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BUILDING A MODEL OF ARTILLERY FIRING EFFICIENCY UNDER CONDITIONS OF LIMITED AMMUNITION RESOURCES AND BARREL WEAR

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This study examines the process of destroying individual and group targets by an artillery battery under conditions of limited ammunition of a given quality and varying degrees of barrel bore wear.

The task addressed relates to the revealed lack of a quantitative model that would make it possible to assess the effectiveness of aimed or planar fire in the presence of resource limitations on shots and barrel bore wear, which could facilitate determining the threshold conditions for the feasibility of continuing fire.

A discrete stochastic model of the effectiveness of firing an artillery unit has been proposed. The model is designed to assess the effectiveness of aimed fire under conditions of limited resource of high-quality ammunition. The model also takes into account the presence of guns in the unit, the barrels of which have different levels of wear. It is proposed to divide shots from each gun in the artillery unit into four different classes, which takes into account the quality of the shot elements: the projectile and the propellant charge, as well as the condition of the barrel.

It is proposed to consider the successive states of the artillery unit's guns as elements of the Markov chain. It was established that it is possible to achieve a firing efficiency coefficient that lies within the range of 0.63–0.83. The limiting estimate of the firing efficiency coefficient of the artillery unit on a separate target was obtained, which is equal to 0.5.

The results showed that when the limiting value of the firing efficiency coefficient is obtained, aimed fire degenerates into planar target destruction with enlarged scattering ellipses. This state leads to an almost twofold increase in the cost of shots and the time of firing, which sharply increases the risk of counter-battery damage to the battery.

Such results are attributed to the combination of the accepted classification of shots by quality, taking into account the increased wear of the barrel bores due to a decrease in the probability of hitting, and the analysis of a set of realistic gun firing strategies.

The model could be offered for performing tactical calculations when choosing a fire mode, the structure of using available ammunition, and assessing the feasibility of switching from aimed to plane fire

Keywords: artillery battery, limited resources, barrel bore wear, firing efficiency coefficient, firing threshold conditions, counter-battery damage

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1. Introduction

The three-year combat experience of the War for Independence of Ukraine showed that artillery, despite the transition to remote warfare or "drone warfare", remains the main significant component of the strike force of units during ground operations. More to the point, over the three years of hostilities, the concepts of using barrel artillery have changed radically. 155 mm caliber guns have become the optimal option for equipping artillery units of the Armed Forces of Ukraine [1, 2]. A major role in this process belongs to the armament of Ukrainian artillery with the latest generation barrel systems designed by leading Western manufacturers and allies of Ukraine, supplied in the liberation war. These systems are distinguished by their high rate of fire and high ammunition capacity [3]. Most of these guns are self-propelled artillery systems (SASs) based on high-passenger wheeled chassis, which provides them with high maneuverability on the battlefield under conditions of an extensive road network with paved roads [4–6].

The latest generation of artillery systems have a high level of automation. Built-in control systems make it possible almost in real time to take into account and compensate for both systematic corrections and random disturbances and interferences that arise during the performance of tasks. This enables firing with high accuracy and intensity without the conventional aiming procedure [7–9].

At the same time, a significant part of the artillery installations, which are actually used today in the War for Independence of Ukraine, were manufactured and used before 1991. Such guns are characterized by high firing times, accumulated wear of the barrel bores, and changes in the volume of the charging chamber. In addition, it should be taken into account that ammunition is used after many years of storage in warehouses, which has changed the gerontological characteristics of the shots. Under such conditions, the actual ballistic characteristics of guns and shots differ significantly from the tabulated ones. The main characteristics of the shot, namely the accuracy and aiming of the shot, have significantly deteriorated [10]. However, it should be noted that combat units are also armed with modern, latest-generation barrel systems, which make up a significant percentage of the artillery fleet. These systems are distinguished by their high rate of fire and high ammunition capacity.

Current circumstances dictate new conditions for the use of artillery units. First, combat units are forced to combine modern automated fire control systems with a shortage of high-quality ammunition and varying degrees of wear of artillery barrel channels. For the balanced use of target destruction by all types of guns, quantitative models that make it possible to calculate the effectiveness of fire are of particular importance. The task that has arisen can be solved taking into account the resource of ammunition and barrel wear and justifying the fire mode, either aimed or planar. Research into ways to increase the effectiveness of artillery fire, taking into account the intensive wear of barrel channels and the possible use of ammunition with different technical and ballistic characteristics, is an urgent scientific and practical task.

2. Literature review and problem statement

A series of papers report theoretical studies on the influence of barrel wear on the effectiveness of firing. In [11], an attempt is made to link the level of physical wear of the barrel rifling with the barrel resource; however, the work does not include experimental confirmation of the physical model. The loss of initial velocity of shells depending on barrel wear is tackled in [12]; however, the results were obtained only for small-caliber guns. In [13], a large number of methods for assessing the degree of barrel wear are studied, but without any numerical results. The authors do not show the possibilities of using physical models under combat conditions and it is unclear how such models would affect the tactical effectiveness of the unit. The influence of the use of ammunition with different technical and ballistic characteristics on barrel wear is studied in [14]. In particular, the influence of the gerontological properties of powder charges with a long storage period on barrel wear was investigated. However, the relationships linking the number of shots from the barrel with the initial velocity of the projectile are almost absent in the publicly available literature. An analysis of practical methods for estimating barrel wear based on the initial velocity of the projectile [15] reveals that they cannot always be applied because of the conditions of combat situation. Possible verification procedures are not shown; most attention is paid to the technical aspects of control, rather than their tactical use.

