DESIGN OF A TUMBLING MACHINE (MIXER) USING A STATICALLY DETERMINATE SPATIAL MECHANISM AND DETERMINATION OF RATIONAL GEOMETRIC PARAMETERS

The object of this study is tumbling equipment in which working containers execute a complex spatial motion.

Articulated spatial mechanisms are widely used in various branches of industry, particularly in mechanical engineering, including mechanisms that contain passive constraints in their structure. The presence of passive constraints can cause operational problems and negatively affect equipment reliability. Therefore, an important task is the synthesis of articulated spatial mechanisms through modification of existing structures in order to eliminate passive constraints.

This paper reports the synthesis of a statically determined seven-link articulated spatial mechanism with revolute kinematic pairs. A technique for eliminating a passive constraint in the structure of an articulated spatial mechanism has been proposed, which allows for its static determinacy. As a result, the need to compensate for inaccuracies in geometric relationships between the links by means of clearances in the kinematic pairs is eliminated, making it possible to improve operational characteristics and prolong service life.

The introduction of an auxiliary link into the spatial kinematic chain creates conditions for effective implementation of tumbling technological operations by increasing the amplitude of spatial displacement of the container.

Analytical relationships between the main geometric parameters of the seven-link spatial mechanism that determine its operability have been established. The derived mathematical dependences allow a justified selection of rational geometric parameters at the design stage and provide a basis for calculating key geometric characteristics for further engineering application in industrial practice

Keywords: articulated spatial mechanism, kinematic pair, degree of freedom, mixing of bulk materials

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1. Introduction

In equipment used in various industries, such as pharmaceutical and chemical (mixers) [1, 2], mining [3], automotive [4], and machine-building [5], articulated mechanisms with a spatial structure are applied. In the design of such mechanisms, various kinematic pairs can be used, including pairs with several degrees of freedom (cylindrical, spherical); these mechanisms can also be formed by kinematic pairs with one degree of freedom (rotational and translational). Typically, spatial mechanisms form a combination of different types of kinematic pairs, which greatly simplifies their synthesis. However, such mechanisms are characterized by significant production and operational difficulties, such as the complexity of manufacturing spherical joints of links, the need to use expensive anti-friction materials, high wear intensity of moving parts and, as a result, a limited service life of the equipment.

Given the above, it is more expedient to use spatial mechanisms equipped with only rotational kinematic pairs formed by standard bearings (rolling or sliding). At the same time, as researchers note [6, 7], the use of only rotational kinematic

pairs in spatial mechanisms creates significant difficulties in their rational synthesis and calculation of the necessary geometric relationships.

In addition, the authors of [8, 9] noted that in some spatial mechanisms with rotational kinematic pairs there is a probability of passive constraints. The formation of such passive constraints in structural chains negatively affects the operating characteristics of technological equipment and, in some cases, makes it impossible to design it in compliance with the necessary technological and operational requirements. This emphasizes the importance of a reasonable choice of the type of kinematic pairs and a rational structure of spatial mechanisms in the process of their design.

However, a wide range of mixing and tumbling technological operations (polishing, grinding, separation of parts from sprues, grinding, cleaning from corrosion products, etc.) predetermines the individuality of the approach, which involves using appropriate technological equipment with distinctive design features.

In addition, it is known [7–9] that passive constraints in the structure of spatial mechanisms can worsen the operational properties of the equipment, reduce its durability. The functioning of such mechanisms is possible only at certain clearly defined geometric ratios of the mechanism links and is enabled exclusively by the presence of gaps in kinematic pairs that compensate for the inaccuracy of manufacturing machine parts.

Therefore, when designing mixers and tumbling machines with containers moving along spatial trajectories, one of the key tasks is the synthesis of articulated spatial statically defined mechanisms. It is known [9, 10] that, subject to compliance with this provision, the force loads in the kinematic pairs of the mechanism will be formed exclusively under the influence of technological and dynamic interaction.

Thus, studies aimed at devising methods for the geometric synthesis of articulated spatial statically determined mechanisms with rotational kinematic pairs are relevant in view of the need to improve the reliability, durability, and efficiency of mixers and tumblers used in modern technological processes in various industries.

2. Literature review and problem statement

The results of fundamental research into the geometry of an olloid as a three-dimensional stereometric object and the laws of its motion are reported in [11]. The author considers the olloid from the standpoint of spatial geometry, analyzes its shape, symmetry, and features of movement in space, which determine the unique nature of the motion of a rigid body without slipping. Considerable attention is paid to the transformation of geometric concepts into technical solutions that can be implemented in engineering structures. However, in [11], the formalized apparatus of the theory of mechanisms and machines is not used. The motion of an olloid is not interpreted as a hinged spatial mechanism with defined kinematic pairs and degrees of freedom. Moreover, the monograph does not contain an analysis of the structural determinacy of mechanical systems that could be built on the basis of the geometry of an olloid and does not consider the issues of the possible occurrence of passive constraints in such systems.

In [12], an overview of the design and principle of operation of the "Schatz mechanism" implemented in a mixer with spatial three-dimensional movement of the working capacity is provided. The authors consider the mechanism as a spatial system of the double cardan transmission type, which provides simultaneous rotation and oscillation of the capacity in mutually perpendicular planes. The work describes the kinematic scheme of the mechanism, its main links and general relationships that determine the nature of spatial movement. At the same time, the work is essentially a survey and focuses mainly on demonstrating the technological advantages of mixers of the "Turbula" type, which began to be mass-produced in Switzerland in the 1960s by the "Wilhelm Arnold Bachofen" corporation. During operation of the machine, the working capacity of the mixer repeated the movement of the olloid on the plane – it rotated and, simultaneously, oscillated in all mutually perpendicular planes, which was the key to success in achieving a significant increase in the productivity of technological mixing operations. However, the issue of eliminating the passive constraint in the mechanism structure remains unresolved. The issue of operating the spatial mechanism, which is enabled by the presence of gaps in the kinematic pairs, also remains open.

