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DETERMINING PATTERNS THAT INDUCE THE STRESSED-STRAINED STATE OF A PRINTING PLATE DURING INTAGLIO PRINTING

Tetiana Kyrychok
Doctor of Technical Sciences*
ORCID: https://orcid.org/0000-0002-9639-5486
Olena Korotenko
Corresponding author
PhD, Associate Professor*
E-mail: gushcha.olena@gmail.com
ORCID: https://orcid.org/0000-0001-6439-1192
Vladyslav Korotenko
PhD Student*
ORCID: https://orcid.org/0000-0002-6512-5985
Vladyslav Doroshchuk
PhD Student*
ORCID: https://orcid.org/0009-0000-9658-3455
*Department of Printing Technology
National Technical University of Ukraine
«Igor Sikorsky Kyiv Polytechnic Institute»
Peremohy ave., 37, Kyiv, Ukraine, 03056

One of the main methods for protecting paper banknotes is intaglio printing, whose key quality factor is the durability of the printing plate. This study investigates the process that induces the stressed-strained state of an engraved printing plate obtained by direct laser engraving with different geometric parameters of printing and non-printing elements of printing plate. The task addressed relates to the lack of understanding of the mechanism behind the influence of geometric engraving parameters on the stress concentration and deformation of engraved elements of intaglio printing plates. This causes their premature wear and a decrease in the stability of the printing process.

Based on modelling the contact interaction of the plate and printing cylinders, the results of the study have established quantitative dependences between the geometric parameters of engraving and the values of equivalent stresses and deformations of engraved strokes. It was found that the stress values vary within the range of 15–550 MPa, and the deformations range from 0.01 to 0.06 mm, depending on the characteristic zone of the engraved elements and their geometric parameters. The optimal geometric parameters of engraving have been determined based on the analysis of simulation results.

A notable feature of the results is that the model accounted for the actual printing conditions during intaglio printing, as well as the presence or absence of paint within the stroke. Compiled recommendations for the selection of geometric parameters of engraved strokes could be taken into consideration when designing the engraving of intaglio printing plates. The results of this study are valuable for the industrial production of banknote products and securities

Keywords: intaglio printing plates, direct laser engraving, wear resistance, stressed-strained state, intaglio printing modelling

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1. Introduction

Intaglio printing is one of the key technologies in the modern production of secure printing products, in particular banknotes, securities, and strict reporting documents. Thus, according to [1], 52% of the world market of machines used for printing banknotes belongs to intaglio printing presses. The protective properties of intaglio printing, in particular the ability to reproduce a tactilely controlled highly detailed image, are one of the most effective elements capable of preventing counterfeiting of securities, in particular banknotes. At the same time, approximately 95% of currency management managers use intaglio printing to help the public easily identify the authenticity of banknotes [2].

Importantly, intaglio printing is one of the most complex printing techniques due to the combination of high pressure in the printing contact, sophisticated microgeometry of engraved strokes of the printing plate, the presence of viscoelastic ink, thermal effect and high printing speed [3]. Tactilely controlled relief of prints is formed by ink transfer or inkless embossing on banknote paper in the zone of interaction of the engraved printing plate with the printing cylinder under the action of high pressure [4]. As a result, complex stressed-strained states arise in the printing contact. In addition to the influence of high contact loads, the cyclicity of the pressure action and the interaction with the paint are also added, which leads to gradual wear of the engraved elements. These changes directly affect both the quality of the reproduction of the elements and the wear resistance of the printing plates, which entails significant economic and time costs for their replacement [5].

The task of improving the technological process of plate manufacturing using the direct laser engraving (DLE) method as one of the main techniques for manufacturing intaglio printing plates is becoming particularly relevant [6, 7]. Despite the fact that this technology is actively evolving and improving, the issue of ensuring the wear resistance of printing plates and the mechanism of their wear is not sufficiently systematized and researched.

Studying the stressed-strained state of engraved elements of a printing plate would make it possible not only to explain the mechanism of plate wear but also form a basis for increasing and predicting plate wear resistance. Such results have direct practical significance for enterprises producing banknote products and securities as improving the wear resistance of intaglio printing plates leads to a reduction in the costs of their production. At the same time, the use of wear-resistant printing plates leads to a decrease in technological failures in the production process, which has a positive effect on the quality of finished printed products. This, in turn, implies an increase in the level of protection against counterfeiting and the stability and efficiency of the circulation of banknotes in society, which ultimately improves the national financial security of the state.

Therefore, research on the mechanics of contact interaction, stressed-strained state, and improvement of geometric parameters of engraved elements in intaglio printing plates are urgent tasks of national importance.

2. Literature review and problem statement

An overview of the principles of direct laser engraving is given in [7]. In particular, the paper reports the results of research into laser optics and material selection to achieve optimal quality of engraved lines. The established process allows the implementation of true three-dimensional elements with control over the depth and profiling of lines, regardless of their width, while simultaneously reducing the time required to produce secure documents. However, the analysis did not include the study of the mechanical behavior of engraved lines under load, stress and wear characteristics.

In [8], an overview of laser processes in intaglio printing and flexography is given with an emphasis on the precise micro structuring of intaglio printing elements using direct laser ablation. Direct laser engraving on metal surfaces is performed using a powerful Nd:YAG laser system. In this case, the precise modulation method of the system makes it possible to engrave a full cell with each individual laser pulse and independently and freely determine the diameter and depth of each cell. Although the cited work is a valuable source for understanding the technological foundations of engraving printing plates, it did not provide an analysis of the mechanical state of the engraved structures. There is also no analysis of how the geometric parameters of the strokes correlate with the stressed-strained state and wear resistance of printing plates under load during operation.

Work [9] highlights a structured approach to assessing the quality of the technological process of laser engraving of intaglio printing plates, which is an important component in the standardization of their manufacturing process. In particular, the work presents software for full or partial automation of the process of evaluating laser engraving results and establishing with its help the correspondence of the cross-section of the engraved stroke of the plate to the designed one. The work is based on real experimental data, which enhances the practical value of the research conducted. However, the authors analyzed only the geometric correspondence of the plate parameters to the designed technical parameters, without taking into account the mechanical aspects of its further use. The impact of the research results on the control of wear resistance and durability of printing plates, which are key practical characteristics of the plates, has also not been demonstrated.

In work [10], a comprehensive study of the quality of reproduction of intaglio printing plates obtained by direct laser engraving with subsequent vacuum application of a protective coating on their surface was performed. The work highlights the influence of the designed technological parameters of laser engraving, laser power, and pulse frequency on the dimensions and shape of the finished printing elements of the plate. The analysis is aimed at establishing the dependence of the quality of engraving of lines on the designed profile parameters and engraving modes. A useful empirical data set is provided for the selection of geometric parameters for engraving and parameters of laser equipment, which is a valuable source of data for engineers and researchers. However, as in the previous work, there is no analysis of the mechanical consequences of the formed geometry of the surface of the printing plate. The authors' conclusions are of a correlational, not causal, nature, which limits the possibility of optimizing processes.

The authors of [11] investigated peculiarities in the influence of the geometric profile of engraved strokes of a printing plate made by the method of direct laser engraving on the graphic and gradation accuracy of intaglio prints made using these plates. The cited work gives a solution to a topical problem that makes it possible to improve the modes of processing plates by the method of direct laser engraving and expand its use in the production of protected printing products. However, the work did not analyze the printing plates during operation, in particular the stressed-strained state when operating under pressure and its effect on wear resistance.

Conventional analytical methods for assessing the strength of printing plates do not allow for the full consideration of local concentrations of stresses and deformations that arise in engraved strokes in the contact zone. Therefore, the use of the Finite Element Method (FEM) is proposed, which is a promising direction for a deeper analysis of the mechanics of processes in the contact zone of intaglio printing.

Paper [12] shows the effectiveness of the finite element method for the analysis of contact processes in printing, in particular in the pad printing process. The results have methodological value in the application of this method for the analysis of the mechanics of the interaction of contacting surfaces in the printing process but cannot be directly used for the analysis of intaglio printing plates.

Study [13] investigates mechanical processes in the printing contact zone and analyzes in detail the distribution of contact stresses between cylinders during flexographic printing. The work also uses the finite element method to analyze stress distribution in the contact zone. The paper is based on numerical modeling and experimental studies. However, it examined flexographic printing, in which the printing plate is made of elastic polymer materials, and, accordingly, it experiences different deformation stresses than the intaglio printing plate, which is practically rigid. This necessitates the use of a different approach to numerical modeling of printing plates in intaglio printing.

Available studies are focused more on the description of physical processing mechanisms, methods for quality control of printing plates, macroscopic models of contact interaction or on the rheology of inks during ink transfer. However, the mechanical behavior of engraved elements under the action of external pressure, the influence of the geometry of the profile of engraved strokes on the stressed-strained state of the strokes, and, consequently, the wear resistance of the plates, remain insufficiently studied. An important aspect of investigation is also the influence of liquid filling (paint) of engraved plate elements on the stressed-strained state of the printing plate during printing.

All this allows us to state that it is advisable to conduct a study on a comprehensive analysis of the influence of parameters of direct laser engraving of printing elements of intaglio printing plates on their stressed-strained state under real working load conditions. It is advisable to perform the study using numerical modeling methods, which will make it possible to establish regularities in the redistribution of stresses and deformations inside plate strokes, and to simulate the presence and absence of ink in engraved strokes.