This applies even to advanced acoustic methods [16]. To assess the level of barrel wear, it is recommended to focus on its service life before write-off, which is given in specialized literature. The service life of a rifled gun barrel of artillery systems of ground artillery is understood as the maximum number of shots, after which the initial velocity of the projectile decreases by more than 10%, which means its limit wear [17]. This indicator varies significantly depending on the gun model. In particular, for the M777 howitzer, according to [18], it is 2650 shots. In [19], the value of 2800 shots is indicated. This unprincipled difference is probably due to the fact that the M777 gun is produced in different modifications. In the above three studies, the authors did not present a quantitative model of firing efficiency under barrel wear conditions. In [20] the characteristics of guns by leading European manufacturers are analyzed. The resource for the PzH2000 SAC is 4500 rounds, for the M109 SAC – 6000 rounds. The paper draws a conclusion for the strategic and operational level of gun wear taking into account the number of rounds, but formal modeling of firing taking into account the quality of ammunition and the degree of barrel wear is not the subject of analysis.

Paper [21] provides a justification for two procedures for selective quality control over ammunition equipped with propellant charges of different quality. The authors consider the impact of shot control on tactical effectiveness but do not consider how to implement the results of selective control into the firing model.

A productive mathematical apparatus for the effectiveness of artillery firing is the theory of Markov chains. In [22], a complex Markov model is constructed that assesses the effectiveness of the artillery unit's actions taking into account its maneuvers and changes in tactics. However, the proposed model does not take into account the wear of the barrel channels and the presence of shots of different quality. The Markov model [23] considers the organization of actions regarding the use of an artillery unit in battle. The theory of Markov chains regarding artillery firing has been successfully applied in the model of ground artillery firing [24]. However, the study did not address the aspects of the limited resources of the artillery unit.

Study [25] showed the possibility of optimizing the use of artillery, but the optimization function does not include parameters related to the resource of barrels and the quality of ammunition. Papers [26–29] report studies on critical infrastructure facilities and control systems under uncertainty based on stochastic models and reliability calculation methods. However, the authors do not consider the problem of assessing the effectiveness of artillery fire.

Our review of the literature [1–6, 18, 20] demonstrates the modeling of artillery combat operations at the operational and strategic levels. At the same time, the specified models do not take into account both the wear of barrel channels and the shortage of ammunition. Studies [11–16, 18] report materials on barrel wear and ammunition degradation but leave out of consideration the issue of operational use of the battery under conditions of limited resources. The analysis of ammunition quality control methods is given in [14, 15, 21], but the control results are considered theoretically and are not integrated into the work of the artillery unit. The use of stochastic and Markov models in [22–25] proved the possibility of modeling artillery fire. However, these studies do not take into account the quality of ammunition and the degree of wear of the barrel bores. It should be noted that none of the studied sources solved the problem of finding any threshold conditions for the feasibility of aimed and plane fire.

Analysis of related work allowed us to identify a scientific problem. This is the lack of a model for the effectiveness of hitting targets by an artillery unit. The model should take into account resource constraints on quality shots and different degrees of wear of the barrel bores. In addition, such a model could make it possible to determine the threshold conditions for the feasibility of aimed and plane fire, taking into account the probability of counter-battery damage.

3. The aim and objectives of the study

The purpose of our study is to build a discrete stochastic model of the effectiveness of hitting a single and group target by an artillery unit under the simultaneous action of two resource constraints: the supply of ammunition of a certain quality and the residual service life of the barrels. The model is intended to determine the conditions for the transition from the aimed to the planar fire mode according to the criteria of effectiveness and time of hitting the target.

To achieve the goal, the following tasks must be solved:

– to formalize the statement of the task of assessing the aiming mode of hitting a target by an artillery unit under simultaneous resource constraints, namely the number of shots of a certain quality and the residual service life of the barrels;

– to determine the indicator of quantitative assessment of the limit of use of the aiming mode of hitting a single target under the simultaneous action of two resource constraints, namely the supply of shots of a certain quality and the residual service life of the gun barrels;

– to investigate the planar mode of hitting a group target as a possible alternative to the aiming mode.

4. The study materials and methods

The object of our study is the process of fire damage to a single and group target by an artillery unit under conditions of simultaneous limitations of the resource of ammunition of a given quality and the residual service life of gun barrels.

The subject of the study is a discrete stochastic model in the form of "state-control-transition" and a method of numerical evaluation of strategies for distributing shots between guns with different states of the residual service life of the barrels, including the rules of transitions, efficiency criteria, and the condition for transition from the aiming to the planar mode of target defeat.

