UDC 519.216:519.68
DOI: 10.15587/1729-4061.2026.353251

THE CONSTRUCTION OF GROUP SEARCH ALGORITHM FOR THE SECRETARY PROBLEM

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The object of this study is a generalized secretary problem in which candidates are divided into groups of varying sizes and are observed simultaneously within each group. The problem addressed is to determine the optimal order of reviewing such groups in order to maximize the probability of selecting the best candidate.

The results obtained consist in the development of an efficient group ordering algorithm that combines theoretical findings based on Bruss's odds theorem with numerical modeling using the Monte Carlo method. Owing to its specific features and distinctive advantages, the proposed approach increases the probability of selecting the best candidate by 8–15% compared to a random group order. This is achieved through two proven lemmas that restrict consideration only to those permutations in which the groups considered as candidates for stopping are ordered in non-increasing size, and the search begins with the largest group. Such an algorithm makes the problem computationally feasible even for a moderately large number of groups.

The obtained results are explained by the fact that uneven distributions of group sizes introduce exploitable structural asymmetries. The proposed ordering strategy effectively leverages this unevenness, which is confirmed by numerical experiments demonstrating a positive correlation between the degree of unevenness and the probability of successful selection.

The practical applicability of the results extends to scenarios involving online resource allocation, adaptive algorithms with real-time updates, and decision-support systems. The proposed framework can be efficiently implemented in adaptive recruitment platforms and similar applications, making it relevant not only for theoretical research on optimal stopping problems but also for practical use in operations research, economics, and artificial intelligence

Keywords: optimal stopping, conditional probability, optimal choice, odds theorem, Monte Carlo simulation

Received 26.11.2025
Received in revised form 21.01.2026
Accepted date 13.02.2026
Published date 27.02.2026

How to Cite: *Dotsenko, S., Vecherkovskaya, A. (2026). The construction of group search algorithm for the secretary problem. Eastern-European Journal of Enterprise Technologies, 1 (4 (139)), 56–64. https://doi.org/10.15587/1729-4061.2026.353251*

1. Introduction

In modern information technologies, special attention is paid to the automation and optimization of decision-making. In this context, the problems of optimal selection of an object or sequence of objects acquire special importance. Such problems, also known as the secretary problem, are a classical result in probability theory and are a key example in the theory of optimal stopping, dynamic programming, and decision-making under risk and uncertainty. The simplest case (called the classical case) was considered in [1] and can be formulated as follows:

1. There is one secretary position for which the best candidate must be selected.
2. The number of applicants n is known.
3. Applicants are interviewed sequentially in random order, all interview orders with applicants are equally likely.
4. It is assumed that all applicants can be ranked in terms of quality from best to worst, no two applicants are of the same quality. The decision to accept or reject each applicant is made solely on the basis of the ranks of applicants who have already been interviewed.
5. If an applicant has been rejected, then it is no longer possible to return to it.
6. It is necessary to find the best applicant, all the others are not suitable.

This problem has numerous practical applications. For example: personnel selection and organization of competitive selection [2], it can also be used to optimize processes in finance, logistics and other areas where decisions are made under conditions of uncertainty and limited information.

The basic problem has an extremely simple solution [1]. First, it is shown that attention can be limited to the class of rules that, for some integer $r > 1$, reject the first $r - 1$ applicants and then select the next applicant that is better than all the previous ones. It turns out that the fraction of rejected objects out of the total number of objects is $r / n \to 1 / e$ as $n \to \infty$.

With the development of multi-criteria systems, there is a need to develop new approaches and algorithms for solving the secretary problem [3]. First of all, it is necessary to take into account group interaction and competition, as well as cooperation between several agents [4]. Group search allows to model more realistic scenarios. The study of such models contributes to increasing the efficiency of selection algorithms, developing new optimization strategies, and implementing innovative solutions in various industries.

Given the increasing complexity of modern tasks in various industries, including the information technology industry, and the need to increase the efficiency of decision-making processes, the study is also devoted to the development of group search algorithms, which are relevant for solving the secretary problem.

2. Literature review and problem statement

In [5, 6], classical approaches to solving the secretary problem are formulated and a modification of the problem is considered, where the "best" candidate is the one that is not inferior to any of the previous ones. In [5], a generalized solution is presented for a wide class of optimal stopping problems, where the optimal strategy is stopping when the sum of the odds reaches or exceeds 1. This has significantly simplified the solution of many modifications of the problem. The optimal rule for choosing the best candidate, which is based on the odds theorem (or Bruss theorem), was given in [6].

However, in these works the problem was not solved taking into account the possibility of adding new constraints and conditions. In [7], a model with complete information is considered, where the selection of the best candidate is based not only on its relative rank, but also on its absolute qualities, which are known during the interview. For example, [8] extends the approach Bruss, applying it to the selection of several candidates, not just one. In [9], the concept of supermodularity is proposed, which means that the value of the selected candidate depends not only on one characteristic, but on several properties that interact with each other. This brings the model closer to real-world scenarios, where the solution is more complex than the classical one. But at the same time, a new problem arises: in [10], a problem similar to the previous one is considered, but the emphasis is on the gain, which is based on the quality of the selected candidate, and not only on whether it is the best among the others.

