

This study investigates the motion of melt jets, droplets, and prills of mineral fertilizers in the working space of a prilling tower. A specific feature of the process is the presence of a rotational velocity component caused by the rotation of the vibropriller basket.

At the design stage of prilling equipment, the influence of basket rotation on the motion of jets, droplets, and prills, as well as their aerodynamic interaction with the air flow, is considered only to a limited extent in most existing models of prilling systems.

A prilling tower with an internal diameter of 24 m, a prill flight height of 80 m, and a melt load of 175 t/h was adopted as the calculation model. The results showed that a basket rotation speed of 180 rpm provides the most effective radial expansion of the melt plume. Droplet trajectories were constructed; the horizontal and vertical velocity components were determined for different vibropriller basket configurations.

The results were obtained by numerically solving a system of differential motion equations considering initial melt outflow conditions, geometric parameters, and basket rotation speed. A quantitative relationship between basket rotation parameters and aerodynamic conditions of particle motion through changes in relative phase velocity was established.

The adopted approach could be applied at the design stage of high capacity prilling equipment to select basket configuration and rotation regimes. The resulting correlations make it possible to predict prill trajectories and prevent adhesion of non-crystallized melt to the internal tower surfaces. Elimination of secondary droplet breakup conditions reduces dust formation and stabilizes the particle size distribution of the product

Keywords: prilling tower, vibropriller, basket, melt plume, rotational motion, relative velocity

IDENTIFICATION OF THE INFLUENCE OF THE ROTATIONAL MOTION OF A VIBROPRILLER BASKET ON MELT JETS AND DROPLETS OF MINERAL FERTILIZERS IN A PRILLING TOWER

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Received 27.11.2025

Received in revised form 26.01.2026

Accepted date 12.02.2026

Published date 27.02.2026

How to Cite: Sklabinskyi, V., Karutskyi, A. (2026). Identification of the influence of the rotational motion of a vibropriller basket on melt jets and droplets of mineral fertilizers in a prilling tower.

Eastern-European Journal of Enterprise Technologies, 1 (1 (139)), 51–58.

<https://doi.org/10.15587/1729-4061.2026.353110>

1. Introduction

In the production of mineral fertilizers in prilling towers, rotating vibropillers (RVPs) have proven effective. These are granulators whose operation is based on the process of destruction of melt jets (liquid) flowing from the holes of the vibropiller basket [1].

The general view and layout of RVP are shown in Fig. 1. It is known that the melt inside the vibropiller basket is affected by mechanical vibrations. These vibrations also affect the melt jets flowing from the vibropiller basket and the drops that have already formed, and the jets that have not yet broken up into separate drops. The diagram shows that these jets and drops are additionally affected by an air flow.

The evolution of industrial prilling towers was accompanied by the transition from small-diameter installations to modern high-performance systems with prilling towers with a diameter of more than 20 meters. In early designs, static granulators or granulators with a rotation speed of up to 50–60 rpm were used. The geometry of the basket was determined mainly by the productivity of the equipment and the

possibility of arranging the required number of holes. With an increase in the diameter of the towers, the need arose to form a wider granulation flame, which led to an increase in the rotation speed and the choice of the shape of the basket surface. This is justified by the fact that to obtain a wide flame of prills and the distribution of these prills over the entire cross-section of the prilling tower, the vibropiller basket can rotate at a speed of up to 220 rpm.

To calculate the granulation process and obtain a granulation product of the desired composition and the required strength with the required final temperature of the prills, it is necessary to take into account the interaction between jets and melt drops with moving air. The complexity of the process of calculating the diameter of the droplets obtained after the melt jet collapses on a drop relates to the possibility of repeated collapse of the droplets on smaller droplets – satellite droplets. The formation of so-called satellite droplets significantly worsens the quality of the final product, leads to inhomogeneity, and increases dust emission.

Fig. 2 shows a general scheme of the RVP design [2, 3].

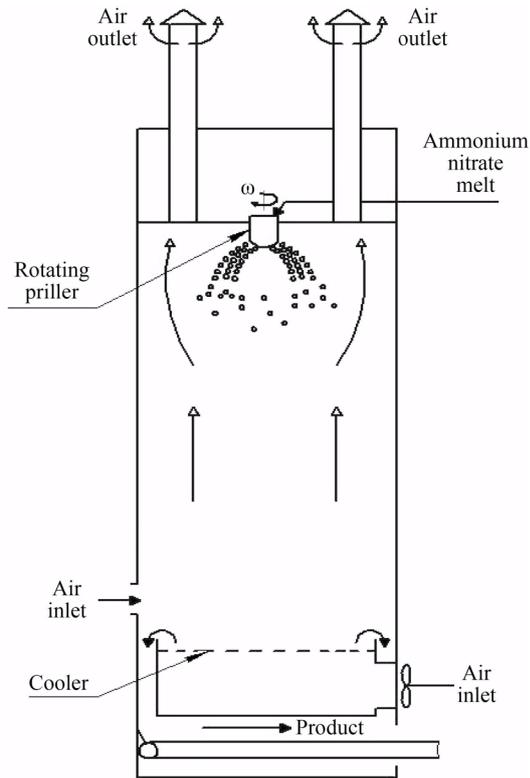


Fig. 1. General view of a prilling tower with a vibropiller [2]

