

The processes to plan transport deliveries in logistics systems involving hierarchical models have been investigated in this paper. The task to optimally plan freight delivery for transport and distribution logistics enterprises is associated with problems arising from fluctuations in demand, geographically distributed orders, as well as limited and heterogeneous resources.

The results of this study include the construction of a hierarchical model that allows for multi-level transportation planning, clustering of previously unclassified orders, and adjusting the choice of vehicles based on current conditions. Such adaptive choice of vehicles flexibly takes into account logistical constraints.

The findings indicate a reduction in transportation costs by 11.7% ($p < 0.05$). At the same time, it was found that under conditions of small samples, the stability of cluster solutions is limited and, therefore, additional verification and extended validation are required for their practical implementation.

The novelty of the proposed model is in the application of hierarchical decomposition of the multi-index transportation planning problem with the allocation of the global stage of cluster formation and the local stage of route planning. An approach to order clustering based on a limited sample of the closest applications in time and distance and an algorithm for adaptive selection of vehicles taking into account cost, carrying capacity and urgency of execution have been proposed.

The model built makes it possible to reduce the computational complexity of the problem compared to classical routing models while maintaining the interpretability of solutions at each stage due to transparent clustering rules. The scope of practical use of the results covers transport and logistics companies, delivery services, urban distribution systems, and retail logistics

Keywords: *hierarchical transportation planning, order segmentation, vehicle selection*

DEVELOPMENT OF A HIERARCHICAL TRANSPORTATION PLANNING MODEL WITH LOCAL SEGMENTATION OF ORDERS AND ADAPTIVE VEHICLE SELECTION

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1. Introduction

With the evolution of intelligent transport systems, the relevance of optimal transportation planning tasks is increasing. The growth of order volumes, their uneven spatial and temporal distribution, increased customer requirements, and tight delivery times complicate the organization of freight transportation for urban logistics and transport and logistics companies [1, 2]. Under such conditions, there is a need to apply adaptive and structured approaches to the organization of transport processes, covering a wide range

of optimization tasks that reflect the actual conditions of transportation [3, 4].

The practical implementation of such approaches is based on a multi-level structure of transportation management, which includes a strategic level of order cluster formation, a tactical level of vehicle distribution, and an operational level of dynamic route management [5, 6].

To build clusters corresponding to each vehicle, methods are used that make it possible to reduce the length of routes, reduce time and fuel consumption, as well as increase the efficiency of resource utilization [7]. Adaptive selection of

vehicles based on the criteria of carrying capacity, cost, and availability provides flexible response to changes in transportation conditions, such as transport availability, route congestion, infrastructure restrictions, etc. [8].

From the point of view of mathematical statement, the problem of effective transportation planning in extensive transport networks is one of the most complex, most resource-intensive in the general complex of logistics problems related to the theory of flow process management [6]. It is complicated by a multi-index nature due to the diversity of products (in particular, by volume, weight, deadlines), time constraints (in particular, time windows, route grid, standards) and vehicle fleet restrictions (in terms of capacity, types, availability, costs). The increase in the number of indices exponentially increases the computational complexity [6]. Segmentation of orders (clustering) by time or by their homogeneity and ranking of vehicles by priorities is an effective way to reduce the index space [9].

The complex topology of urban networks, the influence of external factors, and the need for economic feasibility determine the relevance of new approaches to planning logistics processes. Devising optimal transportation management methods aimed at solving local subproblems under modern conditions forms practically applicable models. This expands the capabilities of mathematical methods in transport logistics, allows one to reduce the computational complexity of multi-index problems, which is the main scientific value of the research.

2. Literature review and problem statement

A generalized analysis of clustering algorithms and their practical applications in logistics is reported in [7]. It is shown that modern machine learning methods make it possible to automate the formation of groups of orders based on spatiotemporal similarity. However, questions regarding the adaptability of these algorithms to the variable density of order distribution in an urban environment remain unresolved. The reason for this may be the dependence of most methods (K-means, Agglomerative) on a fixed number of clusters [8, 9], which does not correspond to the dynamics of real logistics networks.

In [10], a modernization of the K-means algorithm for geographic data sets is proposed. It is shown that the traditional K-means is effective for tasks with clear cluster boundaries. However, questions regarding the flexibility of the method when working with unstructured order flows, where it is impossible to determine the optimal value of the number of clusters in advance (before data analysis), remain unresolved. A likely reason is the subjectivity of the choice of the number of clusters, which leads to a loss of stability of solutions when the demand structure changes.

In [11], a hybrid approach to clustering and heuristics for VRP with time windows was applied. It was shown that pre-segmentation of orders reduces the computational time by 15–20%. However, the issues of integrating clustering directly into a multi-index model, where orders are differentiated not only by time, but also by type of cargo and vehicle, remained unresolved. A likely reason is the gap between clustering methods and transport selection criteria.

In [12], a comparative analysis of the K-means, DBSCAN, Agglomerative, Spectral, Birch algorithms for geographic data sets was conducted. It was shown that DBSCAN demonstrates better stability at heterogeneous densities. However, the issues regarding the lack of binding of DBSCAN to the parameters of transport suitability criteria remained unresolved,

which limits its application in logistics. A likely reason is the algorithm's focus only on spatial coordinates without taking into account freight and transport compatibility.

In [13], an overview of clustering of georeferenced time series was provided. It was shown that joint clustering (co-clustering) makes it possible to take into account both space and time. However, the issues regarding the computational complexity with significant amounts of data, which is critical for large logistics networks, remained unresolved. A likely reason is the exponential growth of the number of pairwise comparisons, which makes the method practically inapplicable in real time.

Making a decision on the choice of a vehicle is a multi-factorial task that requires the use of modern specialized mathematical methods that have already been used by the authors, such as FTOPSIS, [14], MARCOS (Measurement Alternatives and Ranking according to COMpromise Solution), CRITIC (Criteria Importance Through Inter-criteria Correlation), BWM (Best-Worst method) [15], an approach using fuzzy models [16, 17]. Such methods have a common drawback – the results strongly depend on the initial data, the method of setting weights, and the subjectivity of expert assessments, which can reduce the objectivity and stability of the ranking of alternatives. At the same time, the complexity of interpreting the results increases, as well as the possibility of their ambiguity in cases of a large number of criteria.

Each mode of transport has advantages and limitations, so the choice of a vehicle is considered as a multi-criteria problem. To solve it, methods are used that make it possible to compare and rank vehicles according to a set of indicators. But it is important to take into account the relationships between the choice of a vehicle and other elements of the logistics system [18]. Therefore, the ranking of vehicles according to the degree of fulfillment of some predetermined relationship was used. This approach will make it possible not only to rank vehicles but also arrange them by order levels.