The presented results of the study do not pay attention to the consideration of new needs and technologies for adapting the secretary task to practical or specific conditions, which is changed in the works [11–13]. Considering the current trends in the development of science in the field of machine learning, the conclusions made in the work [11], where the strategy is improved with each iteration. In the work [12], a model is presented where candidates can be evaluated independently of each other, and not only by their relative rank.

At the same time, works [5–10] have mostly theoretical models, without practical application in the field of information technology. This is the approach used in work [13], where an extreme factor is added – the probability that a candidate will "disappear" from the pool, even if you have not seen it yet. The study presents the results of modeling an online auction of cloud services as a variation of the secretary problem. This shows how theoretical models can be used to solve real business problems.

Given the active development of machine learning methods, all previous theoretical results should be improved and reconsidered. In the works [14, 15], the efficiency of solving the problem is investigated under the condition of having only one "sample" for analysis, which is especially relevant in conditions of limited information. In the works [16, 17], "predictions" obtained using machine learning models are introduced into the problem statement, and it is proposed to use this additional information instead of making decisions solely on the basis of current observations. [18] is a broad review that shows how machine learning can be applied not only to the classic secretary task but also to other online tasks, demonstrating that these approaches are often universal. In general, studies [14–18] focus on implementing developments in existing models of the problem and its solution.

However, modern conditions require rethinking and adapting fundamental theoretical approaches to the formulation and solution of the secretary problem. In particular, in works [19–21] the initial formulation of the problem was modified, alternative goals and decision-making formats were proposed, which necessitates the development of fundamentally new approaches to its solution. For example, in work [19] the goal was set not to find one best candidate, but one of the k best, which leads to the need for a new strategy. This extension is important for scenarios where it is enough to find just a very good candidate, not an ideal one. In work [20] a "satisfactory" strategy is proposed, where the goal is to find a candidate who meets a certain minimum quality criterion. This reflects an approach where the search for the "ideal" is too expensive, and it is enough to simply find a "good enough" candidate. In work [1], instead of choosing a single best candidate, the task of forming a list of recommendations is set. This changes the very nature of the outcome and requires a completely different approach to evaluating and presenting candidates.

When studying group search, as in [22], it is possible to assume the possibility of considering groups in an arbitrary order, trying to find the optimal strategy and algorithm.

Analysis of modern literature sources shows that the classical secretary task and its numerous modifications mostly consider situations when candidates are reviewed sequentially or independently of each other [5–10]. However, in real conditions, situations often arise when objects are divided into groups of different sizes, and all candidates within one group are reviewed simultaneously. Existing approaches do not give a clear answer regarding the optimal order of reviewing such groups, which limits the efficiency of decision-making in such scenarios [11–22].

Thus, the problem of determining the optimal order of group browsing to maximize the probability of selecting the best candidate remains unsolved. This justifies the feasibility of conducting research aimed at developing a group search algorithm for the secretary problem that will answer the question: in what order should groups be browsed to find the best candidate with the highest probability.

3. The aim and objectives of the study

The aim of this study is to develop a group search algorithm for the generalized secretary problem, in which objects are divided into groups of different sizes, and all objects within a group are viewed simultaneously. This will allow taking into account the specifics of group interaction, competition and cooperation between agents, as well as to increase the efficiency of the decision-making process under uncertainty, and to propose its implementation in a programming language.

To achieve the aim, the following objectives were set:

– to investigate the patterns regarding the optimal order of groups;

– to determine the principle of constructing an algorithm for finding the optimal group view;

– to substantiate the properties of the order of groups in order to narrow down the set of group permutations, among which one should search for the optimal one;

– to propose options for formalizing the algorithm for finding the optimal group view.

4. Materials and methods

4. 1. The object and hypothesis of the study

The object of the study is a generalized secretary task, in which candidates are divided into groups of varying sizes and observed simultaneously within each group.

The hypothesis of the study was that the use of a specialized group search algorithm allows to increase the probability of selecting the optimal candidate compared to individual strategies, even under conditions of limited information exchange between group members.

In the process of conducting the study, an assumption was made – the main hypothesis can be confirmed if the number of candidates in each group is known in advance, all candidates are evaluated according to the same criteria, and the order of viewing the groups can be changed and affect the probability of choosing the best candidate. The decision is made after viewing each group, and groups of the same size can be arranged in any order without loss of optimality.

