A belt conveyor has been investigated in this work. The effect of the angle of the belt axis relative to the drum axis on the lateral runout of the belt was considered. The influence of the belt speed, the load at the point of contact of the belt with the generator drum, the curvature of the generator drum, and the linear mass of the belt on the lateral runout of the belt were also examined. The friction coefficient of the belt sliding on the rollers in the running section of the belt on the drum, the belt tension at the point of running on the drum were investigated.

It was established that lateral runout always occurs in the absence of perpendicularity of the belt axis relative to the drum axis. The belt speed during lateral runout is maximum at the beginning of the transition process and decreases as the displacement increases. During lateral runout of the belt in the zone of its contact with the generator drum, tangential and normal loads occur.

The stationary state is achieved when the belt axis becomes perpendicular to the drum axis.

Tangential loads in the absence of slippage do not depend on normal loads and are caused by the non-perpendicular location of the belt axis relative to the drum axis. Tangential load is proportional to the lateral displacement of the belt. The transient process of belt slippage on a drum with a slight curvature of the generator is described by an equation corresponding to an aperiodic link of the first order.

This study make it possible to determine optimal parameters when designing a system for automatic belt centering on a drum with a curved generatrix. Thus, in the development of the end station of a belt conveyor (Patent of Ukraine No. 98378), a hydraulic pump NSH-10E was used. The magnitude of the drum curvature is limited by the resulting unevenness of belt tension across the width, as well as the possibility of belt slippage, and significantly depends on the stiffness of the belt

Keywords: belt conveyor, lateral belt runout, drum, mathematical model, transient process

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DEFINING THE TRANSIENT PROCESS OF BELT LATERAL RUNOUT FROM A CONVEYOR DRUM WITH SLIGHT CURVATURE

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1. Introduction

Transport schemes include loading points, linear part, unloading points. Regarding raw materials in their natural state or allowed for processing, a number of process violations occur at the specified stages of transportation:

a) sticking, hanging on the elements of loading points (overloading), subsequent collapse failures, filling of the point, belt drift (belt drift), overload blockage;

b) belt drift leads to displacement of the material relative to the belt axis, its spillage from the belt along the length of transportation, increased labor intensity of transport maintenance.

Lateral belt runout at the loading point uniquely determines the subsequent spillage from the belt on all transportation routes. This is jamming of the conveyor flight, movement of the lower branch of the belt not along the rollers, material spillage, an increase in its friction coefficient by 8–10 times.

Therefore, it is a relevant task to carry out studies on investigating lateral belt runout at the loading point.

2. Literature review and problem statement

Paper [1] reports the results of studies in which the skew of the tail drum is considered. It is shown that the axis of the drum is not perpendicular to the center line of the conveyor. It is assumed that the rules of the theory of elasticity applied to steel are also valid for conveyor belts. A belt guided by a drum, the axis of rotation of which is not perpendicular to the center line of the conveyor, will deviate in the direction of lower belt tension. Flat conveyor belts moving by a conical drum were also considered. The deformations of the belt were recorded by a camera, as well as by conventional potentiometric distance sensors. However, experimental studies could not give an answer to what dependence describes the transient process of lateral displacement of the belt from the drum during its reversal.

In [2], the results of studies on methods for monitoring lateral displacement are given. It is shown that the first method is vibroacoustic analysis of conveyor pulley bearings. The second was based on the analysis of RGB images from a camera to detect moving edges of the belt. The third solution is monitoring the conveyor tension rollers with a thermal imaging camera. The fourth method is based on measuring the compression force on the tensioner drives. The compression force is directly related to the pressure in the hydraulic system. Measuring the pressure and forces in the hydraulic system is the main method for assessing the state of operation of mechatronic systems. However, experimental studies could not give an answer to what dependence describes the transient process of lateral belt runout from the drum during its reversal.

In [3], recommendations are given for eliminating belt slippage from the drum. All pulley centerlines should be at right angles to the conveyor centerline. If this is not the case, this will lead to uneven movement. The belt tends to "move away" from higher tension. If the tension is increased at one edge, it will move in the other direction. It is recommended to use laser alignment methods to establish the baseline. Alternatively, a tensioned wire can be used on or offset from the conveyor centerline, stretched to form a true reference centerline. However, the paper does not provide answers as to what dependence describes the transient process of lateral belt runout from the drum during its reversal.

In [4], the design and development of a self-aligning tray roller support used in a belt conveyor system are shown, and the results of experimental studies are reported. A modeling approach is devised that can be used to simulate lateral displacement of conveyor belts. This method helps determine the location of the roller support assembly that must be installed to prevent the belt from shifting on the drum. However, the author did not show the effect of the roller support on the drum operation. All this gives grounds to argue that it is advisable to conduct a study on the effect of the roller support on the drum operation.

In [5], the basic rules for tracking a conveyor belt are given, which should be followed. All drums and rollers must be installed at right angles to the axis of the belt movement. If one or both drums are not located exactly at right angles to the axis of the belt movement, the belt will inevitably move towards the less tensioned side. To achieve good tracking, the arc of contact on the guide roller should be at least 30°. The center distance between the end drum and the guide roller should be at least twice the diameter of the larger drum. The tracking effect of the inclined rollers on the reverse side is maximized if they are installed on the run-up side in front of the tail drum. To achieve a satisfactory tracking effect, the contact of the belt with the roller should be approximately ¼ of the belt width, and the angle of inclination of the rollers should be between 5° and 10°. The belt tracking is further improved when the inclined rollers are inclined forward by 8–10° at the edges of the belt in the direction of its movement.

Automatic belt control works by detecting the edges of the belt using non-contact sensors.

Work [6] also provides rules for conveyor belt tracking that should be followed. All drums and rollers should be installed at right angles to the axis of the belt.

In [7], similar recommendations are given as in [5, 6] on belt centering but the dependence describing the transient process of lateral belt departure from the drum is not described. All this gives answers to the dependence describing the transient process of lateral belt runout from the drum.

In work [8], the authors proposed a design of a belt conveyor drive drum where the end sections have helical cuts. The work shows the forces between the helical belts. The normal component of the force is applied to the turn of the cut; the tangential component is applied to the belt and moves the belt to the middle of the drum. However, the work does not give an answer to the dependence describing the transient process of lateral belt runout from the drum.

In [9, 10], a mathematical model of the interaction of the belt with a new drum design was built, which allowed the authors to describe the movement of the belt in the transverse direction, taking into account the action of additional dynamic loads and the restoring force. A method of calculation and determination of rational parameters of a new design of drums is proposed, which makes it possible to determine the design parameters of centering sections. Experimental studies of a belt conveyor with specified technical parameters in production conditions were carried out, which allowed the authors to determine the magnitude of dynamic loads during conveyor acceleration, as well as to optimize the start-up time taking into account these loads. Thus, for stationary conveyors with an increase in acceleration time from 10–15 to 24 seconds, dynamic loads can be reduced from 20–35% to 9–10% of the nominal. A comparative assessment of the experimental and calculated values of the magnitude of dynamic loads gives a discrepancy of up to 4%. Meanwhile, all this gives answers to what dependence describes the transient process of lateral deviation of the belt from the drum. How does the deformation of the belt in the incoming section of the belt affect the drum?

In [11], the results of studies of friction forces acting on the conveyor belt and tension rollers using the finite element method are reported. It is shown that tension forces act on the conveyor belt at the point of passage through the drum, as well as the pressure arising from the rotation of the rollers in the tensioning stations. However, the issue of taking into account the friction force of the conveyor belt on the drums and rollers at a given operating speed of the belt remained unresolved. If these parameters were simulated, it would be possible to obtain more realistic and accurate data about the conveyor belt. All this gives grounds to argue that the proposed modeling method is incomplete. The lateral runout of the belt from the drum, when it is turned, was not considered.

Our review of the literature [1–11] showed that to solve the problem of the transient process of the lateral runout of the belt from the drum, it is necessary to have a transition function. The transition function must take into account the angle of the belt axis relative to the drum, the belt speed, the friction coefficient of the belt sliding on the rollers in the incoming section of the belt, the linear mass of the belt, and the belt tension. The transition function will make it possible to define optimal parameters for designing a system for automatic belt centering on the drum, and to design a final station of the belt conveyor.

3. The aim and objectives of the study

The purpose of our work is to determine the transient process of the lateral runout of the belt from the drum. This will make it possible to design automatic belt centering systems on the drum and avoid unforeseen accidents.