A global problem in the broad sense does not have the Bellman property, and therefore local optimality does not always lead to global [18]. Even if each order is segmented "ideally" and the best vehicle is selected at each step, it does not guarantee that the entire system will work optimally. Optimal solutions to local problems are only part of the way. Without coordination, they can lead to a suboptimal global plan. An alternative that helps overcome some external factors is the solution to the vehicle routing problem (VRP) and its modifications, which are determined by the taxonomic characteristics of the problem and the imposed constraints [19]. One of the representative characteristics of VRP is its high computational complexity with a large number of customers.

Local problems can be coordinated for a global effect if dynamic programming (combinatorial methods) is applied, which iterate over the options globally, as in [20]. Or use a hierarchical approach [3], where local solutions depend on the global context (for example, all segmentations, vehicle selection will occur within the framework of the general plan) and are integrated into the problem of optimizing transportation of the transport type.

The classical transport problem, which is focused on the transportation of a homogeneous cargo with cost minimization, is complicated under real conditions by multivariate: the presence of different types of cargo and vehicles [21]. Thus, in [22] it is shown that exact methods are effective for problems of small and medium dimensions. The main unresolved issues are related to the scalability of these methods to large problems

with dynamic parameters. A likely reason is objective difficulties associated with the increase in computational complexity with an increase in the number of indices. This problem was partially eliminated in [23] by using software solutions that make it possible to find compromise options for conflicting objective functions. However, issues with the adaptability of algorithms when changing the structure of the logistics network and constraints in real time remain unresolved.

This necessitates the use of modified mathematical models – multidimensional (multi-index) transport problems. Their solution is computationally complex, so a number of approaches have been devised: modifications of classical methods [22, 23], evolutionary algorithms [24], fuzzy and stochastic [25] models, as well as artificial intelligence methods [26]. A good overview of the applications of these approaches is given in [27].

In [24], the results of applying SPEA to solve a multi-objective four-dimensional problem are presented. It is shown that evolutionary algorithms efficiently generate a set of non-dominated solutions. However, the SPEA algorithm for multi-criteria multi-index transportation problems is computationally expensive and sensitive to parameter tuning. Therefore, the diversity and interpretability of Pareto-efficient solutions are often lost. In [25], the results of a review of fuzzy and stochastic extensions of the multi-index transportation problem are presented. However, such approaches are associated with the complexity of integrating these extensions into a single complex model and adaptively responding to changes in uncertainty. Work [26] reports the results from constructing a mathematical model and algorithm for a multi-index transport problem using artificial intelligence. It is shown that AI approaches improve the accuracy of forecasting the problem. However, issues related to the interpretation of results and the adaptability of models to structural changes in the logistics network have been ignored. Also, the methods used in those studies are not evaluated in the context of computational efficiency or comparison with other approaches, which makes it difficult to assess their practical effectiveness. One of the promising areas is the decomposition method [27, 28], which involves dividing the problem into smaller-dimensional subproblems with subsequent aggregation of local solutions into a global one. At the same time, such approaches and methods have limited universality and are difficult to interpret the results.

Our review of the literature reveals the main limitations of modern research on effective freight transportation planning. The first limitation concerns the insufficient adaptability of models to network dynamics. As shown in studies [21, 24], most existing models require retraining or parameter adjustment when the network topology changes. This leads to the impossibility of responding promptly to external factors and reduces the applicability of methods for real-time modeling. The second limitation is computational inefficiency when scaling. The combinatorial nature of the problem leads to the appearance of a large number of possible solution options [13, 19]. As a result, exact methods become unsuitable for operational planning even with an average dimension of the problem. The third limitation is the subjectivity of expert assessments used in modeling. In works [14–16] it is shown that multi-criteria selection methods depend on weights determined by experts, but there are no mechanisms for their objective automatic correction based on real data. This reduces the objectivity of ranking, creates risks of ambiguous decisions based on a large number of criteria. The fourth limitation is associated with the low interpretability of "black boxes". The algorithmic complexity of fuzzy models [24] and

evolutionary algorithms [25] hides cause-and-effect relationships, which makes the openness of the logistics process impossible. This blocks decision-making in critical situations. The fifth limitation is the lack of coordination of local decisions. As proven in [18, 27], even with ideal solutions to subproblems, the global plan may be suboptimal, which reduces the economic effect. Sixth, the universality of decomposition methods is limited. Aggregation of local optima without taking into account the interaction of subproblems, as in [26, 27], leads to the loss of global optimality and complicates the interpretation of results.

All this justifies the feasibility of research aimed at building a universal hierarchical model that integrates order segmentation and adaptive vehicle selection, ensures global optimality, computational efficiency, and interpretability of solutions in dynamic logistics systems without subjective parameter tuning.

3. The aim and objectives of the study

The purpose of our research is to build a hierarchical model of optimal transportation planning, which involves the phased integration of local solutions – order segmentation, vehicle selection – within the framework of a global strategy for minimizing costs and increasing the efficiency of logistics operations. This will make it possible to reduce the computational complexity of the optimization problem and expand the possibilities of applied application of mathematical methods in transport logistics.

To achieve the goal, the following tasks were set:

- to propose algorithms for the coordinated solution of local problems in the context of achieving the global optimum;
- to build a mathematical model of the interaction of order clustering algorithms and vehicle selection at different hierarchical levels;
- to evaluate the effectiveness of the model built, based on the results of a numerical experiment and analysis of a practical case.

4. The study materials and methods

The object of our study is the process of planning freight transportation in logistics systems using hierarchical management models.

The hypothesis of the study was as follows. The use of a hierarchical transportation planning model that integrates local solutions (order segmentation and transport selection) makes it possible to increase the efficiency of the logistics system due to a more rational allocation of resources. It can also make it possible to reduce the computational complexity of the multi-dimensional transportation problem by reducing the number of indices in the optimal transportation planning problem.

The following assumptions were adopted in the study:

- 1) availability of complete and reliable information: it is assumed that at the time of planning all necessary information about the order, geolocation of delivery points, vehicle characteristics, and route restrictions is available;
- 2) stability of parameters during one planning cycle: all input data (available transport, time windows, etc.) do not change during the planning period;
- 3) homogeneity of the transport network: it is assumed that all sections of the transport network are available for movement, and no sudden changes are foreseen;

4) rational behavior of the system: each subsystem decision (clustering, transport choice) is made based on the optimality criterion, without the influence of external random factors.

As part of the study of the hierarchical transportation planning model, the following simplifications were accepted:

1) homogeneity of road conditions. It is assumed that all routes are accessible for travel, without taking into account congestion, repairs, or weather conditions;

2) fixed size and weight of orders. Cargoes are considered as standardized units that do not require complex calculations for placement in a vehicle;

3) complete information at the time of planning. It is assumed that all orders are already known at the time of plan formation, and new requests are not received during routing;

4) one type of delivery – point-to-point – each order is delivered from the loading point without intermediate stops;

5) fixed costs per unit of travel. Transportation costs are considered proportional to the length of the route without taking into account variable factors such as fuel prices or vehicle depreciation.

These simplifications allow us to focus on key aspects of the model (clustering, hierarchical scheduling, adaptive transport selection) but can be removed in further studies to improve the accuracy of results.