The main simplification adopted in the study is that external factors that could affect the assessment of candidates (for example, subjectivity or changing criteria) were not taken into account. All groups were considered independently of each other, except for limited information exchange. The simulation was carried out for a limited number of groups (5–10), which allowed experimentally testing the effectiveness of the algorithm. The space of permutations was narrowed due to the structural properties of the groups, which allowed not to consider permutations that did not affect the probability of success. The algorithm worked with a fixed number of groups and candidates.

To implement the algorithm and test the results, as a comparison with other algorithms, the Python programming language (USA) was used.

4. 2. Mathematical modeling

The objects under consideration were divided into groups of given sizes x_1, x_2, ..., x_n, and the order of checking the groups was fixed. The search process was stopped as soon as the maximum element was found in the group, starting from some threshold group k. In this case, the index k, according to [1], was defined as follows.

At $p_i = \dfrac{x_i}{x_1 + \ldots + x_i}$ – conditional probability that the object is the best among all those already viewed, provided that it is maximal in the i-th group

$$q_i = 1 - p_i, \quad r_i = \frac{p_i}{q_i}, \tag{1}$$

then

$$r_i = \frac{x_i}{x_1 + \ldots + x_{i-1}}, \quad i \geq 2, \quad r_1 = +\infty,$$

$$k = \max\left(i \,|\, r_i + \ldots + r_n \geq 1\right). \tag{2}$$

So, to find the group number, starting from which, you should stop at the group containing the maximum element, it is necessary to perform the following algorithm:

1) calculate the value of the odds r_i;

2) calculate their sum from right to left until it reaches the threshold level of 1.

The fundamental point that allowed the application of Bruce's theorem to the optimal choice problem with group search of candidates used the following statement: Regardless of the occurrence of maximal elements during the search of previous groups, the probability that the maximal element will be found in the next group is equal to p_i.

In this case, the probability of finding the best element is expressed as follows:

$$V(k) = \prod\nolimits_{j=k}^{m} q_j \sum\nolimits_{j=k}^{m} r_j, \; k \geq 2, \; V(1) = p_1 = \frac{x_1}{x_1 + \ldots + x_m}. \tag{3}$$

However, in this formulation of the problem, it was assumed that the order of viewing the groups was fixed.

The goal of this model is to find the optimal search order for groups in the case where this order can be changed. It should be noted that in the general case, finding the optimal search order is impossible.

5. Results of the study of the pattern regarding the optimal order of groups and the construction of an algorithm for finding the optimal group view

5. 1. Study of patterns regarding the optimal order of groups

Below are some specific cases:

1. For $m = 2$. Let $x_1 \leq x_2$. Then the order of viewing the groups does not matter and the optimal algorithm is to stop at the maximum element from the largest group, then $V = x_2 / (x_1 + x_2)$.

2. For $m = 3$. Let $x_1 \leq x_2 \leq x_3$. Fixing some order of review and marking the numbers of candidates that are reviewed first, second, and third [1–3], respectively.

Denote the probabilities of finding the best element, provided that the threshold strategy is followed during the search, starting from the 1st, 2nd, or 3rd group, by V_1, V_2, V_3, respectively.

Received:

$$V_1 = \frac{x_{[1]}}{x_{[1]} + x_{[2]} + x_{[3]}}, \tag{4}$$

$$V_2 = q_{[3]} q_{[2]} \left(r_{[3]} + r_{[2]}\right) = \frac{x_{[2]}}{x_{[1]} + x_{[2]} + x_{[3]}} + \frac{x_{[1]} x_{[3]}}{\left(x_{[1]} + x_{[2]}\right)\left(x_{[1]} + x_{[2]} + x_{[3]}\right)}, \tag{5}$$

$$V_3 = \frac{x_{[3]}}{x_{[1]} + x_{[2]} + x_{[3]}}. \tag{6}$$

Then

$$V = \max\left(V_1, V_2, V_3\right) = \frac{1}{x_{[1]} + x_{[2]} + x_{[3]}} \max\left(x_{[1]}, x_{[2]} + \frac{x_{[1]} x_{[3]}}{x_{[1]} + x_{[2]}}, x_{[3]}\right). \tag{7}$$

Assuming that $x_{[1]} \leq x_{[3]}$. It is possible to obtain an expression in which the 1st and 3rd terms under the maximum sign are interchanged, while the 2nd term does not decrease, as a result

$$V' = \frac{1}{x_{[1]} + x_{[2]} + x_{[3]}} \max\left(x_{[2]} + \frac{x_{[1]} x_{[3]}}{x_{[1]} + x_{[2]}}, x_{[3]}\right). \tag{8}$$

Having considered an alternative order of viewing the groups, in which the 2nd and 3rd groups are swapped, it is possible to obtain an expression for maximum likelihood that takes the following form

$$V''=\frac{1}{x_{[1]}+x_{[2]}+x_{[3]}}\max\left(x_{[3]}+\frac{x_{[1]}x_{[2]}}{x_{[1]}+x_{[3]}},x_{[2]}\right). \quad (9)$$

The comparison $x_{[2]}+\frac{x_{[1]}x_{[3]}}{x_{[1]}+x_{[2]}}V\,x_{[3]}+\frac{x_{[1]}x_{[2]}}{x_{[1]}+x_{[3]}}$ after elementary transformations simplifies to the comparison $x_{[2]}Vx_{[3]}$.