To achieve the goal, the following tasks were set:

– to determine the tangential load of the interaction of the belt with the drum during its lateral runout;

– to determine in the curvilinear coordinate system the lateral speed of belt runout from the drum without taking into account the transverse deformation of the belt running onto the drum;

– to determine, in the Cartesian coordinate system, the lateral speed of belt runout from the drum with a curvilinear generatrix without taking into account the transverse deformation of the belt running onto the drum;

– to determine the speed of the lateral runout of the belt on the drum with a curvilinear generatrix taking into account the transverse deformation of the belt running onto the drum.

4. The study materials and methods

The object of our study is a belt conveyor.

The principal hypothesis assumes that the belt runout is affected by the angle of rotation of the drum relative to the belt axis and the transverse deformation of the belt running onto the drum.

The assumptions adopted in the study imply that the curvature of the drum is not significant.

Experimental studies were performed on an experimental and operating LT-80 conveyor in a mine.

The methodology of the experimental studies was based on measuring such quantities as the angle of rotation of the drum, the magnitude of the longitudinal displacement of the belt on the drum, and the time of belt runout on the drum. The magnitude of the drum deflection angle was changed using a power hydraulic cylinder.

5. Mathematical model of the transient process of lateral runout of a belt from a drum with insignificant curvature

5. 1. Determining the tangential load of interaction between the belt and the drum at its lateral runout

The tension of the belt causes the normal pressure of the belt on the surrounding drum. In order to show that the normal pressure in the absence of slippage does not affect the tangential load of the interaction of the belt with the drum, it is necessary to consider the drum during its reversal.

In order to show the stress and deformations that arise in the zone of contact of the belt with the drum, it is assumed that the load is unevenly distributed along the generatrix of the drum.

It was assumed that the resulting lateral force F_l of some forces provides a uniform translational movement of the belt with a speed V_t directed at an angle to the tangential surface of the drum along the plane xOz (Fig. 1).

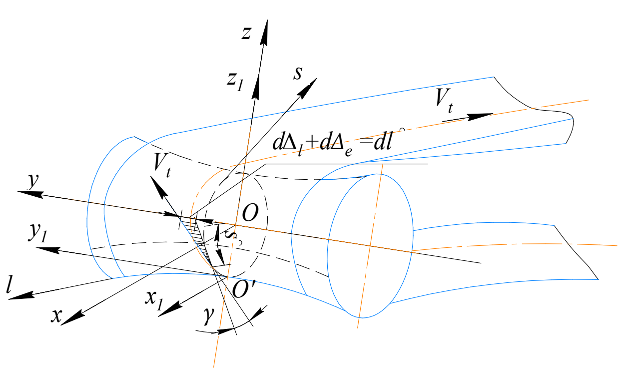

Fig. 1. Principal relations when the belt moves along the drum without slipping: s – moving curvilinear coordinate along the xOz plane, m; l – coordinate along the generatrix of the drum, m

It was also assumed that deformations occur only along the thickness of the contacting lower belt lining and the drum lining within the contact zone. Then the points of the belt belonging to the lower surface, coming into contact with the drum, are shifted along the circumference of the drum and are displaced along its generatrix according to some law $d\Delta_t(l, s)$.

At the same time, the points on the lining surface are also displaced along the generatrix according to the law $d\Delta_e(l, s)$. The relationship between $d\Delta_t$ and $d\Delta_e$ can be found from the condition of equality of tangential stress at the contact boundary of the belt and the drum lining

$$G_{shift.t}\frac{d\Delta_{\tilde{n}}}{\delta_{lining.t}}=G_{shift.e.}\frac{d\Delta_e}{\delta_e},\ \text{N/m}^2, \tag{1}$$

where $G_{shift.t}$ – shear modulus of the belt lining material, N/m^2; $G_{shift.e}$ – shear modulus of the belt lining material, N/m^2; $\delta_{lining.t}$ – thickness of the belt lower lining material, m; δ_e – thickness of the belt lining material, m.

In the absence of belt sliding on the drum, the functions $d\Delta_t$ and $d\Delta_e$ change according to the following law

$$d\Delta_t+d\Delta_e=ds\cdot \text{tg}\gamma,\ \text{m}. \tag{2}$$

The absence of slippage of the belt on the drum is determined by the condition that the tangential voltage at the contact interface does not exceed the specific forces of static friction

$$d\tau_{drum.-t}=G_{shift.t}\frac{d\Delta_t}{\delta l_{ining.t}}(s,l)\le f_0\sigma(s,l),\ \text{N/m}^2, \tag{3}$$

where f_0 – coefficient of static friction; $\sigma(s, l)$ – normal stresses in contact between the drum and the belt, distributed according to a certain law, N/m^2.

From equation (3) it follows that the tangential load of the interaction of the belt with the drum in the absence of slippage does not depend on the normal stress in the contact.

When condition (3) is violated, slippage of the contacting surfaces occurs and the displacement $d\Delta_t$ is determined from the relation

$$d\tau_{drum.-t}=G_{shift.t}\frac{d\Delta_t}{\delta_{lining.t}}(s,l)\le f_0\sigma(s,l),\ \text{N/m}^2. \tag{4}$$

The expression of equation (3) through the shear moduli $G_{shift.t}$, $G_{shift.e}$ and the thickness of the lining $\delta_{lining.t}$, δ_e the thickness of the belt and lining, taking into account equations (1) and (2), leads to the equation

$$d\tau_{drum.-t}=\frac{ds\cdot \text{tg}\gamma}{\frac{\delta_{lining.t}}{G_{shift.t}}+\frac{\delta_e}{G_{shift.e.}}},\ \text{N/m}^2. \tag{5}$$

Taking into account that $dl = ds \cdot \text{tg}\gamma$, and the tangential tension between the drum and the belt arises over the entire width of the belt $\int dl = B_t$, we obtain

$$d\tau_{drum.-t}=\frac{B_t}{\frac{\delta_{lining.t}}{G_{shift.t}}+\frac{\delta_e}{G_{shift.e.}}},\ \text{N/m}^2. \tag{6}$$

Due to the low shear modulus of conveyor belts, it can be assumed that the curvature of the belt axis occurs due to shear deformation of the belt sections parallel to the generatrix of the drum

$$Q_{drum.-t} = -\tau_{drum.-t} \cdot \Delta_l,\ \text{N/m}. \tag{7}$$

In this regard, the tangential load of the interaction of the belt with the drum in the transverse direction in the absence of slippage $Q_{drum.-t}$ is obtained by the proportional displacement Δ_l.

5. 2. Belt displacement in a curvilinear coordinate system without taking into account transverse deformation

Fig. 2 shows a calculation diagram of the movement of the belt on the drum with a generatrix surface in the form of an ellipse, cut off on both sides by the value $L_{drum.}$ / 2, where $L_{drum.}$ – drum length, m.

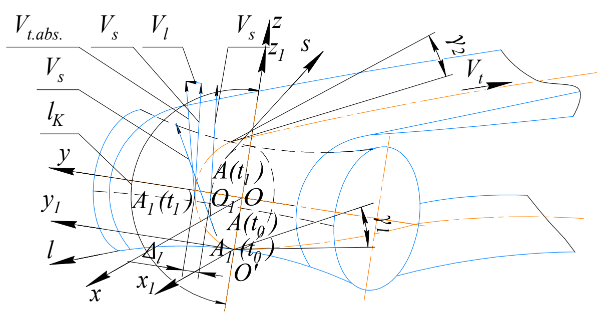

Fig. 2. Calculation scheme for determining the sliding speed along the contact length V_l of the belt with the drums and the speed of the belt leaving the drum $V_{drum.gat.}$

The length of contact of the belt with the drum l_k is taken as the distance between the sections on the surface of the drum lining, limited by the plane zOy.

Point A belongs to the drum lining, point A_1 to the lower belt plate. At the initial time t_0, points A and A_1 are aligned.

When a lateral force occurs or the drum turns relative to the belt axis, the belt is transversely displaced along the drum generatrix.

As a result of transverse sliding, point A_1 is displaced relative to point A by the value Δ_l. At time t_1, the absolute speed of movement of point A_1 $\vec{V}_{t.abc.}$ is equal to the linear speed of movement of point A $\vec{V}_s$ and the transverse speed of belt displacement $\vec{V}_l$ (Fig. 2)

$$V_s = V_t \cos\frac{\gamma_1+\gamma_2}{2} = V_t,\ \text{m/s},$$

as

$$\cos\frac{\gamma_1+\gamma_2}{2} \approx 1,$$

where V_t is the belt speed along the conveyor axis, m/s; γ_1 is the angle of the belt's approach to the drum, degrees; γ_2 is the angle of the belt runout from the drum, degrees

$$\vec{V}_{t.abc.} = \vec{V}_s + \vec{V}_l,\ \text{m/s}.$$

We obtained at given angles of approach γ_1 and runout γ_2 of the belt from a curved drum and its parameters: belt speed along the conveyor axis V, sliding speed along the contact length V_l and speed of runout $V_{drum.gat.}$ of the belt from the drum.