In [13], the structure of the Turbula mixer is considered in detail, emphasizing that this design is an exception to the generally defined rules used in the synthesis of cardan gears. According to the statements in the work, the angle between the axes in each cardan should be no more than 45° and could in no case equal 90°. In contrast, in the spatial mechanism of the Turbula machine the angle between the axes is 180°. The issue of a detailed analysis of the spatial mechanism from a geometric point of view remains unresolved.

The task of eliminating passive constraints in articulated spatial mechanisms is addressed in [14]. It highlights some issues of the mathematical theory of designing statically determined articulated mechanisms. The attention is mostly on the mathematical calculation of geometric constraints imposed on the structure of spatial mechanisms. The considered aspects of the mathematical theory of designing statically determined hinge mechanisms are only conceptually close to the issue of studying the spatial mechanism of the Turbula machine. However, they do not provide a direct analytical basis for the issue of studying the spatial mechanism of the Turbula machine itself.

In work [15], one of the ways of eliminating the passive constraint in a spatial six-link mechanism, which is similar to the spatial mechanism of the Turbula machine, is proposed. Its essence is increasing the degree of mobility of the spatial mechanism by adding another moving link to its kinematic chain. Such a solution once again confirms the general method of eliminating passive constraints in spatial hinge mechanisms: introducing an additional moving link or replacing the kinematic pair with another one with a higher degree of mobility. However, in addition to the general methodology, it is necessary to carry out further analytical studies of the corresponding mechanism, which are absent in the cited work.

In [16], a technique for eliminating passive coupling in the spatial mechanism of the Turbula machine is described, the essence of which is to introduce a slider into the kinematic chain as an additional moving link, which is kinematically connected to the driven shaft and the frame. Additionally, mathematical dependences were derived analytically, which make it possible to calculate the necessary geometric and structural properties of such a design of a tumbling machine, in particular the amplitude of the reciprocating movement of the slider link. However, such a solution leads to the fact that

an additional translational kinematic pair of the 5th class appears in the structure of the mechanism, or a kinematic pair of the 4th class is formed. This will lead to the emergence of additional friction-sliding force, the need to use expensive antifriction materials or faster wear of the working surfaces. All this could lead to objective operational difficulties and an increase in the cost of equipment. Placing a translational kinematic pair "at the end" of the chain of a spatial mechanism could lead to the concentration of dynamic uneven loads on the moving links of the machine and, over time, cause their deformation.

In work [17], for the machine considered in [16], a special drive design was synthesized. The designed drive structure would make it possible to implement a technical solution in which the translational kinematic pair is placed "at the beginning" of the kinematic chain of the spatial mechanism. Accordingly, the torque is transmitted to the shaft, which will move reciprocatingly along the guide. This technical solution significantly complicates the structure of the machine, which affects the increase in the cost of its manufacture and may negatively affect the reliability of the design.

Thus, the authors previously synthesized statically defined mechanisms with only rotational kinematic pairs, the research of which is reported in [8, 17]. In particular, in [8], structures of tumbling machines (mixers) with two working containers that are movably connected to each other were designed. Such structures make it possible to simultaneously perform various technological operations. At the same time, the use of two-tank rolling machine designs is not always rational and significantly narrows the scope of their use. The tank, which is kinematically connected to the driven shaft, moves more intensively than the other tank, which is kinematically connected to the driving shaft. Accordingly, this would create the effect of non-simultaneous completion of the technological operation. The parts in one tank would already be processed, while additional time would be required to process the parts in the other tank.

The production of small metal parts and parts from polymeric materials, the surface quality of which can be determined by the organoleptic method, is largely due to the reduction in the time required to perform tumbling finishing operations. Such finishing operations can take up more than 75% of the total time in the manufacture of parts.

In [18, 19], the authors study in detail the implementation of the mixing processes of loose fine-dispersed substances in the design of the mixer "Turbula". They proved the unconditional advantages of this machine from the point of view of the productivity of the technological process. In [1] it was established that with a transverse model of loading of fractions of the loose mass, their mixing with the achievement of a satisfactory segregation index occurs after 8–10 revolutions of the drive shaft. However, those studies highlight exclusively the technological aspect of the use of equipment, without due attention being paid to the design features of the machine, the features of the functioning of the articulated spatial mechanism underlying it.

In work [20], experimental studies are reported on the change in motion modes in the cylindrical working capacity of an improved design of a tumbling machine, which is equipped with a special drive structure, which makes it possible to implement the necessary law of change in angular velocity on the drive shaft of the machine. However, the experimental research results are characteristic only for the studied machine design and cannot be applied to other structures of tumbling machines that have a different design of the spatial mechanism.

In [19, 21], a methodology for studying the movement of a loose mass inside the cylindrical container of the Turbula mixer is described using modeling in a special computer-aided design system. At the same time, studies using computer-aided design systems that would describe the spatial movements of the moving parts of the mixer mechanism are not reported.

In [22], the results of research on the synthesis of spatial kinematic chains with spherical kinematic pairs are described. It is shown that the use of analytical and numerical methods makes it possible to solve nonlinear algebraic systems and determine admissible geometric relations for chains with spherical pairs. However, the issues related to the practical implementation of such mechanisms in tumbling machines, in which the working containers move along a complex spatial trajectory, have remained unresolved. The reason for this is the lower reliability of spherical kinematic pairs compared to rotational kinematic pairs of the 5th class. In particular, spherical kinematic pairs are prone to more intense wear, require complex manufacturing, and often require the use of expensive antifriction materials, which complicates operation and increases maintenance costs. An option for overcoming these difficulties may be the use of mechanisms with exclusively rotational kinematic pairs of the 5th class, which provides increased reliability, durability, and simplifies technological manufacturing.

In [23], focus is on the analysis of the kinematic and dynamic accuracy of spherical mechanisms. In turn, this actually confirms their high sensitivity to geometric errors. Thus, achieving high accuracy in mechanisms with spherical pairs is possible only if they are manufactured to increased accuracy tolerances. In addition, spherical kinematic pairs complicate the implementation of effective lubrication and heat removal. All this once again confirms the rationality of using spatial mechanisms with kinematic pairs of the fifth class, mainly of the rotational type.