The proposed hypothesis of our study assumes that due to the presence of two resource limitations, the quality of ammunition and the residual service life of the barrels, there is a limiting range of firing parameters. In it, the expected effectiveness of aimed fire and/or the expected time of target defeat change to such a level that the aiming mode becomes tactically impractical compared to the use of the planar mode. It is assumed that the specified limit range could be quantitatively identified on the basis of a discrete stochastic model with the Markov property of transitions between states. In the model, one of the states determines the success of hitting the target, and the other state determines the remainder of the shots and the remaining service life of the barrels.

The study adopts a system of assumptions. An artillery unit of four guns is considered: two guns belong to the group with a high residual service life of the barrels, which provides the predicted ballistic properties of the barrel, and two guns belong to the group with a low residual service life. The effect of wear is represented by a decrease in the probability of a successful hit and an increase in dispersion. When implementing the current target engagement mode, the degree of barrel wear does not change, i.e., the decrease in the residual service life of the barrels during the target engagement mode is not modeled.

According to the level of substantiation of the constancy of ballistic characteristics, shots are divided into two categories: first, those that have passed the full verification procedure for the specified properties, and those that have passed the shortened selection procedure and have increased uncertainty regarding the constancy of ballistic characteristics. In the model, the change in ballistic characteristics is reflected through different conditional probabilities of effective target damage.

Four types of shots are given as combinations of the shot category and a group of states with a residual service life of the barrels. Each type is assigned a conditional probability of effective target damage p_j, which is the result of the total impact of the shot quality and barrel wear. Therefore, the limitation of the residual service life of the barrels is introduced not through a physical model of erosion processes but through a probabilistic decrease in the effectiveness of the shot for the group with a low residual service life.

For a single target, a requirement is introduced to achieve a predetermined number of effective hits n^*, which is a condition for effective target destruction. Within one strategy, parameters p_j are considered constant, and the reduction in the residual service life of the barrels during the time of target destruction is not taken into account, since a model constraint in the form of a quasi-stationary approximation is accepted. Such a simplification establishes the extent of the influence of factors that limit resources and the structure of the strategy on the expected effectiveness and time of destruction.

For planar firing at a group target, the task is formalized as covering a rectangular area of terrain at a significant range, for example, $D = 12000$ m. The dispersion of gaps when hitting a target is modeled by a scattering ellipse with semi-axes specified for each type of shot and group of conditions with a residual service life of the barrel. For barrels with a reduced residual service life of the barrels, increased semi-axes are taken, which reflects the deterioration of accuracy. The evaluation criteria for planar firing at a group target are consistent with the criteria for a single one: shot consumption and task completion time.

The mathematical basis of the study is a discrete stochastic model with the Markov property. The system model is represented by states that describe the current process of effective target engagement by the number of effective hits, the remaining shots of each category, the belonging of the guns to the group of states with the remaining serviceable barrel resource, which are determined by the available probabilities p_j. The control problem is solved by choosing the gun and the type of shot at each step of the shot execution.

The transition between states occurs discretely at the time of the shot execution. The transition probability is determined by the selected gun and type of shot, the conditional probability of effective hit for this combination, and the remaining resources. The modeling process stops when one of the conditions is reached, either the target is engaged by achieving the requirement n^*, or the resource required to use the selected strategy is exhausted.

To compare the strategies, the efficiency coefficient K_{eff} and the expected engagement time T_{fire} were used. The efficiency coefficient K_{eff} is defined as the ratio of the expected number of effective hits to the expected consumption of shots when evaluating the current strategy. The time of execution of the defeat T_{fire} is determined by the total number of shots and the rate of fire of the guns of the corresponding groups with the remaining serviceable resource of the barrel. The boundary condition for the transition to the plane mode is formed as a parameter range in which further aimed fire does not provide acceptable K_{eff} and/or leads to an unacceptable increase in T_{fire} compared to the plane mode.

5. Results of the study of artillery firing under conditions of limited resources

5. 1. Stating the problem of modeling the aiming mode of defeat and determining the boundary conditions of its feasibility

To achieve the goal of the study, the first step is to formalize the statement of the problem of evaluating the aiming mode of defeating a target by an artillery unit under simultaneous resource constraints, namely the number of shots of a certain quality and the residual service life of the barrels. The goal is to build a discrete stochastic model of the "state-control-transition" type and determine the K_{eff} and T_{fire} indicators, on the basis of which the boundary condition for changing the feasibility of the aiming mode is set.

An artillery unit with m guns is considered, which performs the task of defeating a single target in a limited time. The unit has shots of two categories, which differ in the level of confirmation of the constancy of ballistic characteristics, which are subsequently taken into account in the model through different conditional probabilities of effective hit.

Shots are divided into two categories, those verified by the full random control procedure, and those that have undergone accelerated random control and have a higher uncertainty. The category is then taken into account by the parameters of the probability of a successful hit.

The practical feasibility of the model lies in the fact that an artillery unit carries out the process of fire destruction under conditions of strict time constraints and uncertainty of ammunition quality, as well as different residual service life of gun barrels. This requires comparing shot distribution strategies according to the criteria of expected effectiveness and expected time of destruction, and not only by ammunition consumption. That is, an artillery unit, when performing a given combat mission, follows the action of "shoot-and-scoot" [25].