3. The aim and objectives of the study

The aim of our work is to establish regularities in the plateation of the stressed-strained state of the printing plate in an intaglio printing process depending on the geometric parameters of engraving and the conditions for contact interaction. This will make it possible to predict and control the wear resistance of intaglio printing plates.

To achieve the goal, the following research tasks were set:

– to build a three-dimensional model of the contact interaction between the plate and printing cylinders;

– to investigate the influence of various geometric parameters of engraved strokes (width, depth, angle of inclination of the side walls of engraved strokes, distance between them) on the distribution of stresses and deformations in an intaglio printing plate;

– to derive recommendations for improving the geometry of the printing plate in order to increase the wear resistance of intaglio printing plates.

4. The study materials and methods

The object of our study is the process that induces the stressed-strained state of an engraved printing plate obtained by direct laser engraving at different geometric parameters of printing and space elements.

The hypothesis of the study assumes that the geometric parameters of engraved strokes obtained by direct laser engraving significantly affect the nature of the distribution of stresses and strains in an intaglio printing plate. It is assumed that changing the width, depth of engraving, the angle of inclination of the side walls, and the distance between the strokes leads to a redistribution of local concentrations of stresses and strains. It is expected that the revealed patterns could make it possible to compile practical recommendations that would contribute to increasing the wear resistance and durability of intaglio printing plates.

In order to simulate the influence of the geometry of the engraved strokes profile on the stress distribution and deformation in an intaglio printing plate when pressure is applied, 3D models were designed in the SolidWorks CAD system (USA). The 3D models built are fragments of the contact zone of two cylinders. Considering that the width of the contact line of the plate with the printing cylinder significantly exceeds the width of the engraved strokes [7], the printing and forging cylinders are considered as elastic planes; on the surface of the latter an engraved printing plate is fixed. The surface of 33 types of engraved strokes was modeled with variable parameters of stroke depth $d \in \{30, 60, 100\}$ μm, width $b \in \{30, 60, 90, 120, 140, 180, 210\}$ μm, distance between strokes $l \in \{25, 50, 100, 150\}$ μm, and angle of inclination of side walls $a \in \{60, 75, 90\}$ degrees. The proposed models correspond to the actual geometric parameters of engraving intaglio plates used in production [14] (Fig. 1).

A feature of the model was the consideration of three adjacent strokes:

– the central stroke was filled with ink, which imitates the intaglio printing workflow;

– the two outer strokes remained empty, which imitates colorless embossing by intaglio printing.

This approach makes it possible to determine the influence of the presence of ink on the pressure distribution and the nature of stress distribution in the ink-filled and empty plate elements.

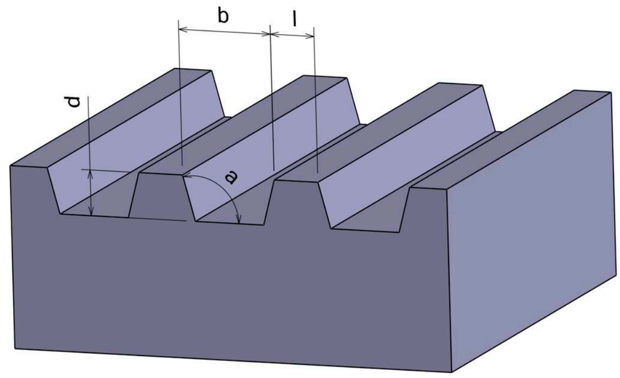


Fig. 1. Calculated geometric 3D model of the engraved surface of the printing plate with the parameters of stroke depth *d*, width *b*, distance between strokes *l*, and angle of inclination of the side walls *a*, which were used in numerical modeling

The modeling was carried out using the finite element method. The ANSYS Mechanical package [15] was used for simulation, in which a pressure of 256 MPa was applied between the contact planes, which corresponds to the conditions of real printing contact during intaglio printing. The distribution of stresses and deformations in the printing zone was investigated using static analysis.

The following assumptions were made during simulation:

– the material of the intaglio printing plate is considered homogeneous and linearly elastic;

– thermal and dynamic effects were not taken into account;

– the ink layer in the central stroke was modeled as a region with hydrostatic pressure.

The basic principles of applying the finite element method in this case can be summarized as follows.

The geometry of the cylinders (both the plate and the printing) is divided into separate elements with nodes – the geometry is discretized. For the zone with engraved strokes, various geometric parameters are taken into account (width, depth, angle of inclination of the walls, distance between strokes).

Differential equations describing the equilibrium of forces and moments are written on each element. In this case, approximating (shape) functions are used to discretize the displacement field [16–18].

A local stiffness matrix is built for each element, which, after integration over the volume of the element, takes the following form

$$K_e = \int_V B^T \cdot D \cdot B \mathrm{d}V, \quad (1)$$

where K_e is the element stiffness matrix, which determines the relationship between the displacements of the element nodes and the internal forces and is obtained as a result of integrating the local properties of the element over its volume; D is the matrix of material properties (constitutive matrix), describing how the material reacts to deformations, that is, it sets the relationship between stresses and deformations (according to Hooke's law); B is the strain-displacement matrix, which converts the displacements in the element into the corresponding deformations and is derived from the shape functions of the element and contains the derivatives of these functions with respect to the coordinates; B^T is the transposed matrix [16–18].

The appropriate boundary conditions are imposed on the model, and a force load is applied, which generates a maximum stress of 256 MPa in the area of the strokes with their subsequent fixation.

After compiling the global stiffness matrix, the system of equations is solved by the method of numerical integration, which makes it possible to obtain the distribution of displacements, stresses, and deformations throughout the structure.

To simulate the ink in the central stroke, a special 3D hydrostatic fluid element (HSFLD242) is used to model the liquid (ink) [19]. Through APDL (the ANSYS parametric programming language), an additional node is built, which acts as a pressure node, and a surface load is applied (via the ESURF command) to the elements representing the interface between the liquid and the solid. This approach makes it possible to combine the behavior of the liquid and solid parts of the model, ensuring the correct transfer of stresses between them.

The HSFLD242 element is designed to simulate the hydrostatic behavior of liquids in 3D space with the following assumptions for such an element:

– there are no shear deformations in the liquid, and the only type of stress is normal stresses described by hydrostatic pressure;

– the density of the liquid (ρ) is used to calculate the mass of the element, which is important for possible dynamic calculations.

Formally, the stress tensor for a fluid is written as

$$\sigma_{ij} = -p \cdot \delta_{ij}, \qquad (2)$$

σ_{ij} – components of the stress tensor (the indices i and j denote directions in space); p – hydrostatic pressure; δ_{ij} – Kronecker delta, which takes the value 1 if $i=j$, and 0 if $i \neq j$, and ensures isotropic pressure – the same in all directions.

For a fluid element, the energy potential can be written, which takes into account the interaction of pressure and volumetric deformations

$$\Pi = \frac{1}{2}\int_V \frac{p^2}{K}\mathrm{d}V - \int_V p\left(\nabla \cdot u\right)\mathrm{d}V, \qquad (3)$$

where Π is the potential energy of the fluid element, which determines the total energy stored in the element and consists of the energy associated with pressure and the work done during deformation; $\int_V \frac{p^2}{K}\mathrm{d}V$ is the integral over the volume, which calculates the accumulated energy associated with pressure over the entire volume of the element; $\int_V p\left(\nabla \cdot u\right)\mathrm{d}V$ is the integral of the work done by pressure when volumetric deformation occurs, integrating over the volume of the element.

The element is formulated as a mixed-type problem, where in addition to the displacements of the nodes, an additional variable is taken into account – pressure. This ensures a correct reflection of the interaction between the fluid and the solid.

When discretizing the element, the standard scheme for constructing the stiffness matrix is used, but with the features of the hydrostatic state according to equation (1), while for a hydrostatic fluid matrix D takes the following form

$$D = K \cdot I, \qquad (4)$$

where I is the identity matrix and K is the volume stiffness.

Thus, the interaction between pressure and volume strain is taken into account through the following relation

$$p = K \cdot \varepsilon_v, \qquad (5)$$

where p is the pressure in the liquid; K is the compressibility coefficient (bulk modulus) of the liquid. It determines the resistance of the liquid to volume changes; $\varepsilon_v = \nabla \cdot u$ is the volumetric deformation, which determines how much the local increase or decrease in volume occurs [16–18].

5. Results of investigating the influence of engraving geometry on the generation of stressed-strained state of an intaglio printing plate

5. 1. Three-dimensional model of contact interaction between the plate and printing cylinders

We simulated the contact interaction between the printing and intaglio printing plate cylinders (the engraved printing plate is fixed on the latter) using the finite element method in a three-dimensional setting. The method involves the use of a dense mesh in the contact zone and in the area of the engraving stroke, which makes it possible to correctly reproduce the stress concentrations inside the plate (Fig. 2).

The model constructed reproduces the real geometry of the plate taking into account the profile of the engraved strokes, in particular the width, depth of the engraved strokes, the angle of inclination of the side faces, and the distances between the strokes (Fig. 3). The influence of the ink was taken into account in the form of hydrostatic pressure applied to the bottom and side faces of the profile of the average engraved stroke.