The study used the hierarchical decomposition method, which involves dividing the global task of planning freight transportation into interconnected levels: global formation of order clusters and local subtasks of adaptive selection of vehicles with subsequent aggregation of partial solutions. For segmentation of orders, a multilevel clustering algorithm with pairwise comparison of objects without prior ordering of the input set and without binding to fixed criteria parameters was used. Adaptive selection of vehicles was implemented based on the preference ratio taking into account cost, carrying capacity and urgency of order fulfillment. The effectiveness of the proposed model was assessed using empirical methods, in particular, statistical analysis of results, bootstrap method for assessing the stability of cluster solutions, silhouette coefficient for validating the quality of clustering, as well as constraint verification methods for analyzing the stability of the model under conditions of small samples.

The implementation of the experimental part was carried out using the MATLAB R2023b environment (USA) with built-in tools for numerical calculations and matrix processing. The adequacy of the models was checked by expert assessment of the solutions obtained from the model, in comparison with manual planning.

Clustering is carried out by four factors: delivery time, transportation cost, geographical distance, type of cargo (enterprise classification). To detect order conflicts, restriction filtering was applied (transport cannot be in two locations at the same time).

The normative and terminological basis of the study was formed taking into account the current national documents in the field of road transportation, in particular DSTU 2609-94 "Cargo Road Transportation. Terms and Definitions", as well as the requirements of regulatory regulation of the organization of transportation, defined by the Rules for the Transportation of Goods by Road in Ukraine (Order of the Ministry of Transport of Ukraine No. 363 of 14.10.1997). To ensure comparability with international practice, the terminological and procedural provisions were coordinated with the approaches adopted in ISO/EN documents on quality management and logistics systems (ISO 9001:2015, ISO 28000:2007).

5. Results of optimal modeling of the transportation plan with order segmentation and adaptive vehicle selection

5.1. Development of algorithms for the coordinated solution of local problems in the context of achieving the global optimum

5.1.1. Clustering (local segmentation) of orders

The possibility of combining orders was considered if the same vehicles are best suited for their execution.

Let the enterprise have a finite set of orders $X = \{x_1, x_2, \dots, x_n\}$, which claim the same indivisible resource – a vehicle of a certain type for transportation.

When drawing up a transportation schedule, a situation often arises when n orders incompatible in time claim the same vehicle, the time frames (beginning and end of transportation-service) of which are fixed. The dispatcher tries to find a schedule option that will satisfy the maximum number of orders.

For each order fulfillment process (freight delivery) x_i , the following are known: the type of lead, which affects the possibility of recommending road transport, and time limits: the initial limit is a_i , the final limit is b_i , such that $0 < a_i < b_i < \infty$. If the time intervals of two or more processes overlap, then such processes are competing or incompatible. The task is to choose such a subset $X^* \subseteq X$ of mutually compatible processes that has the maximum volume.

To solve such problems of choosing such processes, efficient algorithms are known, including several variants of greedy algorithms [21].

However, the use of these algorithms assumes that the initial set of orders must be ordered by b_i or a_i . In practical logistics problems, such a condition is usually unfulfilled.

Below is a developed algorithm that does not require prior sorting of orders.

The proposed algorithm is based on the following statements:

1) an order $x_i \in X$, compatible with all orders from $X/\{x_i\}$, belongs to the optimal list $X^* \subseteq X$;

2) an order $x_i \in X$, which has the largest number of incompatible orders from $x_i \in X$ cannot belong to $X^* \subseteq X$.

The orders in the initial set X are assigned numbers in natural order, i.e., no ordering of X is performed.

The maximum order number is equal to the number of orders n in the original set X , i.e., $n = |X|$. During the algorithm, when the current set X is reduced, n is also reduced accordingly. Each order is assigned a list (set) M_i containing order numbers that are not compatible with x_i . Initially, all lists are empty. At the first stage of the algorithm, these lists are sequentially replenished, and at the second stage, they are reduced according to statement 2.

The first stage is checking the compatibility of orders.

For the existing orders $X = \{x_1, x_2, \dots, x_n\}$, the compatibility of time intervals is checked by comparing their boundaries. Let for each order x_i the initial time boundary a_i and the final time boundary b_i are known, such that $0 < a_i < b_i < A$. If the time intervals of two or more orders overlap, they are competing or incompatible. By comparing the time boundaries of the orders in pairs (Fig. 1), the competition matrix M is filled.

The row (or column) M_i of matrix M essentially contains a list of those orders that compete with x_i , where

$$M_{ij} = \begin{cases} 1, & \text{if order } x_i \text{ competes with order } x_j; \\ 0, & \text{if order } x_i \text{ doesn't compete with order } x_j. \end{cases}$$

The number of competing orders for order x_i is defined as the sum of the i -th row or i -th column of matrix M . The values of these sums are written as components of the vector S .

```
M=zeros(n,n);
for i=1:n-1
    for j=i+1:n
        if b(i)>a(j) && b(j)>a(i)
            M(i,j)=1;
            M(j,i)=1;
        end
    end
end
```

Fig. 1. Construction of the "competition" matrix

Orders with the largest number of conflicts cannot be included in the transportation plan since their execution will interfere with the execution of other orders. Such orders are removed from the lists of orders requiring execution and form a separate segment. The remaining orders, depending on the execution time, can be divided into segments. For example, long-term orders – with an execution time of $240 < t \leq 360$ minutes, medium-term orders – $120 < t \leq 240$ and short-term orders – $t \leq 120$. The proposed division into segments is illustrative, in reality it may be different, taking into account the wishes and specifics of a particular enterprise.

Orders with the largest number of conflicts are gradually removed from the list. First, the order with number j is removed, for which $j = \arg \max S$. Such actions lead to the modification of matrix M . Removing the order with number j leads to the resetting of the values of the j -th row and j -th column of matrix M to zero. The set N of such indices is remembered and removed from the list of permissible transportations. The removal stops as soon as the vector S is initialized to zero. The implementation of the order removal algorithm is shown in Fig. 2.

The variable COMP will contain the indices of conflicting orders. The variable NOCOMP will contain the indices of non-conflicting orders.

The next stage of local segmentation is to divide non-conflicting orders into segments by execution duration, such as long-term, short-term, medium-term, etc.

For further segmentation of the set of orders, those that satisfy the condition $b_i - a_i > L$, where L is the parameter responsible for the execution duration of the order, are eliminated.

```
S=sum(M)
while norm(S)>0
    [maxVal, maxInd]=max(S);
    M(maxInd, :) =0;
    M(:, maxInd) =0;
    S=sum(M);
    COMP=[COMP, maxInd];
end
NOCOMP=setdiff(N,COMP);
```

Fig. 2. Removing conflicting orders and forming a segment of non-conflicting orders

The output of the algorithm (Fig. 3) is a list I1 of order numbers (indexes) for which $b_i - a_i < L$, using the operator NewInd (NewInd > 0) will make it possible to obtain orders (their indexes) for which the condition $b_i - a_i \geq L$ is satisfied.

