*This study investigates a gas generator for a hydrogen storage and supply system with a heat-shielding coating. The subject is the properties of the heat-shielding coating of the gas generator for a hydrogen storage and supply system under fire conditions. The properties of such a heat-shielding coating are taken into account by the thermal protection error of the thermodynamic system, which includes the heat-shielding coating and the gas generator wall.*

*To describe the dynamic properties of the thermodynamic system in the frequency domain, amplitude-phase frequency characteristics and their components – amplitude-phase and phase-frequency characteristics – are used. The frequency characteristics of the thermodynamic system have been mathematically described in general form. It is shown that for the characteristic values of parameters for the thermodynamic system, its properties are fully reflected using frequency characteristics in the frequency range $(0 \div 1.0)$ $s^{-1}$ at a bandwidth of $0.02$ $s^{-1}$. The thermal effect of fire on the thermal state of the cavity of the gas generator of the hydrogen storage and supply system is taken into account using exponential-type correlation functions.*

*Thermal interference is "white noise". Under these conditions and using the amplitude-frequency characteristics of the thermodynamic system, mathematical models of the components of the error of the thermal protection of the gas generator were built in general form. As an indicator characterizing the error of the thermal protection of the gas generator, its root mean square value was used. It is shown that for real fire conditions and operation of hydrogen storage and supply systems, the root mean square error of the thermal protection of the gas generator is 5.14°C. In practice, the existence of such an error estimate opens up the possibility for improving the reliability in determining the thermal protection for a gas generator*

*Keywords: hydrogen storage and supply system, thermal protection, thermal protection error*

# DETERMINING A PRIORI ESTIMATE OF ERROR IN THE THERMAL PROTECTION OF GAS GENERATOR IN A HYDROGEN STORAGE AND SUPPLY SYSTEM

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## 1. Introduction

Among all the "green" alternatives, hydrogen, due to its large quantity and various production technologies, is becoming an increasingly viable environmentally friendly energy source [1]. The main key to the full development of hydrogen energy is safe, compact, and efficient hydrogen storage [2]. For this purpose, hydrogen storage systems (HSSs) of various

types are used [3], one of the main requirements for which is to ensure their safe operation. The options for enabling the safe operation of HSS include the use of thermal protection of gas generators of such systems under conditions of fire.

Determining the efficiency of thermal protection for gas generators in a hydrogen storage and supply system necessitates the use of indicators that characterize the level of this efficiency. Such indicators include the error of thermal protection of the gas generator in a hydrogen storage and supply system under the action of thermal factors of a fire. As a result, there is a need to obtain an estimate for the error of thermal protection of the gas generator in the hydrogen storage and supply system. Obtaining such an assessment is of particular importance at the design stage of hydrogen storage and supply systems.

Therefore, research on the thermal protection of gas generators in hydrogen storage and supply systems is relevant.

## 2. Literature review and problem statement

The peculiarities of HSSs are that their number is growing while the areas of their application have a steady tendency to expand. These features are most evident in on-board HSSs. For example, the number of registered cars in North Korea on hydrogen fuel cells (FCEV) is 1 million [4]. Intensive development of trains based on hydrogen systems is underway in Poland, South Korea, the USA, the Czech Republic, Canada, and China [5].

The main problem of hydrogen systems is the issue of safety of hydrogen processes, which is dual. The duality of this problem is due to the fact that the safety of hydrogen systems includes two components – safety associated with the properties of hydrogen and safety associated with the operation of hydrogen systems [6]. The first component is the most studied [7]; it is based mainly on the use of experimental research methods. In particular, this concerns the determination of the level of flammability or the level of explosivity depending on the conditions of use of hydrogen [8]. As a rule, such studies are conducted under laboratory conditions, which are not always adequate in relation to actual conditions.

When operating hydrogen systems, various factors are considered – design defects, damage to system elements, personnel errors, external factors in the form of impacts or collisions [9]. It should be emphasized that when implementing the option of preventing explosions and fires during the operation of hydrogen systems, effective risk assessment is crucial. In [10], the results of the fire hazard analysis of FCEV vehicles using a method based on combining failure mode and effect analysis (FMEA) with a risk assessment matrix are reported. This method is based on the use of hydrogen system decomposition and analysis of subsystems or components according to the required level of analysis. The FMEA results are analyzed at a qualitative level using a risk assessment matrix, which provides visualization for assessing the severity (weight) of each risk. However, such an analysis does not take into account the human factor, the environment and the interaction between subsystems or components.

In [11], based on the method of accident risk assessment for industrial systems (ARAMIS) and computational fluid dynamics (CFD) methods on the example of HSS, which represents a filling station, risk estimates were obtained that do not exceed the level of $5.8 \cdot 10^{-5}$. It should be noted that such

derivation of quantitative risk assessment (QRA) does not provide identification of dangerous factors. Such an approach to obtaining QRA of hydrogen systems is integral and can be used to compare several hydrogen systems by risk level. It should be noted that when defining QRA, it is necessary to take into account the frequency of risks. This circumstance is a significant problem since statistical data on the vast majority of hydrogen systems are practically absent. One of the ways out of this situation is that the frequencies of risks for hydrogen systems can be estimated using data on chemical plants [12].

But there is no justification for such use of statistical data, which necessitates a critical attitude to the results obtained with their help. For HSSs, the most common way to store hydrogen is to use cylinders. Basically, vehicles use cylinders with a working pressure of 35 MPa or 80 MPa [10]. In particular, the Toyota Mirai is equipped with two cylinders with a pressure of 70 MPa. In [10], the results of studies on the consequences of rupture of hydrogen storage cylinders under conditions of fire are reported. Studies on this type are exclusive in nature, and recommendations for the safe operation of cars are intuitive in nature. In [13], data on the conditions under which explosions can occur are given, but a quantitative assessment of the probability of their occurrence is not provided. The methodology for obtaining a quantitative assessment of the risk of rupture of a hydrogen cylinder is described in [14]. Two cylinder charge (SoC) options are considered – SoC = 99% and SoC = 59%, and the number of fatalities per vehicle per year, the cost of each incident and the fire resistance rating (ERR) of the hydrogen tank are used as risk indicators. This approach to obtaining a risk assessment does not make it possible to obtain information regarding the fire and explosion hazard situation, which is characterized for motor vehicles with a temperature of 103°C and a fire growth rate of 5.5 W s$^{-2}$ (according to the concept of the t-squared fire growth curve) [4]. For the risk analysis of FCEV vehicles, all studies mainly use FMECA (Failure Mode, Effects, and Criticality Analysis), FFMEA (Functional Failure Mode and Effects Analysis), FTA (Fault-Tree Analysis), and HAZOP (Hazard and Operability) methods, most of which are qualitative risk analysis based on fuzzy mathematics.

