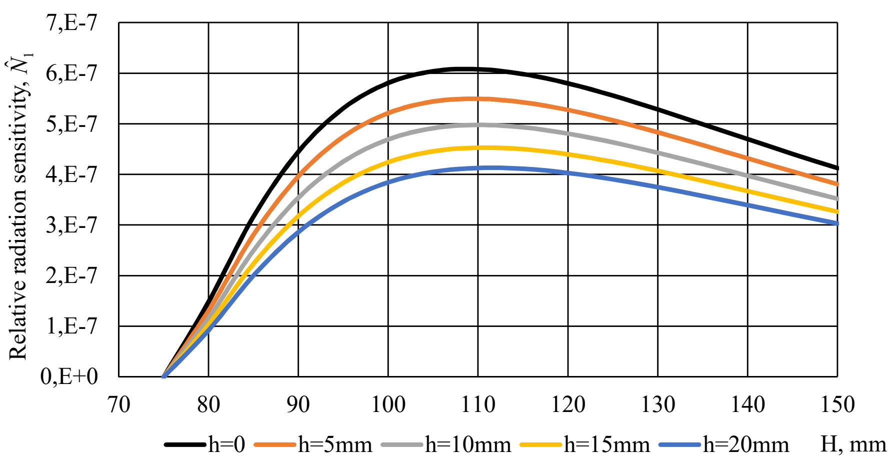

Fig. 4. Dependence plots of the relative radiation sensitivity on the distance of γ-radiation source to the rock at different distances of the detector to the upper edge of the protective container of the γ-radiation source

Analysis of the plots shown in Fig. 4 reveals that they are extreme in nature, i.e., they reach their maximum value at a certain distance from the γ-radiation source to the ore sample. At the same time, there is a decrease in the relative sensitivity of the radiation with increasing distance from the detector to the upper edge of the protective container of the γ-radiation source. The greatest value of the relative sensitivity is observed when the distance from the detector to the upper edge of the protective container of the γ-radiation source is zero (h = 0). It should also be emphasized that the maximum values of each of the curves shown in Fig. 4 are achieved at a constant distance from the γ-radiation source to the rock, which is 110 mm. However, the values of the relative sensitivity of the radiation change.

Table 2 gives the results of calculating the maximum relative sensitivity of the radiation depending on the distance from the detector to the upper edge of the protective container of the γ-radiation source. Calculations were performed using the Maximize program.

Table 2

Dependence of the maximum relative radiation sensitivity on the distance of the detector to the upper edge of the protective container of the γ-radiation source

h, mm	0	5	10	15	20
$\hat{N}_{1.max}$	6.08E-07	5.495E-07	5.5E-07	4.98E-07	4.53E-07

Fig. 5 shows a graphical representation of Table 2, as well as an approximation of the data from Table 2 by a straight line.

Equation of linear approximation of data from Table 2 takes the form

$$\hat{N}_{1.max} = -7.2 \cdot 10^{-9} \cdot h + 6.04 \cdot 10^{-7}. \quad (24)$$

The correlation coefficient of the calculation results and their linear approximation was

$$r = 0.974. \quad (25)$$

According to the Chaddock qualitative scale, since, according to (25)

$$0.9 < |r| < 1, \quad (26)$$

then there is a "very high" strength of the connection between the calculation results and their linear approximation [22].

Conducting a computational experiment in mathematical modeling of the centrally-displaced geometry of γ-irradiation of rock makes it possible to show the convergence of the results of mathematical modeling with the calculated (experimental) data. Using the data on the parameters of the device under study, which were used above, the relative sensitivity was measured in γ-irradiation of rock. In this case, the distance from the detector to the upper edge of the protective container of the γ-radiation source was set to $h = 15$ mm. The measurement results are given in Table 3.

Fig. 6 shows the results of processing experiments to determine the relative sensitivity of rock to γ-irradiation.

According to the data from Table 3, this correlation coefficient took the value of

$$r = 0.981. \quad (27)$$

On the Chaddock scale, based on (27) and according to formula (26), the strength of the relationship between the dependences in question is "very high".