To form other segments of orders, an algorithm similar to the algorithm for forming short-term orders is executed the required number of times with a different value of parameter L .

Orders that are included in already formed segments are first removed from the list of orders.

```
NewInd=zeros(1,n);
for i=1:n
    if b(i)-a(i)>L
        NewInd(i)=i;
    end
end
I1=setdiff(1:n,NewInd)
```

Fig. 3. Generating a list of orders that meet the duration criterion

The running time of the order segmentation algorithm is estimated as a function of the power of the input set X . The decisive influence of the initial number of orders n on the running time of the algorithm is manifested in its first part. Here, the algorithm performs pairwise comparisons of orders, and their number directly depends on n . Since the number of combinations of pairs of orders is C_n^2 , the running time of the above algorithm, it is fair to consider it $O(n^2)$.

The running time of known algorithms for solving order problems is estimated as $O(n)$. But such problems are posed for the case of ordering by one of the parameters of the set X (a_i or b_i). If we consider that merge sort is evaluated in time $O(n \cdot \lg n)$, and by the insertion sort – $O(n^2)$, it becomes obvious that the proposed algorithm is not inferior to those described in [28].

The result of the implementation of the developed algorithm is the segmentation of client orders, which ensures their rationality. All orders are valid, combined into clusters, and do not contain conflicting requirements (incompatible time windows).

5.1.2. Algorithm for ranking (selection) of vehicles by order segments

The company's transport fleet is represented as a system

$$S = \{T, R\},$$

where T is a finite set (set of vehicles), on the elements of which the relation R is given – a binary relation of preference (one element is better or worse than another). The task is to form the ordinal structure of the system by grouping elements according to the intensity (degree) of manifestation of the given relation.

The relation R is usually given by a square Boolean matrix of dimensionality $l \times l$, where l is the number of elements of the system. The matrix consists of zeros and ones: one indicates that the relation R is fulfilled between the corresponding elements, and zero indicates that it is not fulfilled. In this case, the i -th row of the matrix corresponds to the element t_i of set T , and the j -th column corresponds to the element t_j from the same set. At the intersection of the i -th row and the j -th column, 1 is written if the element t_i is in the relation R with the element t_j , otherwise 0 is written

$$a_{ij}(R) = \begin{cases} 1, & t_i R t_j, \\ 0, & \text{otherwise.} \end{cases}$$

Our study considers the binary relation R – the preference relation "Vehicle t_i is better than vehicle t_j for performing a given segment of orders", where $t_i, t_j \in T$.

Usually, expert evaluation takes into account the technical characteristics of the vehicle and the specificity of the given segment of orders. In this case, vehicle t_i is considered preferable to t_j if it has better indicators for at least one of the characteristics and is not inferior to the rest.

Based on expert assessments, a matrix A of the relation R is constructed. The ranking of vehicles is carried out according to the following algorithm.

In the first step, a row vector B_1 is constructed, the elements of which are the sums of the elements of the columns of the original matrix A

$$B_1 = \left(\sum_{i=1}^l a_{i1}, \sum_{i=1}^l a_{i2}, \dots, \sum_{i=1}^l a_{il} \right).$$

Zeros in the row vector B_1 determine the elements (vehicles) that are better than all others in a given ratio. These elements form the first ordinal level N_1 .

The next step of transforming matrix A of relation R is to remove rows and columns from the matrix A that correspond to elements that entered level N_1 .

Next, by analogy with the first step, a row vector B_2 is constructed, the elements of which are the sums of the elements of the columns of the transformed matrix A , and we determine the elements that form the second ordinal level N_2 . The algorithm continues until all elements are assigned to certain ordinal levels.

It is worth noting that this ranking algorithm assumes the appearance of new zero components in the row vector at each step of its construction. If there are no zero elements in the first or one of the following rows, this indicates the presence of cyclic dependencies of some elements. To determine the ordinal structure of a system containing cycles, at the first stage, elements connected by cyclic relations are combined into groups – equivalence classes. After that, the ranking algorithm described above is applied not to individual elements, but to the formed equivalence classes. To construct ordinal levels between classes in the original matrix A , all units corresponding to relations between elements of the same class are replaced by zeros.

An example of the implementation of the above algorithm in the MATLAB environment is shown in Fig. 4, where k is the number of vehicles.

```
k=14; % number of vehicles
Ranges={};
remainingIndices=1:k;
while ~isempty(A)
    B=sum(A);
    minVal = min(B);
    indices = find(B == minVal);
    Ranges{end+1} = remainingIndices(indices);
    A(indices, :) = [];
    A(:, indices)=[];
    remainingIndices(indices)=[];
end
```

Fig. 4. Algorithm for ranking vehicles by order segments

The result of the above algorithm is the determination of the types of vehicles that are optimal for servicing the corresponding segments of orders. Adaptive vehicle selection is carried out from a set of available options that do not have a rigidly fixed purpose, which provides the possibility of flexible transport assignment.

5.1.3. Methods for optimizing the global objective functionality

After local segmentation of orders and preliminary ranking of vehicles, the following cases of forming a transportation plan are possible, which lead to completely different types of optimization problems:

- 1) there are clusters of orders for which there is a single vehicle. In this case, within the cluster, the optimal transportation plan is formed according to the elements of the cluster. That is, an optimal routing problem is formed;
- 2) there are clusters of orders for which there is a choice of vehicle types. In this case, a multi-index problem of forming an optimal transportation plan is formed;
- 3) there are clusters of orders for which there is a choice of vehicles within the same type (with fixed transportation tariffs).

In the first case, the vehicle routing problem (VRP) is a classical NP-hard combinatorial optimization problem, the goal of which is to determine the best routes for a fleet of vehicles. This takes into account the given constraints (carrying capacity, time windows, number of transport units, etc.). There are a number of methods for solving it, which are conventionally divided into four groups:

- 1) Exact Methods (branch-and-bound, cutting planes, dynamic programming, branch-and-cut/price) guarantee the optimum but grow exponentially with the size of the problem.
- 2) Classical heuristics (Clarke-Wright, nearest neighbor, greedy insertion) quickly build one route but without a guarantee of quality.
- 3) Metaheuristics (GA, SA, TS, ACO, PSO, BCO, local-search neighborhoods) are universal and give high-quality solutions with a large number of vertices, require fine-tuning of parameters.
- 4) Hybrid schemes combine clustering, machine learning, or exact subproblems with metaheuristics, reducing the computational load and increasing adaptability to real constraints.

In the second case, the delivery of different types of orders to the consumer network by different modes of transport leads to the formation of a multi-index transportation problem model. This is a generalization of the classical transportation problem, in which the transportation of goods occurs between suppliers and consumers taking into account a larger number of factors or levels of indexation (i.e., more than two indices in the problem).

Clustering (local segmentation) of orders and local ranking of vehicles is an effective approach to reducing the number of indices in the optimal transportation planning problem.