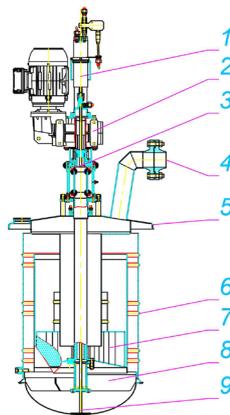


Fig. 2. Diagram of a rotating vibropiller:
1 – vibrator; 2 – motor – gearbox; 3 – bearing assembly;
4 – melt inlet; 5 – plate; 6 – basket; 7 – melt distributor;
8 – blades; 9 – rod [2]

The use of RVP makes it possible to obtain a more uniform distribution of prills by size and a smaller amount of dust in a wide range of flow rates than when using other types of granulators. This has been proven by field tests and makes this method of producing mineral goods one of the most promising.

Further development of this direction is an urgent scientific and technical task aimed at ensuring the stability of vibropiller operation and improving the granulation composition. One of the factors affecting the stability of vibropiller operation in the prilling tower is the rotational movement of the basket. Therefore, studies on the influence of the rotational movement of the vibropiller basket are relevant.

2. Literature review and problem statement

Modeling of pilling processes in towers usually includes three interrelated modules: jet disintegration, the movement of the dispersed phase in the gas flow, and heat and mass transfer during cooling and crystallization. For high-performance rotating vibropillers, the emergence of a circumferential velocity component, which is formed by the rotation of the basket and changes the relative velocity of the phases, is decisive. It is the change in kinematic conditions of outflow that affects the trajectories of particle motion and the geometry of the pilling flame. However, under most existing approaches, the rotational nature of the outflow source is taken into account to a limited extent, which complicates their use in the design of high-performance towers.

In [4], the capillary disintegration of the jet and the influence of the air flow structure are considered, in particular by means of CFD modeling, and the relationship of the tower aerodynamics with the shape of the flame and dust emission is shown. At the same time, the initial velocity conditions of the outflow are set independently of the kinematics of the vibropiller, and the relative velocity of the phases is determined mainly by the gas flow. Ignoring the circumferential component of the velocity does not make it possible to establish a connection between the parameters of the basket rotation and the conditions of the movement and destruction of the drops. The probable reason is the focus on the gas dynamics of the tower, dust generation and flare opening with a simplified description of the outflow source.

In [5] it is shown that the disintegration of a liquid jet in a transverse gas flow is determined by the interaction of aerodynamic and inertial forces and the relative velocity of the phases. Most of the analyzed models consider stationary sources of outflow, where the relative velocity is formed mainly by the parameters of the gas flow. For rotating vibropillers, there is no quantitative assessment of how the rotation of the basket changes the relative velocity and the regime of jet disintegration. This is due to the fact that the model is designed for classic nozzle systems and does not take into account the kinematics of rotating equipment.

In [6], a comparison of the pilling process using rotating vibropillers with other pilling techniques is given. In this case, the aerodynamic effect on jets and drops is considered to a limited extent. The probable reason is the focus of the work on technological indicators, and not on the kinematics of the basket movement.

In [7], a mathematical model of the pilling process is proposed that describes the heat and mass transfer during the formation of ammonium nitrate prills. The model is focused on technological parameters and the thermodynamics of granule cooling. This is important for predicting the final state of the product. However, the work does not take into account the change in the relative velocity of the phases caused by the rotational motion of the vibropiller basket and does not assess the effect of the peripheral velocity on the flight time and spatial distribution of particles. Therefore, the application of this approach to RVP in large towers remains limited. The reason is the dominance of the thermodynamic description with simplified kinematics of motion.

In [8], the analysis of the stability of the jet in a transverse gas flow was performed and the conditions of initial instability were determined. The model is based on the assumption of a stationary leakage source; therefore, the influence of rotational kinematics and additional peripheral velocity was

not taken into account. As a result, the instability limits and conclusions require correction for the case of high-performance RVPs, when the relative velocity of the phases is also formed by rotation. The reason is the classical statement of the problem without rotational kinematics.

