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CONSTRUCTION OF THE DISCRETE-CONTINUOUS MATHEMATICAL MODEL OF A HYSTERESIS DAMPER IMPACT DEVICE

This study investigates the process of interaction between the impact device tool and its body elements during an impulse response from the processing medium in the presence of a hysteretic damper of mechanical vibrations. The task addressed is to build a mathematical model with hysteresis damping of oscillations of the impact device elements.

In the mathematical model, the tool is represented by a rod of variable cross-section, and the body parts of the hydraulic hammer are represented by a discrete element with a reduced mass. To damp mechanical oscillations, a rheological model of the hysteresis type is used. The impact interaction of the device elements is modeled by the presence of rigid and dissipative connections. The motion of the impact device elements is described by a system of nonlinear differential equations.

The combination of discrete and continuous types of models has made it possible to solve the task of synthesizing a mathematical model. A comparison for the discrete-continuous model and the discrete model of hysteresis curves justifies their correctness. The proposed model makes it possible to estimate the energy consumption for damping and the distribution of stresses along the length of the tool. When the recoil force changes in the range of 50–500 kN for 1 ms, the energy losses were up to 500 J, and the stress in the conical part of the tool was up to 560 MPa.

To solve the initial-boundary problem, a numerical method is used, which includes the finite difference method and the Euler scheme with linearization. The parameters of the numerical method were determined using a discrete two-mass model. The length step is 0.005–0.01 of the tool length, the time step is 0.001–0.05 ms.

The model could be used in the design of rock development devices and impact systems to increase hydrocarbon production in the oil industry

Keywords: impact device, pulse process, rheological model, energy efficiency, Euler scheme with linearization

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1. Introduction

Technical devices of impulse action with a linear drive are widely used in oil production and construction technology, as well as in the development of rock. The task of activating oil wells and ensuring permissible loads on the elements of the impact device under impulse loads from the processing medium is relevant. The issue of rational use of resources is critical at present.

The rapidity of impulse processes complicates their study by experimental means. The use of mathematical modeling methods makes it possible to find rational solutions when designing such devices (impactors, shock absorbers, etc.).

One of the current design tasks is the choice of a method for damping mechanical vibrations of the device elements during recoil from the processing medium (PM). The issue arises of combining in mathematical models of the technical device the processes of interaction of elements (hammer, tool, housing, PM) with mathematical models of the hysteresis process. It should be noted that rheological hysteresis models reflect the physical process of hysteresis through the mechanical interaction of elastic and dissipative elements (Maxwell, Voigt, Bingham, Kelvin models and their combinations).

Thus, devising models that combine models of the interaction processes of technical device elements with hysteresis models is a relevant issue. When designing pulse impact devices, there is a challenging task to improve efficiency while reducing the recoil reaction on the device body elements.

2. Literature review and problem statement

It is necessary when using impact devices to reduce the recoil reaction and increase their reliability and efficiency. For example, when using an impact system while implementing the "Impulse" technology to increase hydrocarbon production, it is important to achieve an increase in the amplitude of the pulses when acting on the formation system with simultaneous protection from the effects of vibration on the oil well reinforcement [1]. The design of such systems requires construction of mathematical models with effective dampers of mechanical vibrations.

In papers by a number of authors, the study of linear drive devices was carried out using models with discrete elements in the presence of rigid and dissipative connections. The impact force of discrete elements in work [2] was determined according to the Hertz model (in a power dependence relative to the difference in displacements). Such a model is simplified and can be used only in conjunction with continuous elements. In [3], the striker and the tool of the impact device are replaced by discrete elements that have elastic and dissipative connections. Such a model does not make it possible to estimate the distribution of stresses in the tool cross-sections, which arise as a result of the action of the processing medium on the tool. Methods of increasing the efficiency of the impact device were studied in [4]. The given model is built only on the basis of discrete elements, which also limits its efficiency and does not make it possible to solve the specified issues. In such models, low-frequency oscillations obtained by solving a system of ordinary differential equations were studied. The use of only discrete models does not make it possible to determine the parameters of high-frequency oscillations that accompany pulse processes. In addition, it is known that the properties of the processing medium play a significant role.

In [5], a study of the discrete-continuous model of the impact device is carried out. The tool is modeled by a rod of variable cross-section, and the striker by a discrete element. The action of PM on the tool is described by the Voigt model (parallel combination of elastic and plastic elements). The model makes it possible to determine the distribution of stresses in the tool sections. However, the issue of effective damping of vibrations of the impact device elements during an impulse action from PM is not solved. In [6], a discrete-continuous model is described, in which the process of damping of the impact device elements during an impulse reaction from PM is considered. The Voigt model is used as the vibration damper, which can be considered not sufficiently effective.

Mechanical hysteresis models have been studied by the authors of a number of works. In [7, 8], an overview of the properties of hysteretic processes in thermodynamic systems is given. Repulsive clathrates (RCs) and their properties, from which energy efficiency stands out, are considered. The possibility of practical use of such properties is given in [9]. The experiments revealed high efficiency of damping of mechanical vibrations by such systems. The results require theoretical justification based on mathematical modeling. The task of choosing a hysteresis model for its use in the model of a shock device for damping mechanical vibrations is necessary.

In [10], models are considered in which the variable resistance force is determined by analytical formulas with switching conditions. The analytical formula is an approximation of experimental data. The disadvantage of such a model is the absence of a mechanism of interaction between elements that characterizes the physical process. In the work, the analytical expression is based on the given dependence "force-displacement derivative" in the form of a polynomial. This approach only simplifies the formulas, but the indicated disadvantages remain. In [11], a hysteresis model is described, the basis of which is the Ishlinsky operator. The apparatus of fractional derivatives is also used. One should note the complexity of implementing such a model in practical tasks. A comparison of the effectiveness of such models with linear and nonlinear dampers is given in [12]. A functional approach is implemented in determining the dependence of the resistance force on displacement, and the effectiveness of hysteresis damping of oscillations is shown. The methods are based on elements of the mathematical theory, which is given in work [13], where the operator approach is used to build hysteresis models. These studies are mainly qualitative in nature and their use in practice requires the construction of rather complex algorithms. In general, this approach is formal in nature, the physical nature of hysteresis remains unclear.

The authors of [14] studied the model of infiltration of a non-wettable liquid into a nanoporous medium. The model uses analytical methods of the percolation theory. The model allowed them to establish a relationship between the parameters of the porous medium and macroscopic characteristics. It should be noted that the use of percolation theory models is quite complicated for practical application in the calculation schemes of technical devices. Models of technical devices have a mechanical structure (consisting of discrete and continuous elements, elastic and plastic connections), therefore a similar structure of hysteresis models is also desirable.

In [15], the rheological model of a repulsive clathrate and an example of its application in automobile shock absorbers are considered. Such a model reflects the physical process of hysteresis and therefore makes it possible to for the interpretation of the process of intrusion and extrusion of liquid into the pore channels of the matrix. Only the kinematic model is considered, the task of adapting this model to the general calculation scheme of the interaction of the main elements of a technical device for the purpose of damping mechanical vibrations remains unsolved.

The use of only discrete elements in the model does not make it possible to determine the stresses in the cross-sections of the tool and take into account high-frequency oscillations. The use of analytical formulas in hysteresis models requires the determination of coefficients from experimental data. This approach lacks a physical mechanism of interaction between the elements of the impact device. The use of vibration damping models with a linear damper (Voigt model) does not make it possible to significantly increase the efficiency of vibration damping. Hysteresis models are built on the basis of the theory of percolation, fractional derivatives, operator theory, and require rather complex algorithms for use in practical tasks.