In [15], data on studies of on-board HSS using HAZOP methods in conjunction with FMEA methods are given. Four scenarios are considered: hydrogen refueling, hydrogen storage, hydrogen supply and pumping. It is characteristic of these studies that no data is provided on the reliability of risk assessments. In addition, the influence of hydrogen leakage, diffusion, combustion and modeling conditions, grid size and boundary conditions on the accuracy of numerical models is not considered. The question of the probability of a fire-hazardous situation in HSS is open. One way out in this case is to establish a relationship between fire safety indicators and reliability indicators [16]. In particular, the probability of failure-free operation of its main elements can be used to determine the probability of a flammable environment in HSS. The peculiarity of this approach is that data on HSS reliability indicators are practically absent [17]. One way out of this situation is to use innovative methods to obtain estimates of reliability indicators.

Thus, in [18], the probability of failure-free operation of the main element of the hydrogen storage and supply system – the gas generator – is determined using its amplitude-frequency characteristic. But this approach is charac-

terized by multi-stage, as a result of which errors occur at each stage of its implementation, the values of which are integrated. In [19], to obtain reliability indicators of the gas generator of the hydrogen storage and supply systems, its amplitude-frequency and phase-frequency characteristics are used. Reliability indicators are estimated only at the time of gas generator start-up. These shortcomings are absent if the dependence of pressure on operating time at a random moment in time is used to determine the level of fire hazard of the gas generator of the hydrogen storage and supply system through its reliability indicator [20].

The reliability indicator is determined using the Laplace function, the argument of which is the rate of change of pressure in the gas generator cavity. The magnitude of this rate depends on several factors, in particular, the temperature, which is not controlled. The need to implement the option of controlling thermal processes that occur during the operation of hydrogen systems based on metal hydrides is justified in [2]. Regarding hydrogen systems of other types, it is necessary to conduct appropriate research. Controlling the temperature or pressure in the gas generator cavity of hydrogen storage and supply systems involves the implementation of options for controlling these parameters. Controlling the parameters that determine the technical condition of hydrogen storage and supply systems opens up new opportunities for increasing their fire safety.

Paper [21] provides data on testing and controlling such systems using fire safety subsystems, based on the results of which the fire-safe state of hydrogen storage and supply systems is restored. However, the results of these studies have not been brought to the construction of appropriate algorithms. One of the indicators used in risk assessment is the fire resistance index (limit). Examples of such use are given in [9, 11], in which the task of risk analysis in hydrogen systems is solved. The issue of choosing methods and means that ensure risk reduction in hydrogen systems is not resolved. One way out of this situation is the use of heat-shielding coatings – thermal protection systems (TPSs) [22]. The most significant results in this direction were obtained in the rocket and space industry [23]. TPSs have been designed, which are oriented to use under conditions of exposure to temperatures at the level of (1800÷1900) K and heat fluxes at the level of 100 W·cm$^{-2}$. In 2013, the UN put into effect the GTR#13 fire resistance test protocol, which includes a number of standards [24].

However, the reproducibility of GTR#13 is difficult to ensure, especially for new HSS samples. This circumstance necessitates the determination of effective methods for ensuring the safety of hydrogen systems. Regarding experimental methods, it should be noted that they require large financial costs and cannot always provide an idea of the heat transfer processes. In order to save expensive investments in experimental studies, attention has recently been focused on simulation studies. In [24], an example of such an approach is given using a 3D fire safety model of a hydrogen system, which combines ANSYS FLUENT and ANSYS Mechanical. It is characteristic of numerical methods that the results of the study depend on the grid size.

Analytical research methods involve the use of the integral Laplace transform to solve the Fourier equation. A feature of this approach is that this solution is given in an operator form that includes hyperbolic functions of an irrational complex argument. This circumstance causes significant mathematical difficulties when using the inverse La-place transform. The complexity of the boundary conditions leads to a significant increase in such difficulties. One of the options for getting out of this situation is given in [25], which involves the use of a transfer function to describe the dynamic properties of the heat-shielding coating of the gas generator of the hydrogen storage and supply system. The solution to this situation is associated with the representation of the transfer function of the heat-shielding coating of the gas generator of the hydrogen storage and supply system in the form of a fractional-rational function of a complex variable. When determining the properties of the heat-shielding coating of the gas generator of the hydrogen storage and supply system, it is necessary to take into account the indicators that characterize its efficiency. First of all, such indicators include the metrological characteristics of the heat-shielding coating. In the vast majority of cases, computational and experimental methods are used to determine fire resistance. Generalized data on such methods are given in [26].

It should be noted that when using these methods to obtain fire resistance indicators, the error of the discrepancy between the results of numerical and experimental studies is determined. In fact, the effectiveness of the numerical method for determining fire resistance is defined, but the effectiveness of thermal protection is not specified. The need to conduct experimental studies to determine the error of the discrepancy makes it impossible to use such an approach to determine the effectiveness of thermal protection, in particular, the gas generator of the hydrogen storage system and supply at the design stage.

The results of our analysis provide grounds for conducting research aimed at further developing options for obtaining an estimate of the error of thermal protection of the gas generator of the hydrogen storage system and supply under conditions of fire.

## 3. The aim and objectives of the study

The purpose of our study is to obtain an a priori estimate of error in the thermal protection of a gas generator in a storage and supply system under conditions of fire. In practice, the presence of such an error estimate opens up the possibility for increasing the reliability in determining the thermal protection of a gas generator in a hydrogen storage and supply system.