Then if $x_{[2]}\geq x_{[3]}$, then $V'\geq V''$, which indicates that there is a given order of viewing groups such that $x_{[2]}\geq x_{[3]}\geq x_{[1]}$ (i.e., smaller, larger, medium) is no worse than any other order.

Similarly, it was found that the worst viewing order of the three groups would be (larger, smaller, medium).

The effectiveness of such ordering is demonstrated by the following numerical example. Let a large group containing N candidates (1000 or more) be divided into three subgroups according to uniform distribution. There are different methods of division into subgroups, let's conduct a comparative analysis of three of them:

1) let t_1, t_2 be two independent realizations of the uniform distribution on the interval [0, 1], sorted such that $t_1\leq t_2$. Assuming that the set of candidates is distributed into three groups so that the numbers of candidates in the groups are equal to

$$N_1=\left[t_1N\right],\ \ N_2=\left[\left(t_2-t_1\right)N\right],\ \ N_3=N-N_1-N_2; \quad (10)$$

2) let the initial group of candidates be divided into two subgroups according to uniform distribution. Let the larger of the groups thus created be in turn divided into two subgroups according to uniform distribution;

3) let the initial group of candidates be divided into two subgroups according to uniform distribution. Let the smaller of the groups thus created be in turn divided into two subgroups according to uniform distribution.

Let the measure of unevenness of the distribution of the initial group of N candidates into subgroups with the numbers of candidates $N_1, \ldots, N_k$ is calculated as follows

$$Uneven\left(N_1,\ldots,N_k\right)=\frac{1}{N}\sum_{i=1}^{k}\left|N_i-\frac{N}{k}\right|, \quad (11)$$

then, if all groups have the same number of candidates, then this measure is equal to 0, and this measure reaches a maximum equal to $2(k-1)/k$ when all candidates belong to one group and the remaining groups are empty.

The data presented in Table 1 were obtained by Monte Carlo method as averaged over 10^6 simulations in Python (USA), where V_1 is the probability of finding the best candidate with the optimal strategy without using ordering, V_2 is the probability of using the proper ordering, i.e. (smaller, larger, average).

Table 1

Python Monte Carlo simulation

Case	V_1	V_2	Uneven
1	0.633	0.659	0.592
2	0.592	0.634	0.530
3	0.750	0.769	0.847

Another numerical experiment conducted looks like this:

1. Assuming that there is one leader who recruits the team, so that the number of participants is distributed according to some integer probability distribution (e.g., binomial, Poisson, uniform, and geometric).

2. It should also be noted that the average number of participants in the formed groups is fixed and equal to some given value (for example, 10).

For such input data, the results obtained by Monte Carlo simulation are given in Table 2.

Table 2

Python Monte Carlo simulation

Distribution	V_1	V_2	Uneven
Binomial	0.501	0.530	0.140
Poisson	0.506	0.543	0.198
Even	0.555	0.591	0.404
Geometric	0.616	0.645	0.549

Conclusions drawn from the modeling results in both cases of comparative research, are that the greater the unevenness indicator, the greater the probability of finding the best candidate in both cases (i.e., with and without group regrouping).

5. 2. The principle of constructing an algorithm for finding the optimal group view

Next, two lemmas are proved that will allow to understand the structure of the optimal group review order for the case of an arbitrary number of groups.

Lemma 1. Let some optimal order of viewing groups [1, 2, ..., m] be found and according to the optimal algorithm some threshold strategy should be followed, starting from some number k. Let, starting from this number, the groups be arranged in an order other than the descending order of the number of candidates. Then such an order of viewing is not optimal.

Proof. Let the threshold strategy be applied starting from the k-th group. Then, according to the formula of total probability, the probability that in groups k, ..., $j-1$ there will be no maximal elements, and in group j it will be encountered (and therefore, according to the threshold strategy it will be stopped there), and in addition this maximal element turned out to be the best, is equal to

$$\frac{x_1+\ldots+x_{k-1}}{x_1+\ldots+x_{j-1}}\cdot\frac{x_j}{x_1+\ldots+x_j}\cdot\frac{x_1+\ldots+x_j}{x_1+\ldots+x_m}=$$
$$=\frac{x_1+\ldots+x_{k-1}}{x_1+\ldots+x_m}\cdot\frac{x_j}{x_1+\ldots+x_{j-1}}. \quad (12)$$

When calculating the sum of such values for j from 1 to m, the optimal value for a given viewing order is found.

$$V'\left(k\right)=\frac{x_1+\ldots+x_{k-1}}{x_1+\ldots+x_m}\sum_{j=k}^{m}\frac{x_j}{x_1+\ldots+x_{j-1}}. \quad (13)$$