Projection of the difference $\vec{V}_t$ and $\vec{V}_s$ in speed onto the axis O_1l produced the transverse sliding speed. We shall consider the movement of the belt along the drum in the curvilinear coordinate system lO_1s. The shape of the belt axis is obtained by the function $l(s, t)$, determined with an accuracy of an arbitrary constant value.

The transverse sliding speed V_l of point A_1 and the speeds of the entire set of points belonging to the belt, equal to it, are determined by the full derivative of the function $l(s, t)$. Here t serves as an argument (partial from a complex function).

Neglecting longitudinal sliding, we obtained

$$V_l = \frac{dl}{dt} = \frac{\partial l}{\partial t} + \frac{\partial s}{\partial t}\cdot\frac{\partial l}{\partial s},\ \text{m/s},$$

where $V_s = \frac{\partial s}{\partial t} = V_t$ – belt speed along the conveyor axis, m/s;

$$V_l = \frac{\partial l}{\partial t} + V_t \cdot \frac{\partial l}{\partial s},\quad \text{m/s}, \tag{8}$$

The partial derivative with respect to t at fixed s is the speed of the belt leaving the drum

$$V_{trom.gat.} = \frac{\partial l}{\partial t} = V_l + V_{trom.geomet.},\quad \text{m/s}, \tag{9}$$

where $V_{trom.geomet.} = -V_t \frac{\partial l}{\partial s}$ is the geometric component of the runout velocity, due to the non-parallelism of the belt axis and the s axis during the longitudinal movement of the belt, m/s.

During the time t of being in contact, point A_1 is displaced along the l axis relative to point A by an amount

$$\Delta_l = \int_0^t V_l \cdot \mathrm{d}t,\ \text{m}, \tag{10}$$

where $t = s$ / V_t, s; s is the length of the arc from point $A(t_0)$ to point $A(t_1)$, m.

From expression (10) we get

$$\frac{\partial \Delta_l}{\partial s} = \frac{V_l}{V_t}. \tag{11}$$

Substituting expression (11) into (7), we obtain the differential form of the law of tangential force transfer in the contact of elastic bodies with constantly changing contact areas

$$\frac{\partial Q_{drum.-t}}{\partial s} = -\tau_{drum.-t}\frac{V_l}{V_t},\ \text{N/m}^2, \tag{12}$$

At a constant sliding speed along the length of contact, a linear law of change $q^{drum.-t}_{tang.load}(s)$ is obtained from (12).

By the physical nature of the forces of interaction between the belt and the drum are dissipative. The energy accumulated in the belt coating and the drum lining during contact is dissipated after leaving it.

From the equilibrium condition of a belt element of length ds with a slight curvature of the drum, it follows that

$$\frac{\partial P_l}{\partial s} = q^{drum.-t}_{tang.load},\ \text{N/m}, \tag{13}$$

where P_l is the projection of internal forces in the cross section of the belt onto the l axis, N; $q^{drum.-t}_{tang.load}$ – distributed tangential load during the interaction of the belt with the drum along the l axis, N/m.

It has been experimentally established that, neglecting bending, the belt deformation equation can be written in the form

$$\frac{\partial l}{\partial s}=-\frac{P_l}{S_t+G_{shift.t}B_t}, \quad (14)$$

where S_t – belt tension (longitudinal force in the belt), N; B_t – belt width, m; $G_{shift.t}$ – conveyor belt shear modulus (N/m of belt width); for a 2U type belt with five linings, the dynamic shear modulus $G_{shift.t}=(45–55)\cdot10^3$N/m, for a 2K-300 belt with five main linings – $(70–80)\cdot10^3$N/m; the values of static moduli are 1.5–1.8 times lower.

Substituting equation (13) into (14) and performing the transformation, the differential equation of belt deformation is obtained

$$\frac{\partial}{\partial s}\left[\left(S_t+G_{shift.t}B_t\right)\frac{\partial l}{\partial s}\right]=-q^{drum.-t}_{tang.load},\ \text{N/m}. \quad (15)$$

Since the inertial forces are not significant, they can be neglected.

The equation of motion of the belt on the drum is obtained by substituting equation (8) into (11)

$$\frac{\partial \Delta_l}{\partial s}=\frac{1}{V_t}\left(\frac{\partial l}{\partial t}+V_t\frac{\partial l}{\partial s}\right). \quad (16)$$

From (7)

$$\partial Q_{drum.-t}=-\tau_{drum.-t}\cdot\partial\Delta_l,\ \text{N/m}.$$

In turn

$$\partial Q_{drum.-t}=q^{drum.-t}_{tang.load},\ \text{N/m},$$

hence

$$\partial\Delta_l=-\frac{q^{drum.-t}_{tang.load}}{\tau_{trom.-t}},\ \text{m}. \quad (17)$$

Substitution (17) in (16) gives

$$-q^{drum.-t}_{tang.load}=\frac{\tau_{drum.-t}}{V_t}\left(\frac{\partial l}{\partial t}+V_t\frac{\partial l}{\partial s}\right)\partial s,\ \text{N/m}. \quad (18)$$

Substituting (18) into (15) and performing the transformation, the equation of the movement of the belt along the drum is obtained:

$$n^2_{trom.}=\frac{\tau_{drum.-t}}{S_t+G_{shift.t}B_t},\ \text{m}^{-2}, \quad (19)$$

$$\frac{\partial^3 l}{\partial s^3}-n^2_{trom.}\cdot\frac{\partial l}{\partial s}=\frac{n^2_{trom}}{V_t}\cdot\frac{\partial l}{\partial t},\quad \text{m}^{-2}. \quad (20)$$

The essential feature of this equation is that it is not invariant with respect to the reference direction of the current coordinate and in this sense differs from the known equations of mathematical physics. This property follows from the specificity of the forces of interaction of the belt with the drum. The process of transverse motion occurs with energy dissipation.

Two boundary conditions of equation (20) are obtained from the condition of deformation of the extreme sections:

$$\frac{\partial l}{\partial s}(0)=\gamma_1,\ \text{rad},$$

$$\frac{\partial l}{\partial s}(l_K)=\gamma_2,\ \text{rad}, \quad (21)$$

or taking into account(14):

$$\frac{\partial l}{\partial s}(0)=-\frac{P_{over.}}{S_t+G_{shift.t}B_t},\ \text{rad},$$

$$\frac{\partial l}{\partial s}(l_K)=-\frac{P_{run.}}{S_t+G_{shift.t}B_t},\ \text{rad}, \quad (22)$$

where P_{over}, P_{run} – forces in the oncoming and descending sections, N; l_k – length of contact of the belt with the drum along the axis s, m.

The additional third boundary condition is obtained from the differential equation (15), taking into account the following physical fact: the elements of the belt falling on the drum are not affected by tangential forces of interaction, since the forces appear only after the displacement of these elements along the drum:

$$\left.\begin{aligned} &q^{drum.-t}_{tang.load}(0)=0,\ \text{N/m},\\ &\frac{\partial^2 l}{\partial s^2}(0)=0,\ \text{rad}. \end{aligned}\right\} \quad (23)$$

The values of γ_1,$\gamma_2$were considered constant. A partial solution to equation (20) corresponding to a steady process was obtained

$$\frac{\partial l}{\partial s}(s,t)=\frac{\partial l}{\partial s}(s),\ \text{rad}, \quad (24)$$

If we assume that $\frac{\partial l}{\partial s}$ is some function of s(24), then

$$l=\int_{s_o}^{s}\frac{\partial l}{\partial s}\text{d}s+\psi(t)=\varphi(s)+\psi(t),\ \text{m}. \quad (25)$$

where $\psi(t)$ is some arbitrary function of t.

Expression (25) is substituted into equation (20).

The partial derivatives $\frac{\partial l}{\partial s}$ and $\frac{\partial^3 l}{\partial s^3}$ depend only on s, and the entire left-hand side depends only on s.