To achieve this goal, it is necessary to solve the following tasks:

– to obtain a mathematical description of the thermodynamic system, which includes a heat-shielding coating and a wall of the gas generator in a hydrogen storage and supply system, in the frequency domain;

– to build mathematical models for the components of error in the thermal protection for a gas generator in a hydrogen storage and supply system under conditions of fire; to obtain an estimate for this error.

## 4. Materials and methods

The object of our study is the gas generator in a hydrogen storage and supply system with a heat-shielding coating. The subject of the study is the properties of the heat-shielding coating in a hydrogen storage and supply system under conditions of fire. The principal hypothesis assumes that the

heat-shielding coating is considered as an element of a dynamic system, the thermal impact on which is taken into account using the correlation function of the thermal factor of the fire.

The basic assumption is that the dynamic properties of a thermodynamic system are formalized using transfer functions that belong to the class of fractional-rational functions of a complex argument.

To describe the dynamic properties of a thermodynamic system, methods of technical cybernetics and methods from the theory of a complex variable function are used. To transition from the time domain to the complex frequency domain, the integral Laplace transform was used. When constructing mathematical models of errors in the heat-shielding coating for a gas generator in a hydrogen storage and supply system, methods from the theory of stationary random functions, correlation analysis, and methods of statistical dynamics are used. Dynastic or fireclay brick is used as the material for the heat-shielding coating of the gas generator in a hydrogen storage and supply system. The wall of the gas generator in a hydrogen storage and supply system is made of composite material.

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## 5. Results of research on determining a priori assessment of the thermal protection of a gas generator under conditions of fire

### 5. 1. Mathematical description of a thermodynamic system in the frequency domain

The thermal protective coating of a gas generator in a hydrogen storage and supply system together with its wall represents a thermodynamic system, the structural and dynamic diagram of which is shown in Fig. 1.
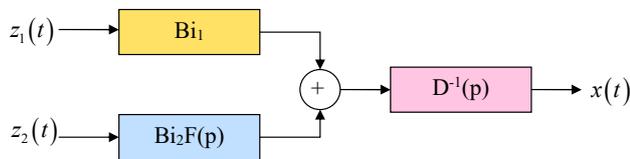


Fig. 1. Structural-dynamic diagram of a thermodynamic system

The signal $z_1(t)$ reflects the influence of thermal factors of fire on the heat-shielding coating, and the signal $z_2(t)$ corresponds to the thermal state of the gas generator cavity. The reaction of the thermodynamic system to the input signals is described by the signal $x(t)$. In this structural-dynamic diagram, $Bi_1$ and $Bi_2$ are the Biot criteria for the heat-shielding coating and for the gas generator wall, respectively; functions $F(p)$ and $D(p)$ take the following form:

$$F(p) = \sum_{i=0}^{2} a_i p^i; \tag{1}$$

$$D(p) = \sum_{i=0}^{3} b_k p^k, \tag{2}$$

where $p$ is a complex variable; $a_i$, $b_k$ are parameters.

For signal $x(t)$, there is an expression

$$x(t) = L^{-1}\left[\sum_{m=1}^{2} W_m(p) z_m(p)\right], \tag{3}$$

where $L^{-1}$ – Laplace inverse transform operator:

$$W_1(p) = Bi_1 D^{-1}(p); \quad W_2(p) = Bi_2 F(p) D^{-1}(p); \tag{4}$$

$$z_m(p) = L\left[z_m(t)\right], \tag{5}$$

$L$ is the Laplace integral transform operator.

The dynamic properties of a thermodynamic system in the frequency domain are determined by its amplitude-phase-frequency characteristics $W_1(j\omega)$ and $W_2(j\omega)$, which, according to (2) and (4), take the following form:

$$W_1(j\omega) = Bi_1\left[\sum_{k=0}^{3} b_k (j\omega)^k\right]^{-1} = N_1(\omega) + jM_1(\omega); \tag{6}$$

$$W_2(j\omega) = Bi_2\left[\sum_{i=0}^{2} a_i (j\omega)^i\right]^{-1}\left[\sum_{k=0}^{3} b_k (j\omega)^k\right]^{-1} = N_2(\omega) + jM_2(\omega), \tag{7}$$

where $j$ – imaginary unit; $\omega$ – circular frequency;

$$N_1(\omega) = Bi_1\left(b_0 - b_2\omega^2\right)\left[\begin{array}{c}\left(b_0 - b_2\omega^2\right)^2 + \\ +\omega^2\left(b_1 - b_3\omega^2\right)^2\end{array}\right]^{-1}; \tag{8}$$

$$M_1(\omega) = -Bi_1\omega\left(b_1 - b_3\omega^2\right)\left[\begin{array}{c}\left(b_0 - b_2\omega^2\right)^2 + \\ +\omega^2\left(b_1 - b_3\omega^2\right)\end{array}\right]^{-1}; \tag{9}$$

$$N_2(\omega) = Bi_2\left[\begin{array}{c}\left(a_0 - a_2\omega^2\right)\left(b_0 - b_2\omega^2\right) + \\ +a_1\omega^2\left(b_1 - b_3\omega^2\right)\end{array}\right] \times$$
$$\times\left[\left(b_0 - b_2\omega^2\right)^2 + \omega^2\left(b_1 - b_3\omega^2\right)^2\right]^{-1}; \tag{10}$$

$$M_2(\omega) = -Bi_2\omega\left[\begin{array}{c}\left(a_0 - a_2\omega^2\right)\left(b_1 - b_3\omega^2\right) - \\ -a_1\left(b_0 - b_2\omega^2\right)\end{array}\right] \times$$
$$\times\left[\left(b_0 - b_2\omega^2\right)^2 + \omega^2\left(b_1 - b_3\omega^2\right)^2\right]^{-1}. \tag{11}$$

For characteristic values of parameters that are equal to, based on [25]:

$Bi_1 = 0.2$; $Bi_2 = 0.3$; $a_0 = 1.2$; $a_1 = 4.5$ s; $a_2 = 4.9$ s$^2$;

$b_0 = 0.56$; $b_1 = 33.1$ s; $b_2 = 97.2$ s$^2$; $b_3 = 90.1$ s$^3$.