In study [9], the disintegration of the jet in a transverse air flow and the characteristics of the formed droplets were considered. At the same time, the study does not take into account the features of the pilling equipment and the kinematics of the rotating leakage source. Therefore, the results of the study do not allow for a direct assessment of the influence of peripheral velocity on the trajectories and interaction of droplets with the gas flow in the prilling tower.

In [10], the mechanisms of secondary fragmentation of droplets are generalized and the modes of their destruction under the action of aerodynamic forces are classified. The authors showed the determining role of the Weber criterion in the transition between the destruction modes. However, the initial conditions of leakage are considered to be given and not related to the design of the liquid supply source. Therefore, the results of the work are difficult to apply to the selection of the basket rotation speed in RVP. The model is generalized and does not take into account the kinematics of the leakage source.

In study [11], the aerodynamic deformation and destruction of droplets when the relative velocity of the phases changes and the influence of the flow velocity on the nature of fragmentation are shown. It is demonstrated that an increase in the flow velocity changes the nature of fragmentation, but the influence of the rotational motion of the liquid supply system on the formation of the initial velocity of droplets is not considered. Therefore, it is impossible to assess how the rotation of the basket affects the transition of droplets to the secondary fragmentation modes in an RVP flame. This is due to the consideration of a drop in a given flow without taking into account the kinematics of the equipment.

The fundamental principles of instability of liquid jets are summarized in [12], in which it is shown that the decay is determined by the initial conditions of the outflow and the flow parameters. However, the presented approaches are formed for stationary sources of outflow and do not take into account the appearance of the circumferential velocity component.

In [13], the secondary fragmentation of drops at moderate values of the Weber criterion was investigated and characteristic fracture modes were determined. However, the model is based on the classical statement of the problem for an isolated drop in a gas flow and does not connect the fragmentation parameters with the kinematics of the vibropiller.

Experimental results [14] show the influence of the transverse flow on the trajectories of drops. The circumferential velocity component characteristic of RVP is not taken into account in the work. Therefore, the influence of the circumferential velocity on the spatial distribution of drops in the tower is not estimated. This is due to the fact that the experiments were performed for stationary nozzle systems.

Study [15] considers the modes of secondary crushing and droplet size distribution for different liquids. It is shown that the fragmentation intensity is determined by the flow parameters; however, the connection between the Weber criterion and the rotational parameters of the liquid supply has not been established. The reason is the use of specified speeds without modeling the source of their formation.

In work [16], the influence of the mutual arrangement of several rotating vibropillers on the spatial structure of flares in the prilling tower was investigated. Most attention was paid to the layout solutions and operating conditions of the equipment that determine the efficiency of using the working volume of the tower. The proposed approach is focused on the geometric consistency of the flares and the technological reliability of the installation. At the same time, in [16], the initial velocity conditions of the jets are considered simplified, without analyzing the change in the relative velocity of the phases and the influence of the circumferential component of the velocity caused by the rotation of the basket. This limits the possibility of evaluating the aerodynamic modes of the droplet motion and their interaction with the gas flow. The reason is the priority of layout solutions over the detailing of kinematic conditions of outflow.

Our review of the literature demonstrates that existing studies describe in detail the mechanisms of jet disintegration, secondary fragmentation of drops and their interaction with the transverse gas flow. Attention is also paid to the influence of aerodynamic conditions of the tower on the shape of the pilling flame and product dispersion. At the same time, in most works the outflow source is considered as stationary or kinematically simplified, and the initial velocity conditions are set regardless of the design of the vibropiller.

Under such assumptions, there is no quantitative relationship between the rotation of the basket and the change in the relative velocity of the phases. It has not been determined how the circumferential component of the velocity affects the trajectories of jets, drops, and prills in the working space of the prilling tower. The conditions under which rotation can cause the transition of drops to secondary crushing modes have also not been sufficiently studied. This limits the possibility of using existing models at the design stage of high-performance pilling systems.

Thus, the unsolved task is the lack of a consistent computational approach that makes it possible to quantitatively link parameters of the basket rotation with the initial velocities of jets, drops, and prills, and their subsequent movement in the gas flow, taking into account the design limitations of the prilling tower. Solving this task requires research aimed at establishing the influence of the rotational motion of the basket in the formation of the geometry of the pilling flame, changing the trajectory of the prills depending on the design features of the basket and the conditions of secondary crushing.

3. The aim and objectives of the study

The purpose of our study is to establish the influence of the rotating motion of an RVP basket on the aerodynamic conditions of the movement of jets, drops, and prills in the prilling tower. The study is aimed at quantitatively assessing the role of the circumferential component of the velocity in the formation of particle trajectories and determining the conditions for secondary crushing of drops. Results should provide justification for the vibropiller operating parameters in the formation of a stable flame, reducing dust emission and eliminating the adhesion of melt on the surface of the tower.