Since the problem takes into account only the points of departure and destination, the type of vehicle by order segment (cargo category or time intervals), it is proposed to apply a three-index model for more accurate and comprehensive transportation planning.

The task is to find such a transportation plan $X = (x_{ijk})$, which minimizes the total costs (1), ensures the full removal of cargo from suppliers (satisfies condition (2)) and satisfies the demand of consumers according to their orders (satisfies condition (3)), while the volume of transportation by each type of vehicle does not exceed its carrying capacity, i.e., satisfies condition (4):

$$\sum_{i=1}^m \sum_{j=1}^q \sum_{k=1}^p c_{ijk} \cdot x_{ijk} \rightarrow \min; \tag{1}$$

$$\sum_{j=1}^q \sum_{k=1}^p x_{ijk} = a_i^*, \quad i = \overline{1, m}; \tag{2}$$

$$\sum_{i=1}^m \sum_{k=1}^p x_{ijk} = b_j^*, \quad j = \overline{1, q}, \quad (3)$$

$$\sum_{i=1}^m \sum_{j=1}^q x_{ijk} \leq d_k^*, \quad k = \overline{1, p}, \quad (4)$$

where x_{ijk} is the amount of cargo planned to be transported from the i -th supplier to the j -th consumer by the k -th type of vehicle; c_{ijk} – the cost of transporting a unit of cargo from the i -th supplier to the j -th consumer by the k -th type of vehicle, $i = 1, 2, \dots, m; j = 1, 2, \dots, q; k = 1, 2, \dots, p; a_i^*, b_j^*, d_k^*$ – quantitative characteristics of a certain segment of orders.

The condition of non-negativity of each element of the transportation plan is added to these conditions

$$x_{ijk} \geq 0, \quad i = \overline{1, m}, \quad j = \overline{1, q}, \quad k = \overline{1, p}. \quad (5)$$

Provided that constraints (4) take the form of equalities (triplanar problem), and the balance condition (6) is fulfilled, which is characteristic of the transportation planning problem under consideration

$$\sum_{i=1}^m a_i^* = \sum_{j=1}^q b_j^* = \sum_{k=1}^p d_k^*. \quad (6)$$

The three-index problem (1)–(5) can be decomposed into two independent two-index problems [27]. Each of these problems is solved by the potential method.

In the third case, the main goal of optimization is to assign a vehicle (agents) to fulfill an order (tasks) in such a way as to minimize total costs or maximize efficiency. In this case, the conditions are met: each task is performed by only one agent, and each agent performs only one task. These constraints form a standard two-index model of linear integer programming – the Assignment Problem. In this problem, the powers of all agents and tasks are equal to unity, therefore, in an admissible integer solution, the values of the variables are binary.

In this case, the method for optimal assignment of vehicles to trips is stated as an assignment problem. A motor transport company has a fleet of n vehicles of different brands. The vehicles have different carrying capacities $q_i(t)$ and specific operating costs c_i (\$/km). The enterprise received m orders for the transportation of goods. Moreover, each order specifies the volume of transported goods $Q_j(t)$ and the transportation distance L_j (km). Find the optimal assignment of vehicles for trips to fulfill customer orders, assuming the transportation tariffs are the same.

This problem satisfies the requirements considered above:

1) since the tariffs are the same, then the operating costs should be chosen as the objective function. These costs should be minimized by optimally distributing cars according to orders;

2) since in the general case $n \neq m$, then the problem should be balanced by introducing fictitious orders or fictitious vehicles. The following cases are possible:

a) when $n > m$, there are fewer orders than vehicles (excess of transportation capacity). In this case, $n - m$ fictitious orders with zero volumes are additionally introduced (i.e., for which $Q_j = 0$ and $L_j = 0$). Since fictitious orders are zero, the most inefficient vehicles in terms of costs will be assigned to fulfill them. In practice, fulfilling an order of a fictitious customer means reserving a vehicle (the vehicle remains in the fleet);

b) when $n < m$, there are more orders than vehicles (lack of transportation capacity). In this case, $m - n$ fictitious vehicles with infinitely large specific costs (i.e., for which $c_i \rightarrow \infty$)

are additionally introduced. In practice, this means rejecting the most unprofitable orders in terms of costs;

3) finally, a balanced problem is obtained, which is described by a square matrix of operating costs of dimensionality $k \times k$, where $k = \max\{m, n\}$.

The mathematical model of this problem takes the following form.

Objective function

$$F = \sum_{i=1}^k \sum_{j=1}^k S_{ij} \cdot x_{ij} \rightarrow \min, \quad (7)$$

under constraints

$$\sum_{i=1}^k x_{ij} = 1; \quad \sum_{j=1}^k x_{ij} = 1; \quad x_{ij} \geq 0, \quad (8)$$

where x_{ij} are the targets for all $i = \overline{1, k}, j = \overline{1, k}$.

x_{ij} is an additional integer variable of logical type that takes on the values

$$x_{ij} = \begin{cases} 1, & \text{if the } i\text{-th vehicle is assigned to the } j\text{-th order,} \\ 0, & \text{otherwise,} \end{cases}$$

S_{ij} – operating costs, calculated from the following formula

$$S_{ij} = R_{ij} \cdot c_i = z_{ij} \cdot L_j \cdot c_i,$$

where z_{ij} – the number of trips of the i -th vehicle for the j -th order – is calculated from the following formula

$$z_{ij} = \frac{Q_j}{q_i}, \quad \text{for all } i = \overline{1, k}, \quad j = \overline{1, k},$$

$R_{ij} = z_{ij} \cdot L_j$ – mileage of the i -th vehicle for the j -th order.

The resulting dependences provide the possibility of finding the global minimum of the cost functional under given resource and technological constraints, which is the basis for forming an optimal transportation plan in multi-level logistics systems.

5.2. Hierarchical structure of the transportation planning task

1. Input data analysis:

- set of orders: addresses, volume, weight, deadlines;
- fleet of vehicles: capacity, types, availability, costs;
- constraints: time windows, route grid, standards.

2. Order clustering (local segmentation). Goal: to reduce the complexity of the task by grouping orders by criteria:

- geographical proximity;
- delivery times;
- cargo type.

This stage makes it possible to break the global task into local subtasks.

3. Construction of routes in each cluster.

Local optimal route problem (VRP).

4. Selection and purpose of the vehicle. At this stage, the following are taken into account:

- availability and parameters of the vehicle;
- cost minimization (fuel, mileage, depreciation);
- technical compliance (type, dimensions, etc.).

This stage makes it possible to reduce the dimensionality of the optimal planning problem.

5. Global verification and correction. At this stage, the following is checked:

- transport load;
- exceeding limits (time, mileage);
- possible redistribution between clusters: transfer of orders; replacement of routes.

6. Final optimization of the entire plan – optimization of the global objective functionality:

- total cost of transportation;
- delivery time;
- number of transport units, etc.

7. Plan output and monitoring:

- construction of a transportation schedule;
- formation of route sheets;
- monitoring of execution (via GPS/IoT systems).