Fig. 2 shows graphical dependences (6) and (7), in which the parameter is the circular frequency, the value of which belongs to the range of $(0 \div 2.0)$ s$^{-1}$.

The amplitude-phase-frequency characteristics $W_m(j\omega)$ of a thermodynamic system combine its amplitude-frequency $A_m(\omega)$ and phase-frequency $\varphi_m(\omega)$ characteristics. This combination is described by the following expression

$$W_m(j\omega) = A_m(\omega)\exp\left[j\phi_m(\omega)\right]; \quad m = 1,2, \tag{12}$$

in which:

$$A_m(\omega) = \left[N_m^2(\omega) + M_m^2(\omega)\right]^{0.5}; \tag{13}$$

$$\phi_m(\omega) = \mathrm{arctg}\left[M_m(\omega) N_m^{-1}(\omega)\right]. \tag{14}$$
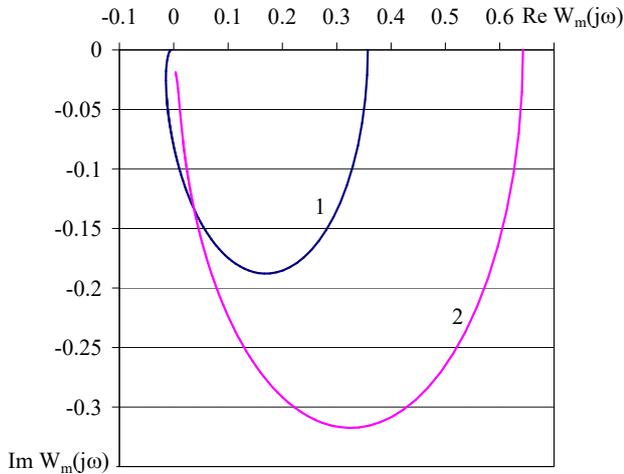
Fig. 2. Amplitude-phase frequency characteristics of a thermodynamic system: $1 - W_1(j\omega)$; $2 - W_2(j\omega)$

After combining (8)–(14) for the frequency characteristics of a thermodynamic system, the following expressions will be valid:

$$A_1(\omega) = Bi_1\left[\left(b_0 - b_2\omega^2\right)^2 + \omega^2\left(b_1 - b_3\omega^2\right)^2\right]^{-0.5}; \qquad (15)$$

$$A_2(\omega) = Bi_2\left[\begin{array}{l}\left[\left(a_0 - a_2\omega^2\right) + \left(a_1\omega\right)^2\right]\times \\ \times\left[\left(b_0 - b_2\omega^2\right)^2 + \omega^2\left(b_1 - b_3\omega^2\right)^2\right]^{-1}\end{array}\right]^{0.5}; \qquad (16)$$

$$\phi_1(\omega) = -\operatorname{arctg}\left[\omega\left(b_1 - b_3\omega^2\right)\left(b_0 - b_2\omega^2\right)^{-1}\right]; \qquad (17)$$

$$\phi_2(\omega) = -\operatorname{arctg}\left[\omega\begin{bmatrix}\left(a_0 - a_2\omega^2\right)\left(b_1 - b_3\omega^2\right)- \\ -a_1\left(b_0 - b_2\omega^2\right)\end{bmatrix}\times \right.$$
$$\left. \times\left[\left(a_0 - a_2\omega^2\right)\left(b_0 - b_2\omega^2\right) + a_1\omega^2\left(b_1 - b_3\omega^2\right)\right]^{-1}\right]. \qquad (18)$$

From (15) it follows that

$$A_1(0) = Bi_1 b_0^{-1}, \qquad (19)$$

and from expression (16) it follows that

$$A_2(0) = Bi_2 a_0 b_0^{-1}. \qquad (20)$$

Expressions (19), (20) represent expressions for the transfer coefficients of the thermodynamic system at the first and second inputs, respectively. Physically, these transfer coefficients characterize the efficiency of thermal protection of the thermodynamic system from thermal factors of fire and the thermal state of the gas generator cavity in the steady state.

For parameters $a_0$, and $b_0$, the following holds:

$$a_0 = 1 + Bi_1; \qquad (21)$$

$$b_0 = Bi_1 + Bi_2 + Bi_1 Bi_2, \qquad (22)$$

As a result, expressions (19), (20) are transformed as follows:

$$A_1(0) = \left[1 + Bi_2\left(1 + Bi_1^{-1}\right)\right]^{-1}; \qquad (23)$$

$$A_2(0) = \left[1 + \left[Bi_2\left(1 + Bi_1^{-1}\right)\right]^{-1}\right]^{-1}. \qquad (24)$$

From (23), (24) it follows that the efficiency of thermal protection of the gas generator in a hydrogen storage and supply system is determined by two parameters – Biot criteria $Bi_1$ and $Bi_2$. The nature of the influence of criteria $Bi_1$ and $Bi_2$ on the values of transmission coefficients $A_1(0)$ and $A_2(0)$ is determined by the graphical dependences, which are shown in Fig. 3, 4, respectively.
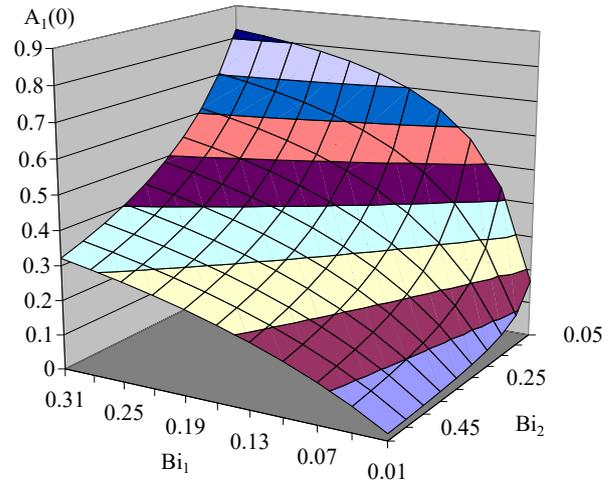