An artillery unit performs the task of destroying a single target. The state of the gun barrel is represented by two groups of values of the residual service life of the barrels, namely, high or low. The quality of the shot is determined by belonging to one of two categories, namely those that are verified by the full procedure of selective control, and those that have passed accelerated selective control. The combination of gun barrel channel state groups and shot quality control category determines the conditional probability of a successful hit for the current shot.

This paper uses a generalized probabilistic characteristic of a shot. It implies the conditional probability of a successful hit p_j that is influenced by the quality of the shot and the value of the residual serviceable state of the barrel resource. Specific procedures for classifying shots and barrel states, including measurements, used in [26], are considered as a source of parameterization p_j, but are not included in the mathematical statement of the problem.

4 types of shots $j \in \{I, II, III, IV\}$ are introduced as combinations of the residual serviceable state of the barrel resource and the shot category, for which conditional probabilities of a successful hit p_j are given. Shot types:

– I: high residual service life of the barrel and shots verified by the full procedure of random control;

– II: high residual service life of the barrel and shots that have passed the accelerated random control;

– III: low residual service life of the barrel and shots verified by the full procedure of random control;

– IV: low residual service life of the barrel and shots that have passed the accelerated random control.

For the calculation experiment, the set of values of the probabilities of a successful hit $\{p_I, p_{II}, p_{III}, p_{IV},\}$ for each type of shot at a fixed range $D = 12000$ m is given; $p_I = 0.9$, $p_{II} = 0.7$, $p_{III} = 0.6$, $p_{IV} = 0.5$. The specified values are scenario parameters and are used to compare strategies under the same conditions.

For a single target, the requirement is set to achieve n^* effective hits, which is the condition for completing the hit. The modeling results are interpreted through the mathematical expectations of the number of shots and the time to achieve this condition. Rounding, if necessary, is applied only at the stage of representing the results but is not included in the definition of probabilistic transitions.

The time to perform the hit is determined by the total number of shots t. This allows us to consider the model as discrete. Let h_t be the number of effective hits from t shots; a_t is the choice of the shot type. Since the probability of the next state depends only on h_t and the current choice of the shot type a_t, the model has the Markov property. Then the conditional probability of a successful hit p_j can be represented as:

$$P\left(h_{t+1} = h_t + 1 \mid a_t = j\right) = p_j,$$
$$P\left(h_{t+1} = h_t \mid a_t = j\right) = 1 - p_j. \quad (1)$$

The choice of strategy by an artillery unit is defined as the choice of gun and type of shot depending on the current state of the system. The state is described by the vector

$$s_t = \left(h_t, r_I(t), r_{II}(t), r_{III}(t), r_{IV}(t)\right), \quad (2)$$

where $r_j(t)$ is the remainder of available shots of type j after the t-th shot.

The control is the choice of a_t. The strategy θ specifies the rule $a_t = \theta(s_t)$.

Resource update:

$$r_j(t+1) = \begin{cases} r_j(t) - 1, \text{ if } a_t = j, \\ r_j(t), \text{ if } a_t \neq j, \end{cases} \quad r_j(t) \geq 0. \quad (3)$$

The process is absorbing. The simulation ends when the number of shots fired is $t = \tau$ if one of the following conditions is met:

– the target is hit: $h_\tau \geq n^*$;

– the resource is exhausted: for all j we have $r_\tau(t) = 0$ or it is impossible to implement the current strategy due to resource constraints.

The Markov chain diagram corresponding to the battery strategy is shown in Fig. 1. The last state of the Markov chain occurs when one of the conditions is met: "target is destroyed", "shots of the 1st category are spent", or both conditions together. In the event of the target being destroyed, the state T "change of firing position" occurs. The transition probability $P_{i,i+1}$ is determined from formula (1).

For each strategy θ, the mathematical expectations M_θ of the corresponding quantities, $M_\theta(h_\tau)$, $M_\theta(\tau)$, $K_{eff}(\theta)$ and $T_{fire}(\theta)$ are estimated by the method of simulation modeling or by the method of absorbing Markov chains using the fundamental matrix.

The generalized efficiency of the aiming mode for strategy θ is defined as the ratio of mathematical expectations $M_\theta(h_\tau) / M_\theta(\tau)$

$$K_{eff}(\theta) = M_\theta(h_\tau) / M_\theta(\tau). \quad (4)$$

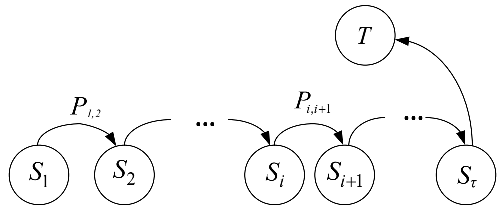

Fig. 1. Generalized scheme of an absorbing Markov process for a given shot selection strategy under an aiming mode

The second criterion is the expected hit time T_{fire}, which is determined by the expected number of shots and the rate of fire. For types of shots fired by guns with a high residual service life of the barrel, the rate is set to $v_I = v_{II}$; for guns with a low residual service life of the barrel – $v_{III} = v_{IV}$. In the calculation experiment, we apply $v_{I,II} = 6$ shots/min and $v_{III,IV} = 3$ shots/min.