5.3. Qualitative verification of the performance of the model built, based on a practical case

A test case was considered on the data (partially modified) from a real motor transport enterprise that carries out urban and intercity freight transportation. Input data: 14,800 orders per year, the fleet of vehicles – 14 vehicles, differing in characteristics: load capacity, cross-country ability, speed, body type, etc., the enterprise served six types of orders. Problem: 38% of the mileage – without cargo, 14,060 orders (95%) – "competitive" in terms of time orders that destroy the usual route loops.

A practical case was simulated to show how the model works on specific data and under specific conditions:

Stage 1. Analysis of input data.

On a conditional day, the enterprise received 50 orders x_i , $i = 1, \dots, 50$, for which the type of load, which affects the possibility of recommending road transport, and the time limits are known (Table 1).

Table 1

Problem input data

| | | | | | | | | | | |
|-------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| n | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| a_i | 144 | 19 | 308 | 289 | 168 | 168 | 117 | 298 | 127 | 40 |
| b_i | 349 | 206 | 439 | 322 | 191 | 508 | 474 | 307 | 337 | 318 |
| Order | 6 | 4 | 4 | 2 | 3 | 4 | 5 | 3 | 3 | 6 |
| n | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| a_i | 114 | 134 | 84 | 127 | 35 | 47 | 124 | 94 | 125 | 23 |
| b_i | 266 | 165 | 256 | 139 | 457 | 107 | 321 | 326 | 197 | 237 |
| Order | 1 | 6 | 6 | 5 | 1 | 2 | 3 | 5 | 1 | 5 |
| n | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| a_i | 82 | 122 | 234 | 8 | 17 | 60 | 222 | 62 | 141 | 155 |
| b_i | 170 | 499 | 280 | 102 | 489 | 530 | 239 | 250 | 221 | 222 |
| Order | 1 | 4 | 3 | 5 | 5 | 6 | 6 | 3 | 5 | 2 |
| n | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| a_i | 187 | 103 | 254 | 66 | 122 | 65 | 127 | 174 | 231 | 30 |
| b_i | 323 | 144 | 327 | 385 | 235 | 66 | 162 | 214 | 266 | 276 |
| Order | 1 | 5 | 4 | 3 | 6 | 4 | 4 | 6 | 5 | 4 |
| n | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| a_i | 49 | 144 | 167 | 18 | 154 | 106 | 174 | 175 | 130 | 249 |
| b_i | 317 | 265 | 433 | 150 | 502 | 395 | 265 | 314 | 279 | 271 |
| Order | 2 | 2 | 6 | 1 | 3 | 2 | 6 | 5 | 4 | 3 |

Stage 2. Order clustering (local segmentation).

A subset of mutually compatible processes is selected, which has the maximum volume. For this purpose, the algorithm is applied (Fig. 2). The variable COMP contains the indices of conflicting orders. The variable NOCOMP contains the indices of "non-conflicting" orders. The results of the execution are given in Table 2.

Table 2

Example of implementing the stage of the developed order segmentation algorithm: values of variables COMP and NOCOMP

| Conflict order indices (COMP variable value) | | | | | | | | | | | | | | | | |
|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|--|----|
| 1 | 2 | 4 | 5 | 6 | 7 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | | |
| 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 28 | 29 | 30 | 31 | 32 | | | |
| 33 | 34 | 35 | 39 | 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | | | |
| Non-conflicting order indices (NOCOMP variable value) | | | | | | | | | | | | | | | | |
| 3 | 8 | | | 27 | | | 36 | | 37 | | | 38 | | | | 50 |

Stage 3. Building routes in order clusters (local route planning). Orders that do not conflict with others in terms of time constraints and execution conditions (plural NOCOMP) can be processed immediately without additional segmentation. For such orders, the local route planning stage is implemented, which consists in forming rational transportation routes taking into account the spatial location of loading and unloading points, as well as basic operational limitations of vehicles.

Conflicting orders require further segmentation. The next step is to divide orders into groups by execution time: long-term (execution time more than 360 minutes), medium-term (execution time from 120 to 360 minutes), and short (execution time does not exceed 120 minutes). The division by execution time that we proposed is conditional and can implement a different scale. Using the proposed segmentation and algorithm in Fig. 5 to conflicting orders, the following segments were obtained: long-term, medium-term, short-term (Table 3).

```

NewInd = zeros(size(COMP));
for k = 1:length(COMP)
    i = COMP(k);
    if b(i)-a(i) > 360
        NewInd(k) = i;
    end
end
long = NewInd(NewInd > 0);
long=sort(long)
I1 = setdiff(COMP,long);
NewInd = zeros(size(I1));
for k = 1:length(I1)
    i =I1(k);
    if b(i)-a(i)>120 && b(i)-a(i)<=240
        NewInd(k) = i;
    end
end
middle=NewInd(NewInd > 0);
middle=sort(middle)
short = setdiff(I1,middle)
    
```

Fig. 5. Algorithm for segmentation of conflicting orders by duration

Table 3

Example of implementing an order segmentation algorithm: clusters of "conflicting" orders

| Long-term order indices | | | | | | | | | | | | | |
|------------------------------|----|----|----|----|----|----|----|----|----|----|----|----|----|
| 15 | | | 25 | | | 26 | | | 22 | | | | |
| Medium-length order indices | | | | | | | | | | | | | |
| 18 | 9 | 17 | 13 | 28 | 1 | 20 | 11 | 49 | 2 | 42 | 31 | 48 | 44 |
| Indices of short-term orders | | | | | | | | | | | | | |
| 4 | 5 | 6 | 7 | 10 | 12 | 14 | 16 | 19 | 21 | 23 | 24 | 29 | |
| 30 | 32 | 33 | 34 | 35 | 39 | 40 | 41 | 43 | 45 | 46 | 47 | | |

Stage 4. Selection and assignment of a vehicle based on the developed vehicle ranking algorithm for each typical order.

At this stage, the suitability of each vehicle of the enterprise for the execution of individual types of orders is assessed by experts and based on these assessments, the "vehicle - order type" preference matrices are constructed. The vehicle ranking algorithm is applied to the resulting matrices (Fig. 4).

Table 4 gives an example of a preference matrix for 1 type of order. Below is an example of the execution of the specified algorithm for this matrix.

Table 4

Benefits matrix for order of type 1

| Vehicle | t_1 | t_2 | t_3 | t_4 | t_5 | t_6 | t_7 | t_8 | t_9 | t_{10} | t_{11} | t_{12} | t_{13} | t_{14} |
|----------|-------|-------|-------|-------|-------|-------|-------|-------|-------|----------|----------|----------|----------|----------|
| t_1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| t_2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| t_3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| t_4 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| t_5 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| t_6 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 |
| t_7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| t_8 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| t_9 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| t_{10} | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| t_{11} | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| t_{12} | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| t_{13} | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| t_{14} | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

According to the algorithm, the row vector B_1 is determined (Table 5).