Fig. 3. Dependence of transfer coefficient $A_1(0)$ in a thermodynamic system on the Biot criteria $Bi_1$ and $Bi_2$
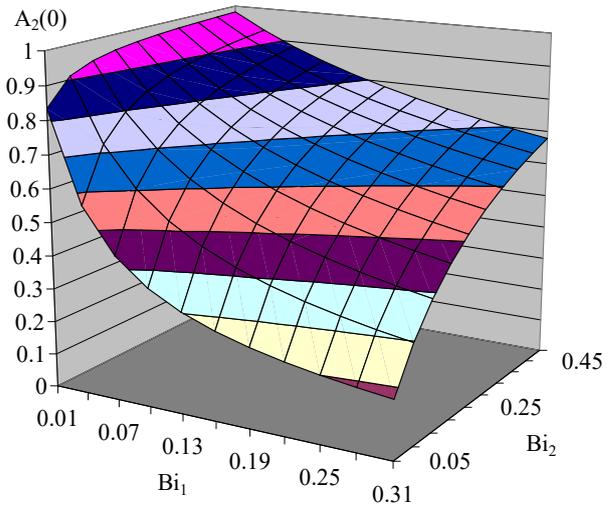


Fig. 4. Dependence of transfer coefficient $A_2(0)$ in a thermodynamic system on the Biot criteria $Bi_1$ and $Bi_2$

Fig. 5 shows graphical dependences (15), (16), and Fig. 6 – graphical dependences (17), (18), which are constructed for the characteristic values of parameters for a thermodynamic system.

Analysis of the frequency characteristics of the thermodynamic system reveals that its dynamic properties are fully determined in the infra-low frequency region with a maximum frequency not exceeding 1.0 s$^{-1}$. The heat-shielding coating of the gas generator in a hydrogen storage and supply system is a low-pass filter with a bandwidth not exceeding 0.02 s$^{-1}$.
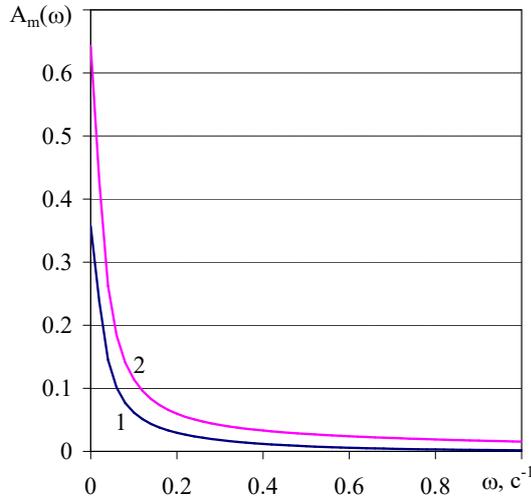
Fig. 5. Amplitude-frequency characteristics of a thermodynamic system: $1 - A_1(\omega)$; $2 - A_2(\omega)$
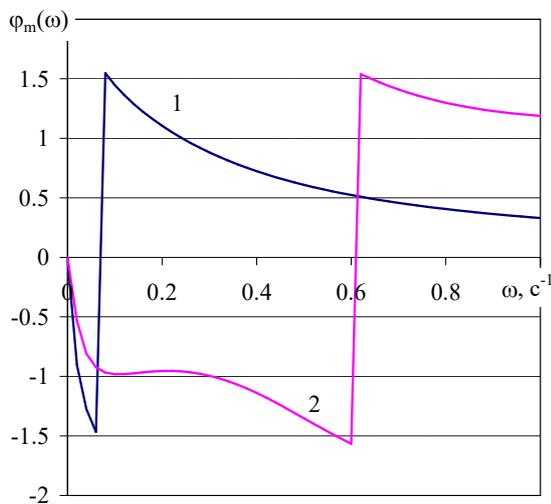


Fig. 6. Phase-frequency characteristics of a thermodynamic system: $1 - \varphi_1(\omega)$; $2 - \varphi_2(\omega)$

The values of transmission coefficients at the first and second inputs of the thermodynamic system are 0.36 and 0.64, respectively. This means that at a generalized fire temperature of $10^{3}$°C, the heat-shielding coating of the gas generator provides a temperature reduction on its wall to 360°C. If the thermal state of the gas generator tank cavity is reflected by a temperature of 60°C, then on the outer side of the gas generator wall there will be a temperature reduction to 38.4°C.

### 5. 2. Mathematical model of the components of error in the thermal protection of a gas generator in a hydrogen storage and supply system under conditions of fire

The influence of thermal factors of a fire can be described using a correlation function, which takes the following form

$$R_1(t) = \sigma_1^2 \exp\left(-\alpha_1 |t|\right), \tag{25}$$

where $\sigma_1$ is the standard deviation of the fire flame temperature; $\alpha_1$ is the parameter.

For the parameters of correlation function (25), we have

$$\sigma_1 = (30 \div 60) \text{ °C}; \ \alpha_1 = (0.3 \div 3.0) \text{ c}^{-1}. \tag{26}$$

Under the condition of stationary thermal influence, the spectral integrity $S_1(\omega)$ is determined using the Fourier transform

$$S_1(\omega) = \int_{-\infty}^{\infty} R_1(t)\exp\left(-j\omega t\right)dt = 2\sigma_1^2\alpha_1\left(\alpha_1^2 + \omega^2\right)^{-1}. \tag{27}$$

The following expression was used as the spectral integrity $S_2(\omega)$, which reflects the thermal state in the gas generator cavity:

$$S_2(\omega) = 2\sigma_2^2\alpha_2\left(\alpha_2^2 + \omega^2\right)^{-1}, \tag{28}$$

where

$$\sigma_2 = (1.8 \div 3.6) \text{ °C}; \ \alpha_2 = (0.5 \div 5.0) \text{ s}^{-1}. \tag{29}$$

It was assumed that the spectral density $S_{10}(\omega)$ of an interference from the side of a heat-shielding coating of the gas generator is a constant value, which is equal to $d_1^2$, and the spectral density $S_{20}(\omega)$ of an interference from the side of a gas generator cavity is a constant value, which is equal to $d_2^2$.

