The object of this study is the complex motion of a particle on the surface of a vertical straight helicoid rotating around its own axis. In screw conveyors, closed helicoids are used as well-known technical helical surfaces. An issue is not the disadvantages of using classical closed helicoids but the limitations of existing mathematical models of particle motion, which essentially reduce engineering research only to this type of surfaces. The lack of a generalized model for other helical surfaces makes their analysis and practical application impossible. The proposed approach expands the class of helicoids under consideration and creates the prerequisites for finding new design solutions.

The derived second-order differential equations describe the trajectory of particle sliding on the surface. Depending on the structural parameters, such a surface can be an open or closed helicoid, as well as a special case of rotation of a horizontal flat disk. That has made it possible to define the parameters of particle motion on different surfaces and compare the results. In particular, the particle sliding trajectories along closed and open helicoids rotating with angular velocity $\omega = 10$ s^{-1} and $\omega = 20$ s^{-1} were constructed. In this case, the friction coefficient $f = 0.3$ and the lift angle $\beta = 15°$ of the outer edge of the surface were assumed at a radius of $R = 0.1$ m of the limiting cylinder. The particle sliding trajectories were constructed within the surface compartment, as well as under the condition that it is not limited by the cylinder.

The practical significance of the results is the possibility of using the model built for designing energy-efficient screw conveyors without an external casing. This makes it possible to reduce the metal content of structures by 15–20% and prevent jamming during the transportation of fractional materials. The resulting analytical dependences make it possible to calculate the optimal screw pitch and shaft radius to ensure a given material movement trajectory

Keywords: vertical helicoid, generalized model, sliding trajectory, complex motion, particle motion

UDC 514.18

DOI: 10.15587/1729-4061.2026.352152

CONSTRUCTION OF A GENERALIZED MATHEMATICAL MODEL FOR PARTICLE SLIDING ON THE SURFACE OF A ROTATING VERTICAL STRAIGHT HELICOID

Tetiana Volina
Corresponding author
Doctor of Technical Sciences, Associate Professor*
E-mail: volina@nubip.edu.ua
ORCID: https://orcid.org/0000-0001-8610-2208

Serhii Pylypaka
Doctor of Technical Sciences, Professor, Head of Department*
ORCID: https://orcid.org/0000-0002-1496-4615

Ivan Rogovskii
Doctor of Technical Sciences, Professor, Dean
Department of Technical Service and Engineering Management named after M. P. Momotenko**
ORCID: https://orcid.org/0000-0002-6957-1616

Mykhailo Kalenyk
PhD, Professor, Dean
Department of Mathematics, Physics and Methods of their Education***
ORCID: https://orcid.org/0000-0001-7416-4233

Vitalii Ploskyi
Doctor of Technical Sciences, Professor, Head of Department
Department of Architectural Structures
Kyiv National University of Construction and Architecture
Povitryanih Sil ave., 31, Kyiv, Ukraine, 03680
ORCID: https://orcid.org/0000-0002-2632-8085

Natalia Ausheva
Doctor of Technical Sciences, Professor, Head of Department
Department of Digital Technologies in Energy
National Technical University of Ukraine "Igor Sikorsky Kyiv Polytechnic Institute"
Beresteiskyi ave., 37, Kyiv, Ukraine, 03056
ORCID: https://orcid.org/0000-0003-0816-2971

Olga Shoman
Doctor of Technical Sciences, Professor
Department of Geometric Modeling and Computer Graphics
National Technical University "Kharkiv Polytechnic Institute"
Kyrpychova str., 2, Kharkiv, Ukraine, 61002
ORCID: https://orcid.org/0000-0002-3660-0441

Vitaliy Babka
PhD, Associate Professor*
ORCID: https://orcid.org/0000-0003-4971-4285

Oleksandr Tatsenko
Senior Lecturer
Department of Transport Technologies
Sumy National Agrarian University
Herasyma Kondratieva str., 160, Sumy, Ukraine, 40000
ORCID: https://orcid.org/0000-0003-1762-8219

Larysa Korzh-Usenko
Doctor of Pedagogical Sciences, Professor
Department of Management of Education and Pedagogy of High School***
ORCID: https://orcid.org/0000-0001-9538-4147

*Department of Descriptive Geometry, Computer Graphics and Design**
**National University of Life and Environmental Sciences of Ukraine
Heroyiv Oborony str., 15, Kyiv, Ukraine, 03041
***Sumy State Pedagogical University named after A.S. Makarenko
Romenska str., 87, Sumy, Ukraine, 40002

Received 28.11.2025
Received in revised form 23.01.2026
Accepted 09.02.2026
Published 27.02.2026

***How to Cite:** Volina, T., Pylypaka, S., Rogovskii, I., Kalenyk, M., Ploskyi, V., Ausheva, N., Shoman, O., Babka, V., Tatsenko, O., Korzh-Usenko, L. (2026). Construction of a generalized mathematical model for particle sliding on the surface of a rotating vertical straight helicoid. Eastern-European Journal of Enterprise Technologies, 1 (7 (139)), 61–69. https://doi.org/10.15587/1729-4061.2026.352152*

1. Introduction

Screw conveyors are widely used for transporting bulk technological materials. The material is lifted by rotating the screw surface in a stationary cylindrical casing. If we consider the transportation of a single particle, this process can be divided into two stages: sliding of the particle along the screw surface until it meets the casing and lifting of the particle while simultaneously sliding along two surfaces. Given that the movable screw surface, that is helicoid, can be closed or open, the task to mathematically describe the motion of the particle arises, taking this circumstance into account.

Scientific research into this area is important because it makes it possible to determine the kinematic parameters of the particle at the first stage of its movement along the working body. The results of such studies are needed in practice as they allow us to define parameters of the particle lift at the second stage and compare them for different working surfaces. Screw working bodies can be elastic [1], used for mixing [2], or transporting materials [3]. Despite the widespread use of classical closed helicoids, the lack of a generalized mathematical model of particle motion on other helical surfaces significantly limits the possibilities of finding alternative engineering solutions. In view of this, the related topic of scientific research is relevant.