To achieve the goal, the following tasks were set:

- to determine the most acceptable rotation speed of a vibropiller basket for the given parameters of the prilling tower;

- to determine the influence of the radii of basket holes on the initial velocity of jets and the trajectory of prills;
- to determine the influence of the jet outflow angle on the components of the velocity of the jets, drops, and prills.

4. The study materials and methods

The object of our study is the process of motion of jets, drops, and prills of mineral fertilizers in the working space of a prilling tower. The peculiarity of the motion is the rotational component caused by the rotation of the basket of the vibrating vibropiller.

The principal hypothesis assumes that the rotational motion of the basket of the vibropiller forms an additional circumferential component of the velocity of jets and drops. This determines the relative velocity of the phases, changes the aerodynamic conditions of the movement of particles in the tower, and could lead to secondary crushing of drops when the critical value of the Weber criterion is exceeded.

The work uses theoretical methods based on the analysis of the motion of a single drop in a gaseous medium, taking into account aerodynamic resistance and the action of gravity. The construction of the trajectories of the drop motion was carried out using numerical methods for solving systems of ordinary second-order differential equations.

The choice of an approach based on solving the equations of motion of a single drop is due to the need to directly assess the influence of the initial kinematics of the jet and the parameters of the basket rotation. Unlike complex multiphase models that require significant computational resources, the chosen approach makes it possible to isolate the determining factors of the process and preserve the engineering interpretability of the results. The use of a system of second-order ordinary differential equations provides reasonable accuracy in describing the particle trajectories and the possibility of parametric analysis of the geometry of the basket and its rotation speed.

To enable the possibility of analytical description of the drop motion and reduce the computational complexity of the problem, the following assumptions in the mathematical model are adopted:

1. Droplets are isolated and do not interact with each other.
2. The shape of the drops is assumed to be spherical.
3. The processes of heat and mass transfer between drops and air are not considered within the framework of this study.

The numerical solution to the system of equations was found in the Maple 2025 environment (Waterloo Maple Inc., Canada), which was used to integrate differential equations and construct particle motion trajectories. The processing and visualization of the calculated data was carried out using standard graphical tools of the calculation program.

The repeated fragmentation of drops was determined using the Weber criterion [10]

$$We = \frac{\rho h o_g \cdot V_{vidn}^2 \cdot d_k}{\sigma}, \tag{1}$$

where We is the Weber criterion;

$\rho h o_g$ is the density of the gas (air);

σ is the surface tension coefficient of the liquid in the drop;

V_{vidn} is the relative velocity between the gas (air) and the drop (liquid jet).

Below is a description of the flow of the melt flowing out of the vibropiller basket. Fig. 3 schematically shows how the flow of the melt from the holes of the vibropiller basket looks like. The flow of the melt from the holes of the vibropiller basket occurs at an angle to the horizontal plane due to the fact that the shape of the bottom of the vibropiller basket has a conical, spherical, toroidal, or other shape. The diameter of the vibropiller basket under real conditions of mineral fertilizer production is much smaller than the diameter of the prilling tower. Therefore, to enable uniform distribution of drops and prills over the entire cross-section of the tower, the vibropiller basket rotates. In large prilling towers, this rotation speed can reach 220 rpm. Therefore, in addition to the vertical and horizontal components of the liquid movement from the holes, a fairly large linear velocity of the liquid movement at the exit from the holes in the circumferential direction arises.

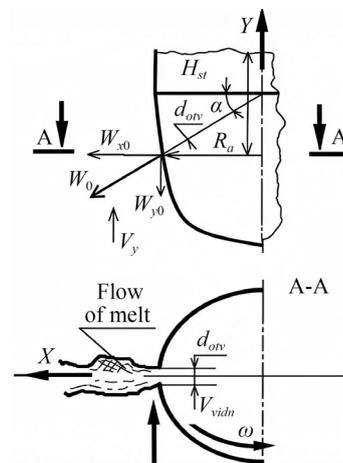


Fig. 3. Scheme of the flow of melt flowing out of the vibropiller basket: R_o – radius at which the hole for the melt outflow is located; d_{orv} – diameter of the holes; α – angle of the flow of the jet; W_{x0} and W_{y0} – initial values of the horizontal and vertical components of the droplet velocity; W_0 – velocity of liquid (melt) outflow from the holes; V_{yidn} – relative velocity of the air flow; X and Y – directions of the corresponding coordinate axis; V_y – velocity of air movement in the prilling tower; ω – angular velocity of rotation of the vibropiller basket; H_{st} – height of the liquid (melt) column above the corresponding hole

The air in the tower in most cases moves only in a vertical direction, and in the circumferential direction its speed is almost zero. Therefore, the jets and drops at the exit from the holes have a relative initial speed, in the plane normal to the axis of the vibropiller. The initial speed is equal to the linear speed of rotation of the corresponding hole in the vibropiller basket ($V_{yidn} = \omega \cdot r$).