For the considered conditions, the root-mean-square deviation $\sigma_p$ of the temperature on the surface of a gas generator will be determined from the following expression

$$\sigma_p = \left(\sigma_{11}^2 + \sigma_{21}^2 + \sigma_{01}^2 + \sigma_{02}^2\right)^{0.5}, \tag{30}$$

where:

$$\sigma_{11}^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty} A_1^2(\omega)S_1(\omega)d\omega; \tag{31}$$

$$\sigma_{21}^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty} A_2^2(\omega)S_2(\omega)d\omega; \tag{32}$$

$$\sigma_{01}^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty} A_1^2(\omega)S_{10}(\omega)d\omega; \tag{33}$$

$$\sigma_{02}^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty} A_2^2(\omega)S_{20}(\omega)d\omega. \tag{34}$$

These expressions, when taking into account (15), (16), (27), (28), are transformed as follows:

$$\sigma_{11}^2 = Bi_1^2\sigma_1^2\left[(b_1 + b_2\alpha_1)(b_2 + b_3\alpha_1) - b_3(b_0 + b_1\alpha_1)\right] \times$$
$$\times \left\{b_0\left[\begin{array}{c}(b_0 + b_1\alpha_1)(b_1 + b_2\alpha_1)(b_2 + b_3\alpha_1) - \\ -b_3(b_0 + b_1\alpha_1)^2 - b_0\alpha_1(b_2 + b_3\alpha_1)^2\end{array}\right]\right\}^{-1}; \tag{35}$$

$$\sigma_{21}^2 = Bi_2^2\sigma_2^2 \times$$
$$\times \left[\begin{array}{c}a_2^2 b_0\alpha_2(b_0 + b_1\alpha_2) - b_0\alpha_2(a_1^2 - 2a_0a_2) \times \\ \times(b_2 + b_3\alpha_2) - a_0^2\left[\begin{array}{c}b_3(b_0 + b_1b_2) - \\ -(b_1 + b_2\alpha_2)(b_2 + b_3\alpha_2)\end{array}\right]\end{array}\right] \times$$
$$\times \left\{b_0\left[\begin{array}{c}(b_0 + b_1\alpha_2)(b_1 + b_2\alpha_2) \times \\ \times(b_2 + b_3\alpha_2) - b_3(b_0 + b_1\alpha_2)^2 - \\ -b_0\alpha_2(b_2 + b_3\alpha_2)^2\end{array}\right]\right\}^{-1}; \tag{36}$$

$$\sigma_{01}^2 = 0,5Bi_1^2 d_1^2 b_2\left[b_0(b_1b_2 - b_0b_3)\right]^{-1}; \tag{37}$$

$$\sigma_{02}^2 = 0{,}5Bi_2^2 d_2^2 \left[ b_0 b_1 a_2^2 - b_0 b_3 \left( a_1^2 - 2a_0 a_2 \right) + b_2 b_3 a_0^2 \right] \times$$
$$\times \left[ b_0 b_3 \left( b_1 b_2 - b_0 b_3 \right) \right]^{-1}. \tag{38}$$

It should be noted that for $Bi_1 = Bi_2$, $\sigma_1 = \sigma_2$ and $\alpha_1 = \alpha_2$, expressions (35), (36) coincide if $a_0 = 1.0$, $a_1 = a_2 = 0$, and expressions (37), (38) coincide if $Bi_1 = Bi_2$, $d_1 = d_2$ and $a_0 = 1.0$, $a_1 = a_2 = 0$.

Expressions (35) to (38) are a generalized parametric formalization for the components of the mean square error of the thermodynamic system, which is due to the thermal effect of the fire and the internal thermal state of the gas generator. Such a formalization is represented through the generalized parameters of the thermodynamic system $Bi_i$, $a_i$ and $b_k$, as well as through parameters $\sigma_i$, $\alpha_i$ and $d_i$ of the models that characterize this thermal effect.

Fig. 7 shows dependence $\sigma_{11} = \sigma_{11}(\sigma_1, \alpha_1)$, and Fig. 8 – dependence $\sigma_{21} = \sigma_{21}(\sigma_2, \alpha_2)$. These graphical dependences are given for the characteristic values of parameters for the thermodynamic system under conditions (26), (29), respectively.
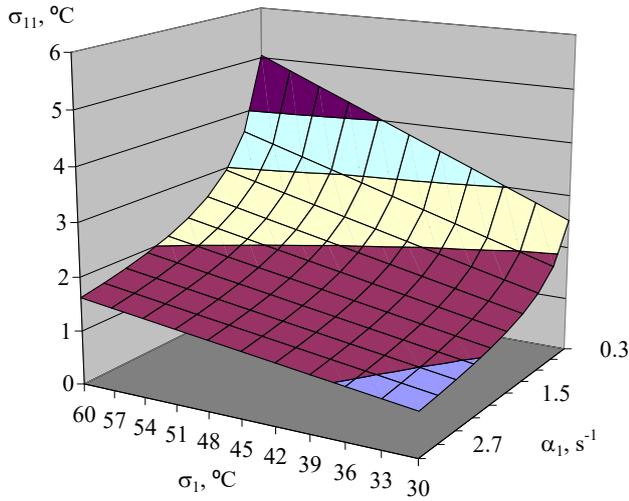


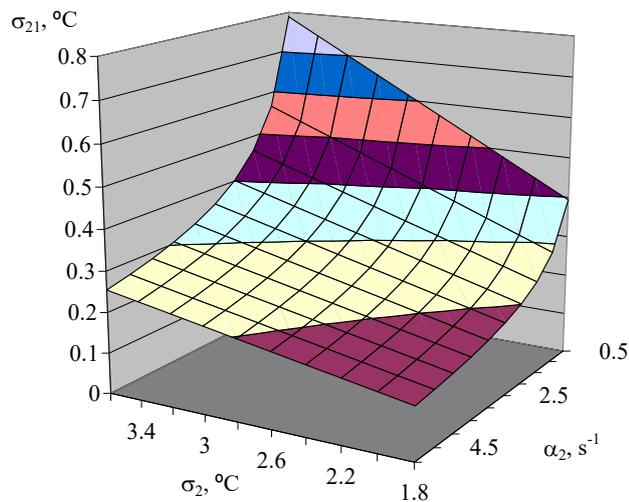Fig. 7. Dependence of the mean square deviation of temperature $\sigma_{11}$ on parameters $\sigma_1$ and $\alpha_1$