Expected hit time for the model

$$T_{fire}(\theta) = M_\theta\left[\sum_{t=1}^{\tau}\frac{1}{v(a_t)}\right], \tag{6}$$

where $v(a_t) \in \{v_{I,II}, v_{III,IV}\}$ is the rate of fire corresponding to the shot type selected at step t and taking into account mixed shot types.

The strategy comparison problem is two-criteria and consists of maximizing $K_{eff}(\theta)$ and minimizing $T_{fire}(\theta)$. For practical selection, a preference rule is introduced, after which a subset of the best strategies for given resource parameters is determined.

The resulting relations (1), (3) to (5) form the problem statement for the aiming hit mode, which consists of probabilistic transitions, resource changes, termination conditions, and strategy evaluation criteria. On this basis, it is possible to determine the limiting value for parameters at which the aiming mode loses its feasibility compared to the transition to the planar mode of hitting the target.

5. 2. Assessing the limit of using the aiming hit mode and the conditions for transition to the planar mode

The second step in achieving the goal is to quantitatively assess the limit of use of the aiming mode of hitting a single target under the simultaneous action of two resource constraints: the supply of shots of a certain quality and the residual service life of the gun barrels. The limit of use is defined as the parameter range in which, according to the given criteria, the best aimed shooting strategy does not provide an acceptable value of K_{eff} and/or leads to an unacceptable increase in T_{fire}, which makes the transition to the planar mode justified.

The search for the best strategy was performed by searching through a finite set of realistic strategies for distributing shots between guns with different residual service lives of the barrels, taking into account the restrictions on the available types of shots. For each strategy, the values of the K_{eff} and T_{fire} criteria were calculated, after which the strategies were compared according to the two criteria. The results are given in Tables 1–3.

The best strategy is chosen as follows. Let Π be the set of considered strategies. For each $\theta \in \Pi$, $K_{eff}(\theta)$ and $T_{fire}(\theta)$ are calculated. Comparison of strategies can be performed as Pareto-optimal results or using the objective decision-making function, for example, $J(\theta) = w_1\tilde{K}_{eff}(\theta) - w_2\tilde{T}_{fire}(\theta)$, where $w_1, w_2 > 0$ are weighting factors, and $\tilde{K}_{eff}(\theta)$ and $\tilde{T}_{fire}(\theta)$ are normalized indicators.

Table 1 gives the results of calculating a number of typical strategies for an artillery unit with a fixed resource of shots of the 1st category $N_{PHQ} = 10$. For each strategy, the sequence of application of shot types, the expected K_{eff} and T_{fire} values, as well as the sign of the feasibility of the aiming mode according to the adopted comparison rule are given.

Table 1

Typical strategies of an artillery unit with a resource of ammunition of the 1st category $N_{PHQ} = 10$

Str_j	Gun G_1		Gun G_2		Gun G_3		Gun G_4		τ	K_{eff}	T_{fire}
	n_I	n_{II}	n_I	n_{II}	n_{III}	n_{IV}	n_{III}	n_{IV}			
Str_1	10	0	0	3	0	0	0	0	13	0.77	2.5
Str_2	5	0	5	0	0	2	0	2	14	0.71	1.25
Str_3	0	3	0	3	5	0	5	0	16	0.63	1.7
Str_4	2	3	0	5	4	0	4	0	18	0.56	1.33
Str_5	2	3	2	3	3	2	3	0	18	0.56	1.67
Str_6	3	4	3	4	4	0	0	0	18	0.56	1.33
Str_7	2	0	0	0	4	6	4	4	20	0.5	2.0
Str_8	0	5	0	5	5	0	5	0	20	0.5	1.67

According to the obtained K_{eff} and T_{fire} values, it was found that for some strategies in the scenario of a small shortage of resources, the aiming mode does not provide an acceptable ratio of effectiveness to costs and/or leads to an excessive increase in the hit time. Under such conditions, the transition to the planar hit mode is justified by the criteria of efficiency and time.

Additionally, two typical scenarios were considered, namely, with a practically absent shortage of shots of the 1st category ($N_{PHQ} = 15$) and a significantly noticeable shortage of shots ($N_{PHQ} = 8$). The calculation results are given in Tables 2, 3, respectively.