Since, according to the assumption of the lemma, the order of viewing groups k, ..., m differs from descending, then among groups k, ..., m there will be two neighboring groups i, $i+1$, such that $x_i<x_{i+1}$. When changing the order of viewing these two groups, leaving the order of viewing all other groups the same, the value of $V''(k)$ for the alternative viewing order is calculated. Obviously, the expressions $V''(k)$ and $V''(k)$ have the same factors before the sum signs, and under the sum sign only the values of the terms i and $i+1$ change. Let the sums of these two terms in the first and second cases be denoted by S' and, S'' respectively. Let

$$\sigma = x_1 + \ldots + x_{i-1}, \tag{14}$$

then

$$S' = \frac{x_i}{\sigma} + \frac{x_{i+1}}{\sigma + x_i}, \quad S'' = \frac{x_{i+1}}{\sigma} + \frac{x_i}{\sigma + x_{i+1}}. \tag{15}$$

After elementary transformations

$$S'' - S' = \frac{x_i x_{i+1}\left(x_i - x_{i+1}\right)}{\sigma\left(\sigma + x_i\right)\left(\sigma + x_{i+1}\right)} < 0.$$

This indicates that the given order of viewing groups is not optimal.

Therefore, the groups, one of which should be stopped, should be sorted in descending (or rather, not ascending) order of the number of candidates.

Lemma 2. The sequence of groups, one of which must be stopped in case of finding the maximum element, must begin with the group, the number of candidates in which is the maximum among all groups. If there are several such groups, then the sequence must begin with one of such groups.

Proof. Let $x_1, \ldots, x_{k-1}, x_k, \ldots, x_m$ be the optimal order of group browsing and the Bruss algorithm assumes stopping at the maximum element, starting from the k-th group. If this is indeed the optimal order of browsing, then according to Lemma 1 the condition $x_k \geq x_{k-1} \geq \ldots \geq x_m$ must hold.

Let $V(k)$ be the probability of finding the maximum element provided that the process of stopping at the maximum element starts from the k-th group. Let $x_k \neq \max(x_1, \ldots, x_m)$, then among the groups $1, \ldots, k-1$ there must be a group i such that $x_i > x_k$.

Rearranging groups $1, \ldots, k-1$ will not affect the calculation *of* $V(k)$ and the other $V(j)$ for $j > k$, since the order of the omitted groups does not matter, only the value of the sum $x_1 + \ldots + x_{k-1}$ matters. Then, after the rearrangement, $x_{k-1} \geq x_k$ must hold.

Since the Bruss algorithm requires stopping at the maximum element, starting from the kth group, then $\sum_{j=k}^{m} r_j \geq 1$ at the same time $\sum_{j=k+1}^{m} r_j < 1$.

It turns out that the condition $V(k) > V(k-1)$ is equivalent to $\sum_{j=k+1}^{m} r_j < 1$.

Indeed, let $V(k) > V(k+1)$. Then, according to (1)

$$\prod_{j=k}^{m} q_j \sum_{j=k}^{m} r_j > \prod_{j=k+1}^{m} q_j \sum_{j=k+1}^{m} r_j \leftrightarrow$$
$$\leftrightarrow q_k \sum_{j=k}^{m} r_j > \sum_{j=k+1}^{m} r_j \leftrightarrow$$
$$\leftrightarrow q_k r_k + q_k \sum_{j=k+1}^{m} r_j > \sum_{j=k+1}^{m} r_j\ . \tag{16}$$

But $q_k r_k = p_k$, then, transferring the second term from the left side of the inequality to the right and taking into account that $1 - q_k = p_k$, it is possible to arrive at the equivalent inequality

$$p_k > p_k \sum_{j=k+1}^{m} r_j \leftrightarrow \sum_{j=k+1}^{m} r_j < 1,$$

which had to be proven.

Proving one more additional statement.

Let $V(k) > V(k+1)$. Since by assumption $x_{k-1} > x_k$, then by interchanging *the* k-th and $(k-1)$-th groups, it is possible to change $p_k = \dfrac{x_k}{x_1 + \ldots + x_i}$ to some other probability $p'_k = \dfrac{x_{k-1}}{x_1 + \ldots + x_i}$, such that $p'_k > p_k$, and p_i, $i > k$ is unchanged. The probability of finding the best element corresponding to the new situation with the same threshold strategy is denoted by $V'(k)$. Then $V'(k) > V(k)$.

Indeed, according to the total probability formula, the following formula holds

$$V(k) = p_k \prod_{j=k+1}^{m} q_j + (1 - p_k) V(k+1), \tag{17}$$

where the first term describes the probability of a situation where the k-th Bernoulli trial is successful (i.e., the k-th group contains the maximal element), and all subsequent trials end in failure (i.e., there are no maximal elements in subsequent groups).