Fig. 8. Dependence of the mean square deviation of temperature $\sigma_{21}$ on parameters $\sigma_2$ and $\alpha_2$

Analysis of these graphical dependences reveals that for the considered ranges of changes in parameters $\sigma_i$ and $\alpha_i$, the largest values of the mean square deviations of temperature are, respectively

$$\sigma_{11max} = 5.04°C; \; \sigma_{21max} = 0.79°C. \tag{39}$$

For characteristic values, the parameters of the thermodynamic system of dependence (37) and (38) are transformed to the following form

$$\sigma_{01}^2 = 1.1 \cdot 10^{-3} d_1^2; \; \sigma_{02}^2 = 3.6 \cdot 10^{-3} d_2^2. \tag{40}$$

Fig. 9 shows dependence $\sigma_p = \sigma_p(d_1, d_2)$ under conditions (39), (40) at

$$d_i^2 = \left( 0.005 \div 0.015 \right) S_{i\,max} = \left( 0.01 \div 0.03 \right) \sigma_{i\,max}^2 \alpha_{i\,min}^{-1}. \tag{41}$$
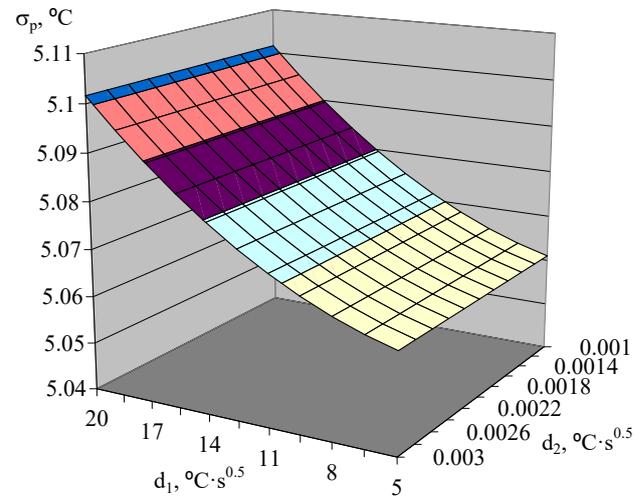


Fig. 9. Dependence of the mean square deviation of temperature $\sigma_p$ on parameters $d_1$ and $d_2$

It should be noted that the temperature interference, which is due to the thermal state of the gas generator cavity, practically does not affect the error value $\sigma_p$. The influence of the interference, which occurs on the outer side of the heat-shielding coating of the gas generator, does not exceed one percent in relation to the $\sigma_{p\,max}$ value. As a result, in the first approximation, expression (30) can be used without taking into account components $\sigma_{0i}^2$, $i = 1,2$. In this case, for the characteristic values of parameters for the thermodynamic system, the maximum value of the mean square deviation of temperature $\sigma_{p\,max}$ is 5.14°C. The maximum value of the absolute error of the thermal protection of the gas generator in a hydrogen storage and supply system with a confidence probability of 0.97 is 15.4°C.

Under the stationary mode, the temperature on the wall of the gas generator is determined by the following expression

$$x_c = \lim_{p \to 0} p \sum_{m=1}^{2} W_m(p) z_m(p) = A_1(0) z_1 + A_2(0) z_2. \tag{42}$$

At $z_1 = 10^3°C$, $z_2 = 60°C$, the $x_c$ value is equal to 398.4°C, as a result of which the relative temperature error on the wall of the gas generator in a hydrogen storage and supply system with a heat-protective coating is 3.9%.

It should be noted that the obtained estimates of errors in the thermal protection of a gas generator in a hydrogen storage and supply system belong to the a priori type of estimates. This allows them to be used already at the design stage of such systems.

## 6. Discussion of results based on the study on obtaining an a priori estimate of error in the thermal protection of a gas generator

The thermal protective coating of the gas generator in a hydrogen storage and supply system is considered as an element of a thermodynamic system, which also includes the wall of this gas generator. This thermodynamic system has two inputs and one output. The first input takes into account the influence of the thermal action of the fire, and the second input – the thermal state of the gas generator cavity (Fig. 1). The output of such a thermodynamic system is the temperature of the surface (wall) of the gas generator, which is determined by the superposition of the reaction of this system to the thermal effect occurring at its inputs. The dynamic properties of the thermodynamic system are determined by two transfer functions, the Hurwitz polynomials of which are of the third order. In the frequency domain, the dynamic properties of the thermodynamic system are described by the amplitude-phase-frequency characteristics, which are represented by the algebraic form of the function of a complex variable. For these frequency characteristics, mathematical expressions were constructed that include the parameters of the transfer functions of the thermodynamic system.

Using the characteristic values of these parameters, which are given in [24], graphical dependences were plotted for the frequency characteristics of the thermodynamic system (Fig. 2). The dynamic properties of a thermodynamic system under such conditions are fully reflected by their frequency characteristics in the frequency range $(0 \div 1.0)$ s$^{-1}$ with a bandwidth equal to 0.02 s$^{-1}$. The presence of the frequency characteristics of the thermodynamic system allows us to obtain indicators that characterize its efficiency under a static mode. Such indicators are the transmission coefficients at the inputs of this system. It was found that the efficiency of thermal protection of the gas generator in a hydrogen storage and supply system is determined by two parameters – Biot criteria for the heat-shielding coating and for the gas generator wall (Fig. 3, 4).