During simulation, special attention was paid to the zone from the bottom of the stroke to the side faces of the engraved element, the middle part of the side faces and the zone between the side faces and the blank elements. Both empty engraved elements (edge elements in Fig. 2, 3) and the engraved element filled with ink (middle element in Fig. 2, 3) were investigated. The calculation was performed under conditions of static load, which was the working pressure of the intaglio printing machine (256 MPa).

As a result of simulation, the distribution of equivalent stresses and the magnitude of elastic deformations for different cases of geometric parameters of the engraving form was determined. Our results confirmed the significant influence of geometric parameters on the stressed-strained state of the printing plate. The devised three-dimensional statement of the problem allowed us to reveal spatial effects, in particular, the complex nature of the distribution of contact stresses and deformations both in the direction of the stroke axis and in the direction of the contact width.

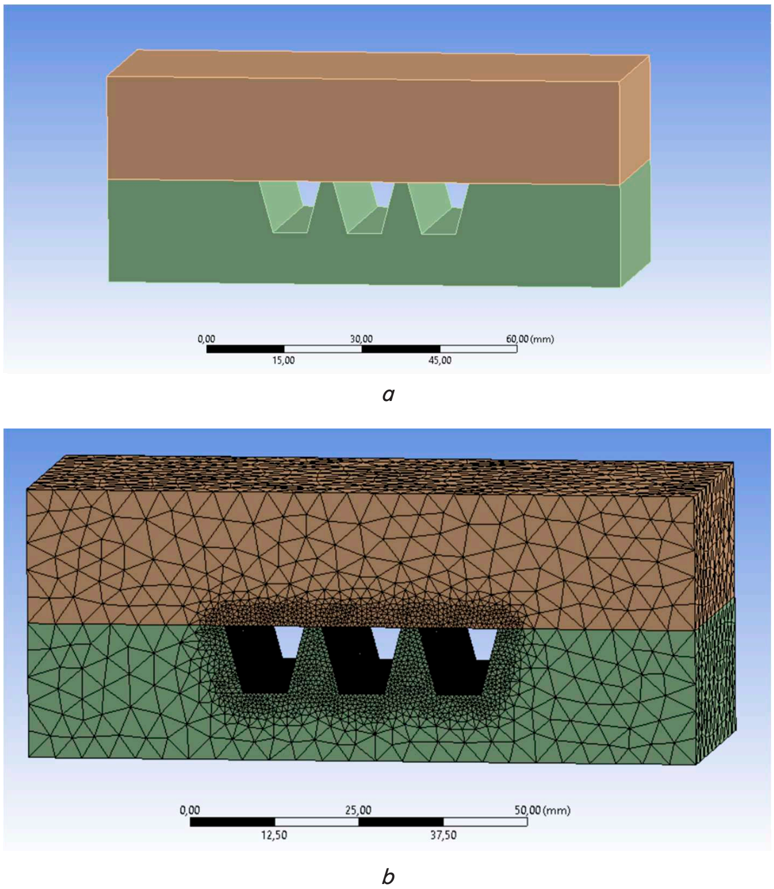


Fig. 2. Simulation of the contact pair "engraved strokes of the printing plate – printing cylinder": *a* – geometric model; *b* – finite element model

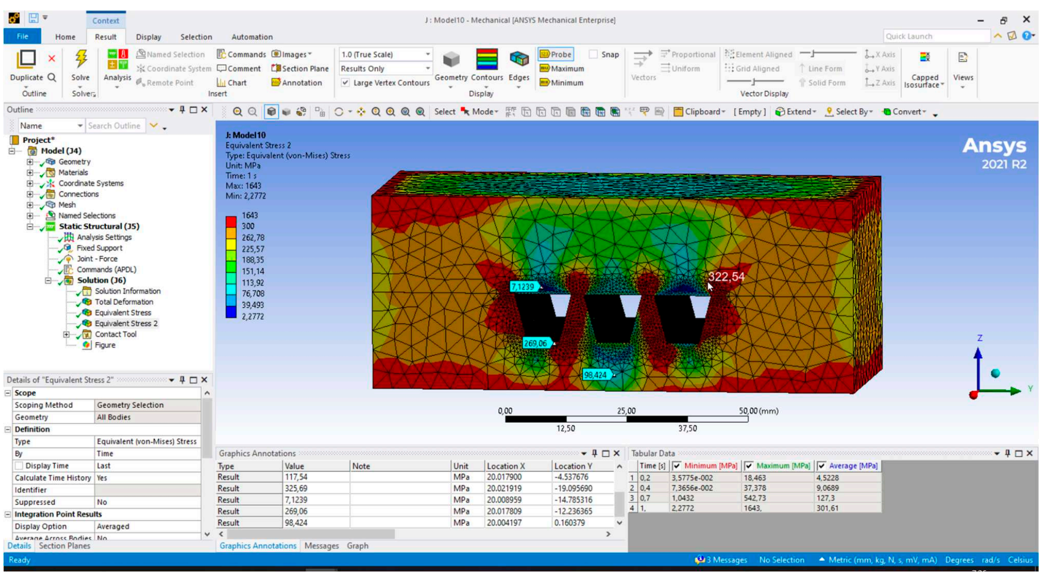


Fig. 3. Three-dimensional model of the contact interaction between the plate and printing cylinders with displayed results from numerical modeling of the distribution of equivalent stresses in the engraved surface, constructed in the ANSYS Mechanica environment

5. 2. Influence of the geometry of engraved strokes on the distribution of stresses and strains in an intaglio printing plate

Fig. 4, 5 show examples of heat maps illustrating the distribution of stresses and strains in an intaglio printing plate under the action of an applied pressure of 256 MPa, taking into account the use of engraved strokes with different geometric parameters.

Diagrams in Fig. 6, 7 show how the stresses and strains inside the engraved stroke of a printing plate depend on its width for different states – with an empty stroke and a stroke with ink – and with different engraving depths.

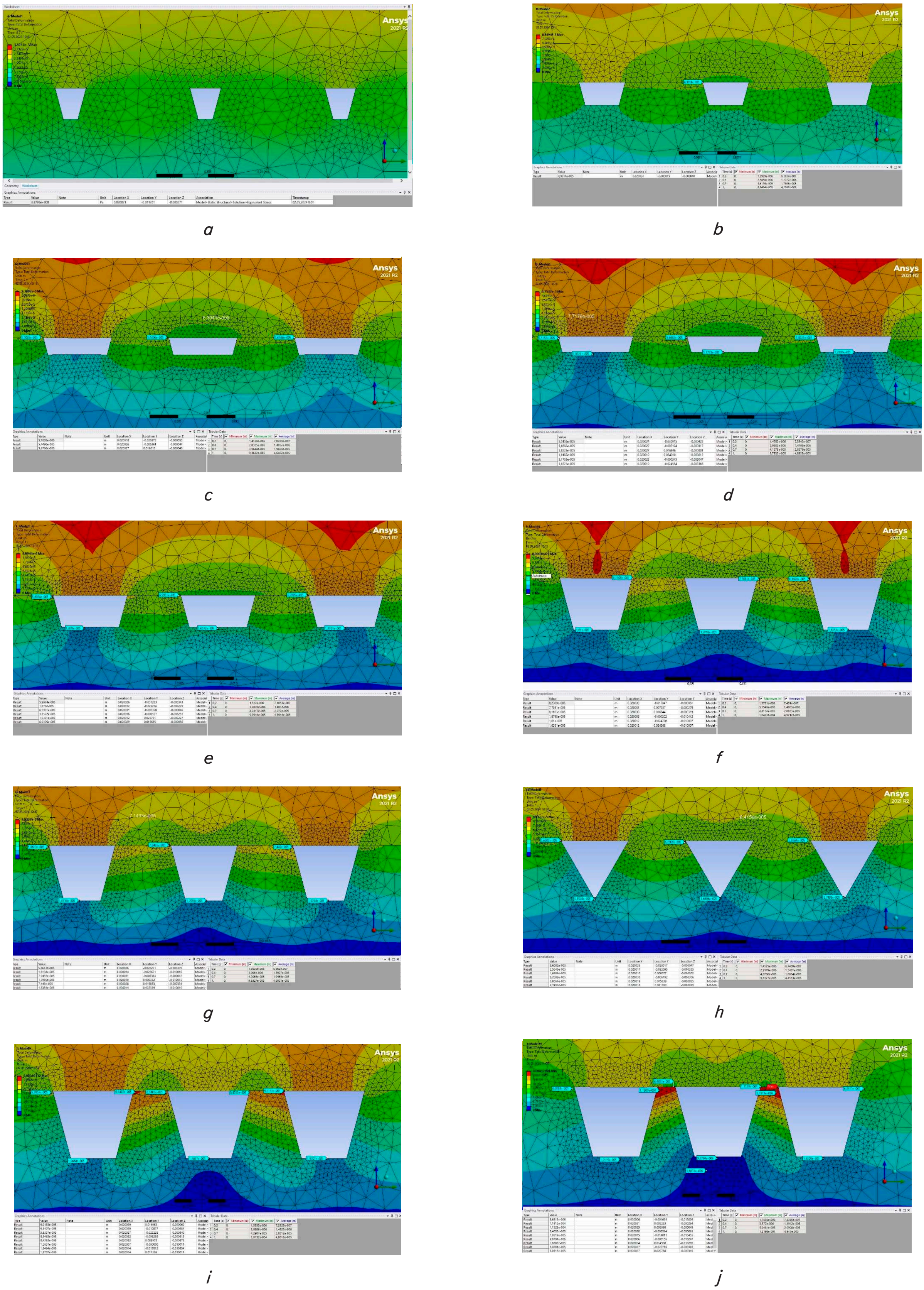


Fig. 4. Deformations of intaglio printing plate elements with different geometric parameters of strokes at a pressure of 256 MPa (width / depth / distance between / angle of inclination of the side walls): *a* — 30 μm / 30 μm / 100 μm / 75°; *b* — 60 μm / 30 μm / 100 μm / 75°; *c* — 120 μm / 30 μm / 100 μm / 75°; *d* — 140 μm / 30 μm / 100 μm / 75°; *e* — 140 μm / 60 μm / 100 μm / 75°; *f* — 140 μm / 60 μm / 100 μm / 75°; *g* — 140 μm / 100 μm / 100 μm / 75°; *h* — 120 μm / 100 μm / 100 μm / 60°; *i* — 120 μm / 100 μm / 50 μm / 75°; *j* — 120 μm / 100 μm / 25 μm / 75°