All this allows us to state that it is advisable to conduct research into construction and improvement of a discrete-continuous model of an impact device with vibration damping by a hysteresis damper.

3. The aim and objectives of the study

The purpose of our study is to construct a mathematical model of hysteretic damping of vibrations of the elements of the impact device during an impulse response from the processing environment. Such a model could make it possible to determine the efficiency of the damper and the dependence of stresses in the tool sections when changing the impulse load from the processing environment.

Achieving the goal requires solving the following tasks:

– to synthesize the calculation scheme "reaction of the PM-tool-elements of the impact device body with a rheological-type hysteretic damper";

– to state the initial-boundary problem with a system of ordinary differential equations and the wave equation under nonlinear contact and boundary conditions;

– to choose a numerical method for solving the initial-boundary problem based on a mixed scheme of the finite difference method, which will make it possible to determine the stresses and energy consumption for damping;

– to build a model problem based on a two-mass discrete system with a hysteresis damper and a numerical method using the Runge-Kutta scheme, which will make it possible to estimate the rational parameters of the finite difference method;

– to conduct computational experiments at different parameters of the impulse load, with the determination of the main characteristics of the damping system and estimates of its efficiency.

4. Materials and methods

The object of our study is the process of interaction between the impact device tool and its body elements during an impulse response from PM in the presence of a hysteresis damper of mechanical vibrations.

The hypothesis of the study is as follows. The process of impulse interaction of the tool with the body elements of the impact device, with an impulse response from the processing medium and hysteresis damping, is described by a nonlinear system of differential equations with contact and initial conditions. Adopting a rheological model as a hysteresis model, which adapts to the discrete-continuous model of the tool with body elements, will make it possible to determine the efficiency of the damper and establish the dependence of stresses in the tool sections taking into account high-frequency oscillations when changing the impulse load from the processing medium.

Basic assumptions:

a) the cross-sections of the rod remain flat during the oscillations;

b) the force of interaction of PM with the end of the tool has an impulse character;

c) contact interaction between the elements of the device during the impact is determined by elastic and plastic elements, the value of the impact force is determined by the power dependence of the difference in their displacements;

d) the hysteresis resistance is determined by the characteristics of the rheological model of RC, adapted to the displacements of a discrete element.

Basic simplifications:

a) the transverse oscillations of the rod are insignificant; therefore, they are not taken into account in the model;

b) the tool has a shape close to cylindrical, with a conical working part;

c) the impulse load on the working end of the tool has only an axial component (determined by the force acting along the axis of the cylinder).

Differential equations of oscillations of the device elements constitute a system of four equations with nonlinear conditions. The connection between the device elements is described by contact conditions. Initial conditions determine the state of the system before the impulse action of the force on the tool end. The system of equations boundary and initial conditions constitute a nonlinear initial-boundary-value problem.

To find a solution to the initial-boundary-value problem, a numerical method is used, built on the basis of a mixed scheme of the finite difference method. A small time interval allows partial linearization of the system of equations without significant loss of accuracy. To estimate the main parameters of the numerical method, a two-mass model is used, taking into account the hysteresis block. Comparison of results over a long time interval makes it possible to estimate the rational parameters of the grid region, which enables the economy and stability of the difference method. The numerical method for the two-mass model, which is nonlinear, was tested using the Runge-Kutta method built into the Mathcad system (USA) with the hysteresis module turned off. The Mathcad system was used to program the algorithms of the numerical methods.

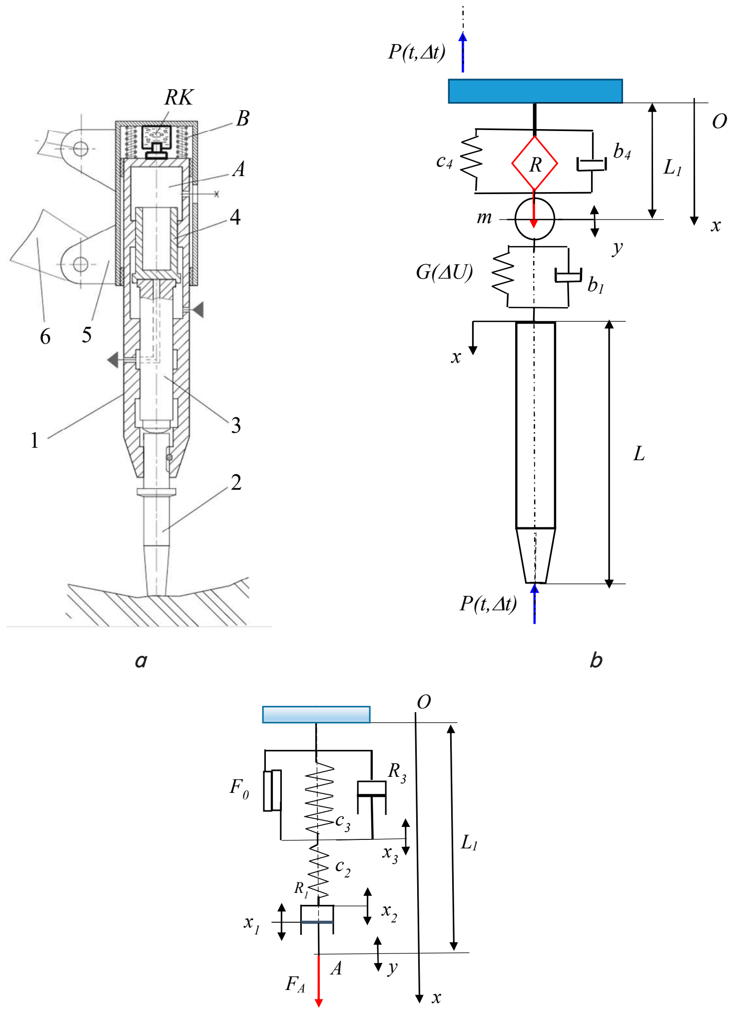
5. Results of the damping process study

5. 1. Construction of the calculation scheme

Fig. 1 shows the design and calculation schemes of an impact device with one striker. The following variable designations are accepted: $u\,(t, x)$ – displacement of the rod cross-section with coordinate x; t – time; $y\,(t)$ – displacement of the center of a discrete element with mass m; E – modulus of elasticity; ρ – density of the rod material; $a = (E/\rho)^{1/2}$ – speed of sound in the rod material; $S = S(x)$ – cross-sectional area of the rod. The use of the repulsive clathrate hysteresis module model as a damper for oscillations of the hydraulic hammer elements was considered. The calculation scheme of the device with a damper is shown in Fig. 1, *b*. In the model, the external one is the impulse load $P(t, \Delta t)$, acting on the tool from the side of the processing medium, the R-module hysteresis. Fig. 1, *c* shows the rheological diagram of the hysteresis module.

The calculation scheme consists of a rod with a variable cross-section, a reduced discrete mass, a hysteresis module that models RC (repulsive clathrate). The listed elements have elastic and dissipative connections. The elements of the device (striker, body, valve) are loaded with a short-term force acting on the working end of the rod-tool from the PM side.