The values of transfer coefficients in the thermodynamic system for the characteristic values of its parameters are 0.36 and 0.64. This leads to the fact that at the temperatures at the first and second inputs of the thermodynamic system of $10^3$°C and 60°C, respectively, the temperature on the wall of the gas generator is 398.4°C. A feature of our mathematical expressions for the frequency characteristics of the thermodynamic system in the form of (6)–(11) or (15)–(18) is that they take a general form. The specification of the dynamic properties of the thermodynamic system is carried out using the parameters of its transfer functions (Fig. 5, 6). This approach opens up the possibility for obtaining mathematical models of errors of the thermal protection of the gas generator in a hydrogen storage and supply system also in a general form. The thermal effect on the thermodynamic system is taken into account using exponential-type correlation functions, and interference is used in the form of "white" noise. As an indicator characterizing the thermal protection

error of the gas generator in a hydrogen storage and supply system, its root mean square value is used. This allows us to determine the components of the total thermal protection error using the amplitude-frequency characteristics of the thermodynamic system (Fig. 8) and the spectral densities of the thermal effect on this system (Fig. 7).

In general, for such components of the thermal protection error, mathematical models are represented by expressions (35) to (38). These mathematical models include parameters of transfer functions for the thermodynamic system and parameters for the corresponding spectral densities. The consequence of such a mathematical description of the components of the thermal protection error of the gas generator in a hydrogen storage and supply system under conditions of fire is its universality. For real fire conditions and operation in a hydrogen storage and supply system with characteristic values of its parameters, the mean square error of thermal protection is 5.14°C (Fig. 9), and the absolute error with a confidence probability of 0.97 is 15.4°C.

The advantage of the above approach to obtaining an estimate of error in the thermal protection of a gas generator in a hydrogen storage and supply system relative to known solutions [25, 26] is the possibility of its a priori use. This approach is of particular importance when used at the initial stage of designing hydrogen storage and supply systems.

The positive aspect of using the above approach to determining the estimate of the error of the thermal protection of the gas generator in a hydrogen storage and supply system is the possibility of obtaining express estimates of the efficiency of using the heat-protective coating of the gas generator. This is due to the fact that the algorithm for obtaining express estimates is reduced to taking into account the numerical values of their parameters in models (35) to (38).

The limitations in determining the estimate of error in the thermal protection for a gas generator in a hydrogen storage and supply system are due to the order of the Hurwitz polynomial in the transfer functions of the thermodynamic system.

The disadvantage of the procedure for determining the error estimate of the thermal protection in a hydrogen storage and supply system generator is that it applies to linear thermodynamic systems "thermal protective coating-gas generator wall".

Further studies may involve simplification of the mathematical description of dynamic properties of the thermodynamic system, which could open up opportunities for simplifying the mathematical models of error components for the thermal protection in a hydrogen storage and supply system gas generator.

## 7. Conclusions

1. We have demonstrated that the combination of a heat-shielding coating and the wall of a gas generator in a hydrogen storage and supply system is a thermodynamic system with two inputs and one output. The first input of such a thermodynamic system takes into account the impact of the thermal action of fire on the gas generator, and the second input – the thermal state of the gas generator cavity. The output of such a system is the temperature on the surface of the gas generator. The properties of the thermodynamic system for each input are described by transfer functions that belong to the class of fractional-rational functions of a complex vari-

able. This approach provides formalization of the dynamic properties of the thermodynamic system in the frequency domain using two additive components – amplitude-phase frequency characteristics. Expressions for such frequency characteristics of the thermodynamic system are represented in the form of real and imaginary additive components, for each of which analytical dependences on the parameters of the transfer functions are obtained. These analytical dependences are universal with respect to the parameters of the transfer functions of the thermodynamic system. For the components of the amplitude-phase-frequency characteristics of the thermodynamic system – amplitude-frequency and phase-frequency characteristics in general form, corresponding mathematical expressions have been constructed. It is shown that the efficiency of thermal protection of the gas generator in a hydrogen storage and supply system is characterized by the transmission coefficients of the thermodynamic system. The values of these transmission coefficients are determined by two parameters – Biot criteria for the heat-protective coating and the gas generator wall. It is established that the dynamic properties of the thermodynamic system are also fully described by the frequency characteristics in the frequency range $(0 \div 1.0)$ s$^{-1}$ with a bandwidth of 0.02 s$^{-1}$. The heat-protective coating of a gas generator provides a temperature reduction from $10^3$°C to $3.6 \cdot 10^2$°C.

2. We have established that the influence of thermal action on the thermodynamic system can be taken into account using correlation functions of the exponential type or through their integral Fourier transform. That allowed us to build a mathematical model of error in the heat-shielding coating of a gas generator in a hydrogen storage and supply system in the form of a dispersion that has four additive components. Each of these additive components is determined by the corresponding amplitude-frequency characteristic of the thermodynamic system and the spectral density of the signal component at each of the inputs of this system. For each additive component of the dispersion that characterizes the temperature of the wall of the gas generator in a hydrogen storage and supply system, mathematical models have been constructed. These mathematical models are represented in a general form and include parameters for the transfer functions of the thermodynamic system and the spectral densities of the signals at its inputs. It is shown that the maximum value of the root-mean-square error of the thermal protection

of the gas generator in a hydrogen storage and supply system is 5.14°C. The maximum absolute error with a confidence probability of 0.97 is 15.4°C. The relative error in the thermal protection of a gas generator in a hydrogen storage and supply system is 3.9% for the stationary mode. The obtained metrological characteristics of thermal protection for a gas generator in a hydrogen storage and supply system are determined a priori and could be used at the design stage of such a system.

## Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study, as well as the results reported in this paper.

## Funding

The study was conducted without financial support.

## Data availability

The data will be provided upon reasonable request.

## Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

## Authors' contributions

**Yuriy Abramov:** Conceptualization, Supervision, Writing – original draft; **Oleksii Basmanov:** Software, Writing – review & editing; **Valentina Krivtsova:** Methodology, Project administration; **Andriy Mikhayluk:** Validation, Investigation; **Oleg Bogatov:** Writing – review & editing; **Vitalii Sobyna:** Formal analysis, Visualization; **Ihor Neklonskyi:** Data curation, Software; **Dmytro Sokolov:** Writing – review & editing; **Roman Chernysh:** Writing – review & editing.

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