In the general case, the speed of flow of melt from the holes of the vibropiller basket occurs under the action of pressure

$$H_a = H_{st} + H_w, \tag{2}$$

where H_{st} and H_w are the static pressure component from the action of the buoyancy column above the hole and the pressure component from the action of centrifugal forces (centrifugal pressure) arising from the rotational motion of the vibropiller basket.

The magnitude of the centrifugal pressure depends on the rotation speed of the vibropiller basket and the radius at which the corresponding hole from which the jet under consideration flows is located

$$H_w = \left(\frac{\pi n}{30} \right)^2 \frac{(2r)^2}{8g}, \quad (3)$$

where n is the basket rotation speed, rpm; r is the radius at which the corresponding hole on the basket surface is located.

To analyze the motion of drops and prills after exiting the holes of the rotating vibropiller, it is necessary to know the numerical values of the droplet velocities, and after the completion of the crystallization process and prills, during their fall in the working space of the prilling tower. Theoretically, such an analysis can be carried out by solving the following equation of motion of drops in a gas stream

$$\begin{cases} \frac{d}{dt} W_x(t) = -\frac{C S_g \rho_{air} (W_x(t) + V_x)^2}{2m}, \\ \frac{d}{dt} W_y(t) = g - \frac{C S_g \rho_{air} (W_y(t) + V_y)^2}{2m}, \end{cases} \quad (4)$$

where τ is time; $W_x(t)$ and $W_y(t)$ are the components of the droplet velocity along the corresponding axes; C is the drag coefficient of the droplet; S_g is the cross-sectional area of the droplet; ρ_{air} is the air density; V_x and V_y are the components of the air velocity along the corresponding axes; m is the droplet mass.

To determine the droplet diameter, researchers recommend the following empirical equation [17]

$$d_k = \frac{6 \cdot d_c}{Re^{0.15}}, \quad (5)$$

where d_k and d_c are the droplet diameter and the liquid jet diameter; Re is the Reynolds criterion.

If, to analyze the motion of droplets and prills, it is necessary to construct the trajectory of their motion, it is possible to use a system of differential equations [16], taking into account the fact that the velocity value in the system of equations (4) is a derivative of time:

$$\begin{cases} \frac{d^2}{dt^2} S_x(t) = \frac{3C_x(t) \rho_{air}}{4d_k \rho_g} \left(\frac{d}{dt} S_x(t) + V_x \right)^2, \\ \frac{d^2}{dt^2} S_y(t) = g - \frac{3C_o(t) \rho_{air}}{4d_k \rho_g} \left(\frac{d}{dt} S_y(t) + V_y \right)^2, \end{cases} \quad (6)$$

where $S_x(t)$ and $S_y(t)$ are the path traveled by the drop along the horizontal and vertical axes, respectively; $C_x(t)$ and $C_y(t)$ are the drag coefficients of the drop along the axes, respectively.

When analyzing the system of equations (4), it was determined that it is currently difficult to obtain an analytical solution to these equations; therefore, these equations can be solved with sufficient accuracy for engineering calculations using numerical methods. In this case, the variable value of the velocity components along both the horizontal and vertical axes depends on time. Therefore, when numerically solving system (3), it is necessary to specify the initial conditions and the values of the floating velocity at the exit from the holes.

The geometry of a typical prilling tower with an internal diameter of the tower of 24 m and a prill flight height of 80 m was taken as the calculation model. The load on the floating urea is 175 t/hour. The largest diameter of the basket holes is 0.5 m.

Defining the maximum permissible rotation speed of a vibropiller basket is predetermined by the maximum permissible opening of the prilling flame at the level of the tower bottom. The opening of the flame should enable a uniform distribution of prills in the cross section of the tower without their contact with the inner surface of the apparatus. Excessive radial expansion of the flame is operationally unacceptable. Additional design limitations are the geometry of the working area of the scraper mechanism in the lower part of the apparatus and the risk of prills being carried out through the air supply windows outside the tower.

Taking the above factors into account, the maximum permissible diameter of the flame opening at the level of the tower bottom is taken to be 4–6 m smaller than the inner diameter of the tower. For a design tower with a diameter of 24 m, the most acceptable flame opening is approximately 19 m.