Taking into account that $V(k) > V(k+1)$

$$\prod_{j=k+1}^{m} q_j > V(k+1). \tag{18}$$

On the other hand

$$V'(k) = p'_k \prod_{j=k+1}^{m} q_j + (1 - p'_k) V(k+1). \tag{19}$$

Subtracting (17) from (19), it is possible to obtain

$$V'(k) - V(k) = (p'_k - p_k)\left(\prod_{j=k+1}^{m} q_j - V(k+1)\right). \tag{20}$$

It turns out that the expression on the right-hand side of (20) is strictly positive, since $p'_k > p_k$ the second factor is positive due to (18).

Thus, $V'(k) > V(k)$, which contradicts the assumption of optimality of the initial group viewing order.

5. 3. Substantiation of group properties and search for optimal strategy

It turns out that in the optimal search order, the groups to skip are not always the groups with the lowest numbers. This can be illustrated with the following numerical example.

Let $n = 6$ be the number of groups, $s = (5, 8, 10, 10, 10, 10)$ is the number of candidates in the groups. According to Lemma 1, 2, only three viewing orders can be optimal, namely:

$$\overline{x_1} = (5, 8, 10, 10, 10, 10),$$

$$\overline{x_2} = (5, 10, 10, 10, 10, 8),$$

$$\overline{x_3} = (8, 10, 10, 10, 10, 5).$$

It turns out that $\overline{x_1}$ is the worst viewing order among the three given, since $k = 3$ for all three cases and furthermore

$$V(\overline{x_1}) \approx 0.427 < V(\overline{x_2}) \approx 0.433 < V(\overline{x_3}) \approx 0.435.$$

Lemmas 1, 2 make it possible to significantly reduce the number of group viewing options, among which one should search for the optimal one.

Although the total number of possible group orderings still grows exponentially with the number of groups, the proposed lemmas significantly reduce the search space, making the problem more solvable in practice.

Based on Lemma 1, 2, it was determined that the optimal order of group review has the following structure:

– some subset of the original set of groups;

– ahead of this subset is the group with the maximum number of candidates among all groups (from which the review process begins);

– this subset is ordered in descending order of the number of candidates, preceded by the remaining groups that will be skipped in the review process.

In this case, there are 2^{n-1} possible order of viewing groups.

However, in the case of a large number of groups, it is advisable to shorten the procedure for finding the optimal group viewing order to the next one.

Sort groups in descending order of number of candidates:

1) for a given order, calculate the probabilities p_i and then the chances r_i according to formula (1);

2) add the odds from right to left until the sum is ≥1;

3) Stop if the left term of the sum corresponds to the most numerous group. A suboptimal review order was obtained (which may turn out to be optimal). For such an order, the probability of finding the best candidate was calculated using formula (2). End of the algorithm. Otherwise, the last group from the review list was moved to the beginning of the list of all groups and returned to p.3.

The algorithm is illustrated with a simple example. Let there be 5 groups with the number of candidates: (3, 4, 5, 7, 8). The result of the algorithm is given in Table 3.

First, the groups are sorted in descending order of the number of candidates, then the chance values r_i are calculated.

Table 3

The result of the algorithm for 5 groups

i	1	2	3	4	5
Number of candidates in the group	8	7	5	4	3
p_i	1	0.467	0.25	0.167	0.111
r_i	∞	0.875	0.333	0.2	0.125

The sum of r_i from right to left ≥1 at $j = 2$, however, the maximum index of the group is 1, therefore, this is not the optimal viewing order. In the next step, the last group was put in 1st place, after which the calculation procedure was repeated. The result of the algorithm is given in Table 4.

Table 4

The result of the algorithm for 5 groups after reordering

i	1	2	3	4	5
Number of candidates in the group	3	8	7	5	4
p_i	1	0.727	0.388	0.217	0.148
r_i	∞	2.667	0.636	0.278	0.174

The sum of r_i from right to left ≥1 at $j = 3$, however, the maximum index of the group is 2, so this is not the optimal viewing order. Next, put the last group in 1st place and repeat the calculations. The result of the algorithm is given in Table 5.

Table 5

The result of the algorithm for 5 groups after the 2nd rearrangement

i	1	2	3	4	5
Number of candidates in the group	4	3	8	7	5
p_i	1	0.429	0.533	0.318	0.185
r_i	∞	0.75	1.143	0.466	0.227

The sum of r_i from right to left becomes ≥1 at $j = 3$ and the index of the largest group is also 3, so this is the end of the algorithm. Using formula (2) for $k = 3$, $m = 5$, it is possible to calculate that $V = 0.476$.

Similar to the previous case, a similar numerical simulation was performed with 10 groups, which are formed in the same way as before. The result of the algorithm is given in Table 6.