2. Literature review and problem statement

Improving the efficiency of screw conveyors is the goal of numerous studies. A significant body of research reports experimental studies of operational parameters. Thus, work [4] describes the results of experimental research into the transportation of agricultural fibrous materials by a horizontal screw conveyor. The authors show the influence of the operating modes of the working body and the characteristics of the material on the productivity of its transportation. However, the questions regarding the analytical description of the motion of individual particles, which could become the basis for further research, remained unresolved. In contrast, in [5], the optimal operating characteristics of a screw conveyor with a vertical axis were experimentally investigated.

The authors of [4, 5] limit themselves to the horizontal or vertical arrangement of the conveyor axis and its structure in the form of a closed helicoid. The reason is the fundamental difference in the patterns of particle motion along the conveyor surface when changing the position of its axis from horizontal to vertical, as well as when changing its geometry.

A stationary screw conveyor with a vertical axis for transporting granular fertilizers is considered in [6]. The paper uses the discrete element method to model particle motion, which makes it possible to evaluate different designs of screw conveyors. The authors emphasize that the geometry of the working body significantly affects the efficiency of material transportation. However, an analytical description of the process of particle motion along the working body was ignored. A likely reason is objective difficulties associated with the chosen research method. An option for overcoming such difficulties is the use of a differential geometry apparatus.

A similar research method was chosen in [7]. The authors identified the discrete element method as an effective tool for determining and optimizing the design and operational parameters of screw conveyors. However, issues related to controlling particle behavior by changing the structural features of the working body remained unresolved. A likely reason is objective difficulties associated with the complexity of such a process using the traditional approach.

The authors of work [8] raise the issue of solving the inverse problem: changing the design of the working body to achieve the required trajectory of particle movement. The problem is solved using the discrete element method, which does not make it possible to obtain an analytical description of the material transportation process. An option for overcoming these difficulties is to construct fundamental mathematical models of particle movement using other methods.

This approach is used in [9, 10], in which a mathematical model of material particle transportation by a stationary vertical screw limited by a coaxial cylinder (casing) was built. In work [9], the upward transportation of material was investigated, and in [10] – downward. The authors derived differential equations of particle motion, but the results cannot be applied to the working body in the absence of a limiting casing or for the case of an open helicoid.

A similar limitation relates to paper [11], which considers the design of a screw chute with a non-expandable (oblique) helicoid as the working surface using the Darboux trihedron. The resulting mathematical apparatus makes it possible to achieve the required particle velocity along the surface of the oblique helicoid by changing its design parameters. However, the results of the study are not extended to other screw surfaces.

In [12], a methodology for calculating the basic design parameters of a screw working body was devised by using analytical and differential geometry methods. The resulting methodology is not generalized but the authors indicate that it could be used in further studies of screw conveyors.

The issues of structural synthesis and reliability are considered in the following series of studies. In [13], the transportation of material by a screw conveyor with a vertical and inclined position of its axis was investigated. However, there is no analytical justification of the material movement at the level of differential equations. This may be due to the fact that the authors' attention was focused on three-dimensional modeling of the transportation process.

Similar in logic, although with a different purpose, is work [14]. It investigated the wear resistance of screw blades and proposed methods for applying a protective coating, but the geometric parameters of the screw surface itself as a factor of efficiency are not considered.

In [15], a generalized mathematical model of the behavior of granular material in pneumatic conveyors was constructed. The results make it possible to assess the efficiency of the process, but they cannot be extended to other structures of working bodies, in particular screw ones. This is due to fundamental differences in the physical nature of the interaction of particles with the working body. However, the analysis of the processes of "hanging" or "sticking" of the material can be used in other mathematical models.

A separate body of research reports experimental studies on the transportation of bulk material by screw conveyors. The study of the dynamic characteristics of a vertical screw conveyor with a variable profile of the screw turns using the discrete element method, which is an analog of the experiment with material particles, was carried out in [16]. In work [17], a special shape of the screw with an inclined periphery in a vertical screw conveyor was investigated. However, no significant advantage over a standard screw was found. An experimental study of the operation of a screw conveyor with a feeder was carried out in [5]. The authors showed that the optimal operating modes depend on both the type of material and the ratio of the speeds of the screw and the feeder.

The conveyor's productivity does not always increase with increasing screw rotation speed. It initially increases to a certain value and then tends to slow down or even decrease at excessively high speeds. For the simultaneous transportation and mixing of bulk materials, a design of a screw conveyor with a rotating casing is presented in [18]. Experimental studies on the influence of various parameters on the performance and energy consumption of a horizontal screw conveyor during the transportation of rice are considered in [19]. With increasing screw rotation speed, the performance initially increased, reaching a maximum, and then began to decrease, which indicates its optimal value. Replacing a traditional screw spiral with a shaft with inclined flat blades is considered in [20]. It is concluded that a shaft with flat inclined blades can be more efficient under some operating modes than a conventional spiral, while the blades are easier to manufacture than a complex screw spiral.

Our review revealed the absence of a generalized mathematical model that describes the sliding of a particle along different types of helicoids: both closed and open. This creates difficulties in optimizing the designs of screw conveyors for specific technological requirements. All this allows us to state that it is advisable to conduct a study aimed at providing the mathematical integrity in the description of screw systems.

3. The aim and objectives of the study

The aim of our research is to build a generalized mathematical model of the complex motion of a particle on the surface of a vertical straight helicoid rotating around its own axis. This will make it possible to optimize the structure of a screw conveyor for specific technological requirements.

To achieve this goal, the following tasks were set:

– based on the law of dynamics of a material point in complex motion, derive differential equations of the motion of a particle on the surface of a vertical straight helicoid rotating around its own axis;

– to perform numerical integration of the resulting equations and construct the trajectories of the particle sliding along the helicoids, including the case when they are stationary.

4. Materials and methods

The object of our study is the complex motion of a particle on the surface of a vertical straight helicoid, which rotates around its own axis. The principal hypothesis assumes that a generalized model of the complex motion of a particle on the surface of a vertical straight helicoid could make it possible to identify the best option among its modifications.

The assumption is that the complex system of differential equations based on the mathematical model can be reduced to a form suitable for numerical integration. The simplification of the study was that when building the mathematical model, the air resistance to the movement of the particle was not taken into account.