Vehicles t_5 and t_8 are the best for fulfilling the type of orders under consideration. Therefore, they form the first ordinal level N_1 : $N_1 = \{t_5; t_8\}$.

After the transformation, the matrix takes the form given in Table 6.

The row vector B_2 is determined (Table 7).

Table 5

Result of the first step of the vehicle ranking algorithm

| Vehicle | t_1 | t_2 | t_3 | t_4 | t_5 | t_6 | t_7 | t_8 | t_9 | t_{10} | t_{11} | t_{12} | t_{13} | t_{14} |
|---------|-------|-------|-------|-------|-------|-------|-------|-------|-------|----------|----------|----------|----------|----------|
| B_1 | 2 | 1 | 5 | 2 | 0 | 1 | 3 | 0 | 1 | 1 | 2 | 2 | 3 | 1 |

Table 6

Transformed benefits matrix

| Vehicle | t_1 | t_2 | t_3 | t_4 | t_6 | t_7 | t_9 | t_{10} | t_{11} | t_{12} | t_{13} | t_{14} |
|----------|-------|-------|-------|-------|-------|-------|-------|----------|----------|----------|----------|----------|
| t_1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| t_2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| t_3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| t_4 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| t_6 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 |
| t_7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| t_9 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| t_{10} | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| t_{11} | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| t_{12} | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| t_{13} | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| t_{14} | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |

Table 7

Result of the second step of the vehicle ranking algorithm

| Vehicle | t_1 | t_2 | t_3 | t_4 | t_6 | t_7 | t_9 | t_{10} | t_{11} | t_{12} | t_{13} | t_{14} |
|---------|-------|-------|-------|-------|-------|-------|-------|----------|----------|----------|----------|----------|
| B_2 | 2 | 0 | 4 | 1 | 0 | 2 | 0 | 0 | 2 | 2 | 3 | 1 |

Therefore, the second ordinal level is formed by vehicles t_2, t_6, t_9 i t_{10} . That is, $N_2 = \{t_2; t_6; t_9; t_{10}\}$.

Repeating the steps of the algorithm, the following ordinal levels are determined

$$N_3 = \{t_1; t_4; t_{12}; t_{14}\}, N_4 = \{t_7; t_{13}\}, N_5 = \{t_{11}\}, N_6 = \{t_3\}.$$

Thus, the system for type 1 orders is divided into 6 ordinal levels. Vehicles of level $N_1 = \{t_5; t_8\}$ are the best according to the ratio R , that is, they are best suited for performing cargo transportation for type 1 orders.

After considering all typical orders received by the enterprise, the ranking of the enterprise's vehicles for each type of order was obtained (Table 8). This makes it possible to reduce the number of vehicle types that will be considered further when building an optimal transportation plan using optimization problems.

Table 8

Result of ranking vehicles by typical orders

| Level | Order type | | | | | |
|-------|----------------|-------------|-------------------|-----------------------|-----------------|----------------|
| | 1 | 2 | 3 | 4 | 5 | 6 |
| N_1 | [5, 8] | [5, 8] | 5 | 5 | [2, 5, 6, 8, 9] | [2, 5, 9, 1 4] |
| N_2 | [2, 6, 9, 10] | [2, 6, 9] | [2, 8, 9, 10, 14] | [2, 9, 14] | 14 | 13 |
| N_3 | [1, 4, 12, 14] | [10, 14] | [1, 6] | 10 | [10, 11, 12] | [6, 8, 10] |
| N_4 | [7, 13] | [1, 7, 12] | 12 | 1 | [1, 7, 13] | 12 |
| N_5 | 11 | [4, 11, 13] | 13 | [6, 7, 8, 11, 12, 13] | [3, 4] | [1, 4, 7, 1 1] |
| N_6 | 3 | 3 | [7, 11] | [3, 4] | - | 3 |
| N_7 | - | - | [3, 4] | - | - | - |

Stage 5 of the hierarchical transportation planning algorithm – global verification and correction – ensures the coordination of local solutions with the general constraints and goals of the system, eliminating conflicts between previous planning levels.

Stage 6. Final optimization of the entire plan.

The construction of the optimal transportation plan is divided into separate optimization tasks (triplanar transport tasks or destination tasks) for each homogeneous cargo (typical order).

Next, the last Stage 7 is implemented – plan output and monitoring of its execution.

Results of the pilot model validation:

1. Filtering revealed 7 non-conflicting orders, which were excluded from further clustering.

2. The developed segmentation algorithm allowed us to form 3 clusters (long-term, medium-term, short-term) for 43 conflicting orders. The initial estimate of the silhouette coefficient [29] of this set is 0.69. Bootstrap (1000 new samples) confirms the stability (silhouette > 0.5) of the clusters only in 68% of cases. Such a low indicator indicates an insufficient sample size and the need to expand the size to $n > 100$ for reliable clustering.

3. Adaptive vehicle selection was carried out in each cluster based on the ranking of the enterprise's vehicles for each of the 6 types of orders. This allowed the four-index transport problem to be reduced to a combination of three-index (triplanar) and two-index problems depending on the available fleet. This significantly simplified the computational complexity of the overall logistics problem.

4. Pilot testing ($n = 50$ orders) confirmed the model's performance: the empty run ratio decreased by 21% (from 32% to 11%), logistics costs by 11.4%. The effect is stationary ($SD = 2.1\%$), the minimum cost reduction is 8.9%. However, due to the limited sample size ($n = 50$) and wide 95% CI (6.2–16.6%), the modeling results are preliminary. For industrial implementation, validation on $N \geq 500$ is required.

5. The model was validated on modified enterprise data ($N = 1500$ orders). Ten-fold cross-validation showed an average reduction in transportation costs of 11.7% compared to the existing system. This value is statistically significant (95% CI: 9.4–14.0%, $t = 2.51$, $p = 0.013$). The standard deviation of the cost reduction between folds is 1.2% (the coefficient of variation is 10.3%). The minimum reduction in transportation costs in individual folds was 10.1%, which is an acceptable result for implementation in logistics systems [2, 9].

6. Discussion of the results of research into the freight planning process using a hierarchical management model

The proposed three-level hierarchical model is fundamentally different from the traditional ones, typical of VRP [19, 21] and their modifications [22, 23]. According to classical methods, the problem is solved monolithically, which leads to an exponential increase in computational complexity with an increase in the number of orders. Alternative decomposition methods [26, 27] use a rigid partition without an adjustment mechanism, which does not guarantee global optimality.

The key difference of the proposed model is the adaptive feedback between the levels: the tactical level (clustering) (Tables 2, 3) (forms groups of orders by geographical and temporal proximity; the operational level (vehicle selec-

tion) (Table 4) assigns specific means taking into account technical compatibility; feedback: if there is no suitable vehicle for the cluster (for example, due to exceeding the dimensions), this information is automatically transferred to the tactical level, which adjusts the cluster boundaries. This is a synergistic effect that is absent in models [5, 6], where clustering and vehicle selection are isolated processes. Thanks to it, interpretability is preserved at each stage: the manager can explain why the order fell into cluster A (temporal proximity) and not B, and why it was assigned vehicle-3 (cost minimization and size compliance). Compared to fuzzy models [25] or evolutionary algorithms [24], where the solution is "black box", our model provides transparency of solutions, which is critical for practical application.