Thus, Fig. 1 shows a scheme that reflects the synthesis of the calculation scheme of the impact device.



a b c

Fig. 1. Schemes of the impact device: *a* — structural diagram; *b* — generalized diagram; *c* — hysteresis resistance module; structural elements: 1 — housing; 2 — tool, 3 — striker; 4 — valve, 5 — adapter, 6 — manipulator; *A* — pneumatic accumulator chamber; *B* — shock absorption module; *RK* (*R*) — hysteresis module of repulsive clathrates; $G(\Delta u)$, c_4 — stiffness coefficients; b_1, b_4 — dissipation coefficients of the connection elements of the tool and housing; *L* — length of the tool-rod; R_1, R_3, F_0 — dissipative elements of the hysteresis module; c_2, c_3 — elastic elements

5. 2. Initial-boundary value problem

The mathematical model that describes the process of interaction of the tool with the body elements during the reaction from the PM side and during hysteretic damping of vibrations is represented by the initial-boundary value problem.

The equations of motion of the rod and discrete element sections are given in the following form:

$$\frac{\partial^2 u(t,x)}{\partial t^2} = a^2\left[\frac{1}{S(x)}\cdot\frac{dS(x)}{dx}\frac{\partial u(t,x)}{\partial x}+\frac{\partial^2 u(t,x)}{\partial x^2}\right],$$

$$0<t\le T,\quad 0\le x\le L, \tag{1}$$

$$m\frac{d^2y}{dt^2} = -F_A(t)+G(\Delta u)\cdot\left(u(t,0)-y\right)+$$
$$+b_1\frac{d}{dt}\left(u(t,0)-y\right)-c_4y-b_4\frac{dy}{dt}. \tag{2}$$

The boundary conditions for the rod reflect the nature of the interaction of the ends with the discrete element and PM:

$$S(0)E\frac{\partial u}{\partial x}(t,0) = -G(\Delta u)\left(y(t)-u(t,0)\right)-$$
$$-b_1\left(\frac{dy}{dt}-\frac{\partial u(t,0)}{\partial t}\right), \tag{3}$$

$$S(L)E\frac{\partial u}{\partial x}(t,L) = -P(t,\Delta t). \tag{4}$$

The condition for the movement of parallel elements of block *R* (block *RC*) was determined by formula [14]:

$$\frac{dx_3}{dt}(t)=$$
$$=\begin{cases}F_m(t)-F_0 sign\left(F_m(t)\right), \text{ if } \left|F_m(t)\right|\ge F_0,\\ 0, \text{ if } \left|F_m(t)\right|<F_0;\end{cases} \tag{5}$$

In equation (5)

$$F_m(t)=F_A(t)-c_3x_3(t). \tag{6}$$

Equation relative to resistance force F_A

$$\frac{dF_A(t)}{dt}=c_2\left(\frac{dy}{dt}(t)-\frac{dx_3(t)}{dt}-\frac{F_A(t)}{R_1}\right). \tag{7}$$

Initial conditions for a rod and a discrete element:

$$u(0,x)=0,\ \frac{\partial u}{\partial t}(0,x)=0, \tag{8}$$

$$y(0)=0,\ \frac{dy}{dt}(0)=0. \tag{9}$$

The dependence of the coefficient of rigid coupling of a discrete element with the end of the rod on the difference in displacements was determined from the formula:

$$G(\Delta u)=\begin{cases}c_1\cdot\Delta u^{\alpha}, \text{ if } \Delta u\ge 0,\\ \tilde{n}_0, \text{ if } \Delta u<0.\end{cases} \tag{10}$$

In formula (10), $u = y(t) - u(t, 0)$, $0 \le \alpha \le 0.5$.

Formula (10) models the contact interaction of a discrete element with the end of the rod. This interaction corresponds to the Hertz model [1].

The action of an external force on the tool during a short period of time Δt was considered:

$$P(t,\Delta t)=\begin{cases}P_0, \text{ if } 0\le t\le\Delta t,\\ 0, \text{ if } t>\Delta t.\end{cases} \tag{11}$$

Therefore, the external force is a consequence of the action of PM on the end of the tool and affects the increase or decrease in the amplitude of oscillations of the end of the tool and the discrete element.

It is advisable to give an explanation for the form of equation (7). The total resistance force of the module if the law of motion $y(t)$ of point *A* is known (Fig. 1*c*) was determined as follows. Local displacements (deformations) of the elements are related to the total displacement of the upper end of the rod by the equation

$$y(t)=x_1(t)+x_2(t)+x_3(t). \quad (12)$$

The resistance force of the hysteresis module is equal to the sum of the forces from the parallel module RC (F_0, c_3, R_3), which is equal to the force from the series-connected c_2 and R_1

$$F_A=F_1=F_2=F_3. \quad (13)$$

Forces are determined from formulas

$$F_A=R_1\frac{dx_1}{dt}=c_2x_2=F_0 sign(F_m)+c_3x_3+R_3\frac{dx_3}{dt}, \quad (14)$$

$$F_A=c_2\big(y(t)-x_1(t)-x_3(t)\big). \quad (15)$$

From equality (15), the time derivative is obtained

$$\frac{dF_A}{dt}=c_2\left(\frac{dy}{dt}(t)-\frac{dx_1(t)}{dt}-\frac{dx_3(t)}{dt}\right)=$$
$$=c_2\left(\frac{dy}{dt}(t)-\frac{dx_3(t)}{dt}-\frac{F_A}{R_1}\right). \quad (16)$$

As a result, a linear first-order differential equation (7) is obtained with respect to the force $F_A(t)$ for a given $y(t)$.

5. 3. Algorithm of the numerical method

The initial-boundary value problem (1) to (10) is approximated by a discrete problem. The basis for choosing a mixed difference scheme is the results obtained in [5, 6]. The partial differential equation is approximated by a two-layer mixed difference scheme with a weighting factor γ:

$$\frac{u_i^{n+1}-2u_i^n+u_i^{n-1}}{\tau^2}=$$
$$=\gamma a^2\left[\frac{1}{S(x_i)}\cdot\frac{S(x_{i+1})-S(x_{i-1})}{2h}\cdot\frac{u_{i+1}^{n+1}-u_{i-1}^{n+1}}{2h}+\frac{u_{i+1}^{n+1}-2u_i^{n+1}+u_{i-1}^{n+1}}{h^2}\right]+$$
$$+(1-\gamma)a^2\left[\frac{S(x_{i+1})-S(x_{i-1})}{S(x_i)2h}\cdot\frac{u_{i+1}^{n}-u_{i-1}^{n}}{2h}+\frac{u_{i+1}^{n}-2u_i^{n}+u_{i-1}^{n}}{h^2}\right], \quad (17)$$

$$i=1,\dots,N-1,\quad n=1,\dots,M-1.$$

To simplify the implementation of the algorithm, an approximation of boundary conditions with the first order of step h is chosen:

$$S(0)E\frac{u_1^{n+1}-u_0^{n+1}}{h}=-G(\Delta u^n)\cdot\left(y^{n+1}-u_0^{n+1}\right)-$$
$$-b_1\left(\frac{y^{n+1}-y^n}{\tau}-\frac{u_0^{n+1}-u_0^n}{\tau}\right), \quad (18)$$

$$S(L)E\frac{u_N^{n+1}-u_{N-1}^{n+1}}{h}=-P(t_n,\Delta t). \quad (19)$$

The discrete element oscillation equations are approximated by the implicit Euler scheme with partial linearization of the stiffness coefficient (the Δu^n value is calculated on the previous time layer)

$$m\frac{y^{n+1}-2y^n+y^{n-1}}{\tau^2}=$$
$$=-F_A^{n+1}+G(\Delta u^n)\left(u_0^{n+1}-y^{n+1}\right)+$$
$$+b_1\left(\frac{u_0^{n+1}-u_0^n}{\tau}-\frac{y^{n+1}-y^n}{\tau}\right)-$$
$$-c_4y^{n+1}-b_4\frac{y^{n+1}-y^n}{\tau}. \quad (20)$$