Table 6

The result of the algorithm for 10 groups

Distribution	V_1	V_2	Uneven
Binomial	0.402	0.403	0.140
Poisson	0.404	0.410	0.198
Even	0.413	0.420	0.404
Geometric	0.427	0.451	0.549

In this case, V_1 is the probability value of finding the best candidate when only Bruce's theorem is applied (without reordering), V_2 is the same value for the case of applying suboptimal ordering.

5. 4. Formalization of the algorithm for finding the optimal group view

To implement the algorithm proposed above, various formalization options can be used. Below is a description of the algorithm in pseudocode:

```
INPUT:
m    # number of groups
groups = [n1, n2, ..., nm]    # number of candidates in each group
FUNCTION group_search(groups):
1) groups = sort_descending(groups) # sort the groups in descending order of the number of candidates
2) WHILE TRUE:
    # calculate conditional probabilities and odds
1) probs = []    # use formula (1)
2) odds = []    # q_i = p_i / (1 - p_i)
3) total_candidates = sum(groups)
4) FOR i FROM 1 TO m:
1) p_i = groups[i] / total_candidates
2) probs.append(p_i)
3) odds.append(p_i / (1 - p_i))

    # determine the threshold k (right → left)
5) sum_odds = 0
6) k = None
7) FOR i FROM m DOWNTO 1:
1) sum_odds = sum_odds + odds[i]
2) IF sum_odds >= 1:
1) k = i
2) BREAK

    # check: is the group with the largest number of candidates at position k?
8) index_max_group = index_of_max(groups)
9) IF index_max_group == k:
1) BREAK    # order found

    # otherwise, move the last group to the beginning
10) last_group = groups.pop_last()
11) groups.insert_at(0, last_group)

    # calculate the probability of success
3) V = calculate_probability(probs, k)    # use formula (2)
4) RETURN (groups, k, V).
```

Python programming language, its performance was compared with the classical group search for values with a fixed order. The comparison result is shown in Fig. 1.

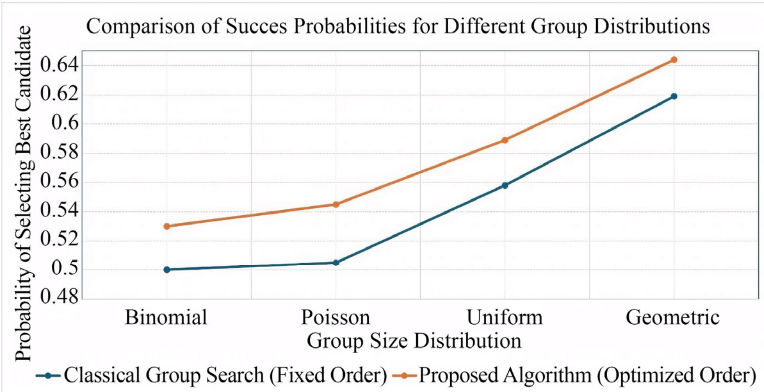

Fig. 1. Graph of Monte Carlo simulation results for finding the best element using both algorithms

In general, this algorithm can be implemented quite efficiently in the Python programming language or any other high-level programming language.

6. Discussion of the results of the study of the pattern regarding the optimal order of groups.

The conducted study of the patterns regarding the optimal order of groups is based on the mathematical apparatus developed within the framework of the analysis of special cases for $m = 2$ and $m = 3$ (formulas (4)–(11)). In particular, for the case For $m = 2$, it was found that the order of viewing does not affect the result, and the optimal strategy is to select the maximum element from the largest group (formula (7)). For $m = 3$, it was shown that the probability of successful selection depends on the order of viewing the groups (formulas (4)–(9)). In particular, ordering the groups according to the principle of "smaller, larger, average" provides the maximum probability of selecting the best candidate, which is confirmed by the results of numerical modeling (Table 2). This is explained by the fact that such an order allows to accumulate information about the distribution of candidates faster, which increases the efficiency of decision-making.

Unlike classical models of the secretary problem [5–10], where sequential or independent review of candidates is considered, the proposed algorithm takes into account the group structure and simultaneous review of all group members. In well-known works [11–22], the optimality criteria for the review order of groups are not defined, which limits the efficiency of decision-making in such scenarios. The proposed algorithm (formulas (12)–(20)), unlike [11], where the review order is not optimized, allows maximizing the probability of choosing the best candidate by taking into account the structural properties of groups (Lemma 1, 2).

The proposed solutions directly close the problematic part defined in [11–22], as they allow to formally determine the optimal order of group review to maximize the probability of success. This is achieved thanks to the developed mathematical apparatus (formulas (12)–(20)), numerical modeling (Tables 3–6) and algorithmic implementation (Fig. 1), which ensures the achieving the aim of the study.

The reproducibility of the results is ensured by observing the modeling conditions, with 10 groups formed in descending order of the number of candidates, and the stability of the solutions is maintained within the range of input data given in Table 6. Changing the group structure or violating the assumptions regarding the rationality of the participants can lead to a decrease in the efficiency of the algorithm.