$\frac{\partial l}{\partial t}$ obtained from (25)

$$\frac{\partial l}{\partial t}=\frac{d\psi}{dt},\ \text{m/s}.$$

If $\frac{\partial l}{\partial t}$ depends on t, then it is impossible to find a solution taking into account the assumption that $\frac{\partial l}{\partial s}$ is a function of s, because the left-hand side of equation (20) depends only on s, and the right-hand side only on t.

Therefore, the solution to equation (15) is possible only if the right-hand side of equation (20) is constant, i.e.

$$\frac{\partial l}{\partial t}=A=\text{const},\ \text{m/s},$$

But from (25)

$$\frac{\partial l}{\partial t}=\frac{d\psi}{dt}=A,\ \text{m/s}. \qquad (26)$$

Therefore, $\psi = At + B$, that is, function $\psi(t)$is not spontaneous, but linear.

Taking into account (24) $\frac{\partial l}{\partial s}=\phi(s)$, then with respect to $\phi(s)$ equation (20) will take the form

$$\phi''_s-n^2_{gat.}\phi=\frac{n^2_{gat.}}{V_t}A,\quad \text{m}^{-2}. \qquad (27)$$

Equation (27) is a second-order inhomogeneous differential equation with constant coefficients.

The general solution of equation (27) consists of the general solution of the homogeneous equation and some particular solution of the inhomogeneous equation

$$\phi=\phi_0+\overline{\phi},\ \text{rad}.$$

A homogeneous equation has the form

$$\phi''_s-n^2_{gat.}\phi=0,\ \text{m}^{-2}.$$

The characteristic equation for this equation will be written in the following form

$$\phi''_s-n^2_{gat.}\phi=0,\ \text{m}^{-2}.$$

Its roots $p_1 = n_{gat.}$, $p_2 = -n_{gat.}$ are real and distinct. Then the general solution of the homogeneous equation has the form

$$\phi_0=C_1\text{sh}n_{gat.}s+C_2\text{ch}n_{gat.}s,\ \text{rad},$$

where C_1 and C_2 are arbitrary constants.

The partial solution is written in the form

$$\overline{\phi}=M,\ \text{rad}, \qquad (28)$$

where M is an unknown quantity.

The expression of equation (27) given equations (28) results in the equation

$$-n^2_{gat.}M=-n^2_{gat.}C=\frac{n^2_{gat.}}{V_t}A,\ \text{m}^{-2}.$$

Hence

$$M=-\frac{A}{V_t},\ \text{rad}, \qquad (29)$$

General solution of an inhomogeneous equation

$$\phi=\phi_0+\overline{\phi}=C_1\text{sh}n_{trom.}s+C_2\text{ch}n_{trom.}s-\frac{A}{V_t}\ ,\ \text{rad}.$$

From the boundary conditions (21), (23) we obtain the unknown constants C_1,C_2 and A:

$$\frac{\partial l}{\partial s}(0)=\phi(0)=\gamma_1,\ \text{rad},$$

$$\frac{\partial l}{\partial s}(l_K)=\phi(l_K)=\gamma_2,\ \text{rad},$$

$$\frac{\partial^2 l}{\partial s^2}(0)=\phi'_s(0)=0,\ \text{rad},$$

$$\phi'_s=C_1n_{gat.}\text{ch}n_{gat.}s-C_2n_{gat.}\text{sh}n_{gat.}s,\ \text{rad},$$

$$\begin{cases}\phi(0)=C_2-\dfrac{A}{V_t}=\gamma_1,\text{rad};\\ \phi(l_K)=C_1\text{sh}n_{gat.}l_K+C_2\text{ch}n_{gat.}l_K-\dfrac{A}{V_t}=\gamma_2,\text{rad};\\ \phi'_s(0)=C_1n_{gat.}=0,\text{rad}.\end{cases}$$

Taking this into account, the following was obtained:

$$\begin{cases}C_1=0,\text{m};\\ C_2=\dfrac{\gamma_2-\gamma_1}{\text{ch}n_{gat.}l_K-1},\text{rad};\\ A=\left(\dfrac{\gamma_2-\gamma_1}{\text{ch}n_{gat.}l_K-1}-\gamma_1\right)V_t,\text{m/s}.\end{cases}$$

Finally, we obtained

$$\phi(s)=\frac{\partial l}{\partial s}=\gamma_1+\frac{\gamma_2-\gamma_1}{\text{ch}n_{gat.}l_K-1}\left(\text{ch}n_{gat.}s-1\right),\quad \text{rad}, \qquad (30)$$

At the same time, the value of the lateral runout of the belt from the drum

$$l_{trom.gat.}=\left(\gamma_1+\frac{\gamma_2-\gamma_1}{\text{ch}n_{gat.}l_K-1}\right)s+ +\frac{\gamma_2-\gamma_1}{\left(\text{ch}n_{gat.}l_K-1\right)}\frac{\text{sh}n_{gat.}s}{n_{gat.}}+At+C_3,\ \text{m}. \qquad (31)$$

From equation (29) the speed of the belt coming off the drum

$$V_{trom.gat.}=\frac{\partial l}{\partial t}=A=V_t\left(\frac{\gamma_2-\gamma_1}{\text{ch}n_{gat.}l_K-1}-\gamma_1\right),\ \text{m/s}. \qquad (32)$$

Substituting the expressions (30) and (32) in (8), the distribution of sliding velocities along the contact length along the s axis is obtained

$$V_l=V_t\left(\frac{\gamma_2-\gamma_1}{\text{ch}n_{gat.}l_K-1}\right)\text{ch}n_{trom.}s,\ \text{m/s}. \qquad (33)$$

Under steady state mode, the slip rate is constant and the same along the entire length of contact and is also proportional to the speed of the belt movement V_t. To analyze the sliding speed and shape of the belt on the drum, we estimate the minimum values of n_{gat}. With a lining thickness of 20 mm, the shear modulus of the drum lining material and the lower belt lining $G_{shift.e} = G_{shift.lining.t} = 250\ \text{N/cm}^2$, the maximum use of the traction force $n_{gat.} = 0.41/\text{cm}$ for the 2U belt with five gaskets and a lining thickness of 3 mm,

$n_{gat.}$ = 0.21/cm for the 2K-300 belt with five main gaskets and a lining thickness of 6 mm.

Fig. 3 shows the $\frac{\partial l}{\partial s}$ distribution plots along the contact length at different lengths of the drum belt girth $l_K = 10$ cm, $l_K = 20$ cm, $l_K = 30$ cm, $l_K = 40$ cm, $l_K = 50$ cm, $\gamma_1 = -\gamma_2 = 0.1$, $n_{gat.} = 0.21$ cm^{-1}.

Fig. 4 shows the $\frac{\partial l}{\partial s}$ distribution plots along the contact length at different lengths of the drum belt girth $l_K = 10$ cm, $l_K = 20$ cm, $l_K = 30$ cm, $l_K = 40$ cm, $l_K = 50$ cm, $\gamma_1 = -\gamma_2 = 0.1$, $n_{gat.} = 0.21$ cm^{-1}.

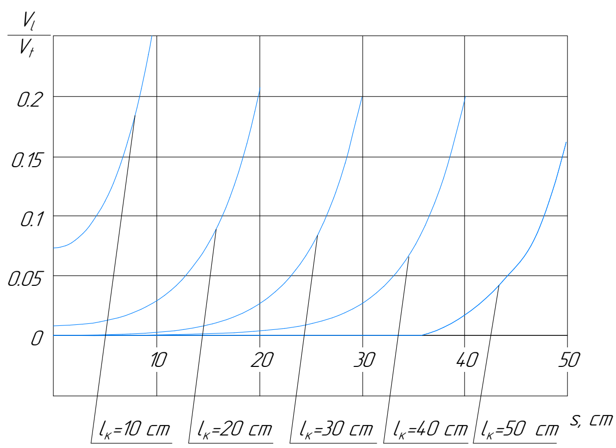

Fig. 3. Distribution plots of V_l / V_t along the contact length

With increasing contact length l_K, the sliding velocity in the approaching section decreases and at $l_K = 50$ cm is practically zero. The largest (in absolute value) V_l value is achieved in the approaching section, where the belt is deformed under the action of the force of the approaching branch.