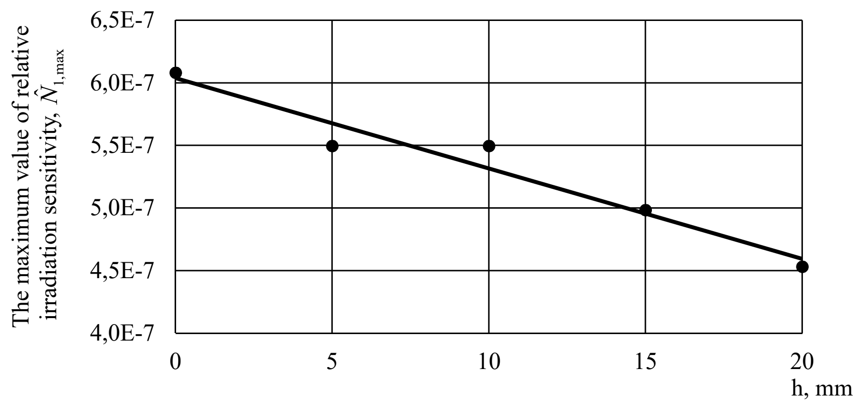

Fig. 5. Graphical representation of the data from Table 2 with their linear approximation

Table 3

Relative sensitivity of rock irradiation in the experiment and calculation according to the mathematical model depending on the distance of the γ-radiation source to the ore sample at a distance from the detector to the upper edge of the protective container of the γ-radiation source equal to 15 mm

Distance from the γ-radiation source to the rock, H, mm	Experiment	Model
	Relative radiation sensitivity of rock, $\hat{N}_{1,exp} \cdot 10^{-7}$	Calculation of the relative radiation sensitivity of rock, $\hat{N}_1 \cdot 10^{-7}$
85	1.8	2.24
95	4.1	3.84
105	4.5	4.46
115	4.45	4.5
125	4	4.25
135	3.8	3.88
145	3.4	3.47

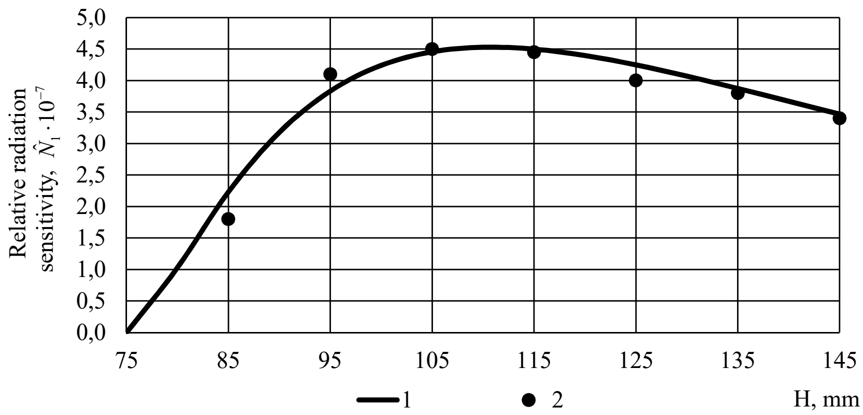

Fig. 6. Results of processing experiments to determine the relative sensitivity of rock to γ-irradiation: 1 – results of calculations using a mathematical model, 2 – experimental data

6. Results of improving the nuclear-physical method for monitoring the iron content in crushed rock mass: discussion and summary

It has been substantiated that, unlike the conventionally used lateral [23] and central geometries [24] (Fig. 1) of γ-irradiation of rock, the centrally shifted geometry of γ-irradiation (Fig. 2) makes it possible to increase the informativeness of the measurement of iron content in rock. This is achieved by ensuring full visibility of the reflected integral flux of γ-irradiation by the scintillation detector.