A straight helicoid can have two modifications: closed (Fig. 1, *a*, *b*) and open (Fig. 1, *c*, *d*). In both cases, the rectilinear generatrix of the surface is perpendicular to its axis.

The parametric equations of an open straight helicoid take the following form:

$$X = r\cos\alpha - u\sin\alpha;$$

$$Y = r\sin\alpha + u\cos\alpha;$$

$$Z = R\alpha\,\mathrm{tg}\beta, \qquad (1)$$

where r and R are the radii of the inner and outer cylinders bounding the surface; β is the angle of elevation of the outer helix (Fig. 2), α and u are independent surface variables. At $r = 0$, equation (1) describes the surface of a straight closed helicoid. The independent variables have a physical meaning: α is the angle of rotation of the radius vector of a point on the surface around the axis of the helicoid; u is the distance along the straight generating surface from the zero value (for a closed helicoid – from its axis).

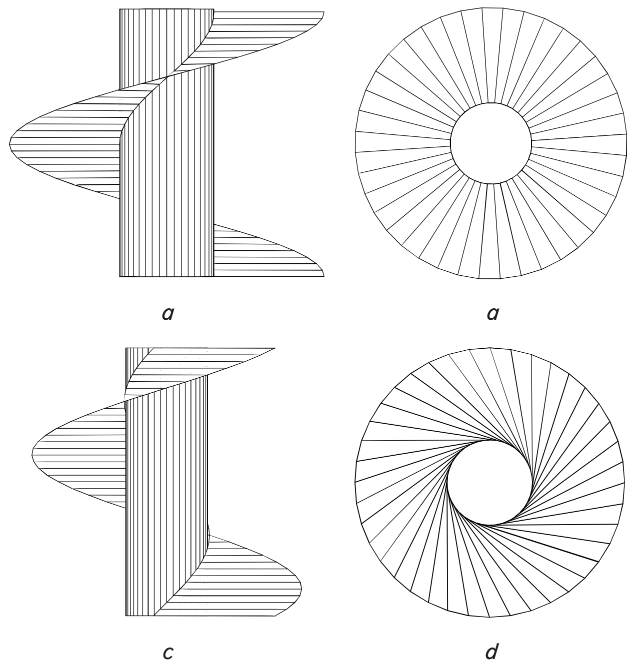

Fig. 1. Modifications of the straight helicoid: *a* — frontal projection of a closed helicoid; *b* — horizontal projection of a closed helicoid; *c* — frontal projection of an open helicoid; *d* — horizontal projection of an open helicoid

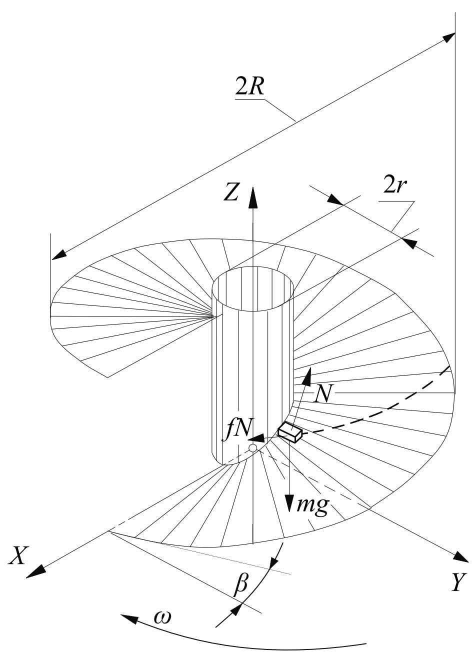

Fig. 2. Designation of the structural parameters of a closed helicoid and the forces applied to the particle

The differential equations of motion of the particle were derived according to the well-known formula, which is written in vector form $m\overline{w} = \overline{F}$, where m is the mass of the particle;

$\overline{w}$ is the vector of absolute acceleration; $\overline{F}$ is the resultant vector of the forces applied to the particle. All vectors will be determined in projections onto the axis of a fixed coordinate system. The vectors of the forces applied to the particle are: the weight force $m{\cdot}g$ ($g = 9.81$ m/s^2 – acceleration of free fall), the surface reaction N and the friction force fN (f – friction coefficient). The weight force is directed downward, the surface reaction – along the normal to the surface, and the friction force – in the opposite direction from the direction of sliding of the particle. The forces applied to the particle are shown in Fig. 2, which also depicts the probable trajectory of sliding of the particle with a dashed line.

The use of a purely theoretical approach at this stage is due to the need to establish fundamental analytical dependences, which are of primary importance for understanding the physics of the process on open helical surfaces. The resulting analytical solutions make it possible to avoid costly field tests at the conceptual design stage, providing a basis for designing new types of energy-efficient conveyors without a casing.

5. Mathematical description of the complex motion of a particle on the surface of a vertical straight rotating helicoid

5. 1. Derivation of differential equations of relative motion of a particle on the surface of a vertical straight helicoid

When establishing a certain dependence between the independent variables of the surface α and u, a line is described on the surface of the helicoid. Let such a dependence be established through parameter t – the time of sliding of the particle on the surface. In this case, the internal equation of the relative trajectory of the particle is described by dependences $u = u(t)$, $\alpha = \alpha(t)$.

The relative trajectory of the particle (that is, the sliding trajectory) will be equations (1) under the condition that $u = u(t)$, $\alpha = \alpha(t)$. These dependences are unknown and must be found as a result of solving the differential equations that need to be compiled.