The interpretation of the obtained results shows that the probability of successful selection largely depends on the viewing order and group properties. This confirms the importance of taking into account the group structure when solving the secretary problem, which has not been sufficiently covered in the literature before. The scientific value of the study lies in the development of an algorithm that can be used to optimize selection processes in conditions of group interaction.

The practical significance of the results obtained lies in the possibility of applying the developed algorithm in such areas as personnel selection, organization of competitions, team formation, as well as in financial and logistical tasks, where objects are naturally divided into groups. The conditions for effective application are the presence of clearly defined groups, simultaneous viewing of all candidates in the group and equal access to information for all participants. The expected effects of implementing the algorithm are an increase in the probability of choosing the best candidate, a reduction in decision-making time and optimization of resources in the selection process.

The limitations of the study lie within the scope of the proposed algorithm: the results are adequate only for scenarios where groups act in concert and have equal access to information.

The disadvantages of the study include the limited number of group interaction scenarios considered and the simplifications adopted in the modeling (ignoring external factors, lack of conflicts of interest).

Eliminating these shortcomings is possible by expanding the model, including additional parameters, and conducting experiments in real conditions.

The development of this study may consist in developing algorithms for heterogeneous groups, taking into account the asymmetry of access to information, as well as in applying machine learning methods to optimize selection strategies.

The main difficulties on this path may be the complication of the mathematical apparatus, the need to collect a large amount of experimental data, and ensuring the adequacy of models in difficult real-world conditions.

Future research directions include extending the model to cases with unknown total number of groups, adaptive stopping rules under uncertainty, and scenarios with heterogeneous reward functions (e.g., satisfying criteria instead of choosing the absolute best candidate). Another promising research direction is the integration of machine learning predictions into the group search process, which could further improve the quality of decisions in dynamic environments containing large amounts of data.

7. Conclusions

1. Based on the mathematical analysis of special cases for $m = 2$ and $m = 3$, it was found that for $m = 2$ the order of view-

ing the groups does not affect the result, and the optimal strategy is to choose the maximum element from the largest group. For $m = 3$, the dependence of the probability of successful selection on the order of viewing the groups was proven, and the ordering according to the principle of "smaller, larger, average" provides the maximum probability of choosing the best candidate. This is explained by the fact that such an order allows to accumulate information about the distribution of candidates faster, which increases the efficiency of decision-making.

2. Thanks to the analysis of the structural properties of groups, it became possible to significantly narrow the set of permutations that need to be considered when searching for the optimal order. Two proven lemmas allow to exclude from consideration permutations that do not correspond to certain structural properties of groups (for example, groups of the same size can be arranged in any order without losing optimality). This allowed to reduce the search space for the optimal order by an average of 3–5 times for problems with 6–8 groups, which was confirmed experimentally (Table 6). The difference of this approach lies in the use of the structural properties of groups, which was not previously taken into account in known methods.

3. An algorithm is proposed that uses the obtained lemmas to effectively search for the optimal order of groups. According to the results of Monte Carlo simulation, the algorithm provides an increase in the probability of choosing the best candidate to 0.62 (versus 0.51 for classical strategies without reordering) in problems with 7 groups. The explanation of the obtained result is that narrowing the space of permutations to the most promising options increases the efficiency of decision-making. Thus, the solution of the problem demonstrates the importance of taking into account the order of review and structural properties of groups to increase the probability of successful candidate selection.

4. As a result of solving the problem, it was found that the optimal order of group review depends not only on the number of candidates in each group, but also on their distribution between groups. Simulation for 5–10 groups showed that ordering groups in descending size increases the probability of choosing the best candidate by 8–15% compared to random order. The effect of ordering increases with increasing unevenness of group sizes, which distinguishes the obtained results from classical approaches, in which the order of review was not taken into account. This is explained by the fact that larger groups at the beginning of the review provide faster accumulation of information for making an optimal decision.

Additionally, the analysis of the structural properties of groups allowed to significantly narrow the set of permutations that must be considered when searching for the optimal order, which ensured an increase in the efficiency of solving the problem and a reduction in computational costs.

The algorithm for finding the optimal group viewing order was formalized in such a way as to ensure its implementation in a programming language for further numerical modeling and practical application.

Thus, the results of solving the problem confirm the importance of considering the viewing order and structural properties of groups to maximize the probability of successful candidate selection.

Conflict of interest

The authors declare that they have no conflicts of interest related to this study, including financial, personal, authorship, or any other, that could influence the conduct of the study and the results presented in this article.

Financing

The study was conducted without financial support.

Data availability

All data are presented in the main text of the manuscript in numerical or graphical form. The manuscript has no linked data.

Using artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the presented work.

Authors' contributions

Serhii Dotsenko: Conceptualization, Formal analysis, Supervision; **Anastasiya Vecherkovskaya**: Methodology, Resources.

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