Analysis of papers [18, 19] revealed that today the spatial movement of the working tanks of the "Turbula" type mixers has been thoroughly studied and the high technological advantages of such equipment have been confirmed [1, 20]. Studies [22, 23] confirmed the prospects and rationality of using spatial mechanisms with kinematic pairs of the fifth class in comparison with kinematic pairs of higher classes. Considering the above, the existing works lack a comprehensive analytical approach to the synthesis of articulated spatial mechanisms of tumbling machines, formed exclusively by rotational kinematic pairs, from the standpoint of their static determinacy. The issues of eliminating passive constraints in the structure of such mechanisms, ensuring their operation without the use of gaps in kinematic pairs, as well as performing geometric analysis of spatial displacements of moving links remain unresolved. This is what limits the possibility of designing simple, reliable, and durable structures of tumbling machines with predictable operational characteristics.

The above considerations allow us to assert that it is advisable to conduct a study aimed at eliminating passive constraint in articulated spatial mechanisms of tumbling machines by synthesizing statically determined mechanisms with rotational kinematic pairs and further performing their geometric analysis.

3. The aim and objectives of the study

The aim of our study is to design a structure of a tumbling machine (mixer) using a statically defined spatial mechanism

and to define rational geometric parameters. This will make it possible to create conditions for reliable and long-term operation of the tumbling machine (mixer) with the subsequent determination of rational requirements for its design.

To achieve the goal, the following tasks were set:
– to synthesize a hinged spatial mechanism containing all kinematic pairs of the rotational type;
– to determine rational ratios of the lengths of the links;
– to derive expressions for the possibility of calculating the main geometric parameters of the machine structure, which could be successfully used by engineers at the design stage of this equipment.

4. Materials and methods

The object of our study is tumbling equipment in which working containers execute a complex spatial movement.

The principal hypothesis assumes that the passive constraint in the articulated spatial mechanism of a tumbling machine of the "Turbula" type (Switzerland) can be eliminated by rational geometric synthesis of a statically defined spatial mechanism. One of the conditions for the synthesis is the use of only fifth-class rotational kinematic pairs in the spatial mechanism. In turn, this could create conditions for increasing the reliability, durability, and operational efficiency of the machine without the need to use additional kinematic pairs of higher classes or compensation for inaccuracies due to gaps in the kinematic pairs.

Before starting the study, the following assumptions were adopted: the geometry of the synthesized spatial articulated mechanism is accurate, with an ideal mutual arrangement of the axes of rotation and compliance with the specified ratios of the lengths of the links; the movement of the mechanism is considered exclusively in the kinematic aspect, without the influence of dynamic factors, such as inertial forces or vibrations.

The following simplifications were accepted in the research process: the use of two-dimensional projections and corresponding additional geometric constructions on them for geometric analysis; the research was carried out without taking into account the aspects of force and dynamic analysis of the synthesized mechanism.

Well-known methods of synthesis and analysis of spatial hinge mechanisms were used. In particular, the synthesis of a statically determined hinge spatial mechanism with kinematic pairs of a rotational type; an analytical approach was also used in the further geometric study of the design of the technological machine.

During the analytical study, additional modeling of the machine itself with the calculated basic geometric parameters was carried out, as well as the determination of some kinematic parameters of the spatial mechanism using the computer-aided design software "SolidWorks – 2024" (France, USA). The study of changes in the positions of moving links was carried out using the application "SolidWorks – Motion" (France, USA).

5. Results of research on geometric synthesis of statically determined spatial mechanism in a tumbling machine

5. 1. Synthesis of a hinged spatial mechanism and design of a tumbling machine structure

Previously, in [6, 8], an analysis of the structure of the hinged spatial six-link mechanism of the industrial mixer "Turbula" ($n = 5$ moving links, 6 rotational kinematic pairs

p_S with one $s = 1$ degree of freedom) was carried out and it was established that in accordance with the Somov-Malyshev formula (1), its degree of freedom is zero

$$W = 6n - \sum_{s=1}^{s=5}(6-s)p_s = 0. \tag{1}$$

According to the Somov-Malyshev formula [24], the mechanism should be inoperable, but its functioning is possible under the condition of the geometrical relationships of the links and the presence of gaps in the kinematic pairs, which will compensate for the inaccuracy of the manufacturing of the machine parts.

It is possible to eliminate the passive constraint by adding another moving link to the kinematic chain of the articulated spatial mechanism. Thus, an auxiliary link (connecting rod) was added to the kinematic chain of the mechanism, which is connected to other links by means of rotational kinematic pairs. In particular, the axis of rotation of this link will pass through the middle of the working capacity. This link is connected to the driven holder by one kinematic pair, and the other by the working capacity itself. Thus, a seven-link articulated spatial mechanism with rotational-type kinematic pairs was designed. The model of the articulated spatial mechanism containing all kinematic pairs of the rotational type is shown in Fig. 1, and the kinematic diagram of the articulated spatial mechanism is depicted in Fig. 2.
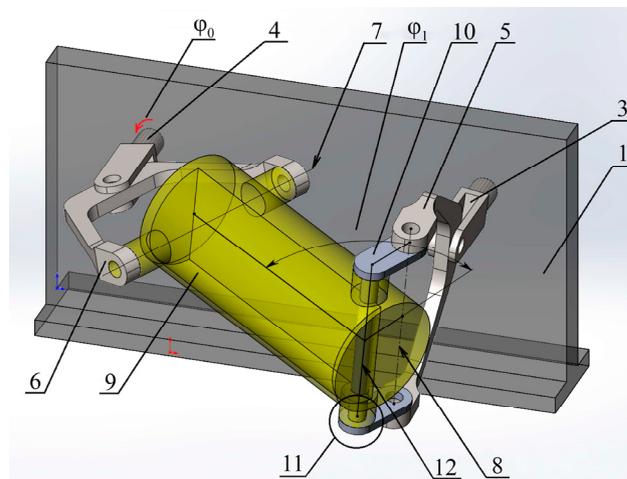
Fig. 1. Model of a hinged spatial mechanism containing all kinematic pairs of the rotational type:
1 — frame; 3 — drive shaft; 4 — driven shaft;
5 — drive holder; 6 — driven holder; 7 — geometric axis of the container attachment; 8 — geometric axis of the auxiliary link attachment; 9 — working container; 10 — auxiliary link;
11 — kinematic pair "auxiliary link-working container";
12 — geometric axis of the container attachment