Differentiated equations (1) with respect to parameter t, taking into account the fact that $u = u(t)$, $\alpha = \alpha(t)$, describe not the surface but the line on it – the trajectory of the particle's sliding. In view of this, the derivatives, which are projections of the particle's relative velocity, are denoted by lowercase letters:

$$x' = -u'\sin\alpha - \alpha'\left(r\sin\alpha + u\cos\alpha\right);$$
$$y' = u'\cos\alpha + \alpha'\left(r\cos\alpha - u\sin\alpha\right);$$
$$z' = R\alpha'\,\mathrm{tg}\beta. \qquad (2)$$

The geometric sum of projections (2) will give the value of the relative velocity

$$V = \sqrt{x'^2 + y'^2 + z'^2} = \sqrt{\left(u'^2 + r\alpha'^2\right)^2 + \alpha'^2\left(u^2 + R^2\mathrm{tg}^2\beta\right)}. \qquad (3)$$

In order for a particle to slide along the surface while simultaneously rising upwards, the surface must rotate clockwise with an angular velocity ω, as indicated in Fig. 2. Let the angle of rotation of the surface be denoted by φ, that is, $\varphi = -\omega t$. Using the known rotation formulas, the parametric equations of the surface after its rotation by an angle φ are written:

$$X = \left(r\cos\alpha - u\sin\alpha\right)\cos\varphi - \left(r\sin\alpha + u\cos\alpha\right)\sin\varphi;$$
$$Y = \left(r\cos\alpha - u\sin\alpha\right)\sin\varphi + \left(r\sin\alpha + u\cos\alpha\right)\cos\varphi;$$
$$Z = R\alpha\,\mathrm{tg}\beta. \qquad (4)$$

After simplifications taking into account $\varphi = -\omega t$, equation (4) takes the form:

$$x = u\sin\left(\omega t - \alpha\right) + r\cos\left(\omega t - \alpha\right);$$
$$y = u\cos\left(\omega t - \alpha\right) - r\sin\left(\omega t - \alpha\right);$$
$$z = R\alpha\,\mathrm{tg}\beta. \qquad (5)$$

If we take into account that $u = u(t)$ and $\alpha = \alpha(t)$, then equations (5) describe a line on the surface. This line is the absolute trajectory of the particle since it includes two motions: the sliding of the particle on the surface and the rotation of the surface. Therefore, the left-hand sides of the equations are denoted by lowercase letters.

The absolute acceleration of the particle can be obtained by successive differentiation of the absolute trajectory (5) with respect to time t. The first derivative is the projection of the absolute velocity of the particle:

$$x' = \left[u' - r\left(\omega - \alpha'\right)\right]\sin\left(\omega t - \alpha\right) + u\left(\omega - \alpha'\right)\cos\left(\omega t - \alpha\right);$$
$$y' = \left[u' - r\left(\omega - \alpha'\right)\right]\cos\left(\omega t - \alpha\right) - u\left(\omega - \alpha'\right)\sin\left(\omega t - \alpha\right);$$
$$z' = R\alpha'\,\mathrm{tg}\beta. \qquad (6)$$

Differentiation (6) makes it possible to obtain projections of absolute acceleration:

$$x'' = \left[\left(\omega - \alpha'\right)\left(2u' - r(\omega - \alpha')\right) - u\alpha''\right]\cos\left(\omega t - \alpha\right) + \left[\left(u'' - u(\omega - \alpha')^2 + r\alpha''\right)\right]\sin\left(\omega t - \alpha\right);$$
$$y'' = -\left[\left(\omega - \alpha'\right)\left(2u' - r(\omega - \alpha')\right) - u\alpha''\right]\sin\left(\omega t - \alpha\right) + \left[\left(u'' - u(\omega - \alpha')^2 + r\alpha''\right)\right]\cos\left(\omega t - \alpha\right);$$
$$z'' = R\alpha''\,\mathrm{tg}\beta. \qquad (7)$$

It is necessary to find the projections of the applied forces. The particle's weight vector is directed vertically downwards. In this case, the unit direction vector of this force in the projections onto the axis of the fixed coordinate system will be written as

$$\left\{0;0;-1\right\}. \qquad (8)$$

The second applied force – the reaction $\overline{N}$ of the helicoid, is directed along the normal to the surface. Its direction of action is determined by the vector product of two vectors tangent to the coordinate lines at the point of the particle. These two vectors are partial derivatives of equations (1):

$$X'_\alpha = -r\sin\alpha - u\cos\alpha;$$

$$Y'_\alpha = r\cos\alpha - u\sin\alpha;$$

$$Z'_\alpha = r\mathrm{tg}\beta;$$

$$X'_u = -\sin\alpha;$$

$$Y'_u = \cos\alpha;$$

$$Z'_u = 0. \qquad (9)$$

In derivatives (9), the subscript below indicates the variable with respect to which differentiation was performed. After vector multiplication of vectors (9), its projections will be written as

$$\{R\mathrm{tg}\beta\cos\alpha; R\mathrm{tg}\beta\sin\alpha; u\}. \qquad (10)$$

It should be borne in mind that the surface normal vector can be directed in one or the opposite direction from the point on the surface depending on the order of alternation of the strips of vectors (9) in the determinant from which the surface normal is determined. It is important to choose the desired direction here. Since the particle's weight is directed downwards, the surface reaction must be directed so that its component in the vertical direction has the opposite sign, that is, is positive.

Vector (10) is applied to the surface provided that it is stationary. Since the surface rotates by an angle $\varphi = -\omega t$ during time t, then vector (10) must be rotated by this angle so that it corresponds to the point of location of the particle on the surface. After rotating vector (10) by the specified angle and reducing it to unity, we obtain:

$$N_x = \frac{R\cos(\omega t - \alpha)\sin\beta}{\sqrt{R^2\sin^2\beta + u^2\cos^2\beta}};$$

$$N_y = -\frac{R\sin(\omega t - \alpha)\sin\beta}{\sqrt{R^2\sin^2\beta + u^2\cos^2\beta}};$$

$$N_z = \frac{u\cos\beta}{\sqrt{R^2\sin^2\beta + u^2\cos^2\beta}}. \qquad (11)$$

The third friction force fN is directed in the opposite direction of the relative velocity V of the particle. This means that the direction of the force is determined by projections (2) taken with the opposite sign. Their division by the velocity modulus (3) gives the unit vector. After rotation by an angle $\varphi = -\omega t$, one can finally obtain its projections onto coordinate axis

$$\left\{\begin{array}{l}\dfrac{(u' + r\alpha')\sin(\omega t - \alpha) - u\alpha'\cos(\omega t - \alpha)}{\sqrt{\left(u'^2 + r\alpha'^2\right)^2 + \alpha'^2\left(u^2 + r^2\mathrm{tg}^2\beta\right)}};\\ \dfrac{(u' + r\alpha')\cos(\omega t - \alpha) + u\alpha'\sin(\omega t - \alpha)}{\sqrt{\left(u'^2 + r\alpha'^2\right)^2 + \alpha'^2\left(u^2 + r^2\mathrm{tg}^2\beta\right)}};\\ \dfrac{R\alpha'\mathrm{tg}\beta}{\sqrt{\left(u'^2 + r\alpha'^2\right)^2 + \alpha'^2\left(u^2 + r^2\mathrm{tg}^2\beta\right)}}.\end{array}\right\} \qquad (12)$$