Equations (5)–(7) were approximated according to Euler's implicit scheme:

$$F_m^{\;n+1}=F_A^{\;n+1}-c_3x_3^{n+1}, \quad (21)$$

$$\frac{x_3^{n+1}-x_3^n}{\tau}=\begin{cases}F_m^{\;n}-F_0 sign\left(F_m^{\;n}\right), \text{ if } \left|F_m^{\;n}\right|\ge F_0,\\ 0, \text{ if } \left|F_m^{\;n}\right|<F_0;\end{cases} \quad (22)$$

$$\frac{F_A^{\;n+1}-F_A^n}{\tau}=c_2\left(\frac{y^{n+1}-y^n}{\tau}-\frac{x_3^{n+1}-x_3^n}{\tau}-\frac{F_A^{\;n+1}}{R_1}\right). \quad (23)$$

The initial conditions for the rod and discrete element were approximated with the first order with respect to t

$$u_i^0=0,\quad \left(u_i^1-u_i^0\right)\cdot\tau^{-1}=0,\quad y^0=0,\quad \left(y^1-y^0\right)\tau^{-1}=0.$$

$$x_i=ih,\quad i=1,2,\dots,N. \quad (24)$$

Here $t_n=n\times t$, $t=T/M$, $x_i=i\times h$, $h=L/N$ are the parameters of the grid region, $u_i^n=u(t_n,x_i)$, $y^n=y(t_n)$ are the grid functions (functions defined only at the grid nodes).

It is expedient to write an algorithm for solving the discrete problem (17) to (24).

From equation (23), the following quantity was derived

$$F_A^{n+1}=\frac{R_1\left(F_A^n+c_2\left(u_N^{n+1}-u_N^n-x_3^{n+1}+x_3^n\right)\right)}{R_1+\tau c_2}. \quad (25)$$

The system of equations (17) to (20) at each time layer $t_n=n\times$ t was solved by the sweep method [16], adapted for a mixed system with nonlinear boundary conditions. Equation (17) was reduced to the form

$$A_iu_{i+1}-B_iu_i+C_iu_{i-1}=-F_i. \quad (26)$$

From equations (17), formulas for coefficients are found A_i, B_i, C_i, and F_i: $i=1, 2, \dots, N-1$:

$$A_i=-\frac{a^2\gamma\tau^2}{h^2}\left(\frac{S(x_{i+1})-S(x_{i-1})}{4S(x_i)}+1\right),$$

$$B_i=-\left(1+\frac{2a^2\tau^2\gamma}{h^2}\right),$$

$$C_i=\frac{a^2\gamma\tau^2}{h^2}\left(\frac{S(x_{i+1})-S(x_{i-1})}{4S(x_i)}-1\right),$$

$$F_i=\begin{bmatrix} -2u_i^n+u_i^{n-1}-(1-\gamma)\dfrac{a^2\tau^2}{h^2}\times \\ \times\left(\dfrac{S(x_{i+1})-S(x_{i-1})}{4S(x_i)}(u_{i+1}^n-u_{i-1}^n)+ \right. \\ \left. +u_{i+1}^n-2u_i^n+u_{i-1}^n \right)\end{bmatrix},$$

$$i=1,2,\ldots,N-1. \quad (27)$$

The algorithm of the sweep method, taking into account boundary conditions and equation (20), was as follows:

1. From the boundary condition (19) and the formula of the sweep method

$$u_i=\alpha_i u_{i+1}+\beta_i, \quad (28)$$

at $i = N - 1$, the system of equations is obtained:

$$\begin{cases} u_{N-1}^{n+1}=u_N^{n+1}+\dfrac{h}{S(L)E}P(t_n,\Delta t), \\ u_{N-1}^{n+1}=\alpha_{N-1}u_N^{n+1}+\beta_{N-1}. \end{cases} \quad (29)$$

So, the formulas for the coefficients of the sweep method are as follows:

$$\alpha_{N-1}=1;$$

$$\beta_{N-1}=\frac{h}{S(L)E}P(t_n,\Delta t). \quad (30)$$

2. The coefficients $\mathbf{a}_i$ and $\mathbf{b}_i$ were calculated using the “inverse formulas” of the sweep method:

$$\alpha_{i-1}=\left(B_i-\frac{A_i}{\alpha_i}\right)(C_i)^{-1},$$

$$\beta_{i-1}=\frac{\beta_i\cdot(B_i-C_i\alpha_{i-1})-F_i}{C_i};$$

$$i=N-1,N-2,\ldots,2,1. \quad (31)$$

3. Boundary condition (18) makes it possible to find the relationship between u_1^{n+1} and y^{n+1}. This relationship is obtained from the system of equations:

$$\begin{cases} S(0)E\dfrac{u_1^{n+1}-u_0^{n+1}}{h}=-G(\Delta u^n)(y^{n+1}-u_0^{n+1})- \\ -b_1\left(\dfrac{y^{n+1}-y^n}{\tau}-\dfrac{u_0^{\,n+1}-u_0^{\,n}}{\tau}\right), \\ m\dfrac{y^{n+1}-2y^n+y^{n-1}}{\tau^2}=-F_A^{n+1}+G(\Delta u^n)\times \\ \times(u_0^{n+1}-y^{n+1})+\dfrac{b_1}{\tau}(u_0^{n+1}-u_0^n-y^{n+1}+y^n)- \\ -cy^{n+1}-b\dfrac{y^{n+1}-y^n}{\tau}, \\ u_0^{n+1}=\alpha_0u_1^{n+1}+\beta_0. \end{cases} \quad (32)$$

As a result of the transformations from (32), a system of two equations was obtained:

$$\begin{cases} y^{n+1}=R_{11}u_1^{n+1}+R_{12}, \\ y^{n+1}=R_{21}u_1^{n+1}+R_{22}. \end{cases} \quad (33)$$

The coefficients in system (33) were determined from the following formulas:

$$R_{11}=\frac{\tau^2S(0)E}{mh\cdot H_1}(1-\alpha_0),$$

$$R_{12}=\frac{\tau^2}{mH_1}\left(-F_A^n+\frac{b_4}{\tau}y^n-\frac{S(0)E}{h}\beta_0\right)+\frac{2y^n-y^{n-1}}{H_1}, \quad (34)$$

$$R_{21}=\frac{\alpha_0(H_0+1)-1}{H_0},$$

$$R_{22}=\frac{\beta_0(H_0+1)}{H_0}+\frac{hb_1(y^n-u_0^n)}{S(0)E\tau H_0}, \quad (35)$$

where:

$$H_0=\frac{h}{S(0)E}\left(G(\Delta u^n)+\frac{b_1}{\tau}\right),$$

$$H_1=\frac{\tau}{m}(\tau c_4+b_4).$$

Thus:

$$u_1^{n+1}=\frac{R_{22}-R_{12}}{R_{11}-R_{21}},$$

$$u_0^{n+1}=\alpha_0u_1^{n+1}+\beta_0. \quad (36)$$

The Mathcad system was chosen to implement the algorithm. In this system, functional autonomous blocks were developed that solved partial problems.