The tumbling machine consists of a frame (position 1), a technological drive (position 2), shafts – the driving (position 3) and the driven (position 4), the axes of rotation of which are parallel to each other and are located in the same horizontal plane. The shafts (positions 3 and 4) are movably connected to the driving holder (position 5) and the driven holder (position 6), respectively, the geometric axes of both holders are perpendicular to each other and movably connected to the working capacity (position 9) and the auxiliary link (position 10), the axis (position 12) of which passes through the middle of the working capac-

ity (position 9). The working capacity (position 9) and the auxiliary link (position 10) are connected by a rotating kinematic pair (position 11). The axis of rotation (position 12) of the kinematic pair (position 11) is parallel to the axis of the mount (position 8). The distance between the geometric axis of the mount (position 7) and the axis (position 12) of the auxiliary link (position 10) may be greater than the distance between the axes of the mount of the leading (position 5) or driven (position 6) holder.

The degree of mobility of the synthesized mechanism was checked using the Somov-Malyshev formulae (1)

$$W = 6n - \sum_{s=1}^{s=5}(6-s)p_s = 6n - 5p_5 = 6\cdot6 - 5\cdot7 = 1,$$

where n is the number of moving links ($n = 6$ links: driving and driven shafts (positions 3 and 4), driving and driven holders (positions 5 and 6), auxiliary link (position 10), working capacity (position 9)); p_5 is the number of kinematic pairs of the 5th class ($p_5 = 7$ rotational kinematic pairs at the joints of the following links: driving shaft – driving holder; driving holder – auxiliary link; auxiliary link – working capacity; working capacity – driven holder; driven holder – driven shaft).

Thus, the degree of mobility according to the Somov-Malyshev formula is equal to one. The redundant connection has been eliminated.

The machine works as follows. When the drive (position 2) located in the bed (position 1) is turned on, the rotational motion is transmitted to the drive shaft (position 3), then to the drive holder (position 5) and through the axis (position 8) to the auxiliary link (position 10), which, through the rotational kinematic pair (position 11), provides spatial motion to the working container (position 9). In turn, the rotational motion, in combination with the spatial motion, of the working container (position 9) is transmitted to the driven holder (position 6) and to the driven shaft (position 4). At the same time, the auxiliary link (position 10) performs an oscillatory motion relative to the working container (position 9). The working container (position 9) performs a complex spatial motion. As a result, the processing medium during treatment intensively moves under a turbulent mode and receives all degrees of freedom, which contributes to the intensification of tumbling operations of processing parts.
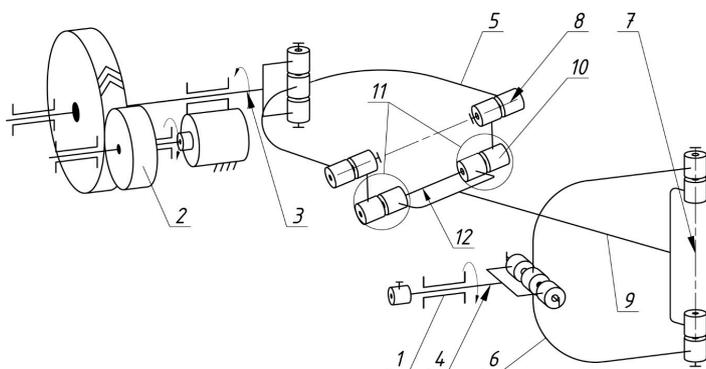

Fig. 2. Kinematic diagram of the articulated spatial mechanism:
1 — frame; 2 — technological drive; 3 — drive shaft; 4 — driven shaft;
5 — drive holder; 6 — driven holder; 7 — geometric axis of
the container attachment; 8 — geometric axis of the auxiliary link
attachment; 9 — working container; 10 — auxiliary link;
11 — kinematic pair "auxiliary link-working container";
12 — geometric axis of the container attachment

5. 2. Determining the rational ratios of link lengths

Despite the fact that the passive constraint has been eliminated in the spatial mechanism of the machine, there remains a need to derive expressions for determining the rational ratios of link lengths. The interaxial length of the auxiliary link is subject to restrictions caused by the design of the spatial mechanism itself. In this regard, the task of determining the range of variation of the interaxial length of this link, which, in turn, depends on the interaxial length of the working capacity, arises.

Further determination of rational ratios of link lengths should begin with the initial data, namely the specified geometric parameters. Depending on the type of technological operations that will be performed on the machine and their productivity, that is, the volume of parts that must be processed in one cycle of the machine, it is necessary to specify the appropriate geometric parameters. These parameters included the center distance of the working capacity – l_{WC}, the diameter of the working capacity – D_{WC}, the center distance of the leading and driven holders – l_H and the "holder diameter", that is, the length of the geometric axis of the fastening of the leading holder with the working capacity or the driven holder with an auxiliary link – D_H, which is determined based on D_{WC}.

Since the machine design uses an auxiliary link, the condition $l_{WC} > l_H$ must be met. It is with this inequality that the basic machine design ("Turbula" [11, 20]) with a passive constraint becomes inoperable. Accordingly, this necessitates the introduction of an auxiliary link into the machine's kinematic chain, which, in fact, frees the machine design from passive constraint.

The machine was considered in a position where the axis (position 7) of the driven holder (position 5) is horizontal, and the axis (position 8) of the driving holder (position 6) is vertical. This position of the machine is shown in Fig. 3. The positions of the links in Fig. 3 correspond to the positions in Fig. 1, 2.