Table 2

Typical strategies of an artillery unit with a resource of shots of the 1st category $N_{PHQ} = 15$

Str_j	Gun G_1		Gun G_2		Gun G_3		Gun G_4		τ	K_{eff}	T_{fire}
	n_I	n_{II}	n_I	n_{II}	n_{III}	n_{IV}	n_{III}	n_{IV}			
Str_1	12	0	0	0	0	0	0	0	12	0.83	3.0
Str_2	6	0	6	0	0	0	0	0	12	0.83	1.5
Str_3	5	0	5	0	3	0	0	0	13	0.77	1.25
Str_4	4	0	4	0	3	0	3	0	14	0.71	1.0
Str_5	4	0	3	0	5	0	4	0	16	0.63	1.33

Table 3

Typical strategies of an artillery unit with a resource of shots of the 1st category $N_{PHQ} = 8$

Str_j	Gun G_1		Gun G_2		Gun G_3		Gn G_4		τ	K_{eff}	T_{fire}
	n_I	n_{II}	n_I	n_{II}	n_{III}	n_{IV}	n_{III}	n_{IV}			
Str_1	8	0	0	5	0	0	0	0	13	0.77	2.0
Str_2	4	0	4	0	0	4	0	4	16	0.63	1.33
Str_3	3	3	3	3	2	0	0	2	16	0.63	1.5
Str_4	4	0	0	5	4	0	0	4	17	0.58	1.33
Str_5	0	5	0	5	4	0	4	0	18	0.56	1.33

Analysis of the results (Table 2) proves that under conditions of practically no shortage of Category 1 ammunition ($N_{PHQ}=15$), the aiming mode can provide high K_{eff} values with an acceptable time of destruction T_{fire} for a certain set of strategies. At the same time, the obtained high indicators of such a scenario are the best since there are practically no resource constraints.

The boundary of feasibility of the aiming mode is defined as the minimum N_{PHQ} value with fixed other parameters, if there is a strategy $\theta \in \Pi$, which simultaneously satisfies two conditions: $K_{eff}(\theta) \geq K_{\min}$ and $T_{fire}(\theta) \leq T_{\max}$.

Analysis of Table 3 reveals that under conditions of a significant shortage of Category 1 ammunition ($N_{PHQ}=8$), the K_{eff} values for the aiming mode of target destruction decrease, and T_{fire} increases for most of the considered strategies. This means that in the considered range of parameters, the aiming mode loses its advantages, and it is advisable to switch to the planar mode of destruction based on the criteria of efficiency and time.

Specific $K_{\min}$ and $T_{\max}$ values are set as scenario requirements and can be selected based on the allowable ratio of "efficiency/cost" and allowable time to complete the fire task. Thus, the boundary of feasibility of the aimed fire mode can be defined as a set of parameters (N_{PHQ}, p_j, $v_{I,II}$, $v_{III,IV}$) at which the best aimed fire strategy according to the comparison rule provides $K_{eff} \geq K_{\min}$ and $T_{fire} \leq T_{\max}$. If the above conditions are not met, the aimed fire mode ceases to be feasible and a transition to the planar attack mode becomes justified.

To complete the comparative analysis, it is necessary to evaluate an alternative mode of application of the artillery unit. Such an alternative is planar fire on a group target, which can provide acceptable performance with less stringent requirements for the quality of shots and, as a result, for the accuracy of hitting, but with different patterns of ammunition consumption and task completion time. That is why, in the next step, it is necessary to formalize a geometric-probabilistic model of target area coverage and implement a numerical evaluation of indicators according to the adopted criteria.

5. 3. Assessing the effectiveness of firing at a group target

The third step in achieving the goal will make it possible to formalize and investigate the planar mode of hitting a group target as a possible alternative to the aiming mode. To do this, it is necessary to estimate the expected cost of shots and the expected time of destruction of the target for planar firing, taking into account two resource constraints. First, the availability of a supply of shots of known quality, and secondly, the existence of a residual serviceable resource of the barrels. The obtained estimates are used further to compare hit modes according to criteria (4) and (5).

The planar mode of target engagement is considered as a generalized model of area coverage without detailing the methods for determining firing positions. The model is used exclusively for comparative analysis of appropriate strategies according to the K_{eff} and T_{fire} criteria.

The assessment of planar firing at a group target in practical applications, as a rule, is based on the calculation of firing elements and a dispersion model for given conditions. Within the framework of this study, computer implementation [27] is considered as a tool for performing model calculations. The starting points for evaluating planar firing are the geometric probability ratios, which will be presented below. This approach to modeling firing elements ensures reproducibility of results and makes it impossible to depend on the interpretation of results from the software implementation used.

The estimation of the required number of shots for planar firing at a group target is performed on the basis of the geometric model of coverage of a given area by scattering and the probabilistic model of shot effectiveness.

A group target at a range D, in the example considered earlier (5. 2) it was proposed that $D=12000$ m, is approximated by a rectangular area of terrain with dimensions L_f along the front and L_d along the depth, then the target area $S_{tar}=L_f \cdot L_d$.

Next, the coverage of the target area by scattering ellipses corresponding to the selected types of shots and groups of states of the remaining serviceable resources of the barrels was considered (Fig. 2).

For each type of shot j, which is a combination of shot quality categories and groups of states of residual serviceable resources of barrels, the desired dispersion of shots on a planar target is modeled by an ellipse with semiaxes a_j and b_j in meters, then the area of the ellipse is $S_{el,j}=\pi \cdot a_j \cdot {}_{bj.}$

In the calculation example, for the given conditions of the range $D=12000$ m, $a \approx 60$ m and $b \approx 40$ m were obtained, which is fully consistent with [27], and hereafter these values will be used as scenario parameters for evaluation.