The unit vectors of the forces applied to the particle are defined, so we can write the differential equations of its motion. It is acted upon by the weight force mg in the direction of vector (8), the surface reaction N in the direction of vector (11), and the friction force fN in the direction of vector (12). The vector equation $mw = F$ must be written in projections onto coordinate axes, taking into account the fact that the projections of the absolute acceleration vector w are given in expressions (7):

$$mx'' = N\frac{R\cos(\omega t - \alpha)\sin\beta}{\sqrt{R^2\sin^2\beta + u^2\cos^2\beta}} - $$
$$-fN\frac{(u' + r\alpha')\sin(\omega t - \alpha) - u\alpha'\cos(\omega t - \alpha)}{\sqrt{\left(u'^2 + r\alpha'^2\right)^2 + \alpha'^2\left(u^2 + r^2\mathrm{tg}^2\beta\right)}};$$

$$my'' = -\frac{R\sin(\omega t - \alpha)\sin\beta}{\sqrt{R^2\sin^2\beta + u^2\cos^2\beta}} - $$
$$-fN\frac{(u' + r\alpha')\cos(\omega t - \alpha) + u\alpha'\sin(\omega t - \alpha)}{\sqrt{\left(u'^2 + r\alpha'^2\right)^2 + \alpha'^2\left(u^2 + r^2\mathrm{tg}^2\beta\right)}};$$

$$mz'' = -mg + N\frac{u\cos\beta}{\sqrt{R^2\sin^2\beta + u^2\cos^2\beta}} - $$
$$-fN\frac{R\alpha'\mathrm{tg}\beta}{\sqrt{\left(u'^2 + r\alpha'^2\right)^2 + \alpha'^2\left(u^2 + r^2\mathrm{tg}^2\beta\right)}}. \qquad (13)$$

Substituting into (13) the absolute acceleration (7) and solving the system with respect to the second derivatives of the unknown functions $\alpha = \alpha(t)$, $u = u(t)$, and $N = N(t)$, gives the result:

$$\alpha'' =$$
$$= -\frac{\cos\beta}{B^2}\frac{u\cos\beta(\omega - \alpha')\left[2u' - r(\omega - \alpha')\right] - Rg\sin\beta}{\sqrt{R^2\sin^2\beta + u^2\cos^2\beta}} - $$
$$-f\frac{A\alpha'}{VB};$$

$$u'' =$$
$$= \frac{Cu(\omega - \alpha')^2 - 2ruu'(\omega - \alpha')\cos^2\beta + Rrg\cos\beta\sin\beta}{B^2} - $$
$$-f\frac{Au'}{VB};$$

$$N = \frac{mA}{B}, \qquad (14)$$

where, for the compactness of writing equations (14), the following designations of individual expressions are adopted:

$$A = ug\cos\beta + R(\omega - \alpha')\left[2u' - r(\omega - \alpha')\right]\sin\beta;$$

$$B = \sqrt{R^2\sin^2\beta + u^2\cos^2\beta};$$

$$C = \left(r^2 + u^2\right)\cos^2\beta + R^2\sin^2\beta;$$

V – according to expression (3). (15)

At $\beta = 0$ and $r = 0$, the partial case of equations (14), which describe the sliding of a particle along a rough horizontal disk rotating around a vertical axis, takes the form:

$$\alpha''=\frac{2u'}{u}\left(\omega-\alpha'\right)-f\frac{g\alpha'}{\sqrt{u'^2+u^2\alpha'^2}};$$

$$u''=u\left(\omega-\alpha'\right)^2-f\frac{gu'}{\sqrt{u'^2+u^2\alpha'^2}};$$

$$N=mg. \qquad (16)$$

Equations (14) are described in the polar coordinate system and are known from the classical examples in the dynamics section of the theoretical mechanics course.

5. 2. Construction of particle sliding trajectories along a straight vertical helicoid: closed and open

The system of three equations (14) is reduced to solving the system of the first two equations, and the third equation (reaction N of the surface) becomes determined after finding dependences $u = u(t)$ and $\alpha = \alpha(t)$. Solving the system of the first two equations (14) requires the use of numerical methods. It is assumed that the friction coefficient $f = 0.3$. In Fig. 3, *a*, the particle sliding trajectories along the surface of a closed helicoid ($r = 0$), which rotates with different angular velocities including a fixed surface ($\omega = 0$), are constructed. The depicted surface section is limited by two radii $r = 0.03$ m and $R = 0.1$ m, with the elevation angle of the outer limiting helix $\beta = 15°$. The sliding trajectories are plotted for $t = 0.1$ s from the moment of the start of movement from the internal limiting helix. During this time, the particle goes beyond the surface compartment except for the particle that descends along the fixed helicoid under the action of its own weight. In Fig. 3, *b*, plots of changes in the Z coordinate for this period are constructed. When sliding along a fixed surface, the particle descends downwards and then stops. This can be verified by constructing a plot of the relative velocity according to dependence (3). This is explained by the fact that as the particle moves away from the surface axis, the angle of greatest inclination of the tangential plane to the surface at the point of its location decreases. When it becomes smaller than the friction angle, the particle stops after inertial movement. On a rotating surface, the particle slides along it, rising upwards, and with an increase in the angular velocity of rotation, the height of the rise increases. However, it should be borne in mind that the rise or fall of a particle depends on angle β and angular velocity ω of the helicoid rotation.

In Fig. 4, the same plots are constructed for a straight open helicoid at r=0.03 m. It should be borne in mind that at the value of $r = 0$ in equations (1) the surface is a straight closed helicoid, and at $r = 0.03$ m – a straight open helicoid. However, in both cases the particle motion begins at a distance of 0.03 m from the axis of rotation, which is given under the initial conditions of integration of equations (14).

From the comparison of Fig. 3, 4, we can conclude that under the same conditions the particle rises to a greater height on the surface of a straight closed helicoid. Table 1 gives values for the kinematic parameters of particle motion along straight closed and straight open helicoids.