Points C and D, which coincide with the centers of the rotational kinematic pairs of the auxiliary link, lie on a conditional circle with the center at point A. Accordingly, for this position of the machine, based on the conditional triangle ABC formed, an expression was derived that defines the distance between the axes of the shafts

$$l_0 = \sqrt{\left(l_{WC} + l_H\right)^2 - l_H^2}. \qquad (2)$$

In a tumbling machine, for the design of which the condition $l_{WC} > l_H$ is valid, and the center distance l_0 is fixed, the distance between the perpendicular axes of the holders will change during operation. In accordance with the position of the links shown in Fig. 3, this distance will be the smallest. On the other hand, in the case when the moving links of the machine occupy the position (Fig. 4) at which the axis of the working capacity is in the vertical plane, the distance between the perpendicular axes of the holders will be maximum – l_{MAX}.

Thus, the maximum distance between the geometric axes of the forks l_{MAX} is defined as

$$l_{MAX} = l_{WC} + l_A, \qquad (3)$$

where l_A is the distance between the axis of the driven holder and the axis of the auxiliary link, which is shown in Fig. 4. Additionally, taking into account the geometric construction, l_{MAX} is defined as

$$l_{MAX} = \sqrt{l_0^2 - 4l_X^2},\qquad(4)$$

where l_X is the distance in the vertical plane between the axis of the tank and the center of rotation of the driven shaft.

Fig. 3. Position of the moving links of the machine when the axis of the driven holder is horizontal and the axis of the driving holder is vertical: 1 — frame; 3 — drive shaft; 4 — driven shaft; 5 — drive holder; 7 — geometric axis of the container attachment; 8 — geometric axis of the auxiliary link attachment; 9 — working container; 10 — auxiliary link; 11 — kinematic pair "auxiliary link-working container"

To be able to derive the formula for calculating l_X, we shall use the projection of the structure onto a plane parallel to the ends of the tank in the position shown in Fig. 4. Accordingly, the representation of the machine structure in the resulting projection is shown in Fig. 5.

The geometric axes of the holder mounting are mutually perpendicular, the extended axis of the container bisects the right angle between the mutually perpendicular axes of the holders, forming an isosceles right triangle with two legs l_X and a hypotenuse l_H. Accordingly, length l_X is defined as

$$l_X = l_H \cos 45°.\qquad(5)$$

The value of expression (5) was substituted into equation (4)

$$l_{MAX} = \sqrt{l_0^2 - 4\left(l_H \cos 45°\right)^2}.\qquad(6)$$

By combining expressions (3) and (6), we define the value of l_A

$$l_A = \sqrt{l_0^2 - 4\left(l_H \cos 45°\right)^2} - l_{WC}.\qquad(7)$$

Based on the distance l_A calculated from formula (7), the following inequality will hold for such a position of the moving links of the machine (Fig. 4, 5):

$$l_{AL} \geq l_A,\qquad(8)$$

where l_{AL} is the center distance of the auxiliary link.

From relation (8), using equalities (2) and (7), we finally obtain

$$l_{AL} \geq \sqrt{\left(l_{WC} + l_H\right)^2 - l_H^2 - 4\left(l_H \cos 45°\right)^2} - l_{WC}.\qquad(9)$$

Thus, expression (9), taking into account the initial geometric parameters, allows one to calculate the minimum permissible center length of the auxiliary link.
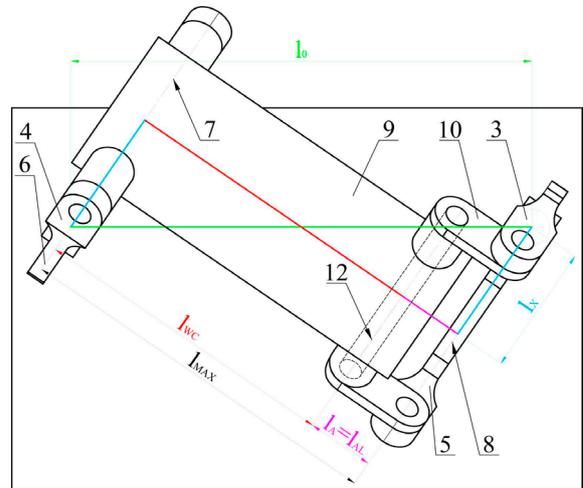
Fig. 4. Position of the moving links at which the axis of the working capacity is in a vertical plane: 3 — drive shaft; 4 — driven shaft; 5 — driving holder; 6 — driven holder; 7 — geometric axis of the capacity fastening; 8 — geometric axis of the auxiliary link fastening; 9 — working capacity; 10 — auxiliary link; 12 — geometric axis of the capacity fastening

Fig. 5. Projection of the structure onto a plane parallel to the ends of the container: 3 — driving shaft; 4 — driven shaft; 5 — driving holder; 6 — driven holder; 7 — geometric axis of the container attachment; 8 — geometric axis of the auxiliary link attachment; 9 — working container; 10 — auxiliary link; 12 — geometric axis of the container attachment

5. 3. Deriving expressions for the possibility of calculating the basic geometric parameters for the machine design

Fig. 3 shows the position of the moving links of the machine at which point *D*, coinciding with the center of the rotational kinematic pair of the auxiliary link with the working capacity, is located "outside" of the driven holder (below the l_{BO} line). Between the axis of the working capacity l_{WC} and the center distance of the auxiliary link l_{AL}, angle φ_1 is formed, which will change depending on the angle of rotation of the driven shaft of the machine φ_0.

Thus, the initial geometric data are given, and we shall calculate the necessary geometric parameters using the previously derived expressions where $l_{WC} = 130$ mm, $D_{WC} = 85$ mm, $l_H = 77$ mm, $l_0 = 192.15$ mm, $D_H = 135$ mm. The adopted numerical values of the geometric parameters l_{WC}, D_{WC}, l_H, l_0 and D_H correspond to the geometric dimensions of the industrial mixer "Turbula T2F", which was used as a structural prototype. The specified parameters enable the correct construction of the spatial kinematic model, making it possible to model (Fig. 1) and examine the design of the tumbling machine in the CAD "SolidWorks – 2024". After that, the designed machine with an auxiliary link was simulated in the CAD "SolidWorks – 2024". The machine model built in "SolidWorks – 2024" is shown in Fig. 1.