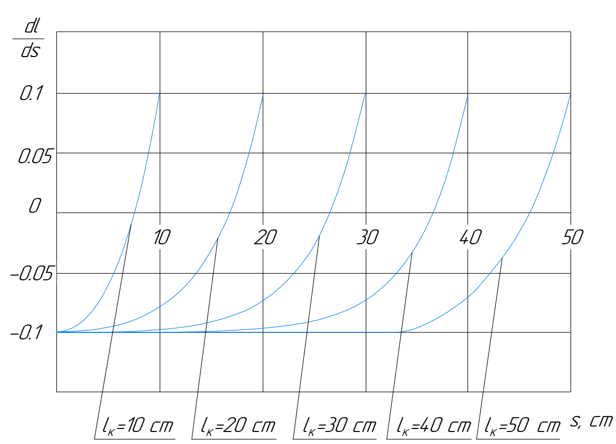

Fig. 4. Distribution plots of ∂l / ∂s along the contact length

The angle of inclination of the tangent to the belt axis at a large contact length retains a constant value over most of the length, changing significantly only around the converging section. At $l_K > (20 \div 30)$ cm, it can be assumed that in the approaching section the sliding speed is zero, and the speed of the belt leaving the drum is

$$V_{trom.gat.} = \frac{\partial l}{\partial t} = -V_t \gamma_1, \text{ m/s}, \quad (34)$$

that is

$$\frac{\partial l}{\partial t} = \frac{dl}{dt}.$$

This dependence persists for most conveyors and is not final.

5. 3. Belt runout in the Cartesian coordinate system without taking into account transverse deformation

The rate of lateral runout of the belt from the drum, which forms the shape of an ellipse cut off at the edges, in the parametric coordinate system

$$V'_{trom.gat.} = \frac{\partial \varphi}{\partial t} = \frac{\partial \varphi}{\partial l} \cdot \frac{\partial l}{\partial t}, \text{ m/s}. \quad (35)$$

The curved coordinate l *is* expressed in terms of angle φ (Fig. 5):

$$\begin{cases} y = a_{ellipse}^{drum.} \sin \varphi, \text{ m}, \\ z = b_{ellipse}^{drum.} \cos \varphi, \text{ m}. \end{cases} \quad (36)$$

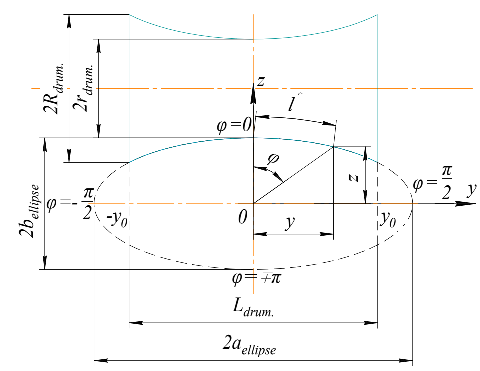

Fig. 5. Calculation scheme for determining the speed of lateral deviation of the belt and the coefficient of curvature of the drum generatrix in the form of an ellipse cut off at the edges

The length of the generatrix of the drum l along coordinate y

$$l = a_{ellipse}^{drum.} \int_0^{\varphi} \sqrt{1 - \left(\varepsilon_{ellipse}^{drum.}\right)^2 \sin^2 \varphi} \cdot d\varphi = a_{ellipse}^{drum.} \cdot E\left(\varphi, \varepsilon_{int}\right), \text{ m}. \quad (37)$$

where $\varepsilon_{ellipse}^{drum.}$ is the eccentricity of the ellipse; $a_{ellipse}^{drum.}$ is half the length of the major axis of the ellipse, m; $E\left(\varphi, \varepsilon_{int}\right)$ is the elliptic integral of the 2[nd] kind in the normal Legendre form.

From equation (37)

$$\frac{d\varphi}{dl} = \frac{1}{a_{ellipse}^{drum.} \sqrt{1 - \left(\varepsilon_{ellipse}^{drum.}\right)^2 \sin^2 \varphi}}, \text{ degree/m}. \quad (38)$$

Taking into account equation (32)

$$\frac{d\varphi}{dt} = \frac{V_t}{a_{ellipse}^{drum.} \sqrt{1 - \left(\varepsilon_{ellipse}^{drum.}\right)^2 \sin^2 \varphi}} \times \left(\frac{\gamma_2 - \gamma_1}{\text{ch} n_{gat.} l_K - 1} - \gamma_1 \right), \text{degree/s}. \quad (39)$$

Velocity of lateral descent of the belt from the drum in the Cartesian coordinate system

$$V''_{trom.gat.}=\frac{\partial y}{\partial t}=\frac{\partial \varphi}{\partial t}\cdot\frac{\partial y}{\partial \varphi},\ \text{m/s.} \quad (40)$$

From equation (36):

$$\frac{\partial y}{\partial \varphi}=a_{ellipse}^{drum.}\cos\varphi,\ \text{m/degree,} \quad (41)$$

$$V''_{trom.gat.}=\frac{\partial y}{\partial t}=\frac{V_t\cos\varphi}{a_{ellipse}^{drum.}\sqrt{1-\left(\varepsilon_{ellipse}^{drum.}\right)^2\sin^2\varphi}}\times$$

$$\times\left(\frac{\gamma_2-\gamma_1}{\text{ch}n_{gat.}l_K-1}-\gamma_1\right),\text{m/s.} \quad (42)$$

Given that $y=a_{ellipse}^{drum.}\sin\varphi$, parameter φ is expressed in terms of y

$$\frac{\cos\varphi}{\sqrt{1-\left(\varepsilon_{ellipse}^{drum.}\right)^2\sin^2\varphi}}=\frac{1}{\sqrt{1-k_{ellipse}^{drum.}+\frac{\left(b_{ellipse}^{drum.}\right)^2}{\left(a_{ellipse}^{drum.}\right)^2-y^2}}},$$

$$k_{ellipse}^{drum.}=1-\left(\varepsilon_{ellipse}^{drum.}\right)^2=\frac{\left(b_{ellipse}^{drum.}\right)^2}{\left(a_{ellipse}^{drum.}\right)^2},$$

where $k_{ellipse}^{drum.}$ – compression coefficient of the ellipse

Hence

$$V''_{trom.gat.}=\frac{\partial y}{\partial t}=\frac{V_t}{\sqrt{1-k_{ellipse}^{drum.}+\frac{\left(b_{ellipse}^{drum.}\right)^2}{\left(a_{ellipse}^{drum.}\right)^2-y^2}}}\times$$

$$\times\left(\frac{\gamma_2-\gamma_1}{\text{ch}n_{gat.}l_K-1}-\gamma_1\right),\text{m/s.} \quad (43)$$

For a practically important case with contact length $l_K>(20\div30)$ cm.

$$V''_{trom.gat.}=\frac{dy}{dt}=-\frac{V_t\cdot\gamma_1}{\sqrt{1-k_{ellipse}^{drum.}+\frac{\left(b_{ellipse}^{drum.}\right)^2}{\left(a_{ellipse}^{drum.}\right)^2-y^2}}},\ \text{m/s.} \quad (44)$$

The curvature coefficient of a drum with a generatrix in the form of an ellipse cut off at the edges $K_{curv.}^{ellipse}$ determines the average value of function

$$\sqrt{1-k_{ellipse}^{drum.}+\frac{\left(b_{ellipse}^{drum.}\right)^2}{\left(a_{ellipse}^{drum.}\right)^2-y^2}}$$

in equation (44)

$$K_{curv.}^{ellipse}\cong\sqrt{1-k_{ellipse}^{drum.}+\frac{\left(b_{ellipse}^{drum.}\right)^2}{\left(a_{ellipse}^{drum.}\right)^2-y^2}}, \quad (45)$$

The average value of function

$$\sqrt{1-k_{ellipse}^{drum.}+\frac{\left(b_{ellipse}^{drum.}\right)^2}{\left(a_{ellipse}^{drum.}\right)^2-y^2}}$$

on the interval $[-y_0; y_0]$ (Fig. 5):

$$y_0=\frac{L_{drum.}}{2},\ \text{m,}$$

$$K_{curv.}^{ellipse}=\frac{2}{L_{drum.}}\int_0^{\frac{L_{drum.}}{2}}\sqrt{1-k_{ellipse}^{drum.}+\frac{\left(b_{ellipse}^{drum.}\right)^2}{\left(a_{ellipse}^{drum.}\right)^2-y^2}}\cdot dy. \quad (46)$$

Taking into account the replacement:

$$y=a_{ellipse}^{drum.}\sin\varphi,\ \text{m},\ y_0=a_{ellipse}^{drum.}\sin\varphi_0,$$

$$\varphi_0=\arcsin\frac{y_0}{a_{ellipse}^{drum.}},\ \text{rad,}$$

$$K_{curv.}^{ellipse}=\frac{a_{ellipse}^{drum.}}{y_0}\int_0^{\varphi_0}\sqrt{1-\left(\varepsilon_{ellipse}^{drum.}\right)^2\sin\varphi}\cdot d\varphi=$$

$$=\frac{a_{ellipse}^{drum.}}{y_0}E\left(\varphi,\varepsilon_{int}\right). \quad (47)$$

For a practically important case with a contact length $l_K>(20\div30)$ cm, the equation of the lateral slip velocity of the belt on the drum, which has the shape of an ellipse cut off at the edges, expressed in the parametric coordinate system through the coefficient of curvature, will take the form

$$V''_{trom.gat.}=\frac{dy}{dt}=-\frac{V_t\cdot\gamma_1}{K_{curv.}^{ellipse}},\ \text{m/s.} \quad (48)$$

Equation of a parabola in polar coordinates (Fig. 6)

$$\rho_{parab.}=\frac{2p_{parab.}\cos\varphi}{1-\cos^2\varphi},\ \text{m,} \quad (49)$$

where $\rho_{parab.}$ is the radius vector with the origin at the focus point of the parabola, taken as the pole, m; $p_{parab.}$ is the parameter of the parabola (distance from the focus to the directrix of the parabola), m.