Analysis of the numerical simulation results given in Table 1 makes it possible to compare the dynamics of particle lift for two types of helicoids. It was found that under the same conditions ($\omega = 0$ s^{-1}, $t = 0.1$ s) the lift height on a closed helicoid is 0.032 m, while on an open one – 0.022 m. The quantitative difference is 31.25%. Increasing the angular velocity to $\omega = 20$ s^{-1} on an open helicoid makes it possible to increase the lift height to 0.058 m, which is 41.4% higher than that of a closed helicoid (0.082 m) due to the lower velocity.

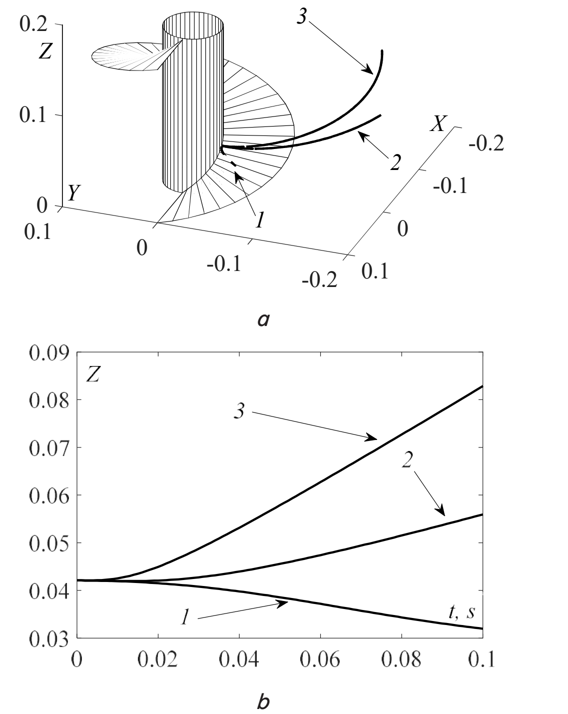

Fig. 3. Kinematic parameters of particle motion along a straight closed helicoid for different angular velocities of its rotation (1 – $\omega = 0$; 2 – $\omega = 10$ s^{-1}; 3 – $\omega = 20$ s^{-1}): *a* – relative trajectory of particle motion (trajectory of sliding on the surface); *b* – plot of particle rise (fall)

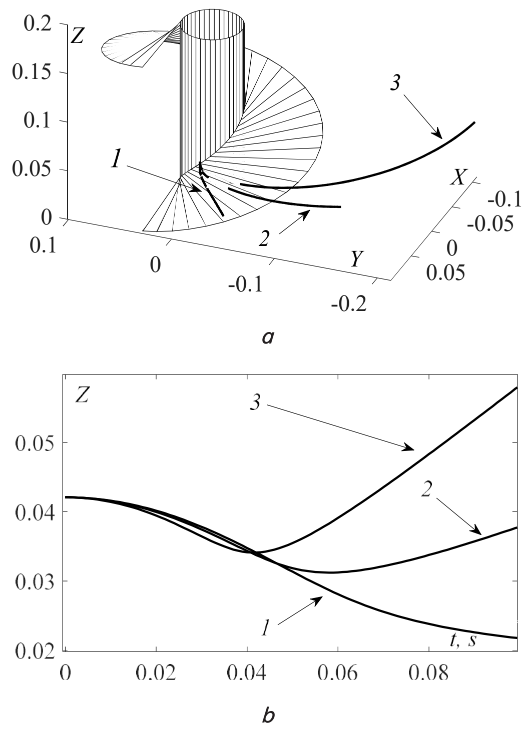

Fig. 4. Kinematic parameters of particle motion along a straight open helicoid for different angular velocities of its rotation (1 – $\omega = 0$; 2 – $\omega = 10$ s^{-1}; 3 – $\omega = 20$ s^{-1}): *a* – relative trajectory of particle motion (trajectory of sliding on the surface); *b* – plot of particle rise (fall)

Fig. 5 plots the trajectories of particle sliding along the surface of open and closed helicoids, which rotate with an angular velocity of $\omega = 20$ s^{-1} for 1 s, provided that they are not limited by a coaxial cylindrical surface.

Table 1

Kinematic parameters of particle motion along straight closed and straight open helicoids

Surface	Motion time, s	External helix elevation angle, degrees	Angular velocity, s^{-1}	Lift height, m
Straight closed helicoid (Fig. 3)	0,1	15	0	0.032
			10	0.056
			20	0.082
Straight closed helicoid (Fig. 3)			0	0.022
			10	0.038
			20	0.058

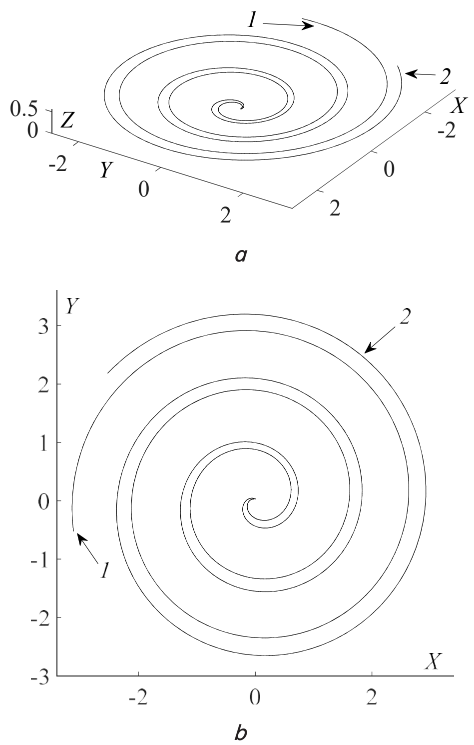

Fig. 5. Trajectories of particle sliding on the surface of straight helicoids rotating with angular velocity $\omega = 20$ s^{-1} (1 – on closed, 2 – on open helicoids): *a* – axonometric image; *b* – horizontal projection

From Fig. 5 we can conclude that there is no fundamental difference in the nature of the particle sliding on the surfaces of closed and open helicoids. However, the main component of the particle lift occurs after its encounter with a stationary cylindrical casing. In this case, the particle slides simultaneously on both surfaces and rises upwards. This process requires separate studies.