5. 4. Construction of a discrete-type model problem

The calculation scheme that forms the model problem is shown in Fig. 2. The system of equations of motion of two discrete elements connected by rigid and dissipative connections taking into account the reaction force of PM:

$$m_1\frac{d^2x}{dt^2}=-P(t,\Delta t)+c_1(y-x)+b_1\left(\frac{dy}{dt}-\frac{dx}{dt}\right), \quad (37)$$

$$m_2\frac{d^2y}{dt^2}=-F_A(t)+c_1(x-y)+$$
$$+b_1\left(\frac{dx}{dt}-\frac{dy}{dt}\right)-c_4y-b_4\frac{dy}{dt}, \quad (38)$$

$$t\in[0,T].$$

Boundary conditions and resistance of block R (Fig. 1, *c*) were determined from the following equations:

$$F_m(t)=F_A(t)-c_3x_3(t), \quad (39)$$

$$\frac{dx_3}{dt}(t)=\begin{cases} F_m(t)-F_0sign(F_m(t)), \text{ if } |F_m(t)|\ge F_0, \\ 0, \text{ if } |F_m(t)|<F_0; \end{cases} \quad (40)$$

$$\frac{dF_A(t)}{dt}=c_2\left(\frac{dy}{\partial t}-\frac{dx_3(t)}{dt}-\frac{F_A(t)}{R_1}\right). \quad (41)$$

The initial conditions were as follows:

$$\frac{dy}{dt}(0)=0,\quad \frac{dx}{dt}(0)=0, \tag{42}$$

$$x(0)=0,\quad y(0)=0,\quad x_1(0)=0. \tag{43}$$

System (37) to (41) models the case when the tool is replaced by a discrete element. Comparison of solutions serves as the basis for the correctness of the choice of the algorithm for solving the difference problem and the parameters of the difference scheme.

The solution to the initial problem was found by a numerical method.

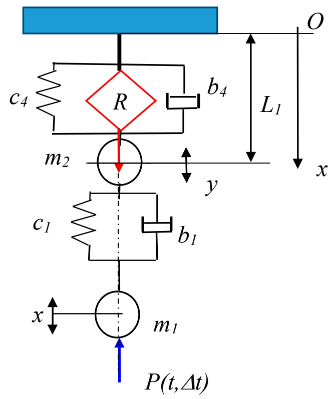


Fig. 2. Calculation scheme of the model problem

The hysteresis module R contains nonlinearities and therefore the use of standard programs is problematic. The second-order equations were approximated by an implicit scheme in which the nonlinearities were linearized:

$$\frac{x^{n+1}-2x^{n}+x^{n-1}}{\tau^2}=\frac{1}{m_1}\left(\begin{aligned}&-P(t_n,\Delta t)+c_1\left(y^{n+1}-x^{n+1}\right)+\\&+b_1\left(\frac{y^{n+1}-y^{n}-\left(x^{n+1}-x^{n}\right)}{\tau}\right)\end{aligned}\right), \tag{44}$$

$$\frac{y^{n+1}-2y^{n}+y^{n-1}}{\tau^2}=\frac{1}{m_2}\left(\begin{aligned}&-F_A^n+c_1\left(x^{n+1}-y^{n+1}\right)+\\&+b_1\left(\frac{x^{n+1}-x^{n}-\left(y^{n+1}-y^{n}\right)}{\tau}\right)-\\&-c_4y^{n+1}-b_4\frac{y^{n+1}-y^{n}}{\tau}.\end{aligned}\right) \tag{45}$$

The algorithm, taking into account the RC block, took the form:

$$x^{n+1}=\frac{\frac{\tau^2}{m_1}\left(-P(t_n,\Delta t)+y^n\left(c_1+\frac{b_1}{\tau}\right)+\left(x^n-y^n\right)\frac{b_1}{\tau}\right)+2x^n-x^{n-1}}{1+\frac{\tau^2}{m_1}\left(c_1+\frac{b_1}{\tau}\right)}, \tag{46}$$

$$y^{n+1}=\frac{\frac{\tau^2}{m_2}\left(-F_A^n+x^{n+1}\left(c_1+\frac{b_1}{\tau}\right)+\left(y^n-x^n\right)\frac{b_1}{\tau}\right)+2y^n-y^{n-1}}{1+\frac{\tau^2}{m_2}\left(c_1+\frac{b_1}{\tau}+c_4+\frac{b_4}{\tau}\right)}. \tag{47}$$

The resistance force F_A was determined using the implicit Euler scheme:

$$F_m^{\,n+1}=F_A^{\,n+1}-c_3x_3^{n+1}, \tag{48}$$

$$x_3^{n+1}=\begin{cases}x_3^n+\tau\left[F_m^{\,n+1}-F_0\,sign\left(F_m^{\,n}\right)\right], & \text{if } \left|F_m^{\,n}\right|\geq F_0,\\ x_3^n, & \text{if } \left|F_m^{\,n}\right|<F_0;\end{cases} \tag{49}$$

$$F_A^{n+1}=F_A^n+c_2\left(\frac{y^{n+1}-y^n-x_3^{n+1}+x_3^n}{1+\frac{\tau c_2}{R_1}}\right). \tag{50}$$

The algorithm was tested with the RC unit turned off. The Runge-Kutta method was used in the Mathcad system.

5. 5. Results of computational experiments

To estimate the parameters of the difference method, which provide acceptable accuracy of the solution, a model problem was used.

Parameters in calculations: $c_1=9\times10^5$ N/m^{1+a}, $b_1=b_4=0$, $m_1=12$ kg, $m_2=100$ kg, $P_0=5\times10^4$ N, $\Delta t=0.001$ s, $T=1$ s, $c_4=5\times10^4$ N/m. For the damper module: $c_2=8\times10^4$ N/m, $c_3=10^4$ N/m, $R_3=5$ Ns/m, $R_1=9\times10^3$ Ns/m, $F_0=500$ N.

Fig. 3 shows plots that demonstrate sufficient accuracy of the numerical method for a nonlinear system. The results with the damper R turned off are close to the results obtained by the Runge-Kutta method in the Mathcad system. The effect of damping oscillations is demonstrated in Fig. 3, *b*, while the Runge-Kutta method solved a linear problem (the damper was not taken into account).

Comparison of the solutions to the model problem with the solutions to the main problem is shown in Fig. 4, 5. Fig. 4 shows the change in time of the displacements of discrete elements and the dependence of the resistance force of the hysteresis module on the displacement of element m_2. It should be noted that additional linear dampers in the system were excluded ($b_1=b_4=0$).

Fig. 5, 6 show solutions to the main problem. The basic parameters: $c_1=9\times10^5$ N/m, $m=100$ kg, $b_1=b_4=0$, $c_4=9\times10^4$ N/m, $T=1$ s, g $=0.5$, $P_0=5\times10^4$ N, $\Delta t=1$ ms. Parameters of the damper module: $c_2=8\times10^4$ N/m, $c_3=10^4$ N/m, $R_3=5$ Ns/m, $R_1=9\times10^3$ Ns/m, $F_0=500$ *N*.

Fig. 6 shows parameters that characterize the operation of the hysteresis block in the main problem.

The change in the parameters of *RC* module over time demonstrates the performance of damping functions by the elements of the system (Fig. 6, *a*). Plot (6) shows that at the first stage the parallel module is moved (displacement x_3). At the second stage the resistance is determined by R_1, the oscillations are damped.

The short-term action of an external force on the rod leads to the effect of high-frequency oscillations of the tool sections, characteristic of impact interaction. Fig. 7 shows high-frequency oscillations of the tool ends, which depend on the magnitude of the external impulse.