Next, using the SolidWorks Motion application, a kinematic study was conducted to determine the value of rotation angle φ_1 depending on the rotation angle φ_0 of the driven shaft of the machine. This dependence plot is shown in Fig. 6.

The position of the moving links, according to which the axis of attachment of the driven holder is horizontal, and the axis of attachment of the driving holder with the auxiliary link is vertical, corresponds to the initial angle of rotation of the driven shaft.

Based on the plot in Fig. 6, it was established that for one full rotation of the driven shaft, the function of changing the value of angle φ_1 will have two periods. At angles of 125° and 235° of rotation of the driven shaft, angle φ_1 will take the form of a deployed one (Fig. 4, 5). Additionally, having rotated the driven shaft by 180°, the working capacity will also rotate by a similar angle, the moving links will take position (Fig. 7), when point D is located "inside" the driven holder (above the l_{BO} line), the driven holder will "cover" the working capacity at a distance greater than its radius. Thus, there was a need to determine such design parameters of the machine at which the working capacity of a given diameter will be able to move freely inside the driven holder.

Let l_P be the distance from the kinematic pair "leading holder – auxiliary link" to its shell, the diameter and shape of which are also specified at the beginning of the design. l_R is the distance from the kinematic pair "leading holder-auxiliary link" to point *K* (Fig. 7).

Fig. 6. Dependence of angle φ_1 formed between the axis of the working capacity l_{WC} and the center distance of the auxiliary link l_{AL} on the angle of rotation of the driven shaft of the machine φ_0
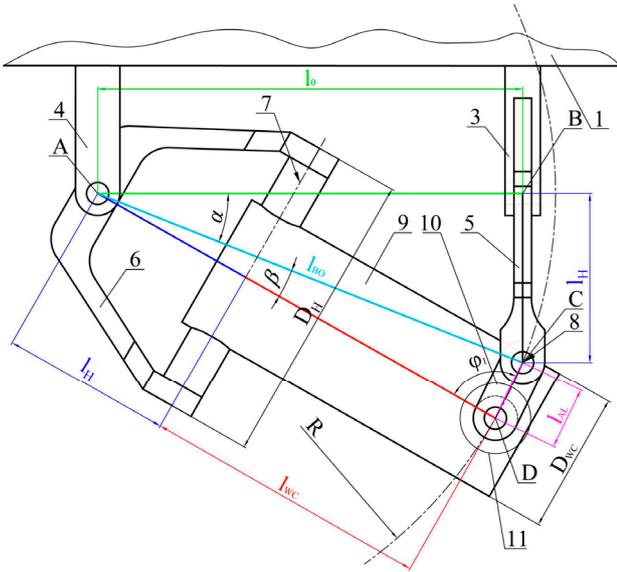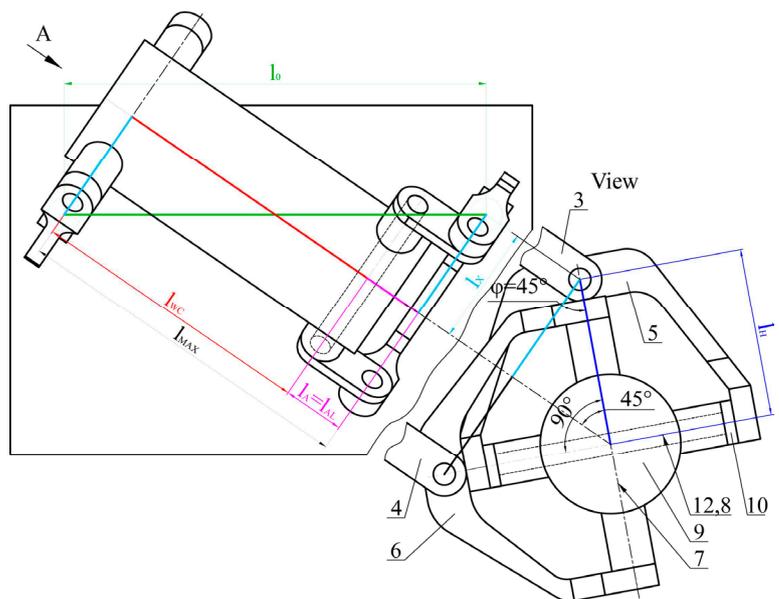

Fig. 7. Position of the moving links of the machine when the axis of the driven holder is horizontal and the axis of the driving holder is vertical: 1 — frame; 3 — driving shaft; 4 — driven shaft; 5 — driving holder; 6 — driven holder; 7 — geometric axis of the container attachment; 9 — working container; 10 — auxiliary link

Obviously, the functioning of the machine was ensured when the following inequality held

$$l_R < l_P. \tag{10}$$

We considered l_R as the sum of distances l_1 and l_2

$$l_R = l_1 + l_2. \tag{11}$$

We wrote down an expression to determine distance l_1

$$l_1 = \frac{R_{WC} + l_B}{\cos \gamma}, \tag{12}$$

where R_{WC} is the radius of the container; l_B is the distance from the kinematic pair "leading holder-auxiliary link", drawn along the normal to the container axis; γ is the angle between lines l_B and l_{AL}.

Also, γ is the angle of inclination of the axis of the working container in the vertical plane, that is, the angle between line l_0 and the container axis l_{WC}. γ can be calculated as

$$\gamma = \alpha - \beta, \tag{13}$$

where α is the angle between lines l_0 and l_{BO}, β is the angle between lines l_{BO} and l_{WC}.

Considering the right triangle ABC, an expression for the possibility of calculating l_{BO} is obtained

$$l_{BO} = \sqrt{l_0^2 + l_H^2}. \tag{14}$$

Equation (13) was represented as follows

$$\gamma = \arccos \frac{l_0}{\sqrt{l_0^2 + l_H^2}} - \arccos \frac{\left(l_H + l_{WC}\right)^2 + l_0^2 + l_H^2 - l_{AL}^2}{2\sqrt{l_0^2 + l_H^2} \cdot \left(l_H + l_{WC}\right)}. \tag{15}$$

The distance l_B can be determined as follows

$$l_B = l_{AL} \cos \delta, \tag{16}$$

where δ is the angle between lines l_B and l_{AL}.