6. Discussion of results based on constructing a mathematical model of particle sliding on the surfaces of closed and open rotating helicoids

In engineering practice, a technical helical surface is understood as a straight closed helicoid (screw). Meanwhile, there are various modifications of helicoids. The proposed equation (1) combines two types of helicoid: closed ($r = 0$) and open ($r \neq 0$). All further calculations were based on the parametric equations of the surface. The direction of the applied forces (the surface reaction N, directed normal to the surface, and the friction force fN, directed in the opposite direction to the direction of particle sliding) depended on them. In works [10, 11], all calculations were oriented to a specific surface – a straight closed helicoid. Accordingly, the reported results concerned this surface.

The advantages of our study are the mathematical description of the surface by parametric equations, as a result of which the surface belongs to two families of coordinate lines. One family of coordinate lines is the surface rectilinear generatrixes – u-lines, and the other – helical lines, or α-lines. Establishing a connection between these coordinate lines in the form of dependences $u = u(t)$ and $\alpha = \alpha(t)$, where t is time, makes it possible to move from the equations of the surface to the equations of the line on it, which is taken as the sliding trajectory. Thus, a transition is made from two independent variables u and α of the surface to one variable t, due to which the equations of the surface are transformed into equations of the line on it. After this, it becomes possible to determine the projections of relative velocity (2), projections of absolute velocity (6), and projections of absolute acceleration (7). These expressions, as well as the expressions of direction vectors (11), (12) of the direction of action of the applied surface reaction forces and friction, are sufficient to compose the differential equations of complex particle motion (14).

The resulting differential equations of the complex motion of a particle on the surface (1) were solved using numerical integration methods. As a result, it was found that the trajectories of the particle sliding on the surface of a closed helicoid (Fig. 3) and on the surface of an open helicoid (Fig. 4) do not have significant differences. This is also confirmed by the trajectories in Fig. 5. The practical significance of our results should be assessed as the first stage of a theoretical study on the sliding of a particle on the surfaces of helicoids. The main result can be obtained at the second stage, when the particle simultaneously interacts with the surface of the helicoid and the cylindrical casing, that is, to find out for which modification of the helicoid the particle lifting speed will be higher. Analysis of experimental studies revealed that they concern screw conveyors with a conventional helical surface or a modified one with the mandatory presence of a casing. They concern the transportation not of a single particle but of their flow. This is the difference between the theoretical studies conducted.

The limitations of our study are within certain limits of the angular velocities of rotation of the screw. Theoretically, the sliding speed of the particle increases with increasing angular velocity of rotation of the helicoid. But, firstly, too high angular velocities are harmful to the dynamics of the conveyor, and secondly, experiments show that the reverse process of reducing the sliding speed may begin.

The disadvantage of the study is that the movement of a single particle does not take into account the interaction with other particles. Advancing our study involves finding the kinematic characteristics of the movement of a particle when it encounters a stationary cylindrical casing. Such a process of interaction of a particle with two surfaces underlies the transportation of bulk material by vertical screw conveyors.

7. Conclusions

1. The helical surface of a straight helicoid is referred to parametric equations in such a way that they describe a

closed helicoid at $r = 0$ or an open one at $r \neq 0$. Establishing a connection between the independent variables of the surface u and α in the form of dependences $u = u(t)$ and $\alpha = \alpha(t)$, where t is time, makes it possible to move from the equations of the surface to the equations of a line on it, which is taken as the sliding trajectory. The system of second-order differential equations is composed in projections onto the axis of a fixed coordinate system. The system was solved by numerical methods, as a result of which dependences $u = u(t)$, $\alpha = \alpha(t)$, as well as the surface reaction $N = N(t)$, are found. If the angular velocity of rotation of the helicoid is zero, then the equations describe the sliding of a particle on a fixed surface under the action of its own weight. At $r = 0$ and $\beta = 0$, where β is the angle of elevation of the surface helix, the parametric equations describe the horizontal plane referred to the polar coordinate system. Accordingly, the differential equations describe the trajectory of a particle sliding along a horizontal disk rotating around a vertical axis.

2. As a result of solving differential equations by numerical methods, we constructed the trajectories of particle sliding along the surface of closed and open vertical helicoids rotating around their own axis. In both cases, the angle of elevation of the outer limiting helix was $\beta = 15°$. The trajectories were constructed at a friction coefficient $f = 0.3$ and angular rotation speeds $\omega = 0$, $\omega = 10$ s^{-1}, $\omega = 20$ s^{-1} for 1 second. It was found that the nature of the trajectories of sliding along open and closed helicoids is similar but there are quantitative differences in the dynamics of lifting. In particular, it was found that on a closed helicoid, due to the initial stage of lowering the particle, the average speed of its upward transport is 5% lower compared to an open helicoid. If the helicoids are stationary, the particle begins to descend down the surface, moving away from the helicoid axis and eventually stops. This is explained by the fact that as the particle moves away from the surface axis, the angle of greatest inclination of the tangential plane to the surface at the point of its location decreases. When it becomes smaller than the friction angle, the particle stops after inertial motion. The convergence of the results of numerical modeling with the classical provisions in the mechanics of bulk media confirms the adequacy of our model and the possibility of its application for optimizing energy-efficient transport systems without a casing, with a potential reduction in the metal content of structures by 15–20%.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study, as well as the results reported in this paper.

Funding

The study was conducted without financial support.

Data availability

All data are available, either in numerical or graphical form, in the main text of the manuscript.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

Acknowledgments

The authors of this paper express their gratitude to the defenders of Ukraine for the opportunity to live and engage in scientific activities.

Authors' contributions

Tetiana Volina: Writing – original draft, Project administration; **Serhii Pylypaka:** Conceptualization, Supervision; **Ivan Rogovskii:** Methodology, Software; **Mykhailo Kalenyk:** Formal analysis, Data Curation; **Vitalii Ploskyi:** Methodology, Writing – review & editing; **Natalia Ausheva:** Resources, Writing – review & editing, **Olga Shoman:** Validation, Writing – review & editing; **Vitaliy Babka:** Investigation, Data Curation; **Oleksandr Tatsenko:** Validation, Visualization; **Larysa Korzh-Usenko:** Writing – review & editing, Visualization.