For planar shooting, it is allowed to shift the aiming points in such a way that the set of dispersion ellipses from several guns provides coverage of a rectangular area of the target. In the first approximation, a geometric estimate of coverage is used as the area of the union of the corresponding ellipses. For the model case, two ellipses from two guns provide close to complete coverage of the target rectangle (Fig. 2).

It is proposed to introduce the geometric component of the shot's effectiveness as the probability of the burst hitting the target area

$$p_{geom,j}=\min\left(1,\frac{S_{\text{cov},j}}{S_{tar}}\right), \tag{6}$$

where $S_{cov,j}$ is the coverage area of a planar target, consisting of the union of the areas of the scattering ellipses for the selected fire configuration.

The probability of effective detonation of shots in the target plane is given as $p_{0,j}=p_j \cdot p_{geom,j}$, where p_j is the conditional probability of shot effectiveness, which generalizes the influence of the quality of the shot and the state of the remaining serviceable resource of the barrel.

To implement a single research idea in order to agree with the efficiency criterion adopted in the previous sections, it is proposed to introduce $K_{eff}^{area} \equiv p_{0,j}$, for the planar mode, that is, the expected number of effective hits in the target area per shot.

If the task requires achieving n_{area} of effective hits in the target area, then the estimate of the expected shot consumption N_j for type j is given by the mathematical expectation

$$M\left(N_j\right) \approx \frac{n_{area}}{p_{0,j}}, \tag{7}$$

a mathematical expectation of the expected execution time, depending on the rate of fire

$$M\left(T_{fire,j}\right) \approx \frac{M\left(N_j\right)}{v_j}, \tag{8}$$

taking into account the number of guns that simultaneously hit the target.

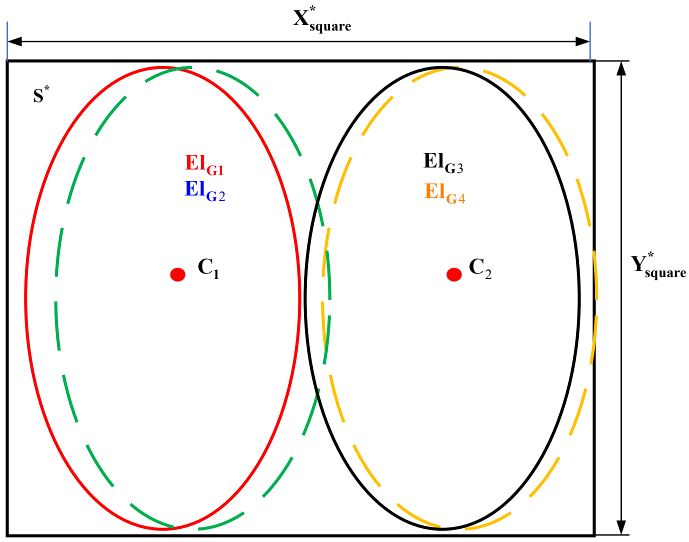

Fig. 2. Covering the area of a group target with shot scattering ellipses

The limited number of shots of a certain quality and a significant deterioration in the remaining serviceable resources of the barrels affect the planar mode of destruction in two ways. First, through a change in the parameters of the expected effectiveness of the shot (p_j), and second, through a change in the dispersion parameters (a_i, b_j), which affect $p_{geom,j}$. Accordingly, in the case of a shortage of high-quality shots and/or with a reduced remaining serviceable resource of the barrels, $M(N_j)$ and $M(T_{fire,j})$ increase, which limits the feasibility of prolonged firing and affects the choice of the destruction mode.

The resulting ratios for $M(N_j)$ and $M(T_{fire,j})$ under the planar mode of destruction of the target provide the possibility of comparing the destruction modes under resource constraints. Such a comparison using the K_{eff} and T_{fire} criteria allows us to determine the values of parameters at which aimed shooting loses its feasibility while the planar mode provides the best compromise between effectiveness, shot costs, and task completion time. Thus, our methodology completes the justification of the transition condition from the aiming to the planar target engagement mode.

6. Research into artillery firing under limited resources: results and summary

Modeling the process of destroying a target by an artillery unit, taking into account different quality ammunition resources and increased barrel wear, allows the commander to adopt a technique for using guns. The impact of the target is affected by the wear of the gun barrels and the quality of the ammunition. The results of the study are given in Table 2.

When choosing a technique for hitting a target, one should not rely on the so-called "commander's intuition" but use the proposed calculation method, the results of which are given in Tables 2, 3. Analysis of the experience of performing combat missions [7, 9, 22, 23] did not reveal models for determining the effectiveness of firing, which take into account the wear of gun barrels and fully control the quality of ammunition. Effective target destruction depends on the quality of the charge and projectile, which are completed in a shot [21] and the wear of the battery's gun barrels [12]. Technical and technological processes that arise to fulfill the conditions for effective target destruction allow guns to be used more effectively. In essence, the proposed strategy is a solution to the problem of finding the best options for performing a combat mission. Such a strategy favorably distinguishes our results from those in [21], where only the quality of shots is taken into account, or from [15–17], in which the wear of gun barrels is taken into account. The obtained best options in Table 1 take into account the quality of charges and shells and the wear of the channels of artillery gun barrels.