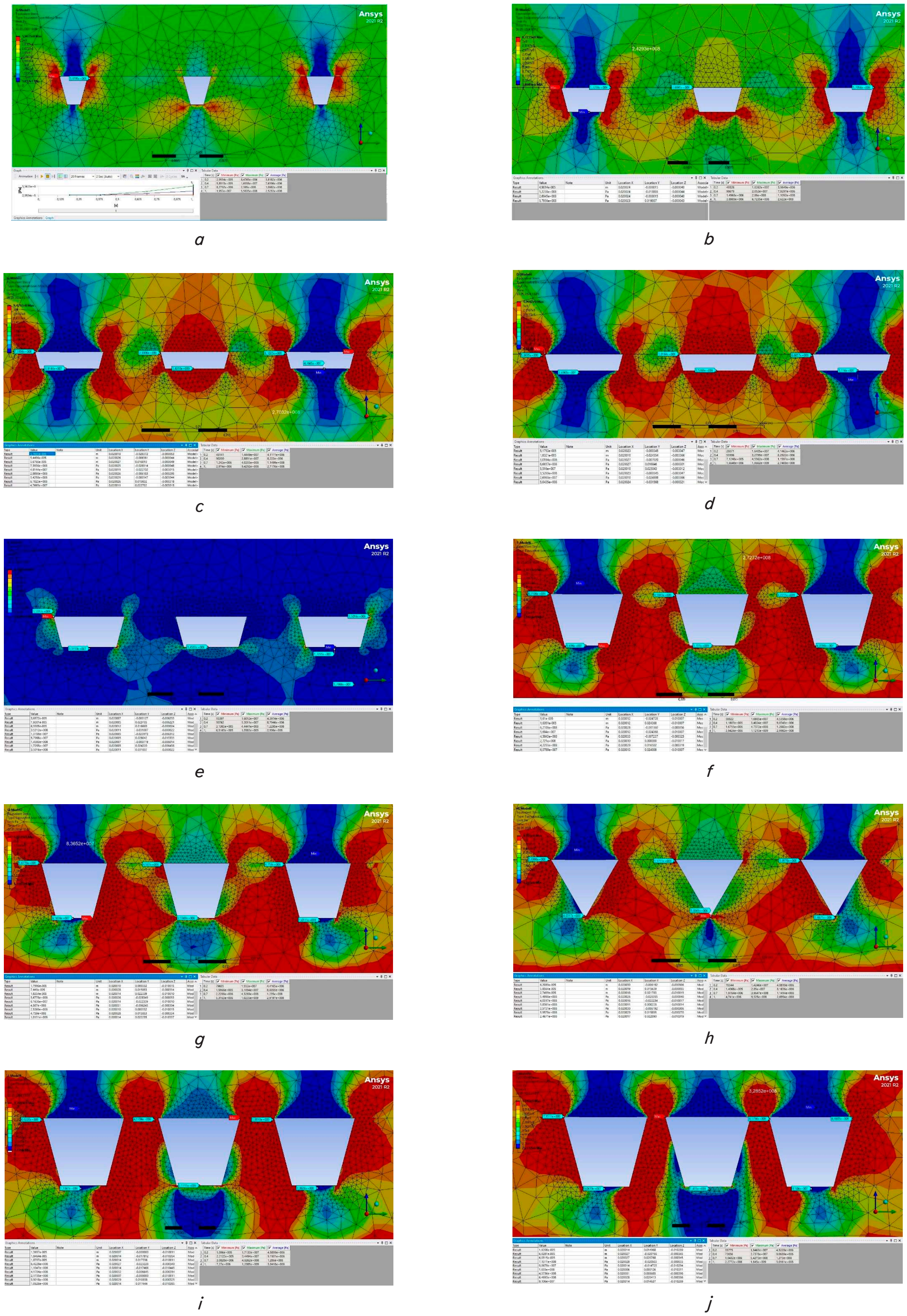
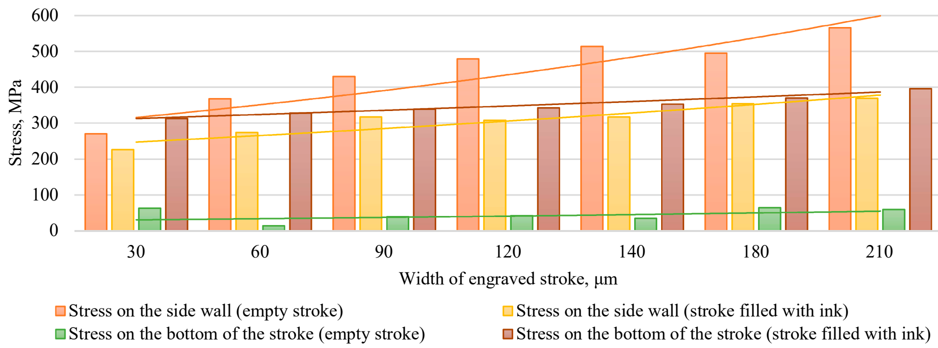


Fig. 5. Stresses of intaglio printing plate elements with different geometric parameters at a pressure of 256 MPa (width / depth / distance between / angle of inclination of the side walls): *a* – 30 μm / 30 μm / 100 μm / 75°; *b* – 60 μm / 30 μm / 100 μm / 75°; *c* – 120 μm / 30 μm / 100 μm / 75°; *d* – 140 μm / 30 μm / 100 μm / 75°; *e* – 140 μm / 60 μm / 100 μm / 75°; *f* – 140 μm / 60 μm / 100 μm / 75°; *g* – 140 μm / 100 μm / 100 μm / 75°; *h* – 120 μm / 100 μm / 100 μm / 60°; *i* – 120 μm / 100 μm / 50 μm / 75°; *j* – 120 μm / 100 μm / 25 μm / 75°



a

b

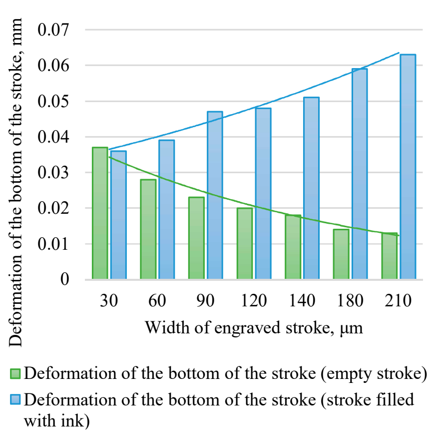
Fig. 6. Dependence of stress distribution on the side wall and bottom of the stroke when applying a pressure of 256 MPa on the stroke width at: *a* — stroke depth of 30 µm, distance between strokes of 100 µm, side wall inclination angle of 75°; *b* — stroke depth of 100 µm, distance between strokes of 100 µm, side wall inclination angle of 75°

As a result of simulation analysis, it was determined that with an increase in the width of the engraved stroke, the stress on the side walls increases for all cases (Fig. 6), including at different values of the engraving depth. The influence of the stroke width on the stress distribution when applying pressure at the bottom of the engraved stroke is more complex, especially in the empty stroke. At a smaller engraving depth (30 µm), the bottom is almost not loaded in the empty stroke, in the presence of ink, the stress at the bottom of the stroke increases sharply with an increase in the width of the engraved stroke. At a greater engraving depth (100 µm), the stress at the bottom of the empty stroke decreases with an increase in the engraving width, in the presence of ink in the engraved stroke, the stress at the bottom steadily increases with an increase in the width of the stroke.

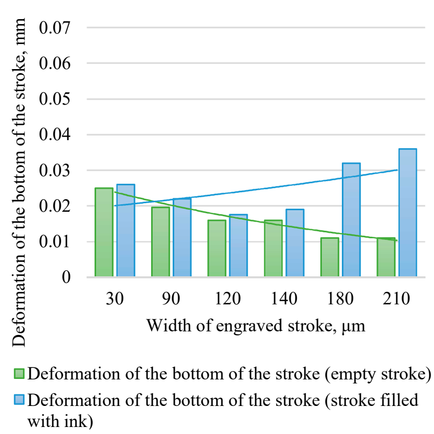
Fig. 7 shows that at different engraving depths, increasing the width of the engraved stroke leads to a decrease in the bottom deformation in empty strokes, and an increase in the bottom deformations of strokes filled with ink. An increase in the engraving depth leads to a decrease in the deformation values in all cases.