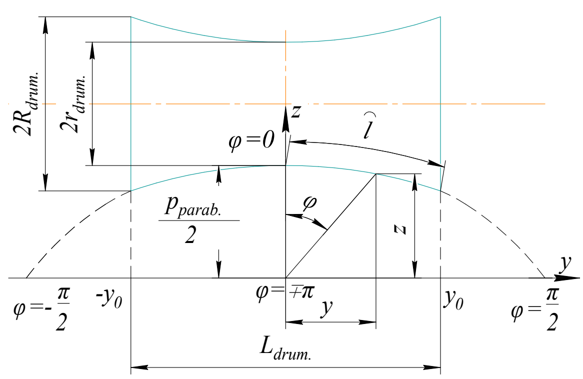

Fig. 6. Calculation scheme for determining the speed of lateral deviation of the belt and the coefficient of curvature of the drum with a generatrix in the form of a parabola

The origin of coordinates is at the focus of the parabola, then the dependence describing the parabola is described by the equation

$$z=-\left(p_{parab.}\cdot y^2+a_{parab.}\right),\ \text{m},$$

where $a_{parab.}=p_{parab.}/2$, m.

The curvilinear coordinate l can be determined through angle φ from the following relationship

$$\begin{cases} y=\rho_{parab.}\cdot\sin\varphi,\text{m}, \\ z=\rho_{parab.}\cdot\cos\varphi,\text{m}. \end{cases} \qquad (50)$$

The expression of equation (50) in terms of equations (49) leads to the equation

$$\begin{cases} y=2p_{parab.}\ \text{ctg}\,\varphi,\text{m}, \\ z=2p_{parab.}\ \text{ctg}^2\,\varphi,\text{m}. \end{cases} \qquad (51)$$

The length of the generatrix of the drum l along coordinate y

$$l=2p_{parab.}\int_0^{\varphi}\cdot\cos ec^2\varphi\sqrt{1+4\text{ctg}^2\varphi}\cdot d\varphi,\ \text{m}. \qquad (52)$$

From equation (52)

$$\frac{\partial\varphi}{\partial l}=\frac{1}{2p_{parab.}\cdot\cos ec^2\varphi\sqrt{1+4\text{ctg}^2\varphi}},\ \text{degree/m}. \qquad (53)$$

Determining the lateral slip velocity of the belt on a drum with a curvilinear generatrix in the form of a parabola in a parametric coordinate system through equations (53) and (34) in (35) leads to the equation

$$V'_{trom.gat.}=\frac{\partial\varphi}{\partial t}=-\frac{V_t\gamma_1}{2p_{parab.}\cos ec^2\varphi\sqrt{1+4\text{ctg}^2\varphi}},\ \text{degree/s}. \qquad (54)$$

Differentiating the equation $y=2p_{parab.}\cdot\text{ctg}\varphi$ with respect to φ, we obtain

$$\frac{\partial y}{\partial\varphi}=2p_{parab.}\cos ec^2\varphi,\ \text{m/degree}. \qquad (55)$$

Substituting dependences (55) and (54) into equation (40) gives

$$\frac{\partial y}{\partial t}=-\frac{V_t\gamma_1}{\sqrt{1+4\text{ctg}^2\varphi}},\ \text{m/s}, \qquad (56)$$

Taking into account that $y=2p_{parab.}\cdot\text{ctg}\varphi$, expressing parameter φ in terms of y, we obtain the lateral slip velocity of the belt on the drum with a curvilinear generatrix in the form of a parabola in the Cartesian coordinate system

$$V''_{trom.gat.}=\frac{\partial y}{\partial t}=\frac{-V_t\gamma_1}{\sqrt{1+y^2/p^2_{parab.}}},\ \text{m/s}, \qquad (57)$$

The curvature coefficient of a drum with a parabolic generatrix $K^{parab.}_{curv.}$ determines the average value of function $\sqrt{1+y^2/p^2_{parab.}}$ in equation (57)

$$K^{parab.}_{curv,}\cong\frac{1}{p_{parab.}}\sqrt{p^2_{parab.}+y^2}. \qquad (58)$$

The average value of function $\frac{1}{p_{parab.}}\sqrt{p^2_{parab.}+y^2}$ on the interval $[-y_0; y_0]$ (Fig. 8)

$$K^{parab.}_{curv,}=\frac{1}{p_{parab.}L_{drum.}}\int_{-\frac{L_d}{2}}^{\frac{L_d}{2}}\sqrt{p^2_{parab.}+y^2}\cdot\text{d}y=$$

$$=\frac{2}{p_{parab.}L_{drum.}}\int_0^{\frac{L_d}{2}}\sqrt{p^2_{parab.}+y^2}\cdot\text{d}y. \qquad (59)$$

After transformations

$$K^{parab.}_{curv,}=\frac{1}{2}\left(\sqrt{1+\frac{L^2_{drum.}}{4p^2_{parab.}}}-1\right)+$$

$$+\frac{1}{L_{drum.}}\ln\left(\frac{L_{drum.}+\sqrt{4p^2_{parab.}+L^2_{drum.}}}{L_{drum.}+2p_{parab.}}\right). \qquad (60)$$

For a practically important case with a contact length $l_K>(20\div 30)$ cm, the equation of the lateral slip velocity of the belt on the drum, which has a generatrix in the form of a parabola, expressed in the parametric coordinate system through the coefficient of curvature, will take the form

$$V''_{trom.gat.}=\frac{dy}{dt}=-\frac{V_t\cdot\gamma_1}{K^{parab.}_{curv,}},\ \text{m/s}. \qquad (61)$$

The speed of lateral descent is not final since it does not take into account the transverse deformation of the belt running onto the drum.

5. 4. The rate of lateral slippage of the belt taking into account the transverse deformation

It has been established that the transient process of slippage of the belt on the drum should be considered taking into account the regularities of its movement on the drum and the regularities of the transverse deformation of the belt running onto the drum [12].

The transverse deformation of the belt running onto the drum depends on the coefficient of friction of the belt sliding on the rollers, the weight, and tension of the belt [12].

Neglecting inertial forces for a flat belt running onto the drum, the equation of the transverse deformation of the belt is written in the following form [12]

$$\theta=\frac{1}{V_t}\cdot\frac{dy}{dt}+\frac{q_t\cdot c_{t.k.}\cdot y\cdot g}{S_t},\ \text{rad}, \qquad (62)$$

where q_t – linear mass of the belt, kg/m; $c_{t.k.}$ – tangent of the angle of inclination of the linear part of the graph of the dependence of the coefficient of friction on sliding, $c_{t.k.}=10\div 30$; S_t – belt tension at the point of contact with the drum, N.

When the drum is turned by angle $\alpha_{turn.}$, the angle between the belt's axis of contact with the drum and the longitudinal axis of the belt along the conveyor takes the value $\theta=\gamma_1+\alpha_{turn.}$ (Fig. 7) [12].