The expression for angle δ can be obtained from the geometric construction of the triangle ACD

$$\delta = 90° - \varepsilon, \tag{17}$$

where ε is the angle between lines l_{WC} and l_{AL}.

Expression (17) was represented taking into account angle ε

$$\delta = 90° - \arcsin \frac{\sin \beta \cdot \sqrt{l_0^2 + l_H^2}}{l_{AL}}. \tag{18}$$

Substituting expression (18) in (16), we obtained

$$l_B = l_{AL} \cos \left(90° - \arcsin \frac{\sin \beta \cdot \sqrt{l_0^2 + l_H^2}}{l_{AL}} \right). \tag{19}$$

We substituted two expressions (19) and (15) into equation (12)

$$l_1 = \frac{R_{WC} + l_{AL} \cos \left(90° - \arcsin \dfrac{\sin \beta \cdot \sqrt{l_0^2 + l_H^2}}{l_{AL}} \right)}{\cos \left(\arccos \dfrac{l_0}{\sqrt{l_0^2 + l_H^2}} - \arccos \dfrac{\left(l_H + l_{WC}\right)^2 + l_0^2 + l_H^2 - l_{AL}^2}{2\sqrt{l_0^2 + l_H^2} \cdot \left(l_H + l_{WC}\right)} \right)}. \tag{20}$$

When designing a machine, special attention should be paid to the calculation and rational determination of the length of distance l_2 since it is a component of l_R. A rationally calculated length l_R enables the functioning of the machine with the corresponding specified geometric parameters.

In turn, length l_R will depend on width h of the container holder and angle γ, which corresponds to the inclination of the axis of the working container to the vertical plane. L_2, taking into account expression (15), can be determined as

$$l_2 = 0.5h \cdot \operatorname{tg} \left(\arccos \frac{l_0}{\sqrt{l_0^2 + l_H^2}} - \arccos \frac{\left(l_H + l_{WC}\right)^2 + l_0^2 + l_H^2 - l_{AL}^2}{2\sqrt{l_0^2 + l_H^2} \cdot \left(l_H + l_{WC}\right)} \right). \tag{21}$$

To calculate the value of distance l_R, equation (11) was written, taking into account the derived formulas (20) and (21)

$$l_R = \frac{R_{WC} + l_{AL} \cos \left(90° - \arcsin \dfrac{\sin \beta \cdot \sqrt{l_0^2 + l_H^2}}{l_{AL}} \right)}{\cos \left(\arccos \dfrac{l_0}{\sqrt{l_0^2 + l_H^2}} - \arccos \dfrac{\left(l_H + l_{WC}\right)^2 + l_0^2 + l_H^2 - l_{AL}^2}{2\sqrt{l_0^2 + l_H^2} \cdot \left(l_H + l_{WC}\right)} \right)} + \\ + 0.5h \cdot \operatorname{tg} \left(\arccos \dfrac{l_0}{\sqrt{l_0^2 + l_H^2}} - \arccos \dfrac{\left(l_H + l_{WC}\right)^2 + l_0^2 + l_H^2 - l_{AL}^2}{2\sqrt{l_0^2 + l_H^2} \cdot \left(l_H + l_{WC}\right)} \right). \tag{22}$$

The functioning of the designed machine structure will be successfully implemented if inequality (10) is fulfilled, as well as with a rational calculation of length l_N, which characterizes the distance drawn normally from the vertical wall of the bed to the kinematic pair "driven shaft-driven holder"

$$l_N = l_{HF} + l_{FN}, \tag{23}$$

where l_{FN} is the distance drawn normally from the vertical wall of the frame to the driven holder, in the position when the axis of the driven holder attachment is in the horizontal plane (taken within 10–20% of l_B, for the purpose of unhindered spatial movement of the links)

$$l_{FN} \approx \left(0.1...0.2\right) \cdot l_H. \tag{24}$$

We wrote down the equation for calculating distance l_{HF}

$$l_{HF} = l_{EF} - l_{EH}, \tag{25}$$

where l_{EF} is the length of the distance from the intersection of the axis of the working capacity and the axis of the driven holder to the outer face of the driven holder; l_{EH} is the length of the distance from the intersection of the axis of the working capacity and the axis of the driven holder to the intersection with line l_0, which connects the kinematic pairs of the driven and driving holders with the driving and driven shafts, respectively.

Next, an equation was written for calculating l_{EF}, taking into account the formed triangle EFG

$$l_{EF} = R_H \cos \gamma, \tag{26}$$

where R_H is half the length of the geometric axis of the driven holder with the capacity.

The equation for calculating l_{EH} is written taking into account the formed triangle AEH

$$l_{EH} = l_H \sin \gamma. \tag{27}$$

Substituting expressions (26) and (27) into equation (25), we obtained

$$l_{HF} = R_H \cos \gamma - l_H \sin \gamma. \tag{28}$$

Substitution into expression (23) of equations (24) and (28) yielded

$$l_N = R_H \cos \gamma - l_H \sin \gamma + \left(0.1...0.2\right) \cdot l_H. \tag{29}$$

At the very end, (29) was written taking into account (15)

$$l_N = R_H \cos\left(\begin{array}{c} \arccos\dfrac{l_0}{\sqrt{l_0^2+l_H^2}} - \\ -\arccos\dfrac{\left(l_H+l_{WC}\right)^2+l_0^2+l_H^2-l_{AL}^2}{2\sqrt{l_0^2+l_H^2}\cdot\left(l_H+l_{WC}\right)} \end{array}\right) -$$

$$-l_H \sin\left(\begin{array}{c} \arccos\dfrac{l_0}{\sqrt{l_0^2+l_H^2}} - \\ -\arccos\dfrac{\left(l_H+l_{WC}\right)^2+l_0^2+l_H^2-l_{AL}^2}{2\sqrt{l_0^2+l_H^2}\cdot\left(l_H+l_{WC}\right)} \end{array}\right) +$$

$$+\left(0.1...0.2\right)\cdot l_H, \tag{30}$$

Thus, equality (30) is obtained, which makes it possible to calculate another important geometric parameter of the designed machine structure – distance l_N.