Based on the physical laws of γ-irradiation of a solid and using integral calculation methods, a mathematical model of the centrally shifted geometry of γ-irradiation of rock was synthesized. This mathematical model makes it possible to determine the intensity of the integral flux of the return γ-radiation depending on the geometric parameters of the γ-irradiation region (12) and the detector (13). The geometric parameters of the modeled device that do not change but affect its functioning were taken into account, which include the dimensions of the protective container of the γ-radiation source, the depth of immersion of the γ-radiation source in the protective container, the dimensions of the detector and the γ-radiation source (18). Special attention was paid to taking into account the geometric parameters that are variables, which include the distance of the γ-radiation source to the surface of the rock and the distance of the detector to the upper edge of the protective container of the γ-radiation source. The radiation sensitivity formula (20) has been derived.

At the same time, it was demonstrated that when conducting computational experiments, it is necessary to choose the relative sensitivity of the backward integral flux of γ-radiation (23), which is recorded by the detector, referred to a unit of albedo, which is due to the impossibility of taking into account the albedo value under the conditions under consideration. Analysis of the results of computational experiments given in Table 1, as well as the family of curves in Fig. 4, which demonstrate the dependence of the relative sensitivity of γ-radiation on the distance between the source of γ-radiation and the irradiated rock at different distances between the detector and the source of γ-radiation, revealed that these dependences are extreme in nature. In this case, the value of the relative sensitivity reaches a maximum at zero distance between the source of γ-radiation and the detector, at a distance of 110 mm between the source of γ-radiation and the surface of the irradiated rock. The computational experiment conducted to assess the strength of the relationship between the results of laboratory tests and the results of mathematical modeling (Table 3) showed (Fig. 6) that the correlation coefficient is equal to 0.981 (27), which, according to the Chaddock scale, characterizes the strength of the relationship between the dependences as "very high".

Conducting computational experiments with a displacement of the protective container of the γ-radiation source from the center by half the detector diameter ensured that the detector controlled all reflected γ-quanta from the controlled rock, which made it possible to implement a centrally shifted irradiation geometry. At the same time, the integral flow of backscattered γ-quanta registered by the detector carries information about the quality of the controlled rock, which also increases the accuracy of operational control over the quality of rock on the conveyor.

The limitations of our study include the need to take into account the size of the crushed ore (in the framework of the experiments conducted – 10–25 mm with a content of fines of 0–5 mm size not more than 5%) and its humidity (3–5%, respectively). In addition, to ensure adequate results, it is necessary to enable leveling of the ore layer on the irradiated surface. The disadvantages of the study also include limited results because of the use of one source of γ-radiation – Am-241. The prospects for further research into this area include the use of other radioactive sources, for example, neutron ones.

7. Conclusions

1. To solve the task of control over the quality of iron ore raw materials, nuclear physical methods with central or lateral measurement geometries are used. Such application leads to a decrease in the informativeness of measurements due to the influence of the protective container of the γ-radiation source and a large scattering angle of γ-quanta, respectively. The combined centrally-displaced geometry of the γ-radiation source makes it possible to eliminate the shortcomings of the specified geometries by ensuring direct visibility of the sensor single crystal and forming a scattering angle close to the optimal value (≈ 90°).

2. The constructed mathematical model has made it possible to identify the main geometric parameters for the system of operational control over the quality of iron ore on the conveyor. These parameters include the distance between the γ-radiation source and the irradiated rock; the distance between the detector and the protective container of the γ-radiation source; the diameters of the single crystal and the protective container; the diameter of the γ-radiation source; the height of the collimation channel. A formula has been derived that linked the geometric parameters for the system of operational control over iron content in ore on

the conveyor using the centrally-displaced geometry of ore irradiation.

3. The results of our computational experiment based on the synthesized mathematical model of the centrally-displaced geometry of γ-irradiation of rock indicate that the maximum relative sensitivity of registration of the integral flux of reverse γ-irradiation at the given parameters is achieved at the minimum distance of the container with the γ-radiation source and the detector single crystal, provided that the distance of the γ-radiation source from the irradiated surface is fixed at 110 mm. The resulting value of a correlation coefficient between the calculated and experimental data is 0.981, which indicates a high level of their consistency.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study, as well as the results reported in this paper.

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Data availability

All data are available, either in numerical or graphical form, in the main text of the manuscript.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

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