5. Results of studies on the influence of a rotational motion of the vibropiller basket on the movement of jets and drops

5.1. Determining the maximum permissible rotation speed of the vibropiller basket for given parameters of the prilling tower

The dependence of the flare opening diameter on the basket rotation speed was obtained by numerical integration of the system of differential equations of drop motion (4) and trajectory equations (6). The initial conditions for the system of equations were determined using the relations (2), (3), which describe the melt outflow speed and the centrifugal component of pressure during basket rotation. The numerical solution was received in the Maple 2025 environment, which allowed us to obtain the trajectories of the prills and the coordinates of their fall at the level of the bottom of the tower. The flare opening diameter was determined by the final horizontal displacement of the prills obtained from the solution to system (6). The radius of the flare was taken equal to the maximum value of the horizontal coordinate of the prill at the moment of reaching the level of the bottom of the tower: $Rf = \max |Sx(t_{end})|$, with $Sy(t_{end}) = H$, where $Sx(t)$ and $Sy(t)$ are the trajectory coordinates determined from equations (6), H is the flight height of the prill, which is 80 m.

Accordingly, the flare opening diameter was determined as $Df = 2Rf$.

Fig. 4 shows the calculated dependence of the flare diameter at the bottom of the tower on the basket rotation speed.

From the obtained dependence it is clear that the basket rotation speed of about 180 rpm provides the flare opening close to the most acceptable value of 19 m for the given geometric parameters of the tower. The obtained value of the rotation speed corresponds to the given design constraints and provides the permissible geometry of the prilling flare for the calculated parameters of the tower.

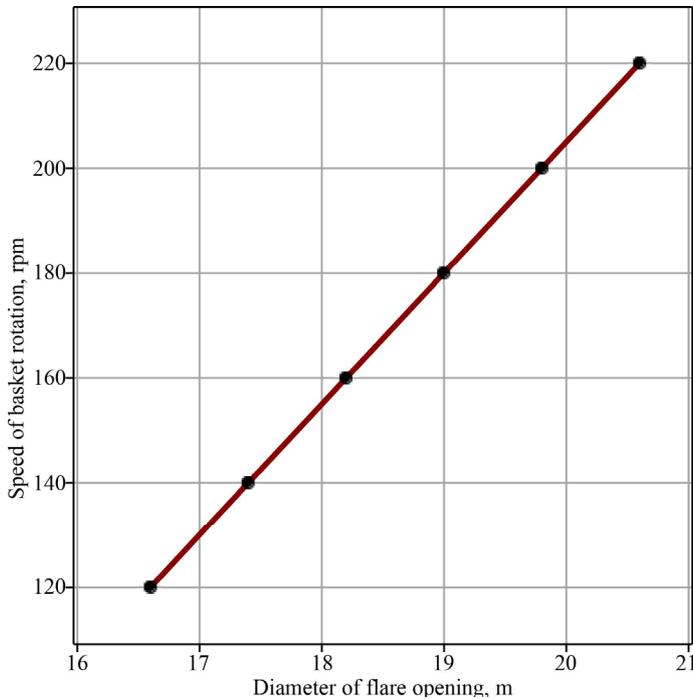


Fig. 4. Dependence of the diameter of the flare at the bottom of the tower on the speed of basket rotation

5.2. Determining the influence of the radii of basket holes on the initial velocity of jets and the trajectory of prills

Fig. 5 shows the calculated trajectories of the prills obtained from numerical integration of the system of equations of motion (4) and the trajectory equations (6) in the Maple 2025 environment. The initial conditions were determined by relations (2), (3), which take into account the influence of the centrifugal component of pressure.

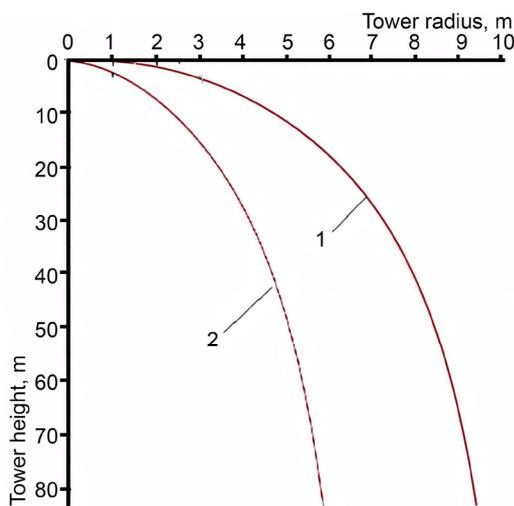


Fig. 5. Calculated trajectories of the movement of prills in the prilling tower at a basket rotation speed of 180 rpm:
 1 – $r = 250$ mm (largest radius in the basket);
 2 – $r = 125$ mm (average radius in the basket)

According to (3), the location of the holes at different radii of rotation leads to different values of the centrifugal pressure and the initial velocities of the jets. The initial circumferential component of the velocity was determined as $V_{vidn} = \omega \cdot r$,

therefore, with an increase in the radius, the horizontal velocity of the particles increases and the trajectory of their movement changes.