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6. Discussion of the results of research of geometric synthesis of a statically defined spatial mechanism of a tumbling machine

A statically determined spatial mechanism with all rotational kinematic pairs has been synthesized, the spatial kinematic scheme of which is shown in Fig. 2, which underlies the tumbling machine, the model of which is depicted in Fig. 1. The corresponding degree of mobility is confirmed by our calculation according to the Somov-Malyshev formula (1).

Thus, when using such a machine in practice, the geometric restrictions that are manifested in the ratios of the lengths of the links imposed by the structure of the statically indeterminate spatial mechanism are removed. The operability of the mechanism will no longer be enabled exclusively by the gaps in its kinematic pairs. A change in the center distances of the moving links becomes permissible. During operation, from a technological point of view, a change in the center distance of the working capacity l_{WC} or a change in the center distance of the auxiliary link l_{AL} may be relevant. By changing the center distance of the working capacity l_{WC}, it is possible to regulate the implementation of the corresponding modes of movement of the array in the middle of the capacity. By changing the center distance of the auxiliary link l_{AL}, it is possible to vary the additional effect of "shaking" the container, which is characteristic of some tumbling technological operations. Additional "shaking" of the container creates conditions for the intensification of some tumbling technological processes, in particular when separating parts from sprues, as well as technological processes of mixing loose substances. When changing the center distance of the auxiliary link l_{AL}, expression (8) should be used, which establishes the limit of the center distance.

For the designed structure of the tumbling machine, mathematical expressions were derived that make it possible to calculate the most important design parameters. Expression (2) makes it possible to calculate the rational distance between the axes of the driving and driven shafts, and expression (9) establishes geometric restrictions on the center-to-center length of the auxiliary link. Using the resulting equations (20)–(22) and (30) will make it possible, at the design stage, to calculate those geometric parameters that will ensure unhindered mutual movement of the moving links. According to the plot shown in Fig. 6, one can see how the mutual angular arrangement between the axes of the working capacity and the auxiliary link will change.

Despite similar studies and synthesis of hinged statically determined mechanisms, which are reported in [6–8], as well as in [15–17], another variant of the synthesis of a statically determined hinged mechanism, different from the previous ones, is proposed in our work. This will make it possible to diversify the range of tumbling equipment for the relevant technological processes. The absence of translational kinematic pairs in the synthesized spatial mechanism will lead to increased reliability of the machine.

The limitations of our research are that the results are adequate within the geometric model of the spatial mechanism of the machine. All analytical dependences were obtained assuming ideal geometry of the links, exact mutual arrangement of the axes. Under these conditions, the synthesis of the mechanism is correct; however, going beyond the geometric limits may require additional clarifications.

The key drawbacks of the study are:
– the lack of experimental verification of the analytical results. The work performed a thorough geometric synthesis and presented analytical dependences, as well as computer simulation in the SolidWorks environment, but the results of experimental or bench tests of the designed machine structure were not described;
– we did not consider the force and dynamic analysis of the synthesized mechanism. The elimination of the passive constraint is justified from the standpoint of structural and geometric certainty, but it is not shown how a change in the structure can affect the distribution of forces in kinematic pairs, the level of dynamic loads;
– there is no assessment of the technological efficiency of the tumbling machine. Conclusions about the possible intensification of the tumbling or mixing process, compared to the basic Turbula design, are hypothetical in nature and are based on the results from previous studies on similar machine designs.

Advancing this research will involve, firstly, experimental or bench tests of the designed machine structure. Secondly, conducting an experimental assessment of the technological efficiency of the tumbling machine during the performance of tumbling technological operations or operations of mixing loose substances. Thirdly, carrying out further force analysis, which will be able to demonstrate how the change in the structure affects the distribution of forces in kinematic pairs, the level of dynamic loads. The force analysis involves determining and comparing the reactions in kinematic pairs of the designed machine structure and the structure of the Turbula machine.

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7. Conclusions

1. A new articulated spatial mechanism has been synthesized, containing all the kinematic pairs of the rotational type, thereby presenting one of the methods for eliminating the passive constraint in the kinematic chain of the articulated spatial mechanism of the Turbula machine. As a result of eliminating the passive constraint, the need to compensate for the inaccuracy of the geometric relationships between the links is eliminated through the gaps in the kinematic pairs, which makes it possible to increase the operational properties of the machine and its durability.

2. Rational ratios of the lengths of the links were determined, in particular, the center distance of the working capacity l_{WC} should be greater than the center distance of the driving or driven holders l_H, the center distance of the auxil-

iary link l_{AL} should be greater than or equal to the distance between the axis of the driven holder and the axis of the auxiliary link l_A. In turn, this ensures the operability of the statically defined spatial mechanism without the use of gaps in the kinematic pairs and makes it possible to increase the amplitude of the spatial movement of the working capacity by 2–3 times compared to the basic Turbula design. Additionally, the conditions for the ratio of lengths l_R to l_P have been determined, which guarantee unhindered mutual movement of the links without jamming.

3. Analytical expressions were derived for the possibility of rational calculation of such basic geometric parameters of the designed machine structure as the distance between the axes of the driving and driven shafts l_0, the center distance of the auxiliary link l_A, the center distance l_{WC} and the radius R_{WC} of the working capacity, the length l_N, which characterizes the distance drawn normally from the vertical wall of the frame to the kinematic pair "driven shaft – driven holder". Resulting expressions make it possible to reasonably choose rational geometric parameters at the design stage, which corresponds to the task of synthesis and rationalization to enable the operability of the machine.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study, as well as the results reported in this paper.

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Data availability

All data are available, either in numerical or graphical form, in the main text of the manuscript.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

Authors' contributions

Mark Zaliubovskyi: Conceptualization, Methodology, Project administration; **Olksiy Volianyk:** Software, Visualization; **Mykola Rubanka:** Validation; **Sergey Koshel:** Writing – review & editing; **Hanna Koshel:** Data curation; **Serhiy Popovichenko:** Formal analysis.

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