The expression of equation $\theta=\gamma_1+\alpha_{turn}$ by equation (62) leads to the equation

$$\gamma_1=\frac{1}{V_t}\cdot\frac{dy}{dt}+\frac{q_t\cdot c_{t.k.}\cdot y\cdot g}{S_t}-\alpha_{turn},\ \text{rad}. \qquad (63)$$

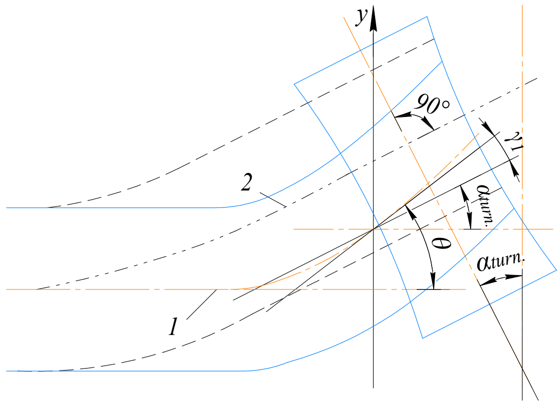

Fig. 7. Diagram of transverse displacement of the belt on the drum: 1 – belt position under the transitional mode; 2 – belt position under the stationary mode

By substituting equation (48) into (63), we obtain a dependence that describes the speed of lateral deviation of the belt on the elliptical drum during its turn, taking into account the transverse deformation of the belt along the conveyor axis, for the practically important case

$$\alpha_{turn}=\frac{1}{V_t}\cdot\frac{dy}{dt}\left(K_{curv.}^{ellipse}+1\right)+\frac{q_t\cdot c_{t.k.}\cdot g}{S_t}y,\ \text{rad.} \tag{64}$$

Dependence describing the speed of lateral deviation of the belt on the drum, with a generatrix in the form of a parabola and taking into account the transverse deformation of the belt along the conveyor axis when the drum turns

$$\alpha_{turn}=\frac{1}{V_t}\cdot\frac{dy}{dt}\left(K_{curv.}^{parab.}+1\right)+\frac{q_t\cdot c_{t.k.}\cdot g}{S_t}y,\mathbb{Z},\ \text{rad.} \tag{65}$$

Equation (65) can be written as:

$$T_0^{el.}\frac{dy}{dt}+y=K_0\alpha_{turn},\ \text{m}, \tag{66}$$

where $T_0^{el.}$ – time constant of the control object (belt), s; K_0 – transmission coefficient by the angle of rotation of the drum, m:

$$T_0^{el.}=\frac{S_t\left(K_{curv.}^{ellipse}+1\right)}{q_t\cdot g\cdot c_{t.k.}\cdot V_t},\ \text{s}, \tag{67}$$

$$K_0=\frac{S_t}{q_t\cdot g\cdot c_{t.k.}},\ \text{m}. \tag{68}$$

Moving on to the operator form of notation, an equation corresponding to the aperiodic link of the first order is obtained

$$\left(T_0^{el.}\lambda+1\right)y=K_0\alpha_{turn},\ \text{m}, \tag{69}$$

where $\lambda = d/dt$ is the differentiation operator.

Equation (67) is written in the form

$$y=u(t)\cdot v(t),\ \text{m}, \tag{70}$$

then $\frac{dy}{dt}=\frac{du}{dt}v+u\frac{dv}{dt}$, m/s.

By substituting these expressions in equation (66)

$$\left(T_0^{el.}\frac{du}{dt}+u\right)v+T_0^{el.}u\frac{dv}{dt}=K_0\alpha_{turn},\ \text{m}. \tag{71}$$

Assuming that

$$T_0^{el.}\frac{du}{dt}+u=0,$$

then

$$\frac{du}{u}=-\frac{dt}{T_0^{el.}}.$$

By integrating, we obtained

$$\ln u=-\frac{t}{T_0^{el.}}.$$

Hence

$$u=e^{-\frac{t}{T_0^{el.}}}. \tag{72}$$

Substituting the found functions in (71), we obtained

$$dv=\frac{K_0\alpha_{turn}}{T_0^{el.}}\cdot e^{\frac{t}{T_0^{el.}}}\cdot dt,\ \text{m/s}. \tag{73}$$

We integrated equation (73) and obtained

$$v=K_0\alpha_{turn}\cdot e^{\frac{t}{t_0^{el.}}}+C,\ \text{m/s}. \tag{74}$$

Substituting (72) and (74) in (70), we obtained

$$y_{gat.}^{el.}=Ce^{-\frac{t}{T_0^{el.}}}+K_0\alpha_{turn},\ \text{m}. \tag{75}$$

At the initial time $t = 0$, the amount of deviation of the belt on the drum $y = 0$. Substituting the boundary conditions into equation (75), we obtain the constant of integration

$$C=-K_0\alpha_{turn},\ \text{m}. \tag{76}$$

By substituting equation (76) into (75), we obtain the value of the belt runout on the elliptical drum in the Cartesian coordinate system, which takes into account the transverse deformation of the belt along the conveyor axis

$$y_{gat.}^{el.}=K_0\alpha_{turn}\left(1-e^{-\frac{t}{T_0^{el.}}}\right),\ \text{m}. \tag{77}$$

Substituting equation (78) into (67), taking into account equations (68) and (69), we obtain the speed of the conveyor belt on the drum with an elliptical generatrix during its turn. The speed of the lateral runout of the belt is obtained for a practically important case with a small coefficient of curvature, when the length of the belt girth of the drum is more than 20 cm

$$V_{trom.gat.}^{el.}=\frac{dy}{dt}=\frac{\alpha_{turn}\cdot V_t\cdot e^{-\frac{t}{T_0^{el.}}}}{K_{curv.}^{ellipse}+1},\ \text{m/s}. \tag{78}$$

Fig. 8–10 show plots of the change in lateral velocity and the amount of belt runout on the drum. The curvilinearity of the generatrix is elliptical.

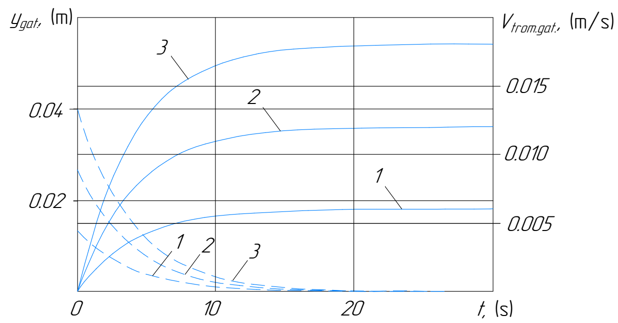

Fig. 8. Plots of change in the speed (solid line) and amount of runout(dashed line) of the belt at $S_t = 400H$, $q_t = 2$ kg/m; $V_t = 0.9$ m/s; $c_{t.k.} = 11.1$; $K^{ellipse}_{curv.} = 1{,}014$, obtained theoretically: 1 – $\alpha_{turn.} = 0.01$; 2 – $\alpha_{turn.} = 0.02$; 3 – $\alpha_{turn.} = 0.03$

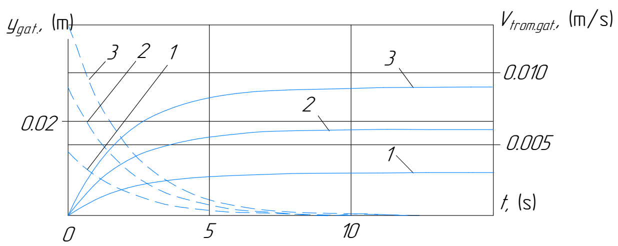

Fig. 9. Plots of change in the speed (solid line) and amount of runout(dashed line) of the belt at $S_t = 200$ N, $q_t = 2$ kg/m, $V_t = 0.9$ m/s, $c_{t.k.} = 11.1$; $K^{ellipse}_{curv.} = 1{,}014$, obtained theoretically: 1 – $\alpha_{turn.} = 0.01$; 2 – $\alpha_{turn.} = 0.02$; 3 – $\alpha_{turn.} = 0.03$

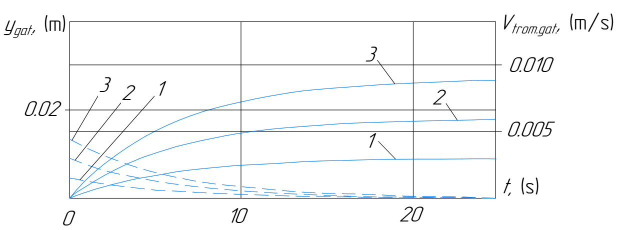

Fig. 10. Plots of change in the speed (solid line) and amount of runout(dashed line) of the belt at $S_t = 200$ N, $q_t = 2$ kg/m, $V_t = 0.3$ m/s; $K^{ellipse}_{curv.} = 1{,}014$, obtained theoretically: 1 – $\alpha_{turn.} = 0.01$; 2 – $\alpha_{turn.} = 0.02$; 3 – $\alpha_{turn.} = 0.03$

With increasing belt tension, belt speed and drum rotation angle, the lateral drift increases.