The trajectories were obtained by numerical integration of the system of differential equations (4) and (6) in the Maple 2025 environment taking into account the initial conditions defined by relations (2), (3). The results show that an increase in the radius of the basket holes leads to an increase in the circumferential component of the initial jet velocity. As a result, jets flowing from larger radii have a larger initial horizontal displacement and form trajectories with a larger radial deviation from the tower axis. For $r = 250$ mm, a significantly larger radial displacement was obtained compared to $r = 125$ mm, which directly affects the geometry of the prilling flame. Thus, the radius of the holes determines the geometry of the prilling flame through a change in the circumferential component of the initial jet velocity.

5.3. Effect of the angle of jet outflow on the components of prill velocity

Fig. 6 shows the results of calculating the horizontal $W_x(t)$ and vertical $W_y(t)$ components of velocity when the liquid jet flows out at different angles of inclination to the horizontal plane α (Fig. 3), which corresponds to the use of different types of baskets. The results were obtained by numerical integration of the equations of motion of the droplet (4) in the Maple 2025 environment at a basket rotation speed of 180 rpm.

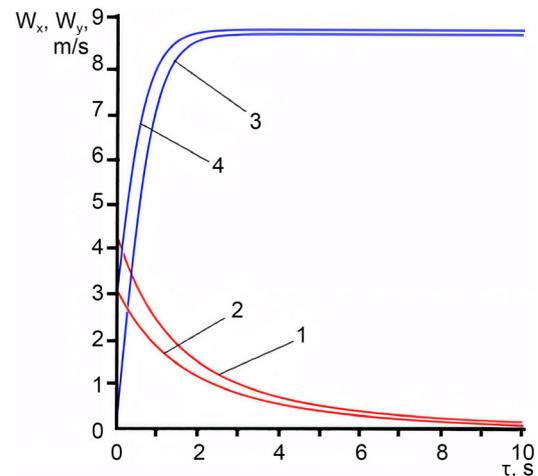


Fig. 6. Calculated values of the velocity of prills during falling in the prilling tower: 1 – $W_x(t)$ at $\alpha = 5^\circ$ (conical basket); 2 – $W_x(t)$ at $\alpha = 45^\circ$ (toroidal basket); 3 – $W_y(t)$ at $\alpha = 5^\circ$ (conical basket); 4 – $W_y(t)$ at $\alpha = 45^\circ$ (toroidal basket)

The change in the outflow angle α , which is determined by the geometry of the basket, leads to a redistribution of the horizontal and vertical components of the initial velocity of the prills. At small angles of inclination, the horizontal component of the velocity prevails, which contributes to a greater radial displacement of the particles, while with increasing α , the vertical component increases and the trajectories of motion acquire a more axial character. The obtained dependences confirm the determining influence of the outflow angle on the kinematic conditions of the movement of prills in the prilling tower.

6. Discussion of results based on the study of the influence of rotating motion of the basket in a rotating vibropiller on the motion of jets and drops

Our results show that the rotating motion of the basket in RVP directly determines the initial conditions of the motion of jets, drops, and prills due to the change in centrifugal pressure and the appearance of a circumferential component of velocity. As follows from relations (2), (3), and the system of equations of motion (4), (6), an increase in the rotation speed of the basket leads to an increase in the outflow velocity and an increase in the horizontal displacement of jets, drops, and prills, which is reflected in Fig. 4 in the form of an increase in the diameter of the flame at the level of the bottom of the tower. Thus, the opening of the flame is determined not only by the geometry of RVP but also by the kinematic conditions of outflow formed by the rotation of the basket.

It has been established that the maximum permissible rotation speed is determined by the design limitations of the tower. On the one hand, it is necessary to ensure sufficient radial opening of the flame to evenly fill the cross-section of the apparatus, and on the other hand, to limit its expansion to avoid contact of not yet crystallized droplets with the walls of the tower. Additional factors limiting the diameter of the flame are the geometry of the working zone of the scraper mechanism in the lower part of the apparatus and the risk of prills being carried out of the tower through the air supply windows. It is these design conditions that determine the permissible value of the flame diameter of about 19 m for a tower with a diameter of 24 m, which corresponds to a rotation speed of about 180 rpm (Fig. 4).

The results shown in Fig. 5 demonstrate that an increase in the radius of the basket holes leads to an increase in the circumferential component of the velocity $V = \omega r$, and, therefore, to a change in the trajectories of prills. Jets flowing from holes located at larger radii have a larger initial horizontal displacement, which forms a wider flame. This confirms that the rotational motion of the basket affects the spatial structure of the flare by changing the kinematic conditions of the outflow, and not only through the geometric parameters of the equipment.