References

1. Diachun, A., Gevko, I., Lyashuk, O., Stanko, A., Pik, A., Omelyanskyi, Y. (2024). Study of fiber deformation of elastic brush-like screws during grain material transportation. INMATEH Agricultural Engineering, 72 (1), 579–588. https://doi.org/10.35633/inmateh-72-51
2. Novitskiy, A., Banniy, O., Novitskyi, Y., Antal, M. (2023). A study of mixer-feeder equipment operational reliability. Machinery & Energetics, 14 (4), 101–110. https://doi.org/10.31548/machinery/4.2023.101
3. Minglani, D., Sharma, A., Pandey, H., Dayal, R., Joshi, J. B., Subramaniam, S. (2020). A review of granular flow in screw feeders and conveyors. Powder Technology, 366, 369–381. https://doi.org/10.1016/j.powtec.2020.02.066
4. Wulantuya, Wang, H., Wang, C., Qinglin. (2020). Theoretical analysis and experimental study on the process of conveying agricultural fiber materials by screw conveyors. Engenharia Agrícola, 40 (5), 589–594. https://doi.org/10.1590/1809-4430-eng.agric.v40n5p589-594/2020
5. Mei, X., Xue, Y., Zhang, L. (2022). Determination of the optimal working performance matching through theoretical analysis and experimental study for a screw conveyor. PLOS ONE, 17 (6), e0266948. https://doi.org/10.1371/journal.pone.0266948
6. Moelder, K., Lillerand, T. (2025). Design and feasbility analysis of vertical static flight screw conveyor usage in granulated fertilizer transportation. 24th International Scientific Conference Engineering for Rural Development Proceedings, 24. https://doi.org/10.22616/erdev.2025.24.tf090
7. Karwat, B., Rubacha, P., Stańczyk, E. (2020). Simulational and experimental determination of the exploitation parameters of a screw conveyor. Eksploatacja i Niezawodność – Maintenance and Reliability, 22 (4), 741–747. https://doi.org/10.17531/ein.2020.4.18

8. Wenwu, Y., Longyu, F., Xiwen, L., Hui, L., Yangqing, Y., Zhanhao, L. (2020). Experimental study of the effects of discharge port parameters on the fertilizing performance for fertilizer distribution apparatus with screw. Transactions of the Chinese Society of Agricultural Engineering, 36 (17). https://doi.org/10.11975/j.issn.1002-6819.2020.17.001
9. Pylypaka, S., Volina, T., Hryshchenko, I., Dieniezhnikov, S., Rybenko, I. (2022). Mathematical Model of Lifting Particles of Technological Material by Vertical Auger. Advances in Design, Simulation and Manufacturing V, 112–122. https://doi.org/10.1007/978-3-031-06044-1_11
10. Pylypaka, S., Babka, V., Hryshchenko, I., Kresan, T. (2018). Mathematical model of moving particle by vertical screw in stationary mode. Machinery & Energetics, 9 (4), 31–36. Available at: https://technicalscience.com.ua/uk/journals/t-9-4-2018/matyematichna-modyel-pyeryemishchyennya-chastinki-vyertikalnim-shnyekom-pri-statsionarnomu-ryezhimi
11. Kresan, T. A. (2020). Calculation of gravitation descent formed by surface of skew closed helicoid. Machinery & Energetics, 11 (2), 49–57. https://doi.org/10.31548/machenergy2020.02.049
12. Klendii, M., Logusch, I., Dragan, A., Tsvartazkii, I., Grabar, A. (2022). Justification and calculation of design and strength parameters of screw loaders. Machinery & Energetics, 13 (4), 48–59. https://doi.org/10.31548/machenergy.13(4).2022.48-59
13. Bidas, M., Galecki, G. (2021). The concept of a screw conveyor for the vertical transport of bulk materials. Mining Machines, 39 (3), 28–33. https://doi.org/10.32056/KOMAG2021.3.3
14. Tarelnyk, V. B., Konoplianchenko, Ie. V., Gaponova, O. P., Tarelnyk, N. V., Martsynkovskyy, V. S., Sarzhanov, B. O. et al. (2020). Effect of Laser Processing on the Qualitative Parameters of Protective Abrasion-Resistant Coatings. Powder Metallurgy and Metal Ceramics, 58 (11-12), 703–713. https://doi.org/10.1007/s11106-020-00127-8
15. Lytvynenko, A., Yukhymenko, M., Pavlenko, I., Pitel, J., Mizakova, J., Lytvynenko, O. et al. (2019). Ensuring the Reliability of Pneumatic Classification Process for Granular Material in a Rhomb-Shaped Apparatus. Applied Sciences, 9 (8), 1604. https://doi.org/10.3390/app9081604
16. Yuan, J., Li, M., Ye, F., Zhou, Z. (2020). Dynamic characteristic analysis of vertical screw conveyor in variable screw section condition. Science Progress, 103 (3). https://doi.org/10.1177/0036850420951056
17. Rademacher, F. J. C. (1974). Some aspects of the characteristics of vertical screw conveyors for granular material. Powder Technology, 9 (2-3), 71–89. https://doi.org/10.1016/0032-5910(74)85011-4
18. Diachun, A. Y., Dmytriv, O. R., Hevko, B. R., Koval, S. O., Tsapyk, R. P. (2024). Experimental automated equipment of the screw conveyor with the rotating casing for bulk materials mixing. Perspective technologies and devices, 1 (24), 38–44. https://doi.org/10.36910/10.36910/6775-2313-5352-2024-24-06
19. Zareiforoush, H., Komarizadeh, M. H., Alizadeh, M. R., Masoomi, M. (2010). Screw Conveyors Power and Throughput Analysis during Horizontal Handling of Paddy Grains. Journal of Agricultural Science, 2 (2). https://doi.org/10.5539/jas.v2n2p147
20. Bulgakov, V., Trokhaniak, O., Holovach, I., Adamchuk, V., Klendii, M., Ivanovs, S. (2022). Investigation of the performance of a screw conveyor with a working body, made in the form of a shaft with inclined flat blades. INMATEH Agricultural Engineering, 67 (2), 406–411. https://doi.org/10.35633/inmateh-67-41