Stress calculations were performed for a variable cross-section of a rod-tool with a conical working part (Fig. 8, *a*). The dynamics of stress distribution along the length of the tool are shown in Fig. 8, *b*. The stresses were determined from the formula

$$\sigma_i^n=E\frac{u_{i+1}^n-u_{i-1}^n}{2h}. \tag{51}$$

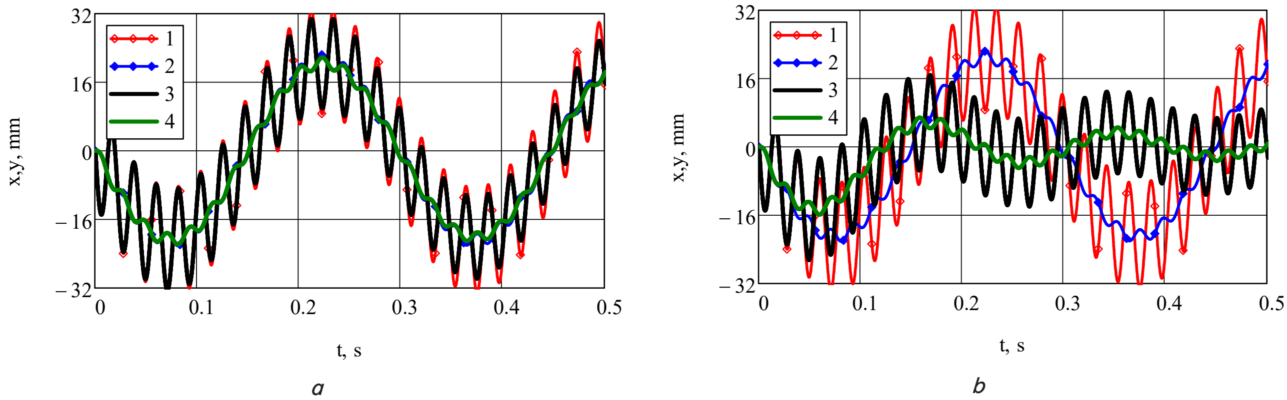


Fig. 3. Comparison of numerical methods: *a* – *R* off; *b* – *R* on; 1, 2 – *x*, *y* (Runge-Kutta method); 3, 4 – *x*, *y* (numerical method); impulse load P_0 = 50000 N, D*t* = 0.001s

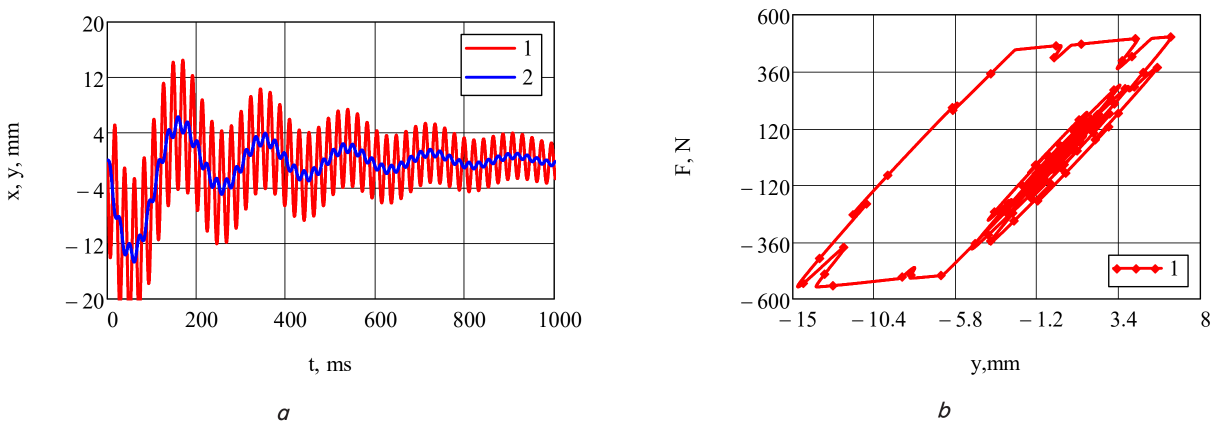


Fig. 4. Solutions to the model problem (for comparison): *a* – oscillations of discrete elements (1 – *x*, 2 – *y*); *b* – dependence of the resistance force of the module of repulsive clathrates on displacement *y*

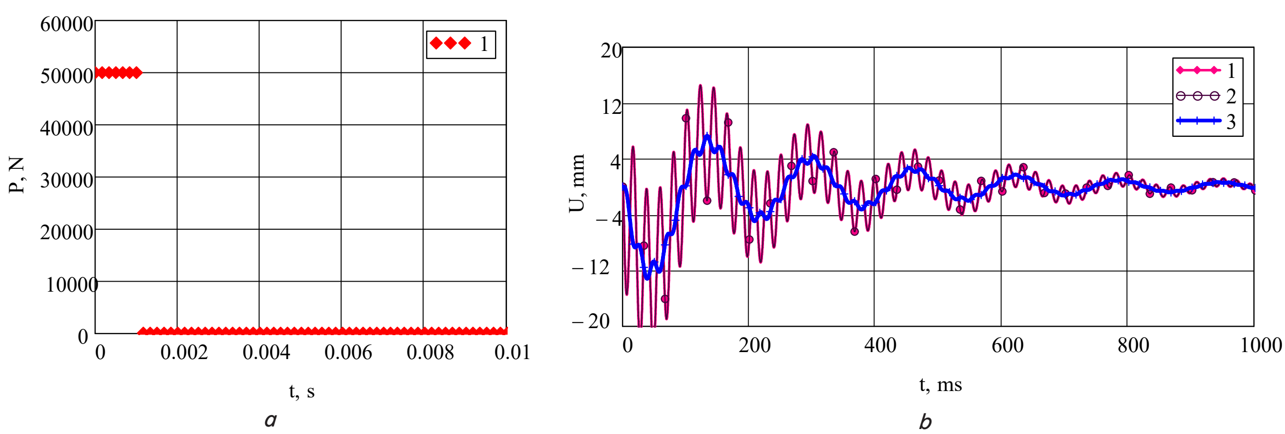


Fig. 5. Load and movement of elements: *a* – imitation of impulse action using an external force; *b* – movement over time; 1 – u_N; 2 – u_0; 3 – *y*

The highest compressive stress was obtained in the conical part of the tool, about 120 MPa (plot 4). The stress distribution along the length of the tool makes it possible to estimate the most loaded sections of the tool at different times.

It should be noted that determining the area bounded by the hysteresis curve in several cycles means determining the energy consumption for damping.

The following parameters were set:

$t = 2$s, $Dt = 0.001$ s, $F_0 = 700$ N, $R_3 = 5$ Ns/m,

$R_1 = 9 \times 10^3$ Ns/m, $c_0 = c_1 = 9{\times}10^5$ N/m, $c_2 = 8 \times 10^4$ N/m,

$c_3 = 10000$ N/m, $m = 150$ kg, $c_4 = 9\ 10^4$ N/m.

With the given parameters, the maximum compressive stress in the conical part of the tool at P_0 = 500 kN reaches a value of 560 MPa, and the damping work is 500 J.