The dependences of velocity components (Fig. 6) indicate that the rotational motion in combination with the outflow angle determines the ratio of the horizontal and vertical components of the prill velocity. At small angles of inclination, the horizontal component of the velocity dominates, which contributes to the radial opening of the flame, while at larger angles the movement of the drops becomes more vertical. This calculation makes it possible to select the optimal basket design for the given tower parameters. Also, knowing the horizontal component of the droplet velocity, it is possible to exclude the possibility of not yet crystallized melt droplets hitting the inner walls of the pelletizing tower at the design stage.

The evaluation of the Weber criterion by formula (1) shows that at a rotation speed of 180 rpm, the peripheral velocity of the liquid can reach about 4.71 m/s, which corresponds to the value $We \approx 1.13$. This value does not lead to intensive secondary crushing of urea droplets, but determining the Weber criterion requires attention for other towers and loads and for granulating other substances.

Unlike approaches [4, 7, 5–15], in which the leakage source is considered stationary, this study shows that for modern rotating vibropillers, the circumferential velocity component becomes a determining factor in the formation of a flame. Similar to work [16], in which the spatial-geometric interaction of flames was analyzed, our results confirm the importance of the

kinematic parameters of the vibropiller and supplement it with a quantitative assessment of the influence of the rotation speed on the initial velocities of jets and the movement of drops.

The results of the numerical study allow us to explain the unsolved issue associated with the lack of a quantitative relationship between the rotation of the basket with the initial velocities of jets, drops, and prills, and their subsequent movement in the gas flow, taking into account the design limitations of the prilling tower. It is shown that it is the rotational motion that forms an additional circumferential component of the velocity, which changes the trajectories of particle motion and the geometry of the flame, which was not taken into account in most known models.

Our results are correct within the accepted assumptions of the mathematical model. The disadvantages of the study include the simplified representation of the shape of the drops as spherical, which can affect the accuracy of determining the aerodynamic drag coefficient. A two-dimensional statement of the motion problem was also used, which does not allow for the spatial asymmetry of the prilling flame to be fully taken into account. The model does not take into account turbulent pulsations of the air flow, which can change the local values of the relative velocity of the phases.

The scope of practical application of our results includes large-diameter prilling towers for the production of urea using rotating vibropillers. The most expedient is the use of the technique in regions with hot climates, in particular in the countries of Asia, Africa, and South America, where natural cooling of prills occurs less intensively. Under such conditions, increasing the diameter of the tower and the speed of rotation of the basket is used to effectively open the melt flame and enable uniform distribution of prills. The chosen approach could also be applied to tower plants for the production of ammonium nitrate, provided that vibropillers with similar kinematic parameters are used.

The limitations of the study include neglecting the processes of heat exchange and crystallization, which affect the mass, shape, and aerodynamic characteristics of the prills.

Further studies may consider three-dimensional numerical modeling of the motion of jets and prills taking into account heat exchange and crystallization, as well as experimental verification of our dependences for different rotation modes and design parameters of vibropillers.

7. Conclusions

1. For a prilling tower used to produce granulated urea with an internal diameter of 24 m, a prill flight height of 80 m and a floating load of 175 t/h, the maximum permissible basket rotation speed of 180 rpm has been determined, at which the flare opening is 19.0 m. In practice, in industrial settings, the flare opening is not equal to the internal diameter of the tower. This is due to the design features of towers.

2. It is shown that the radius of the basket openings is a determining design parameter, which, due to a change in the circumferential component of the initial jet velocity, affects the trajectories of prills and the geometry of the prilling flare. With an increase in the radius, the horizontal component of the velocity of the jets and drops increases, which leads to a greater radial displacement of prills in the working space of the tower. Our results confirm that the spatial arrangement of the holes determines the conditions for the formation of the flame not only geometrically but also kinematically, through a change in the initial outflow velocities.

3. A quantitative dependence of the horizontal and vertical components of the velocity of droplets and prills on the jet outflow angle α , which is determined by the geometry of the vibropiller basket, has been established. It is shown that a decrease in the outflow angle from 45° to 5° leads to an increase in the initial horizontal component of velocity from approximately 3.0–3.2 m/s to 3.8–4.0 m/s (by 20–25%), which directly determines the intensity of the radial opening of the flame. Our dependences make it possible to assess the possibility of contact of uncrystallized droplets with the walls of the tower or the interaction of flames of neighboring vibropillers at the design stage. At the same time, the vertical component of velocity for both configurations quickly stabilizes at 8.5–8.7 m/s, which determines the nature of the interaction of prills with the oncoming air flow and the conditions of their cooling during the fall.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study, as well as the results reported in this paper.

Funding

The study was conducted without financial support.

Data availability

The data will be provided upon reasonable request.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

Authors' contributions:

Vsevolod Sklabinskyi: Conceptualization; Methodology; Validation; Writing – review & editing; Supervision; Project administration; **Andrii Karutskyi:** Formal analysis; Investigation; Resources; Data curation; Visualization; Writing – original draft.

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