Fig. 8, 9 show diagrams of the dependence of the distribution of stress and deformation of strokes when applying pressure on the depth of their engraving.

Analysis of the diagrams (Fig. 8, 9) revealed that the dependence of the distribution of stresses and deformations when applying pressure on the depth of engraving of strokes is quite complex, since different trends are observed with increasing engraving depth for narrow and wide strokes.



a



b

Fig. 7. Dependence of distribution of the deformation of the bottom of the stroke when applying a pressure of 256 MPa on the stroke width at: *a* — stroke depth of 30 µm, distance between strokes of 100 µm, angle of inclination of the side wall of 75°; *b* — stroke depth of 100 µm, distance between strokes of 100 µm, angle of inclination of the side wall of 75°

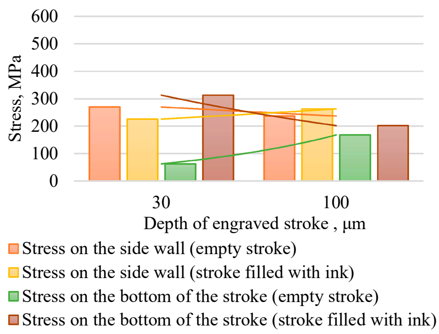
With a small stroke width (Fig. 8, *a*, *c*) with increasing engraving depth, a decrease in stresses on the side walls is observed for both empty strokes and those filled with ink. At the same time, the stresses at the bottom of the empty strokes remain relatively small, but show a tendency to increase, at the bottom of the strokes with ink – they increase, which indicates a gradual redistribution of loads from the walls to the bottom.

For wide strokes (Fig. 8, *b*, *d*) with increasing engraving depth, a decrease in stresses on the side walls of both empty and filled with ink engraved strokes is characteristic. The stresses at the bottom of the strokes with ink also decrease to a greater extent. At the same time, the stresses at the bottom of the empty strokes are moderately low but tend to increase with increasing engraving depth.

Analysis of the diagrams of deformations of the bottom of the stroke when applying pressure demonstrates consistency with the stress distribution. In particular, it is noted that the deformation of the bottom of both strokes with ink and empty strokes decreases with increasing depth of engraving of the strokes.

However, it is worth noting that a clear correlation between the depth of engraving and the distribution of load and deformation is difficult to obtain since the width of the engraving strokes plays an important role in this process. Therefore, Fig. 10 shows the dependences of the distribution of stresses and deformations inside the engraved strokes on the ratio of width to the depth of engraving of the strokes.

From the diagram (Fig. 10) it can be concluded that the ratio of width to the depth of engraving determines the mechanism of formation of the stressed-strained state of the engraved stroke. At small values of this ratio, the load is distributed more evenly between the side walls and the bottom. With an increase in this indicator (in particular, at a value of width/height > 3), the load increases in the side walls of empty and filled strokes and at the bottom of strokes filled with ink, and the load decreases at the bottom of empty engraved strokes. These observations correlate with the deformation of engraved strokes.



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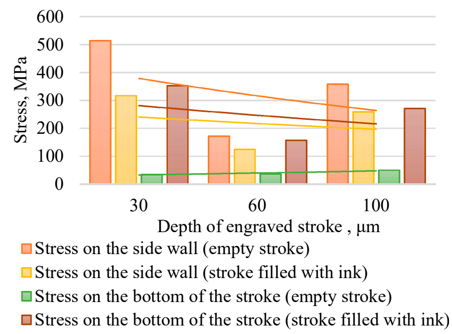

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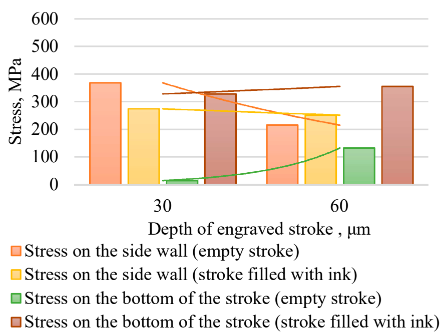
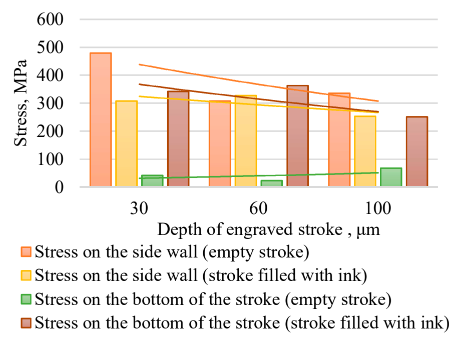


c



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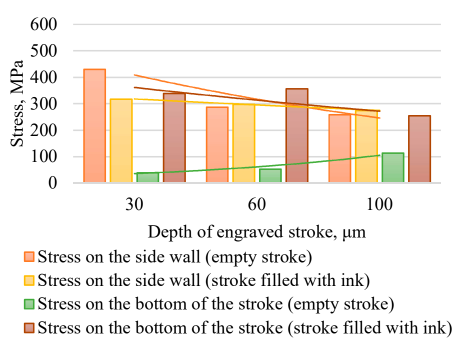


e

Fig. 8. Dependence of stress distribution on the side wall and the bottom of the stroke when applying a pressure of 256 MPa on the depth of the stroke engraving at: *a* – stroke width 30 μm, distance between strokes 100 μm, side wall inclination angle 75°; *b* – stroke width 140 μm, distance between strokes 100 μm, side wall inclination angle 75°; *c* – stroke width 60 μm, distance between strokes 100 μm, side wall inclination angle 75°; *d* – stroke width 120 μm, distance between strokes 100 μm, side wall inclination angle 75°; *e* – stroke width 90 μm, distance between strokes 100 μm, side wall inclination angle 75°

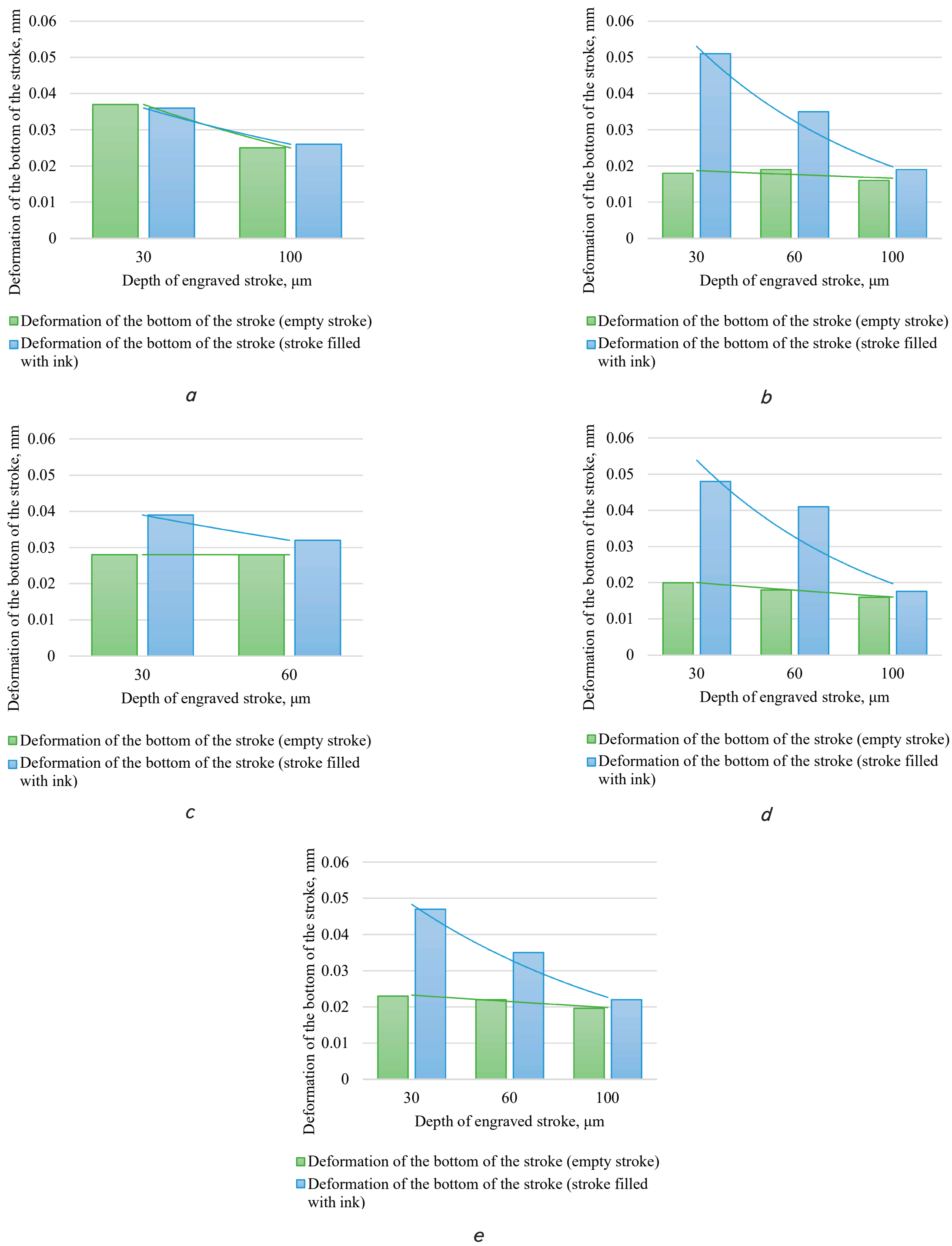


Fig. 9. Dependence of the distribution of deformation of the bottom of the stroke when applying a pressure of 256 MPa on the depth of the stroke engraving at: *a* – stroke width 30 µm, distance between strokes 100 µm, angle of inclination of the side wall 75°; *b* – stroke width 140 µm, distance between strokes 100 µm, angle of inclination of the side wall 75°; *c* – stroke width 60 µm, distance between strokes 100 µm, angle of inclination of the side wall 75°; *d* – stroke width 120 µm, distance between strokes 100 µm, angle of inclination of the side wall 75°; *e* – stroke width 90 µm, distance between strokes 100 µm, angle of inclination of the side wall 75°

Fig. 11, 12 show the dependences of load distribution and deformation when applying a pressure of 256 MPa on the distance between strokes.