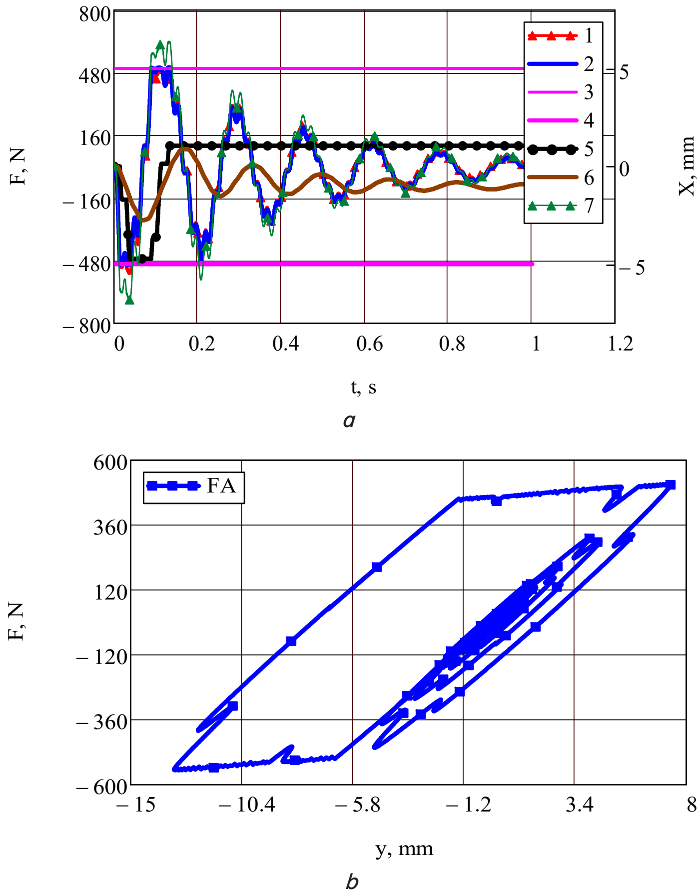


Fig. 6. Damper characteristics: *a* — change in time of basic parameters, 1 — F_A; 2 — Fm; 3 — F_0; 4 — F_0; 5 — x_3; 6 — x_1; 7 — x_2; *b* — hysteresis curve (dependence of force F_A on displacement y)

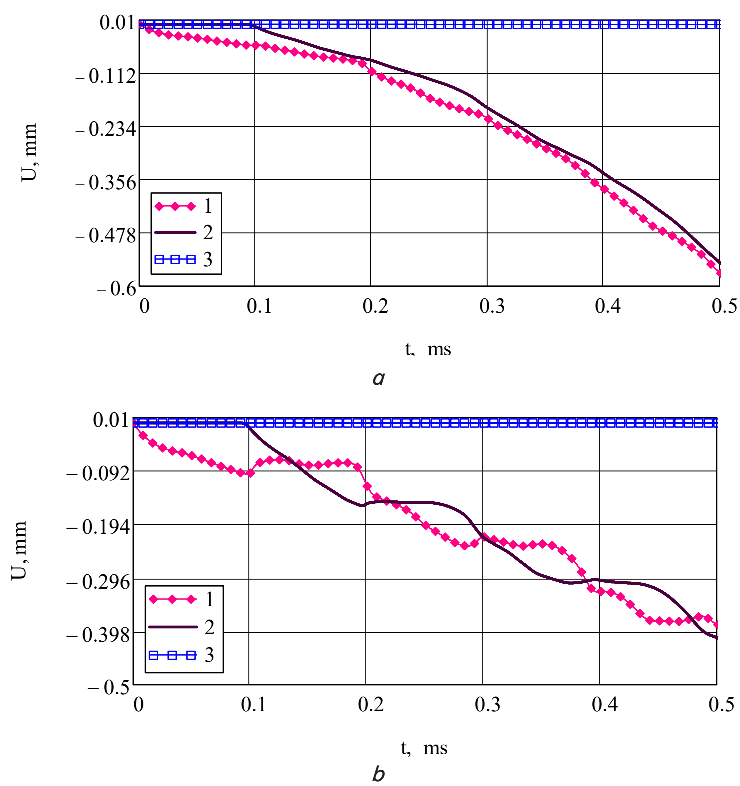


Fig. 7. High-frequency end-face vibrations: *a* — small impulse, $P_0 = 50$ kN, D$t = 1$ ms; *b* — increased impulse, $P_0 = 500$ kN, D$t = 0.1$ ms; 1 — u_N, 2 — u_0, 3 — y

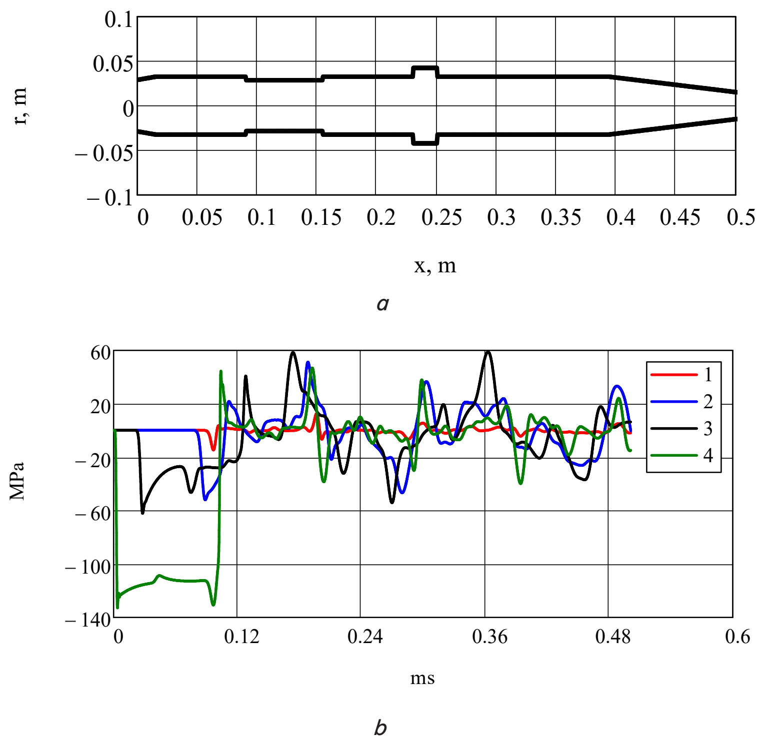


Fig. 8. Dynamics of stress distribution in different sections of the tool: a – tool profile; b – stress in sections; 1 – $x = 5$ mm; 2 – $x = 60$ mm; 3 – $x = 380$ mm; 4 – $x = 490$ mm

6. Discussion of results based on modeling the process of damping vibrations of the elements of an impact device

The impact device, which consists of a tool, striker, and body elements, is reduced to a rod with a variable cross-section, a discrete element, and a hysteresis module (Fig. 1). The hysteresis module is a rheological model with elastic and dissipative elements, which reflects the repulsive clathrate [15, 16]. The model is connected to the discrete-continuous part and provides the process of damping vibrations, which are a consequence of the impulse action from PM. Adaptation of the rheological model to this module makes it possible to obtain stress characteristics, take into account high-frequency vibrations and energy losses for damping.

The discrete-continuous type of impact device model was studied in a number of works [5, 6]. In them, parameters were selected that ensure the stability and efficiency of the numerical method, the basis of which is the finite difference method. When choosing a hysteresis damper model, a rheological model was chosen. This choice turned out to be successful since the components of the model are mechanical elastic and dissipative elements and this corresponds to the analogous elements of the discrete and discrete-continuous models. The numerical method for additional differential equations (5) to (7), which describe the conditions of motion of the elements of the rheological model, was adapted to the numerical method, the basis of which is the finite difference method.