The applied hierarchical decomposition makes it possible to significantly reduce the computational complexity of the multi-index problem, going to a sequence of local subproblems on clusters: optimal routing, triplanar problem (1)–(6), or assignment problem (7), (8). This breaks the exponentially complex problem into a set of smaller problems that can be solved in parallel.

The running time of the clustering algorithm (Fig. 1, 2) is estimated as $O(n^2)$, where n is the dimensionality of the input set of orders. This is due to pairwise comparisons (Fig. 1, Table 2), the number of which is $C(n, 2)$.

Comparison with alternatives:

– K-means [12] has a complexity of $O(n \cdot k \cdot t)$, where k is the number of clusters, t is the iterations. In this case, k is determined a priori, which in dynamic problems requires multiple restarts, increasing the actual time by 2–3 times;

– DBSCAN [12] has a complexity of $O(n \cdot \lg n)$ when using indices, but is sensitive to the parameter ϵ , the determination of which requires expert estimates;

– Agglomerative [12] – $O(n^2)$, like the developed algorithm, but requires $O(n^2)$ memory, while the algorithm presented in this paper works "on the fly" with the current pair.

Thus, the proposed algorithm is not inferior to existing ones in complexity but wins in versatility – it does not require a preliminary determination of the number of clusters or density.

The vehicle selection algorithm based on the preference ratio method (Fig. 4, Table 8) has a complexity of $O(m \cdot l)$, where m is the number of vehicles, l is the number of criteria. This is a linear complexity, which is significantly faster than the evolutionary algorithm [22] ($O(\text{pop-size} \cdot \text{generations} \cdot m \cdot l)$) or fuzzy systems [25] ($O(k^2 \cdot m)$).

The model was tested on a sample of 1500 orders – data (partially modified) from a real motor transport company. The reduction in transport costs by 11.7% ($p = 0.013$) was confirmed by the Student's t -test (DSTU ISO 16269-6:2015). This effect was achieved by reducing empty runs from 32% to 11% (chapter 5 for the results of the pilot model test).

Cluster stability (silhouette > 0.5) was achieved in 68% of cases using the bootstrap method [29]. This indicates (chapter 4 for the results of the pilot model test) the risk of instability in small samples ($n < 100$).

The proposed model demonstrates structural consistency, computational efficiency, and experimentally confirmed economic feasibility, which indicates the possibility of its application in medium and large-scale logistics systems.

Prospects compared to existing solutions:

– comparison of K-means, DBSCAN clustering methods applied to this model according to the criteria of silhouette coefficient, calculation time, and impact on the final solutions of transport models;

- integration of genetic operators [24] to improve stability with small samples;
- hybridization with reinforcement learning methods for dynamic adaptation to network changes based on AI approaches [26].

Critical limitations: the model is static – it assumes full knowledge of the data at the time of planning and does not take into account dynamic changes in the transport network (traffic jams, vehicle failure, new urgent orders). For industrial application, it is necessary to increase the sample to $n > 100$ to stabilize silhouette > 0.8 ; add support for time windows and infrastructure constraints; integrate forecasting of external factors. It is also necessary to take into account the limitations of the structure: feedback increases the computational time compared to rigid decomposition [27, 28] due to the need for iterative correction.

The main disadvantages that are worth noting are the following. Expert assessments of the suitability of each enterprise vehicle for fulfilling individual types of orders and comparison of vehicles introduce subjectivity into the model (Table 4), which reduces the quality of ranking alternatives (Fig. 4). In further studies, it is planned to gradually replace expert assessments with objective data by initializing the model with expert assessments, collecting vehicle operation statistics, adjusting estimates with calculated indicators, and full automation through machine learning. Such a hybrid approach is quite effective in practice [2, 9].

Lack of dynamic adaptation of the model to changes in the logistics network in real time and integration of additional optimization criteria (environmental, social). Further studies will focus on online clustering by mechanisms given in [4], parameter weighting using CRITIC [15] to reduce subjectivity, and the use of approaches that make it possible to take into account parameter uncertainty using methods from [6, 25].

7. Conclusions

1. We have developed interrelated algorithms for local sub-tasks. The algorithm for multi-level clustering (local segmentation) of orders without fixed parameters makes it possible to reduce the complexity of the problem by grouping orders according to a certain criterion or criteria (geographical proximity, delivery time, type of cargo, etc.). The use of this algorithm makes it possible to move from a global problem to local problems on clusters. The algorithm is not tied to specific criteria parameters, does not assume ordering the initial set of orders according to the criteria parameters, which meets the conditions of practical logistics problems. The algorithm for selecting and assigning a vehicle is based on the method of the preference ratio with the criteria of cost, technical compliance, availability, and vehicle parameters. The use of this algorithm makes it possible to reduce the number of indices in the multi-index optimal transportation planning problem. This significantly simplifies the computational complexity of the logistics problem. The key feature is the consistency of the algorithms: the clustering results are the initial data for vehicle selection, which ensures local optimality within the global cost minimization strategy.

2. A hierarchical transportation planning model has been built; the global level (coordination and resource allocation), tactical level (segmentation of orders into clusters), and operational level (adaptive vehicle selection for each cluster) were distinguished. The difference from existing approaches is the presence of feedback between levels: restrictions at the vehicle selection level automatically adjust the clustering parameters, which makes it possible to preserve the interpretability of solutions at each stage.

3. The model's performance has been experimentally proven on a sample of 1500 orders: a reduction in transportation costs of 11.7% was achieved ($p = 0.013$). However, the stability of clusters (silhouette > 0.5) in 68% of cases by the bootstrap method indicates the risk of instability with a small sample ($n = 43$). This indicates the need to increase the sample to $n > 100$ for industrial application and additional consideration of operational constraints (time windows, infrastructure, external factors).

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study, as well as the results reported in this paper.

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Data availability

All data are available, either in numerical or graphical form, in the main text of the manuscript.

Use of artificial intelligence

The artificial intelligence tool ChatGPT (OpenAI, GPT-4-series model) was used to support language editing, improve the readability and academic style of the manuscript. The artificial intelligence tool did not contribute to the scientific content, data analysis, interpretation of results or formulation of conclusions. All methodological decisions, analyses, and final interpretations were made by the authors who bear full responsibility for the content of the manuscript.

Authors' contributions

Ilona Drach: Conceptualization, Methodology, Writing – review & editing; **Oksana Kucheruk:** Formal analysis Investigation Writing – original draft; **Tetiana Kysil:** Software, Data Curation Writing – original draft; **Oleksandr Dykha:** Methodology, Writing – review & editing; **Serhii Matiukh:** Resources, Supervision, Project administration.

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