Analysis of the influence of distance between the engraved strokes on stress state revealed that in general, with an increase in the distance between the strokes, a decrease in the stress on the side walls of both empty and filled strokes is observed. However, there is an influence of the depth and width of the engraving of strokes on the dynamics of dependence of stress in the side walls on the distance between strokes. The most sensitive to changes in the distance between the strokes are narrow strokes, while wide strokes remain more stable when the distance between strokes changes.

At the same time, in the case of deep and wide strokes, the most pronounced increase in the stress on the side walls with a decrease in the distance between the strokes is observed, which also affects the deformation of the side faces. This is visually demonstrated in Fig. 4, *g*, *h*, where it is seen that when the strokes are arranged too close to each other, the phenomenon of cutting off the upper part of the side faces is observed. This phenomenon is associated with the interaction between stress-strain fields in neighboring elements.

There is no clear dependence on the deformation of the bottom of strokes upon application of pressure (Fig. 12). However, it is worth noting that the presence of ink in all cases increases the level of stress and deformation of the bottom, which confirms its key role in transferring the load under the conditions of external pressure.

Fig. 13 shows a diagram of dependence of the stressed-strained state of engraved strokes upon application of a pressure of 256 MPa on the angle of inclination of side walls.

Our results (Fig. 13) indicate that the angle of inclination of the side walls significantly affects the nature of the distribution of stresses and strains in the engraved stroke when pressure is applied. In particular, an increase in the angle of inclination of the side faces of the engraved strokes leads to an increase in stresses and strains on the side walls of the strokes and the bottom of the strokes filled with ink, as well as to a decrease in the load and strains on the bottom of empty strokes.

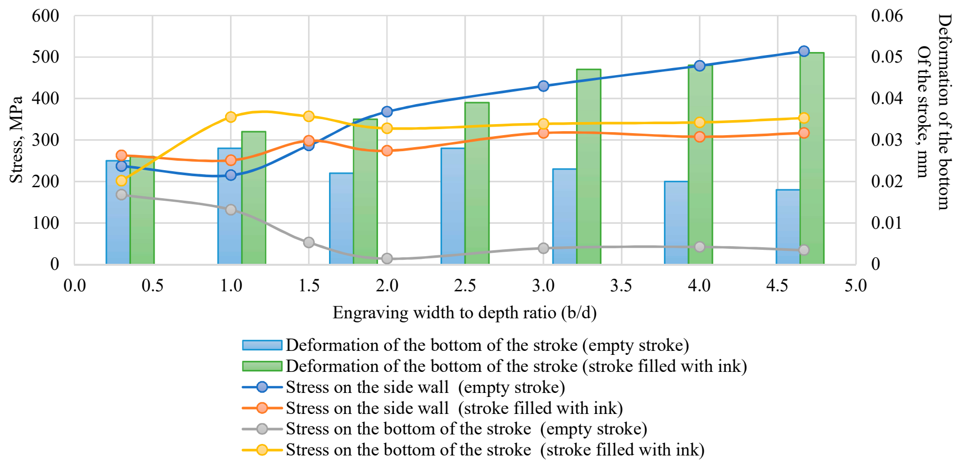
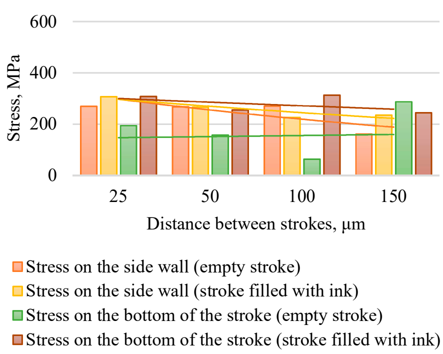
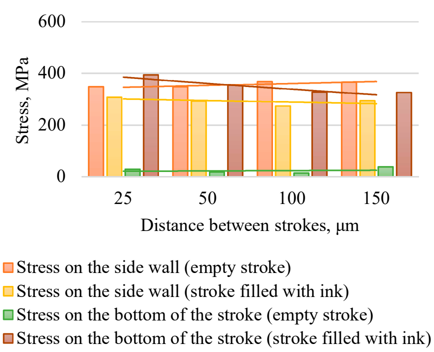


Fig. 10. Dependence of stress distribution in engraved strokes of the plate and deformation of the bottom of the stroke when applying a pressure of 256 MPa on the ratio of width to the depth of engraving of the strokes



a



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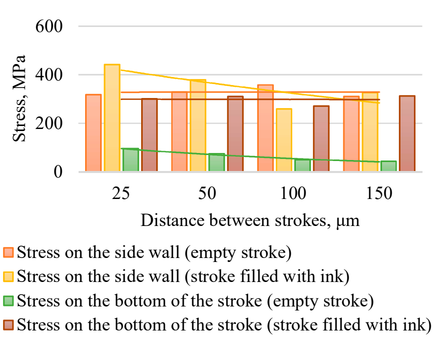

c

Fig. 11. Dependence of stress distribution on the side wall and bottom of the stroke when applying a pressure of 256 MPa on the distance between strokes at: *a* – stroke width 30 μm, stroke depth 30 μm, side wall inclination angle 75°; *b* – stroke width 60 μm, stroke depth 30 μm, side wall inclination angle 75°; *c* – stroke width 140 μm, stroke depth 100 μm, side wall inclination angle 75°

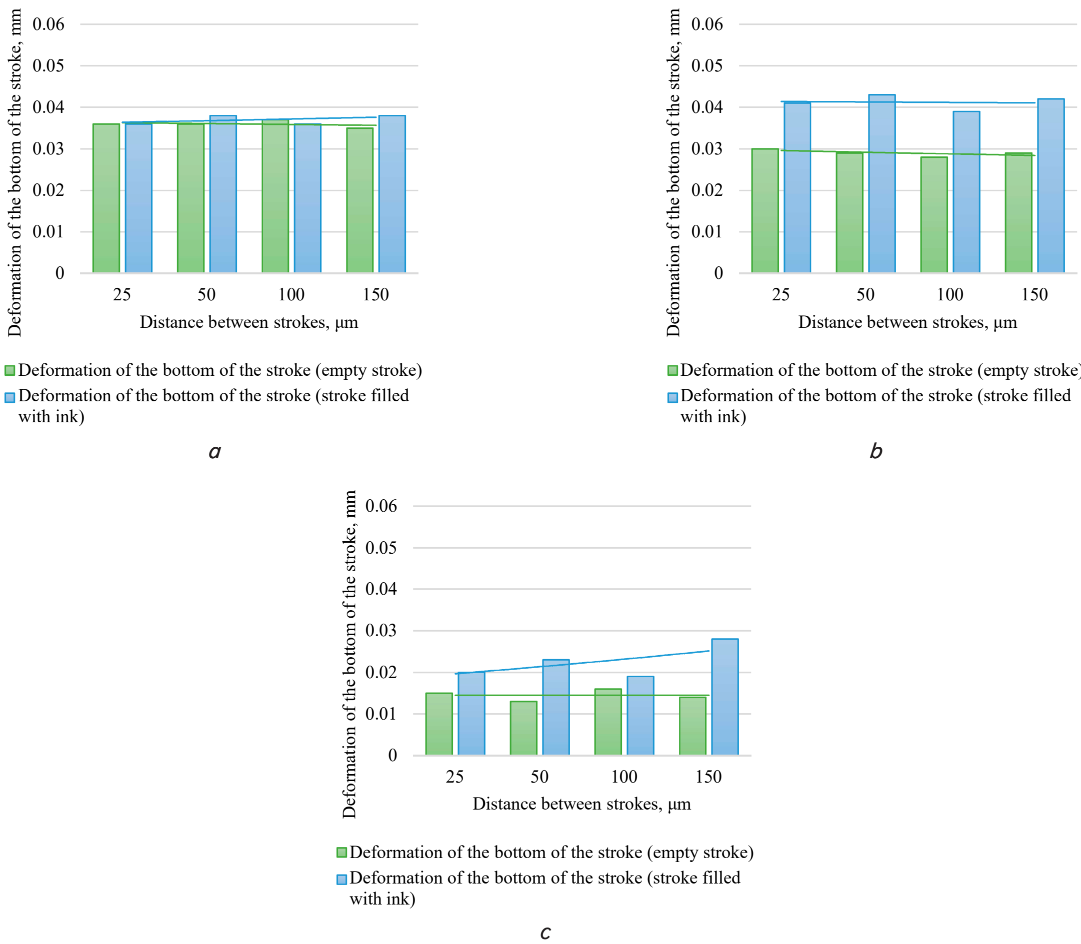

Fig. 12. Dependence of the distribution of deformation of the bottom of the stroke when applying a pressure of 256 MPa on the distance between strokes at: *a* – stroke width 30 µm, stroke depth 30 µm, side wall inclination angle 75°; *b* – stroke width 60 µm, stroke depth 30 µm, side wall inclination angle 75°; *c* – stroke width 140 µm, stroke depth 100 µm, side wall inclination angle 75°