The use of a two-mass discrete model with a module that simulates RC turned out to be necessary for estimating the parameters of the finite difference method scheme. Due to nonlinear conditions, it was not possible to use the standard function of the Mathcad system; therefore, a numerical method was devised, based on an implicit scheme with partial linearization of contact conditions. However, comparing the results with the hysteresis module turned off and on allowed us to conclude that the adopted algorithm is correct (Fig. 3). The nature of the oscillations of discrete elements and the type of hysteresis curves obtained with the same parameters (Fig. 4–6) can serve as a justification for the correctness of numerical methods. Thus, rational ranges of the parameters of the difference scheme were found: $h = (0.005–0.01)\,L$, $g = 0.5$ 0.9, $t = 0.001–0.05$ ms. It should be noted that it was possible to use a mixed finite difference method scheme, since the short time interval over which the oscillatory process was considered was 0.005–2 s, which allowed the use of partial linearization of nonlinear terms under contact conditions.

Regarding the efficiency of the hysteresis damper, it should be noted that the numerical experiments were carried out with the linear dampers turned off ($b_1 = b_4 = 0$). In addition, the permissible range of parameters of the difference scheme and the numerical method was maintained. Thus, only the dissipative force F_A acted on the discrete element, which led to the damping of the oscillations. Over a time period of 2 s, the oscillations almost completely died out and therefore the hysteresis curve was closed. This made it possible to find the area bounded by the hysteresis curves for a certain number of cycles.

By changing the force P_0, which acted over a time interval of 1 ms, the dependence of the work, as well as the maximum normal stress in the conical part of the tool on force P_0 was obtained. In numerical experiments, the main parameters of the rheological model were set, close to those studied in [15], which are consistent with the full-scale experiments conducted by the authors of [9, 15].

It should be noted that in the case of a pulsed process, it is problematic to find a solution to the problem with continuous elements over a large time interval. The energy costs for damping oscillations are easier to find for a two-mass model with equivalent mass characteristics.

Considering the hysteresis curve (Fig. 6, b), two forms of the curve can be distinguished, the first (initial) in the form of a parallelogram, the second (final) in the form of an ellipse. The parameters characterizing the operation of the hysteresis module (Fig. 6, a) show that at the first stage the parallel module (displacement x_3) mainly works, at the second stage R_1, c_2 (displacement x_1, x_2). The type of hysteresis curves significantly depends on the values of the main parameters of the hysteresis module (Fig. 1, c), which consists of chokes R_1, R_2, elastic elements c_2, c_3, and a dry friction element F_0. The possibility of varying these parameters makes it possible to obtain hysteresis curves of different shapes, which will retain their similarity at different values of external loads.

It should be noted that the impulse action of the reaction from PM results in high-frequency oscillations of the tool ends, which are registered only in a short time interval (Fig. 7). The phase lag of the tool ends is determined by the time of passage of the shock wave from the working end to the contact end with the discrete element through the elastic element.

It should be noted that, unlike [6–8, 15], the rheological model is adapted into a system that defines a discrete and discrete-continuous model. In such a general model, nonlinear relationships require the use of a numerical method. The authors of the above works considered only physical and rheological processes in the model; the task of forming a model with dynamic elements (discrete masses, rods, etc.) was not set.

The models considered in [5, 6] are quite close to our model. The role of damping elements belongs to linear dampers, which are less effective than hysteresis ones [9]. The effectiveness of damping mechanical vibrations using a lyophobic system (repulsive clathrates) under impulse loads from the processing medium is known. This is shown in detail, with references to experimental data, in [9, 15]. The proposed hysteresis model makes it possible to increase the decrement of damping of mechanical vibrations to values of 1.4–1.9, i.e. to significantly increase it in relation to linear dampers (Voigt model) under the same loading conditions.

The use of the rheological model of the hysteresis damper significantly complicates the mathematical model and requires improvement of the numerical method. This work is a continuation of studies [5, 6] regarding increase in the efficiency of damping of impulse processes in impact devices.

The limitations of the model are as follows:

1) the model considers only the axial component of PM reaction, the transverse component that arises in real processes and affects the reliability of the device's structural elements is not taken into account;

2) the model can be used for a tool whose shape is close to cylindrical.

The following disadvantages should be noted:

1) the body elements of the device in the model are replaced by a reduced mass, which leads to a decrease in the accuracy of calculations;

2) the parameters of the numerical method are selected using a discrete model, which only approximately guarantees the accuracy and stability of the method;

3) the task of identifying the model with respect to experimental data has not been solved.

The problem of model identification can be solved for specific impact devices under given operating conditions. Such a problem cannot be solved in the general case.

Increasing the accuracy of modeling is possible when using models with several discrete elements.

It should be noted that basically the tools of impact devices have a shape close to cylindrical with a variable cross-sectional area. Therefore, for using the model, the cross-sectional area can be replaced by the area of an equivalent circle.

Taking into account the transverse component of the impulse load from the side of the processing medium requires supplementing the model with a wave equation of transverse vibrations. This complicates the model, so the use of the model is possible with precise tool tuning, which provides a slight deviation from the longitudinal axis.

Possible areas for further development of our research:

– identification of the model with respect to experimental data;

– taking into account the resistance of the processing medium with hysteretic characteristics;

– taking into account transverse loads on the tool and checking the hysteretic type damper during such oscillations;

– increasing the accuracy of the numerical method by means of higher-order approximation and the use of a three-layer difference scheme.

7. Conclusions

1. A calculation scheme "reaction of the processing medium-tool-elements of the impact device body with a rheological-type hysteresis damper" has been constructed. The tool is represented by a rod with a variable cross-section and a conical impact part. The elements of the impact device body are modeled by a discrete element with a reduced mass. The interaction between the rod and the discrete mass is determined by elastic and dissipative elements. A rheological model is used as a hysteresis damper.

2. An initial-boundary value problem has been stated with a system of ordinary differential equations and a partial differential equation. Boundary and contact conditions determine the interaction of the rod end with a discrete mass, the movement of the discrete element with a hysteresis module. The force of the hysteresis resistance is determined depending on the movement of the discrete element.

3. The numerical method of approximate solution to the initial-boundary-value problem has been built on the basis of a mixed scheme of the finite difference method with partial linearization of contact conditions. When solving the system of difference equations at each time layer, the sweep method adapted to the boundary and contact conditions is used. The damping force is determined from the approximate solution to the system of nonlinear equations by the implicit Euler scheme with partial linearization of nonlinear components. The ranges of parameters of the difference scheme that ensure the stability and permissible accuracy of the method are found: length step $h = (0.005–0.01)\ L$, time step $t = (0.001–0.05)$ ms, weight parameter of the scheme $g = 0.5–0.9$.

4. A model problem was constructed based on a two-mass discrete model with a rheological-type hysteresis damper and a numerical method using the Runge–Kutta scheme, which allowed us to estimate the rational parameters of the finite difference method.

5. Computational experiments were conducted at different parameters of the impulse load to determine the main characteristics of the damping system and to estimate its efficiency. With the given elasticity parameters and the absence of linear dampers, the following energy consumption values were obtained: 6–500 J. The maximum stresses in the conical part of the tool were 56–560 MPa with a force change in the range of 50–500 kN, which acted for 1 ms.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study, as well as the results reported in this paper.

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Data availability

All data are available, either in numerical or graphical form, in the main text of the manuscript.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

Authors' contributions

Viktor Slidenko: Conceptualization, Supervision, Writing – review & editing; **Oleksandr Slidenko:** Methodology, Investigation, Writing – original draft; **Oksana Zamaraeva:** Software, Formal analysis, Resources; **Vladislav Tkachenko:** Software, Visualization, Project administration; **Oleksandr Balanyuk:** Validation. Data Curation. Funding acquisition.

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