It is proposed to consider the wear of the barrel of an artillery gun as quasi-constant during the current task. In the next task, a stepwise increase in barrel wear is assumed. The proposed wear model, in contrast to [18, 19], makes it possible to consider the monotonous continuous nature of the wear property as discrete. This allows for an easy transition from one state to another (Fig. 1). The quality model of poorly formalized indicators of the state of an artillery shot is implemented using a Markov chain. This approach was proposed in [22, 23], but such models did not make it possible to consider different firing strategies.

Using the current shot efficiency coefficient (4) and different configurations of the artillery battery (Table 1), which was based on Markov chains (Fig. 1), made it possible to determine the moment of strategy change (5). In [24], a Markov model of ground artillery firing is proposed, but resource constraints on both the quality of shots and barrel bore wear are not considered.

Our model can use poorly formalized properties of both a single shot and the current battery configuration as input data. This distinguishes the model from [12, 13] and provides a firing strategy for obtaining calculated experimental values (Tables 2, 3, Fig. 2). It is the reduction of the model to the Markov model that made it possible to consider barrel bore wear through discrete-continuous states. The quality of charges and shells is considered through the discrete possibility of an effective shot (4). Different strategies for performing a combat mission are considered through discrete probabilities of battery states. The models proposed in [8, 11] do not lead all components to the same property, therefore they do not make it possible to solve the problem under conditions of limited resources.

The inherent limitations of the proposed model are a consequence of the properties obtained precisely through the use of the Markov chain. The Markov model of the battery state in current stochastic states does not make it possible to take into account the stochastic characteristics of the previous state. Such limitations were present in models [22–24]. When modeling the strategies for using an artillery battery to destroy targets with aimed fire or fire along the plane (Fig. 2), there are certain limitations. In particular, they do not make it possible to start the task from one firing position and finish it at another. Simplifying the model to four possible states (Tables 1–3) for performing a combat mission is illustrative. Taking into account the Markov chain (Fig. 1) in addition, the adopted simplifications demonstrate the principle of modeling. To simulate a real combat situation, the number of states should be determined by conducting additional research.

Further development of the model should be considered in three directions. The first direction is to additionally take into account the logistical component, which has stochastic properties, to determine the current coefficient of the battery's readiness for firing when changing the firing position. This component is due to both possible breakdowns during transportation and fire from the opposing side. It is necessary to take into account the human factor which is associated with both maintenance and transportation of the gun. The second direction is to take into account the results of previous shots, which would allow for more probable states in the simulation. Thus, the model should be supplemented with probability calculations at each stage of target destruction, which will reduce the number of states considered and, as a consequence, the number of calculations. The third direction is to adapt the simulation model for training complexes, simulators and, of course, for use in military computer games. It is this direction that should ensure the realism of the modeling and the possibility of personnel training. Such development of the resulting model will provide the basis for designing artillery fire control systems with limited resources of the artillery unit.

7. Conclusions

1. A Markov model of the process of destroying a single target with a gun under conditions of restrictions on high-quality ammunition and residual service life of barrels has been used. Four types of shots were proposed that characterize the shot category and barrel condition, which made it possible to quantitatively estimate the firing efficiency coefficient K_{eff} and the duration of the combat mission. Our results have made it possible to rank the considered target destruction strategies. The best strategies, based on the use of guns with a satisfactory residual service life of barrels and shots that have undergone quality selection during assembly, provide the value of $K_{eff} \approx 0.63...0.83$.

2. The influence of the shortage of shots, complete with charges and shells that have passed the verification procedure of the full random control on the choice of the aimed fire strategy has been studied; threshold conditions for the feasibility of hitting the target were established. An analysis of the adopted typical strategies for given levels of resource availability was conducted. It was shown that with an increase in the shortage of high-quality shots, some of the strategies quickly approach the critical value $K_{eff} \approx 0.5$. This level means that to achieve a given number of hits, the consumption of shots actually doubles, so it is impractical to use strategies with such an indicator. In this case, the commander must make a decision to switch to another fire mode or stop firing at the current target. The threshold level of effectiveness is a formalized criterion that justifies a change in the strategy of the unit's use.

3. The effectiveness of firing at a planar target has been assessed, taking into account dispersion ellipses and the increased risk of counter-battery damage due to an increase in the firing time. It has been shown that the required number of shots from each gun significantly exceeds the needs of aimed fire on a single target. The duration of such fire increases the risk of detection and destruction of one's own battery. Planar firing with high-quality shots under conditions of resource constraints can be justified mainly for the tasks of destroying strategically important targets.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study, as well as the results reported in this paper.

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Data availability

All data are available, either in numerical or graphical form, in the main text of the manuscript.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

Authors' contributions

Maksym Maksymov: Conceptualization, Methodology, Formal analysis, Investigation, Supervision; **Viktor Boltenkov**: Conceptualization, Methodology, Validation, Investigation, Writing; **Yevhenii Dobrynin**: Conceptualization, Methodology, Validation, Investigation, Writing; **Oleksandr Sidelnykov**: Conceptualization, Methodology, Validation, Investigation, Writing.

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