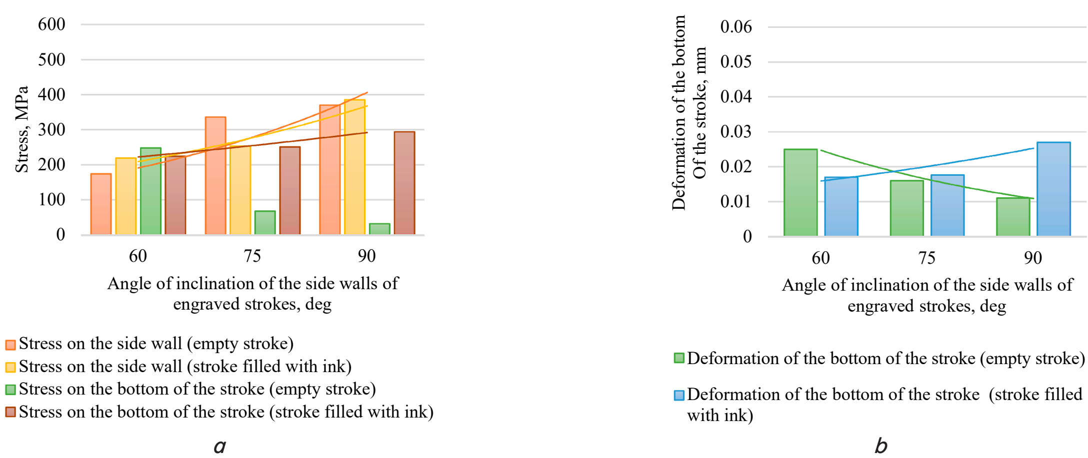

Fig. 13. Dependence of the stressed-strained state of engraved lines when applying a pressure of 256 MPa on the angle of inclination of the side walls of engraved lines with a line width of 120 µm, a line depth of 100 µm, and a distance between lines of 100 µm: *a* – stress distribution in engraved lines of the plate; *b* – deformation distribution of the bottom of the line when applying a pressure of 256 MPa on the angle of inclination of the side walls of a stroke

5. 3. Recommendations for improving the geometry of a printing plate to increase the wear resistance of intaglio printing plates

Analysis of the results of numerical modeling revealed that it is the ratio of the width to the depth of the engraved stroke b/d, and not the absolute values of parameters, that is the determining factor for inducing the stressed-strained state of the engraved strokes of a printing plate in an intaglio printing process. The rational range of the ratio of width to the depth of the engraved stroke b/d has been determined

$$2<\frac{b}{d}<3.$$

In this range, the stresses on the bottom and side walls do not reach critical values, the deformations of the bottom remain relatively low. Thus, the risk of local deformation and premature wear is reduced.

It has been determined that increasing the angles of inclination of the side faces of engraved strokes contributes to an increase in stresses and deformations in engraved elements. This is especially observed at the transition between the bottom and side faces of the stroke. Therefore, it is recommended to use smaller angles of inclination of the side walls within technologically permissible values (without significant deterioration of ink transfer and formation of the thickness of the ink layer).

The small distance between engraved strokes on an intaglio printing plate leads to the overlap of stress zones, which contributes to increased wear of space elements. Therefore, it is important to take into account the level of stresses formed on the side faces caused by other geometric parameters of the strokes. Therefore, it is recommended to provide such a distance between the strokes at which the interaction between the stress zones is minimal. If possible, one should increase the distance between engraved elements if they are deep and narrow. If the design does not allow for increasing the distance between strokes, it is worth ensuring an optimal width-to-depth ratio in these places and reducing the angle of inclination of the side faces of engraved elements.

6. Discussion of results based on investigating the factors that induce the stressed-strained state of the printing plate during intaglio printing

Our simulation results show the complex spatial nature of the stressed-strained state of the printing plate, which occurs when external pressure is applied. From the diagrams of equivalent stresses and strains (Fig. 4, 5) it is clear that their maximum values are localized in the transition zone from the bottom of the engraved strokes to the side walls. This can be explained by a sharp change in the curvature of the surface and the redistribution of contact pressure in this zone. Fig. 13 confirms this: it is clearly seen that an increase in the angle of inclination of the side faces of the engraved strokes leads to an increase in stresses and strains on the side walls of the strokes and the bottom of the strokes filled with ink. This is due to a change in the direction of transmission of contact forces. Therefore, it is recommended to use smaller side wall angles within technologically permissible values, or to add a radius of rounding to these angles when designing the engraving of plates.

Our study revealed the dependence of the nature of distribution of the stressed-strained state on the width (Fig. 6, 7) and depth (Fig. 8, 9) of the engraving of the stroke. In particular, with an increase in the stroke width, an increase in the stresses on the side walls is observed for all the studied options for the engraving depth. This is explained by an increase in the contact area, respectively, by a change in the loading conditions. In this case, the stressed-strained state at the bottom of the stroke depends on both the engraving depth and the presence of ink inside the stroke. At a small engraving depth (30 μm) in the case of colorless embossing (with empty strokes), the bottom is almost not loaded. Whereas when printing (ink-filled strokes), the stresses at the bottom significantly increase with an increase in the width of the engraved stroke. At a greater engraving depth (100 μm), the stresses at the bottom of the empty strokes decrease with an increase in width. This indicates a redistribution of the load between the elements of the engraved stroke. The analysis of deformations reveals similar trends. In empty strokes, increasing the width of the engraved strokes leads to a decrease in the deformation of the bottom, while in strokes with ink – to an increase in the deformation of the bottom. Increasing the engraving depth leads to a general decrease in the deformation of the bottom of the stroke and a decrease in the stresses on the side walls. This indicates an increase in the rigidity and stability of the geometry of engraved shape elements.

It was determined that the determining parameter in generating a stress-strain value is not the absolute values of the width and depth but their ratio. This is explained by the fact that the ratio of width to the depth of the stroke determines the rigidity of the formed profile. At a small ratio of the width to the depth ($b/d < 1$) (narrow deep engraved stroke), the side walls become stiffer, the stresses are distributed more evenly along the height of the walls of the engraved stroke. In this case, the area of the bottom, to which the contact or hydrostatic pressure is directly transmitted, is also small, and, therefore, the total force acting on the base of the stroke is also small. Therefore, low values of stress and deformation are observed when external pressure is applied to such strokes (Fig. 10). In the case when the width and depth of the engraving are equal ($b/d \approx 1$), a geometry is formed in which two deformation mechanisms are superimposed – bending at the bottom and bending-compression on the side faces of the engraved strokes. Therefore, an increase in the resulting loads and deformation of the strokes under the action of external pressure is observed in such engraved strokes (Fig. 10). A further increase in width with a decrease in the relative depth ($2 < b/d < 3$) changes the nature of the interaction – the side walls become lower and stiffer, the stresses are partially redistributed to the bottom of the engraved strokes. In this case, a uniform smooth increase in stresses is observed, where the bending of the bottom is still insignificant, and the influence of the walls is already weakened. It is with this ratio of the width to the depth of the engraving that the most balanced state of the printing plate is determined. Indeed, with a further increase in the stroke width and a relative decrease in the stroke depth, an increase in the values of stress and deformation of the engraved plate elements is observed (Fig. 10). This is explained by an increase in the area of the bottom of the engraved strokes, and, consequently, an increase in its deflection under the action of pressure.

Our simulation analysis also revealed the dependence of the nature of distribution of stressed-strained state on the presence of ink inside engraved strokes. The use of the 3D hydrostatic fluid element HSFLD242 has made it possible to take into account the influence of liquid pressure due to the introduction of an additional degree of freedom and the application of the potential approach. The mathematical model of the element was based on the principles of force balance and potential energy minimization, which ensures an accurate transfer of the relationship between volumetric deformation and pressure. The results of the static analysis showed that the presence of liquid (ink) in the central stroke significantly changes the nature of the fracture.

In the case of hollow engraved elements that imitate colorless embossing, when pressure is applied, the side walls are destroyed. This phenomenon can be explained by the fact that in the absence of ink inside the stroke, the side walls bear the main load under the action of applied pressure, which leads to deformations and stress concentration in these areas.