Experimental studies performed on an experimental (Fig. 11) and on a working conveyor in the tunnel face of Zasyadko mine (Fig. 12) confirm our results.

Tables 1 and 2give the results of calculating the obtained value and the time of the belt runout on a skewed drum with a slight bulge.

The results of the calculations obtained experimentally and theoretically differ by an insignificant amount.

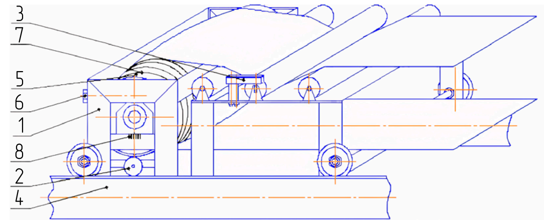

Fig. 11. Structural diagram of the measuring unit for the value and time of belt runout: 1 – trolley frame; 2 – encoder; 3 – force sensor; 4 – stand frame; 5 – drum with a slight bulge; 6 – drum skew screw, 7 – coordinate grid on the drum, 8 – marks on the frame

Table 1

Convergence of theoretical and experimental results for an experimental belt conveyor

Attempt No.	Result obtained experimentally		Result obtained theoretically	
	y, mm	*t*, s	*y*, mm	*t*, s
1	7	24.11	8.8	24
2	28	30.12	26.8	30
3	7	9.99	8.9	10
4	28	12.98	27	13
5	16	47.1	17.6	47
6	50	59.6	53.7	60
7	14	20.88	17.9	21
8	59	23.76	53.9	24
9	10	23.76	8.8	24
10	27	30.1	26.8	30
11	7	10.12	8.9	10
12	27	13.3	27	13
13	16	47.1	17.6	47
14	57	59.89	53.7	60
15	16	20.03	17.9	21
16	50	24.07	53.9	24
17	6	16.71	6.8	17
18	49	21.78	46.7	22
19	25	47.07	25.4	49
20	24	16.69	27	16
21	14	11.0	13.6	12
22	45	29.56	40	29
23	25	23.78	26.9	23
24	25	23.37	26.9	23
25	25	23.21	26.9	24
26	27	23.55	26.9	24

Table 2

Convergence of theoretical and experimental results on a skewed drum with insignificant curvature for a 1LT80 belt conveyor

Angle of drum skew α, rad	Result obtained experimentally		Result obtained theoretically	
	y, mm	*t*, s	*y*, mm	*t*, s
0.008	16	11.68	16.275	11
0.015	30	11.83	30.572	12
0.025	50	11.89	50.953	12
0.031	62	11.92	63.181	12.5

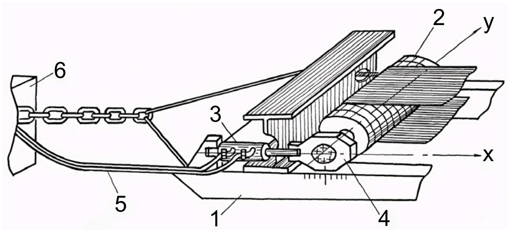

Fig. 12. Diagram of the end station of the belt conveyor 1LT-80 with drum skew adjustment equipment: 1 — end station frame; 2 — drum with applied marks; 3 — power hydraulic cylinder; 4 — sliders; 5 — high-pressure hoses; 6 — tunneling combine

6. Discussion of results of study of a mathematical model of the transient process of lateral belt runout from the drum

The results of our research involve construction of a mathematical model for the transient process of a lateral runout of the belt from the drum with a slight curvature. The influence of the friction coefficient of the belt sliding on the rollers, the belt running mass, the curvature of the generatrix drum, and the belt tension on the lateral runout of the belt from the drum is shown (65). The task addressed is to design an automatic belt centering system on the drum. The dependences established in the study bridge the gap defined [1–11], which is associated with the design of an automatic belt centering system on the drum.

Our studies make it possible to obtain optimal parameters when designing an automatic belt centering system on a drum with a curved generatrix, to produce a belt conveyor terminal station [13].

From an earlier study [12] it was established that with lateral runout of the belt in the zone of its contact with the generatrix drum, tangential and normal loads occur. Tangential loads in the absence of slippage are independent of normal loads and are caused by the non-perpendicular location of the belt axis relative to the drum axis. Due to the low shear modulus of conveyor belts, the curvature of the belt axis occurs due to shear deformation of the belt sections parallel to the drum generatrix. Tangential load is proportional to the lateral displacement of the belt (7).

In a practically important case, in a curvilinear coordinate system, at $l_K > (20–30)$ cm, it can be assumed that in the approaching section, the sliding speed is zero (Fig. 3). The role of transverse sliding in the runout of the belt from the drum is negligible (in contrast to the movement on rollers with a small contact length), the runout is determined only by the non-perpendicularity of the belt axis and the drum axis in the approaching section. The section of the belt on the drum imposes on the belt a connection similar to the ideal non-holonomic connections known in mechanics. This phenomenon is a consequence of the elasticity of belts during shear.

In a Cartesian coordinate system, the speed of lateral runout of the belt from the drum is affected by the curvature coefficient of the drum. Accordingly, the speed of lateral runout is proportional to the belt speed, the angle of incidence of the belt on the drum and inversely proportional to the corresponding curvature coefficient of the drum (61). The magnitude of the drum curvature is limited by the resulting unevenness of the belt tension across the width, as well as the possibility of belt slippage and significantly depends on the stiffness of the belt.

The belt slippage on the drum should be considered taking into account the movement of the belt on the drum and the transverse deformation of the belt running against the drum, namely equation (63). The transient process of belt slippage on the drum should be considered taking into account the regularities of its movement on the drum and the regularities of the transverse deformation of the belt running against the drum [12]. The transient process is described by the equation corresponding to the aperiodic link of the first order (69). It depends on the following parameters: the angle of the drum skew, the speed of the belt, the tension and mass of the belt, the coefficient of friction of the belt sliding on the rollers. With an increase in the belt tension and the angle of rotation of the drum, the magnitude of the lateral slippage increases. As the belt weight increases, the coefficient of friction of the belt sliding on the rollers decreases, and the amount of lateral slip decreases.

In addition, the automatic belt centering system allows one to avoid accidents on a conveyor with a variable length of transportation, during the extension of the conveyor [14].

This research builds on the earlier conducted theoretical study[12]on the design and investigation of a belt stabilization system on the drum at the end station.

The curvature of the drum should be limited. The angle of inclination to the conveyor axis of the forming end sections of the drum did not exceed the angle of friction of the belt on the drum surface [9, 10].

The disadvantage of this study is the average value of the coefficient of friction of the belt sliding on the rollers in the incoming section of the belt on the drum. Determining the coefficient of friction of the belt sliding is possible with additional experimental studies.

The next stage of research implies experimental studies of the transient process of lateral runout of the belt from the drum with slight curvature.

7. Conclusions

1. We have established that the tangential stress between the belt and the rotated drum does not depend on the normal stresses in the contact. The tangential stress along the contact width depends on the width of the belt, the shear modulus, and the thickness of the belt cover material and the drum lining. The tangential load is proportional to the amount of belt displacement on the drum.

2. In a practically important case, with a drum circumference length of $\zeta_K > (20 \div 30)$ cm, the speed of lateral runout of the belt from the drum in a curvilinear coordinate system is proportional to the belt speed and the angle of incidence of the belt on the drum. The transverse deformation of the belt running on the drum was not taken into account.

3. In a Cartesian coordinate system, for a practically important case, the speed of lateral runout of the belt from the drum is proportional to the belt speed, the angle of incidence of the belt on the drum, and inversely proportional to the corresponding coefficient of curvature of the drum. The transverse deformation of the belt running against the drum was not taken into account.

4. The lateral runout, taking into account the transverse deformation of the belt running against the drum, is described by the equation corresponding to the aperiodic link

of the first order. The lateral runout depends on the following parameters: the angle of the drum skew, the speed of the belt, the tension and mass of the belt, the coefficient of friction of the belt sliding on the rollers.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study, as well as the results reported in this paper.

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Data availability

All data are available, either in numerical or graphical form, in the main text of the manuscript.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

Authors' contributions

Alexandr **Gavryukov:** Formal analysis; **Mykhailo Kolesnikov:** Validation; **Andriy Zapryvoda:** Funding attraction**; Oleg Dedov:** Funding attraction.

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