In engraved strokes filled with ink, the destruction occurs mainly at the base of the stroke, that is, in the contact zone between the liquid (ink) and the printing plate material. This indicates that the ink redistributes the stresses in such a way that the load on the side walls is reduced, but the stress concentration at the bottom of the stroke is increased.

It is also important to pay attention to the distance between adjacent engraved strokes (Fig. 11, 12). After all, a decrease in the distance between strokes leads to the overlap of stress zones in the space elements, which contributes to their accelerated wear. This is especially true for deep and narrow elements (Fig. 4, *g*, *h*), where at small inter-stroke distances the phenomenon of side wall shearing can be observed. This effect should be taken into account when optimizing the design of the printing plate.

Thus, modeling has made it possible to establish the nature of influence of the geometric parameters of the engraved elements in an intaglio printing plate on the distribution of stresses and deformations inside it. To evaluate the results, it is advisable to compare them with existing scientific works addressing the quality of laser engraving and wear resistance of intaglio printing plates.

Technical review papers on laser engraving [3, 4] provide a detailed overview of the technological processes of manufacturing engraved printing plates. They are an important source of data for specifying the geometric parameters of printing plate models. However, the cited papers practically do not include an analysis of the mechanical behavior of engraved lines under the action of pressure inherent in the printing process, in particular intaglio printing. In our study, the technological parameters of laser engraving were used as input parameters for a numerical model, which made it possible to assess the influence of actual geometric parameters of engraved elements in an intaglio printing plate on potential areas of plate wear.

Despite the thorough analysis of the process that forms the ink layer and the influence of the geometric parameters of the intaglio printing plates on it, carried out in [5–7], the cited papers do not include an analysis of the mechanical behavior of the printing plate material. In general, the stressed-strained state of engraved elements under pressure is not analyzed, which makes it impossible to assess both local stress concentrations and the overall wear resistance of the plate. In our work, the simulation performed makes it possible not only to assess local stress concentrations but also to investigate the influence of the presence of ink on the distribution of stresses and deformations in engraved elements of the plate when pressure is applied.

In order to be able to simulate the printing contact of intaglio printing, studies using the finite element method were analyzed. In particular, in work [8], the finite element method was used to study the printing contact in the pad printing process. However, the surface geometry in the model is simplified and does not contain the microrelief characteristic of engraved strokes. The influence of geometric parameters on local stresses is not analyzed. In our work, simulation of the printing process of intaglio printing is reported; the simulation using a finite element method was performed taking into account the realistic microgeometry of engraved strokes. This allows for more accurate determination of local stresses and potential areas of wear of printing plates.

In [9], the study considers another printing method, although methodologically the work is a valuable reference point in modeling. However, in the work, the model is focused on the macro level of the printing contact without taking into account the microgeometry of the printing plate. Thus, the cited work does not make it possible to assess the stressed-strained state of individual engraved elements of the plate. In our study, the printing contact is analyzed at the micro level of individual engraved strokes. Our work presents the dependences of distribution of deformations and stresses in the engraved elements when applying pressure, which corresponds to the real intaglio printing process. Such an approach is fundamentally important for intaglio printing, where it is the microrelief that determines the durability of the plate.

The limitations of our study are that the work considers static loading, without taking into account friction in the contact zone. Also, the ink in the engraved stroke is simulated as a liquid that creates uniform hydrostatic pressure without taking into account the real rheological properties of the intaglio printing ink. Our research results are adequate within the range of the given geometric parameters. Therefore, going beyond the proposed parameters will require additional studies.

Among the shortcomings of the proposed research, we should note the lack of experimental verification of numerical results. This is explained by the lack of access to the contact zone and the impossibility of directly measuring the load and deformations under real operating conditions. It is also important to note that simulation involved a single application of pressure, which led to minor local destruction of the elements of the engraved stroke of the printing plate. At the same time, in the actual intaglio printing process, the load is applied repeatedly at high speed, which can cause cyclic deformation. Therefore, even small initial damage could scale and lead to the gradual evolution of destruction and accelerated wear of the surface of the engraved strokes.

Therefore, an area of further research is to take into account the multiplicity and cyclicity of the application of loads when modeling the intaglio printing process. It is also advisable to direct further studies towards the development of an experimental methodology for indirect control over parameters in the contact zone based on the results of plate wear.

7. Conclusions

1. A three-dimensional finite element model of the contact interaction between an engraved printing plate fixed on a plate cylinder and a printing cylinder was constructed. The model took into account the real geometry of engraved strokes used in printing contact during intaglio printing. In particular, the contact interaction between the cylinders under the action of a pressure of 256 MPa was reproduced, options for filling the strokes with ink and their empty state, which simulates colorless embossing, were taken into account. Simulation allowed us to determine the regularities in the formation of the stressed-strained state of the printing plate during intaglio printing.

2. Modeling of the printing process using the finite element method allowed us to determine in detail the distribution of stresses and deformations in the contact zone between the plate and printing cylinders when applying a pressure of 256 MPa, taking into account various geometric parameters of engraved strokes in intaglio printing. Thus, comparison of stress-strain diagrams for the same input parameters confirms that with increasing engraving width, stresses on

the side walls and bottom of the stroke increase. At a depth of 30 μm, increasing the engraving width (30–210 μm) leads to an increase in the side wall stresses within 270–560 MPa in empty strokes and 230–370 MPa in filled strokes. At the same time, an increase in the load on the bottom of the ink-filled strokes is observed (310–400 MPa), at the bottom of the empty strokes the changes are small and do not exceed 70 MPa. Increasing the engraving depth to 100 μm leads to a decrease in the intensity of the load on the side walls and bottom in empty strokes, and an increase in the loads in the engraved strokes filled with ink. Thus, an increase in the engraving width (30–180 μm) leads to an increase in the side wall stresses within 240–340 MPa in empty strokes and 260–420 MPa in filled strokes. At the same time, an increase in the load on the bottom of the strokes filled with ink (200–390 MPa) and a decrease in the load on the bottom of the empty strokes (170–50 MPa) are observed. The analysis of deformations reveals similar trends. Thus, at a depth of 30 μm, an increase in the engraving width causes an increase in the bottom deformation within 0.035–0.065 mm in filled strokes and 0.04–0.01 mm in empty strokes. At a depth of 100 μm, an increase in the engraving width causes an increase in the bottom deformation within 0.025–0.035 mm in filled strokes and 0.025–0.01 mm in empty strokes. This is due to the increase in the contact area and the redistribution of load between the elements of the engraved stroke depending on both the width and depth of the engraving.

It was determined that the ratio of width to the depth of the engraving determines the mechanism of formation of the stressed-strained state of the engraved stroke. At small values of this ratio (0.3–1.5), the load is distributed more evenly between the side walls and the bottom and does not exceed 400 MPa. With an increase in this indicator (especially at values greater than 3), the load increases in the side walls of empty and filled strokes and at the bottom of strokes filled with ink (values reach up to 520 MPa for the samples under study). It was determined that when the strokes are arranged too close to each other (less than 40 μm), the phenomenon of "cutting" of the side walls occurs, which is associated with the mutual influence of the stress-strain fields of adjacent elements.

Increasing the angle of inclination of the side walls of engraved lines from 60° to 90° causes an increase in the loads on the side walls by an average of 190 MPa, and on the bottom of the filled line by 70 MPa. The deformation of the bottom also increases by approximately 0.01 mm. And for empty lines, the opposite trend is observed – increasing the angle of inclination of the side walls of engraved lines from 60° to 90° causes a decrease in the loads by approximately 200 MPa, deformations by 0.015 mm. This is explained by a change in the direction of contact pressure transmission.

Analysis of heat maps has made it possible to derive the following patterns regarding the influence of the presence of ink inside the engraved strokes on the distribution of stresses and deformations of the engraved printing plate when pressure is applied to it. In strokes filled with ink, the bottom collapses, and in empty strokes, the side faces of the engraved strokes. This indicates a significant influence of liquid pressure on the local distribution of stresses.

3. Our results have made it possible not only to assess the mechanism of fracture in the contact zone but also offer recommendations for improving the geometry of the strokes to increase the wear resistance of printing plates. In particular, it is recommended to use the following range of the ratio of the width to the depth of the engraved stroke: $2<b/d<3$. It is also recommended to use smaller angles of inclination of the side walls within technologically permissible values and to ensure such a distance between the strokes at which the interaction between the stressed zones is minimal.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study and the results reported in this paper.

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Data availability

The data will be provided upon reasonable request.

Use of artificial intelligence

The authors declare the use of generative AI in the research and manuscript preparation process. According to the GAIDeT taxonomy, the following tasks were delegated to generative AI tools under full human supervision: during the preparation of the paper, artificial intelligence tools were used exclusively for checking the grammar of individual paragraphs of text, captions for figures, and editing the English translation of the annotation.

Generative AI tool used: *ChatGPT, Grammarly.*

The authors bear full responsibility for the final manuscript.

Generative AI tools are not indicated as authors and are not responsible for the final results.

Authors' contributions

Tetiana Kyrychok: Conceptualization, Supervision, Project administration, Data Curation**; Olena Korotenko:** Methodology, Visualization, Writing – review & editing; **Vladyslav Korotenko:** Formal analysis, Investigation, Writing – original draft; **Vladyslav Doroshchuk:** Resources, Visualization.

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